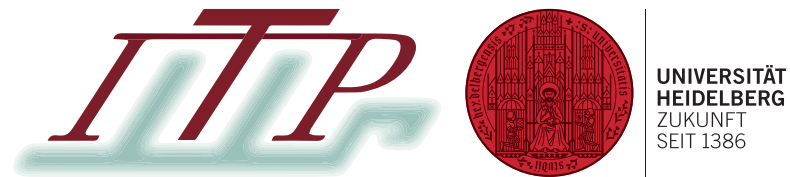


# Probing BSM physics via the trilinear Higgs coupling

Henning Bahl

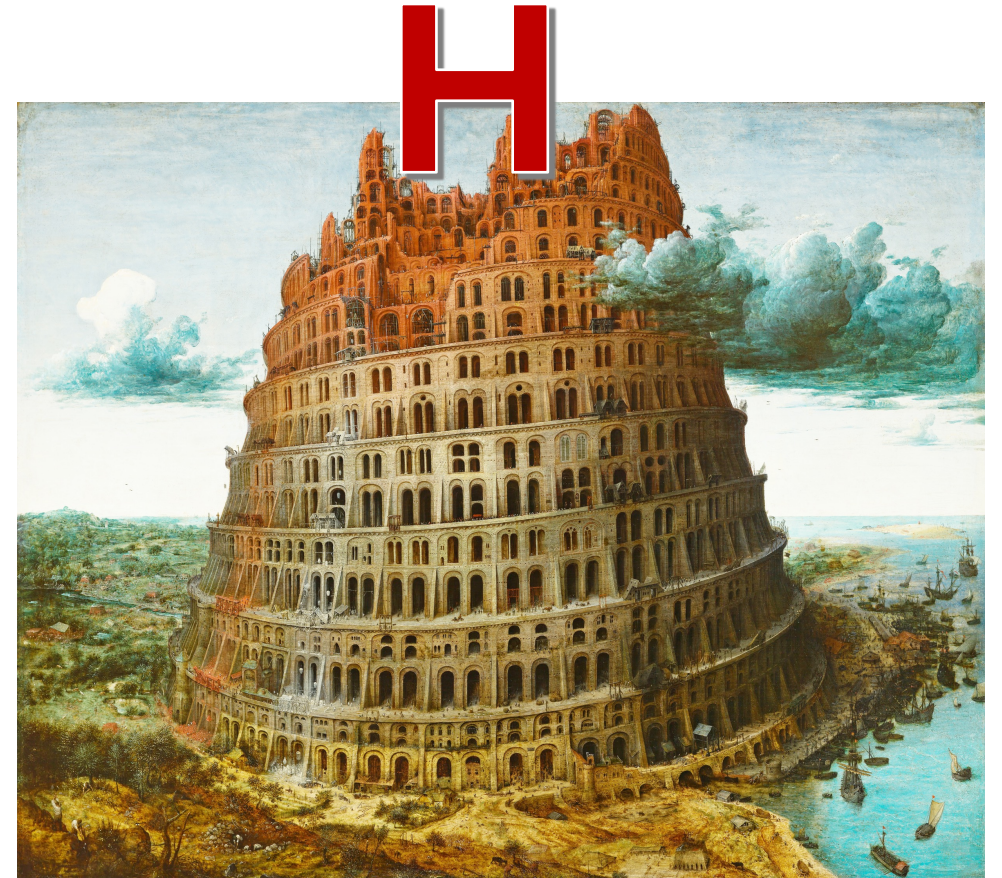


BSM seminar, Karlsruhe, 7.5.2025

# The Higgs discovery

After decades of work, the Higgs discovery was a big success for particle physics.

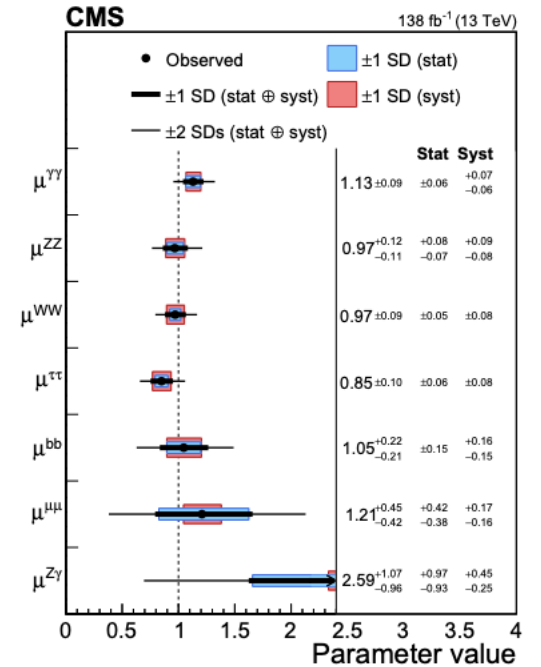
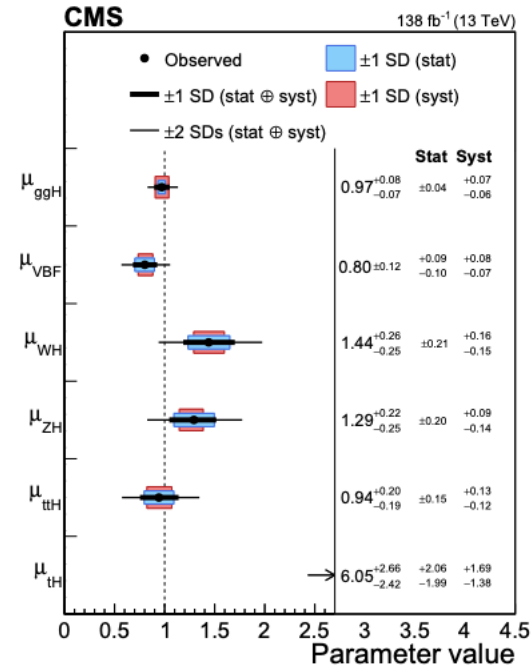
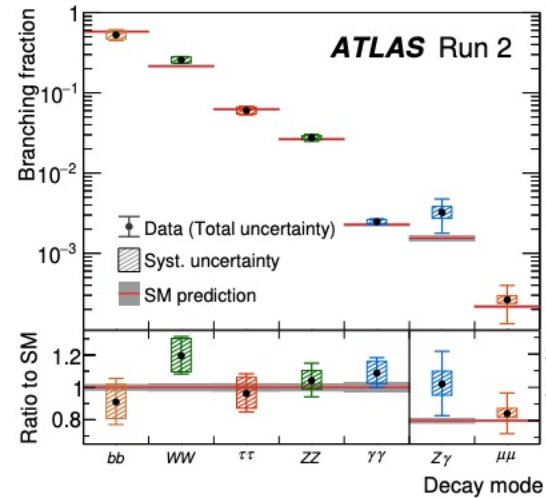
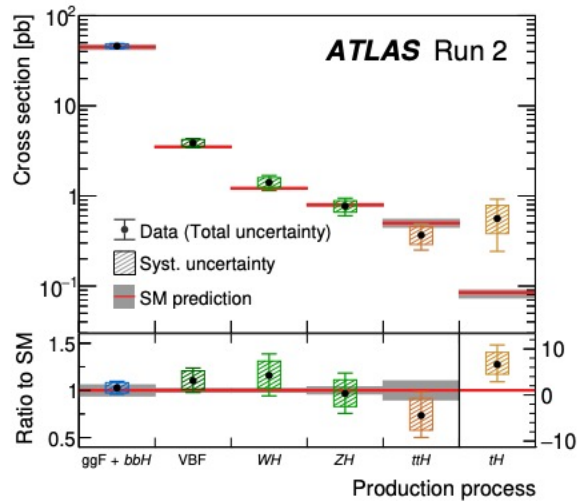
- Where are we more than 10 years later?
- What have we learned about the Higgs in the mean time?
- What is still left to explore?



Tower of Babel ~~the~~ SM

# The Higgs 10 years later

[ATLAS 2207.00092, CMS 2207.00043]



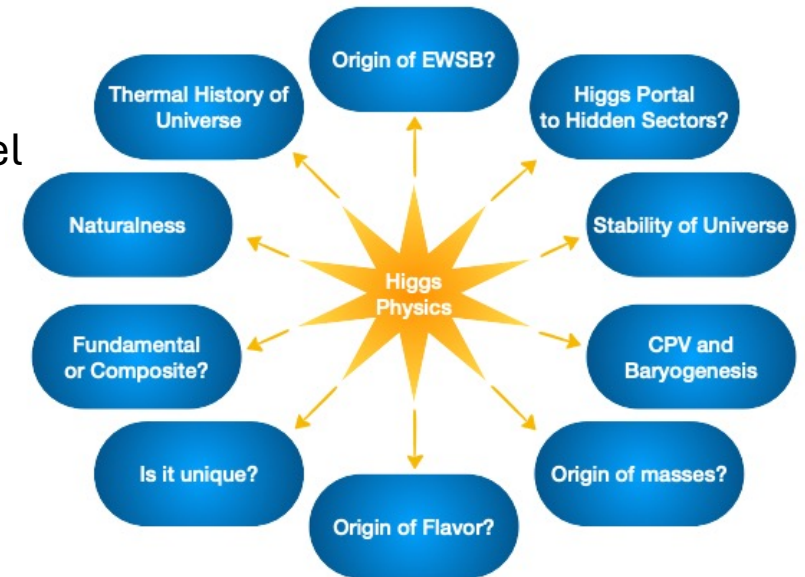
- Ten years later, we have entered the Higgs precision era.
- So far, all Higgs measurements agree with the SM predictions within the experimental and theoretical uncertainties.

So, everything left to do is to confirm the SM with even more precision?

# Motivation for future Higgs measurements

So, everything left to do is to confirm the SM with even more precision? → **No!**

- Most couplings are measured with  $\sim 10\%$  precision.  
→ BSM effects could be hidden within the uncertainties.
- Some Higgs properties are only weakly constrained.
- Existing measurements already provide strong guidance for BSM model building.
- Many types of BSM physics can be linked to the Higgs.



[Snowmass 2209.07510]

⇒ Strong motivation for on-going and future Higgs precision programs.

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Many Higgs properties only weakly constrained, e.g.:

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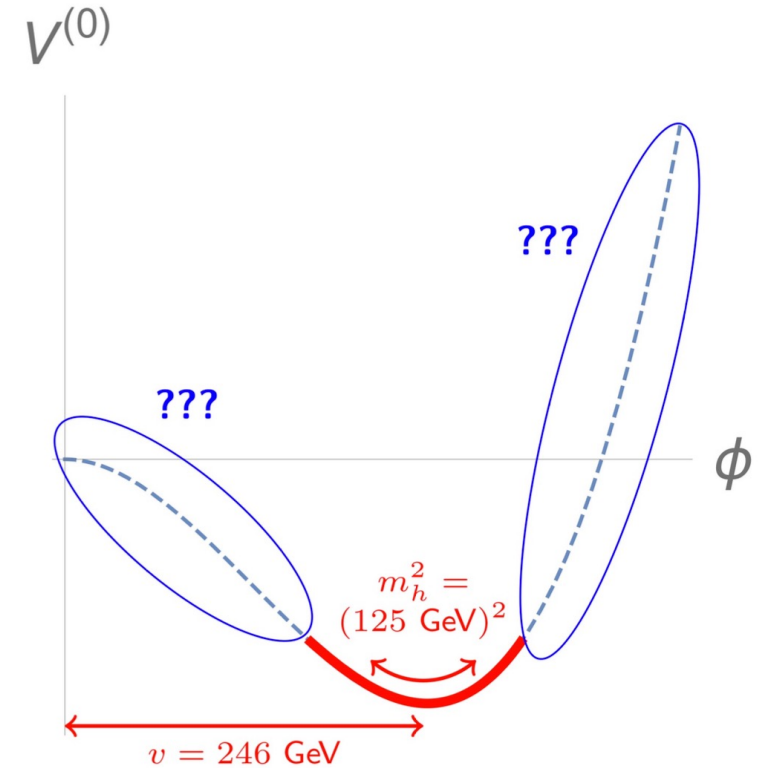
We should test all these predictions!

# What do we know about the Higgs potential?

- After the Higgs discovery, we know
  - the location of the EW minimum:  $v = 246$  GeV
  - the curvature of the potential close to the minimum
- Away from the minimum, the shape of the potential is, however, unknown so far.

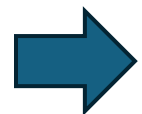
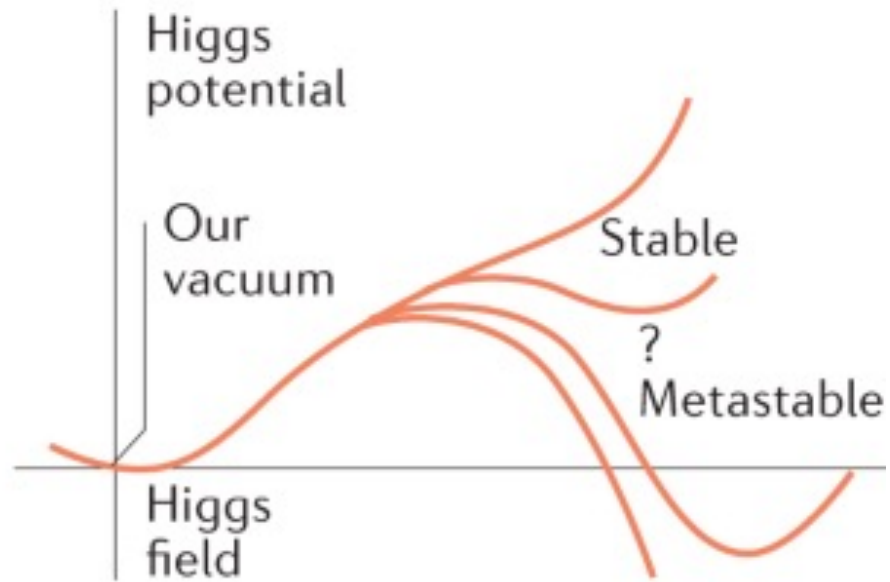


Determination of trilinear Higgs coupling  $\lambda_{hhh}$  crucial



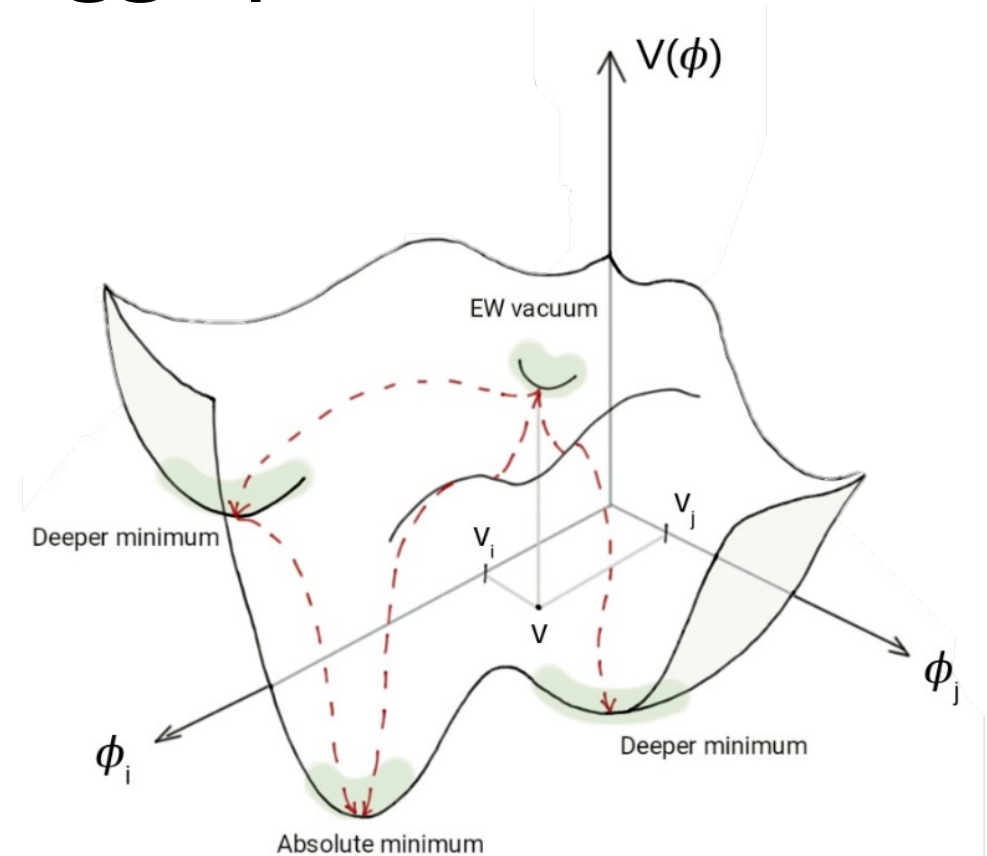
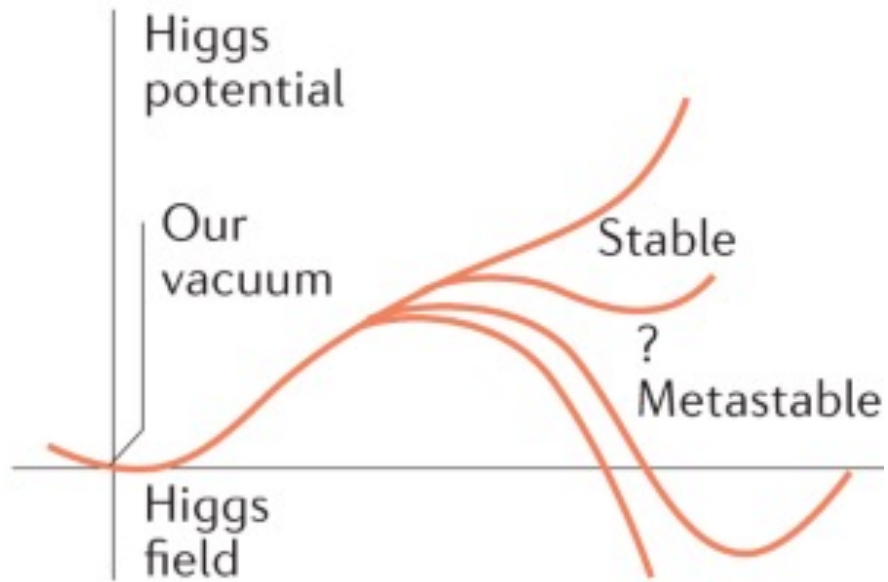
[figure by J. Braathen]

# What is the shape of the Higgs potential?



$\lambda_{hhh}$  closely linked to stability of EW vacuum & nature of EW phase transition

# What is the shape of the Higgs potential?



[figure by K. Radchenko Serdula]



$\lambda_{hhh}$  closely linked to stability of EW vacuum & nature of EW phase transition

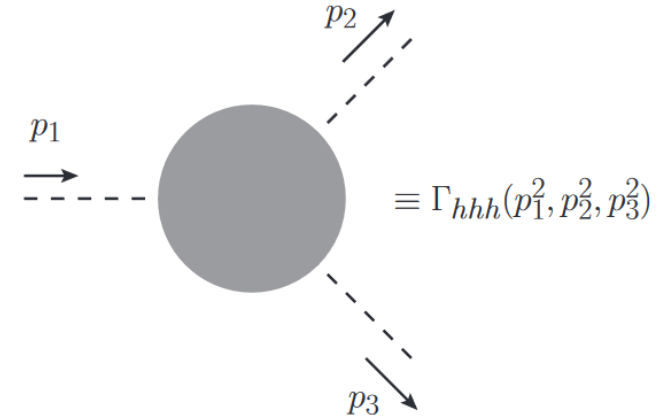
# BSM effects in the trilinear Higgs coupling

How large can they get within existing constraints?

→ focus on BSM extensions of the SM Higgs sector

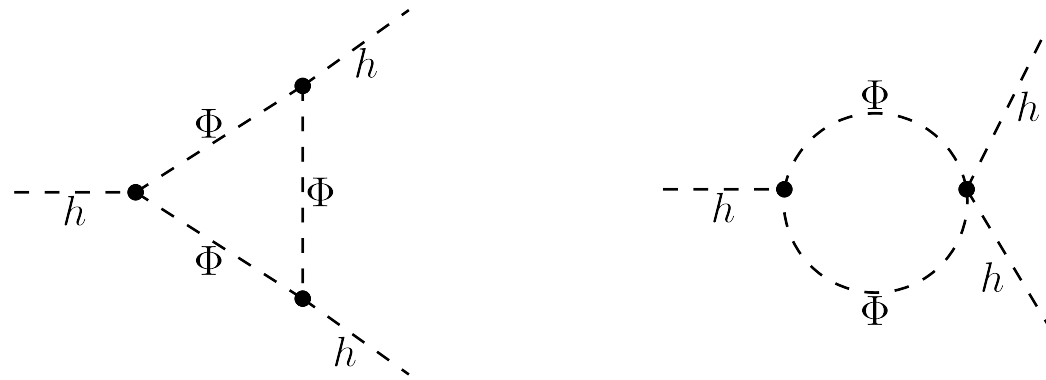
# Calculating BSM corrections to $\kappa_\lambda$

- Need to calculate Higgs three-point function:



- define  $\kappa_\lambda \equiv \frac{\lambda_{hhh}}{(\lambda_{hhh}^{(0)})_{\text{SM}}}$  with  $\lambda_{hhh} = -\Gamma_{hhh}(p_1^2, p_2^2, p_3^2)$

- at the tree-level,  $\kappa_\lambda \sim 1$  due to Higgs precision measurements



- but BSM loop corrections can be large if large couplings of  $h$  to BSM scalars  $\Phi$

# Calculating BSM corrections to $\kappa_\lambda$

- In many models with extended Higgs sectors:

$$m_\Phi^2 = M^2 + \frac{1}{2} g_{hh\Phi\Phi} v^2$$

- $m_\Phi$ : masses of BSM states,
- $M^2$ : BSM mass parameter in the Lagrangian,
- $g_{hh\Phi\Phi}$ : combination of Lagrangian quartic couplings

⇒ Large couplings if large splitting between  $M^2$  and  $m_\Phi^2$  or between BSM scalars

Consider e.g. the case of the 2HDM in the alignment limit:  $\Phi = \{H, A, H^\pm\}$

# 2HDM loop corrections to Higgs trilinear

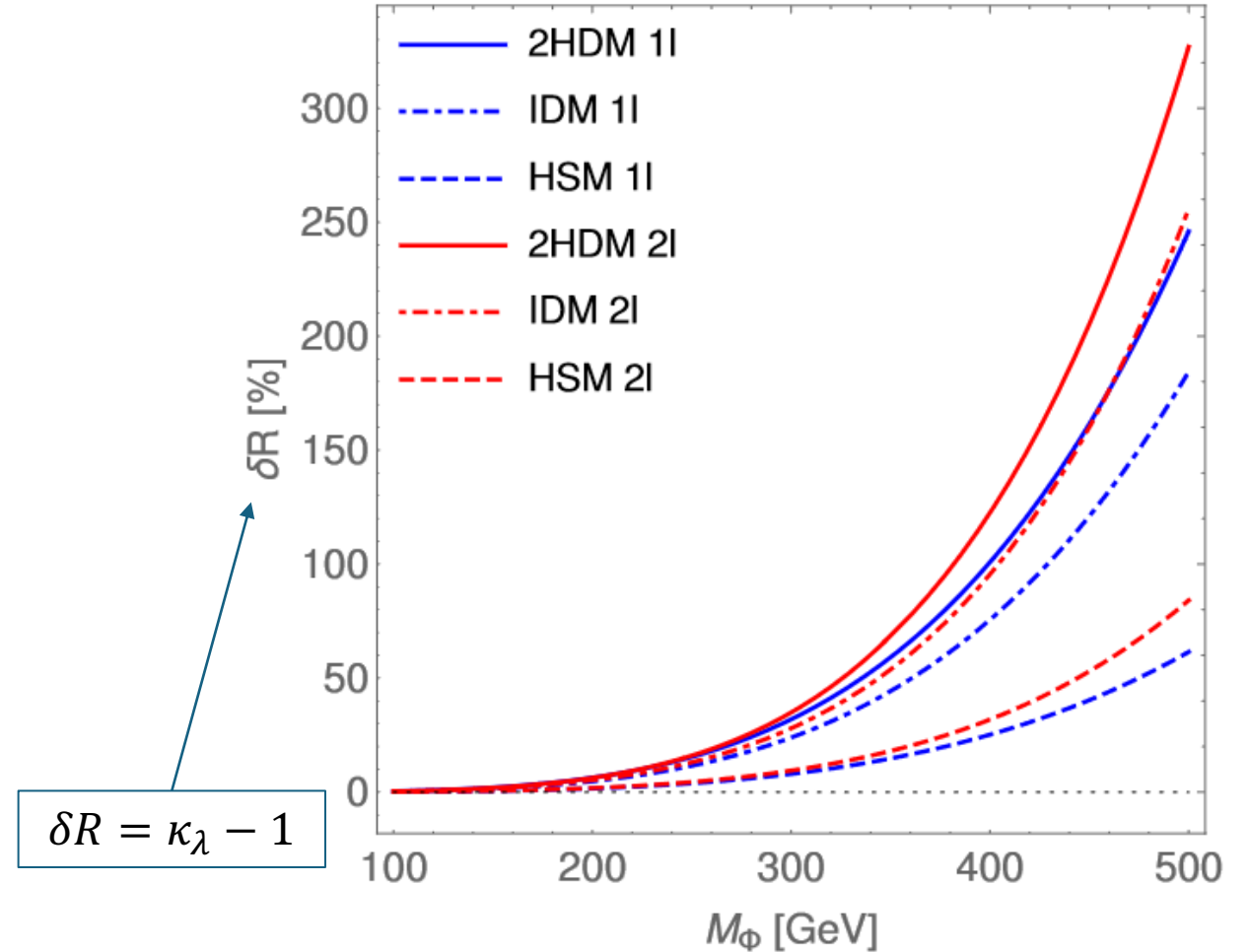
| Coupling/Order                                                                         | 0L | 1L                    | 2L                | 3L                |
|----------------------------------------------------------------------------------------|----|-----------------------|-------------------|-------------------|
| $g_{hhhh}$                                                                             |    | <i>subleading</i><br> | <i>subleading</i> | <i>subleading</i> |
| $g_{(h)h\Phi\Phi}$<br>$\left[ g_{hh\Phi\Phi} = -\frac{2(M^2 - m_\Phi^2)}{v^2} \right]$ | -  |                       |                   |                   |
| $g_{(h)H\Phi\Phi'}$<br>[ $g_{(h)G\Phi\Phi'}$ case similar]                             | -  | -                     |                   |                   |
| $g_{\Phi\Phi\Phi'\Phi'}$<br>[2 BSM scalars of species $\Phi$ , 2 of species $\Phi'$ ]  | -  | -                     |                   |                   |

[table by J. Braathen '24]

# Loop corrections in degenerate limit

[Braathen,Kanemura,1911.11507]

- $\delta^{(1)}\lambda_{hhh} = \sum_{\Phi} \frac{4n_{\Phi}m_{\Phi}^4}{v^3} \left(1 - \frac{M^2}{m_{\Phi}^2}\right)^3$
- all BSM masses degenerate
- $M^2 = 0$
- no violation of perturbative unitarity



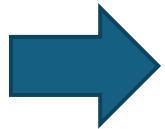
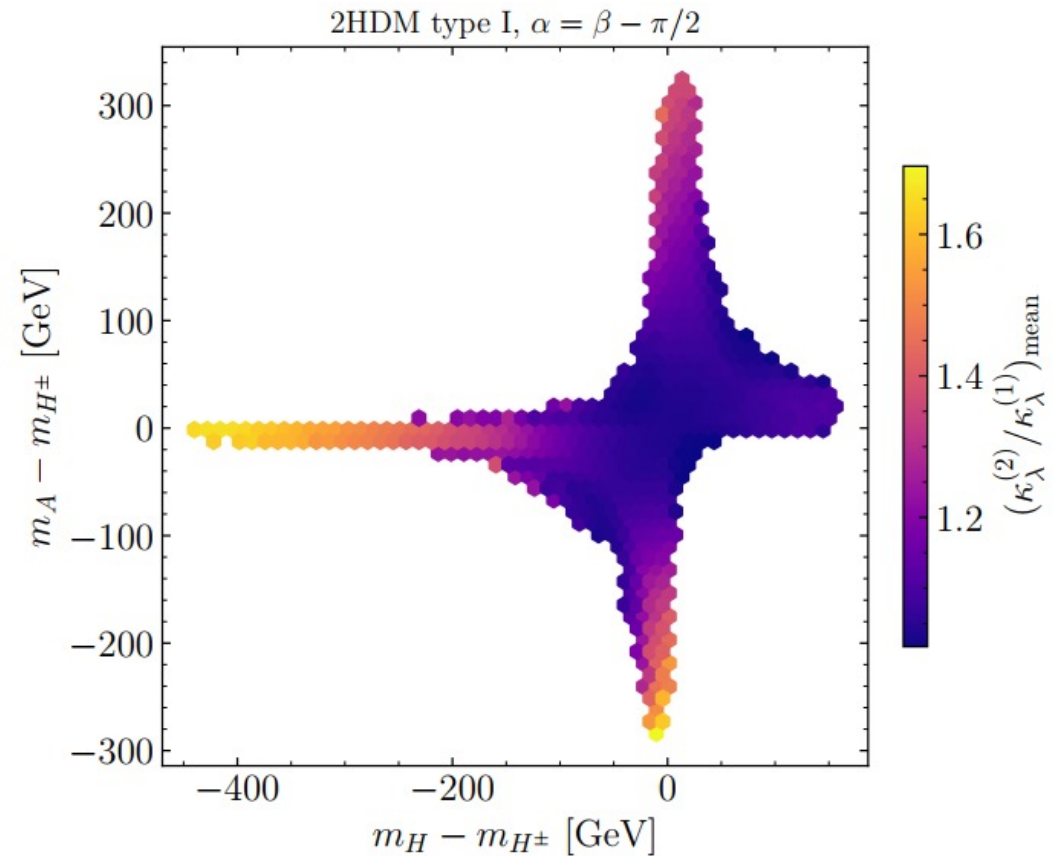
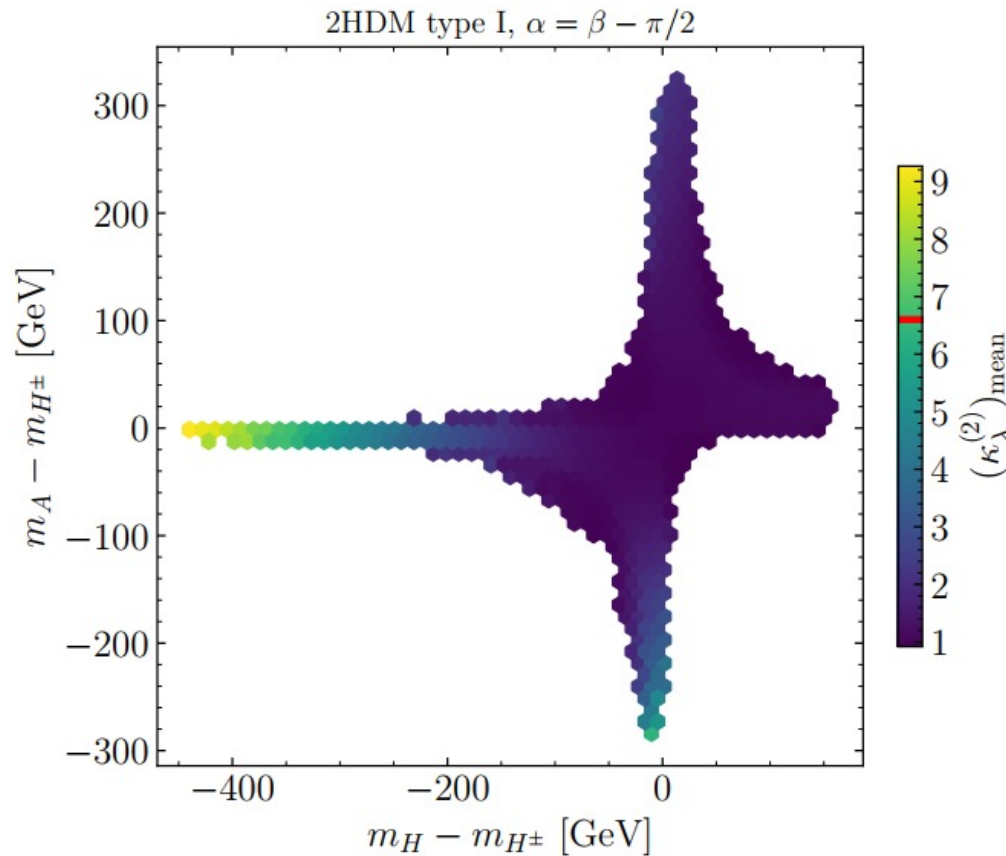
# Full 2HDM parameter scan

[HB,Braathen,Weiglein, 2202.03453]

- Checked for
  - vacuum stability and boundedness-from-below,
  - NLO perturbative unitarity, [Grinstein et al., 1512.04567; Cacchio et al., 1609.01290]
  - electroweak precision observables (calculated at the 2L level using THDM\_EWPOS), [Hessenberger & Hollik, 1607.04610, 2207.03845]
  - SM-like Higgs measurements via HiggsSignals, [Bechtle et al., 2012.09197]
  - direct searches for BSM scalars via HiggsBounds, [Bechtle et al., 2006.06007]
  - b-physics constraints.
- Most constraints checked using ScannerS. [Mühlleitner et al., 2007.02985]
- For each point passing the constraints, calculate  $\kappa_\lambda$  at the 1L and 2L level ( $\kappa_\lambda^{(1)}$  and  $\kappa_\lambda^{(2)}$ ). [Braathen, Kanemura, 1911.11507]

# 2HDM scan results

[HB,Braathen,Weiglein, 2202.03453]



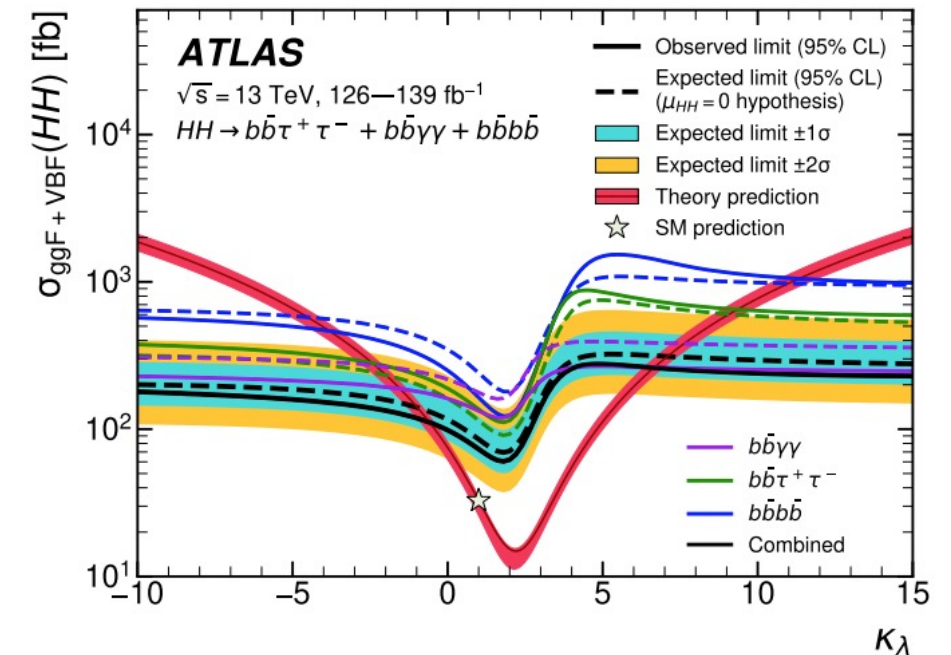
- Largest corrections for  $m_A \simeq m_{H^\pm}$ ,  $m_H < m_{H^\pm}$  and  $m_H \simeq m_{H^\pm}$ ,  $m_A$
- 2L corrections have sizeable impact (up to 70%).

# Experimental bound on $\kappa_\lambda \equiv \lambda_{hhh}/\lambda_{hhh}^{\text{SM}}$

Current strongest limit:  $-0.4 < \kappa_\lambda < 6.1$  at 95% CL [ATLAS 2211.01216].

## Assumptions:

- Simplest analysis assumes that all other Higgs couplings are SM-like.
- Non-resonant Higgs-boson pair production only deviates from the SM via a modified trilinear Higgs coupling (i.e., no heavy resonances).



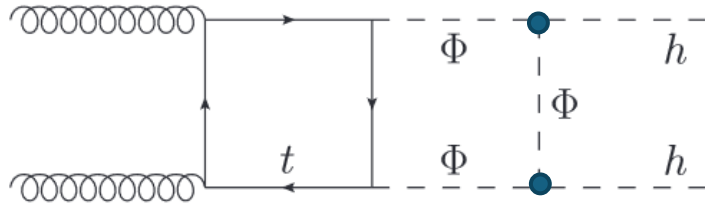
Can we use this seemingly weak limit to constrain the 2HDM?

# Can we apply the experimental constraints on $\kappa_\lambda$ ?

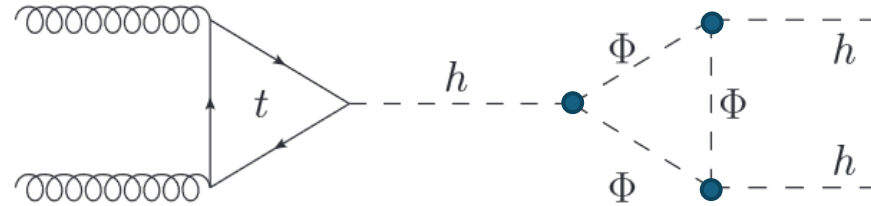
[HB,Braathen,Weiglein, 2202.03453]

Assumptions of experimental bound:

- All other Higgs couplings are SM-like.
  - 2HDM in the alignment limit with heavy BSM masses. ✓
- Higgs-boson pair production only deviates from the SM via a modified trilinear Higgs coupling.
  - No resonant contribution because  $Hhh$  coupling is zero in alignment limit. ✓
  - Other BSM contributions to  $hh$  production?



$\propto \mathcal{O}(y_t^2 g_{hh\Phi\Phi}^2)$  (not included)



$\propto \mathcal{O}(y_t g_{hh\Phi\Phi}^3)$  (included)

- We include the all corrections leading in the large coupling  $g_{hh\Phi\Phi}$  at the NLO and NNLO level. ✓

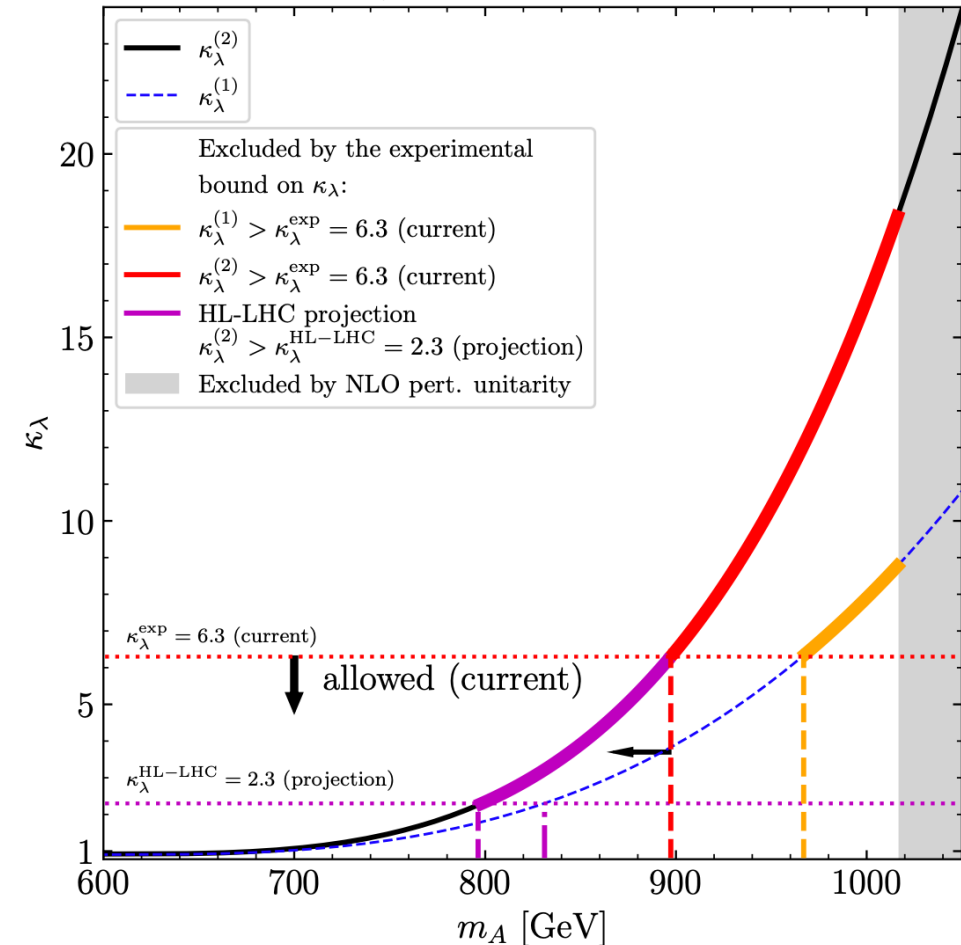
# 2HDM benchmark scenario

[HB,Braathen,Weiglein, 2202.03453]

- $\kappa_\lambda$  bound already now stronger than perturbative unitarity
- 2L corrections to  $\kappa_\lambda$  crucial for accurate bounds
- only high  $m_A$  region constrained by pert. unitarity

Already current experimental limits on  $\kappa_\lambda$  probe so-far unconstrained BSM parameter space!

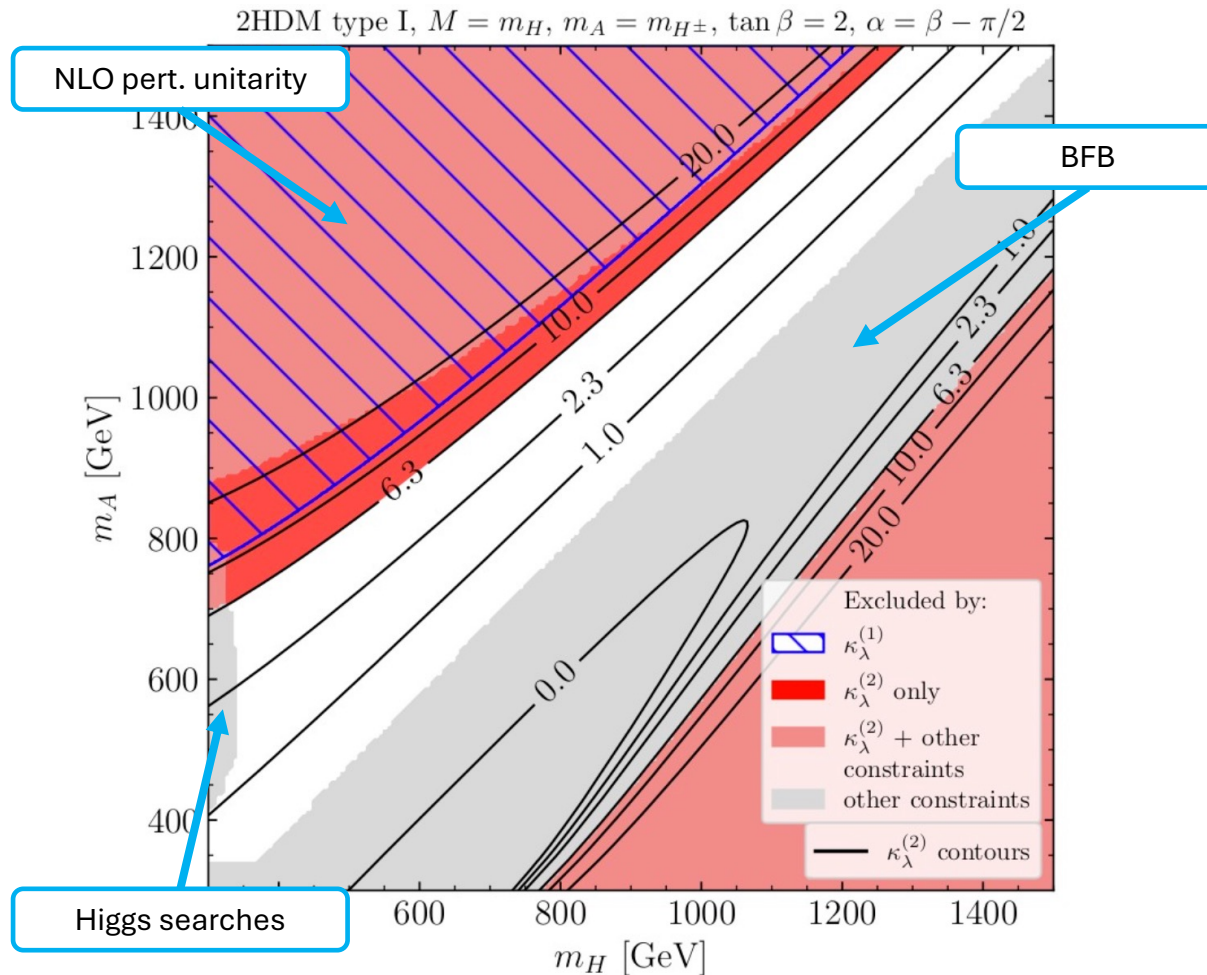
2HDM type I,  $\alpha = \beta - \pi/2$ ,  $m_A = m_{H^\pm}$ ,  $M = m_H = 600$  GeV,  $\tan \beta = 2$



[HB,Braathen,Weiglein, 2202.03453]

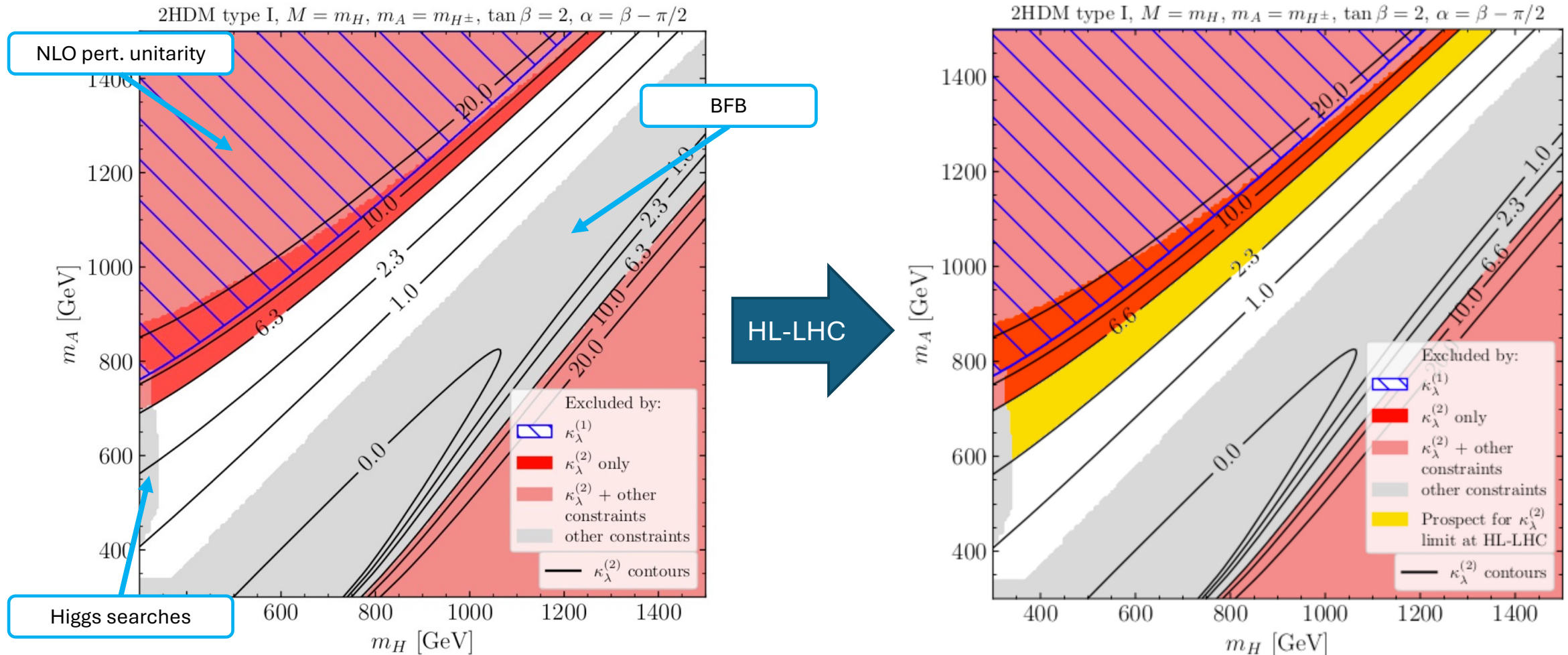
# Constraints on $\kappa_\lambda$ — benchmark plane

[HB,Braathen,Weiglein, 2202.03453]



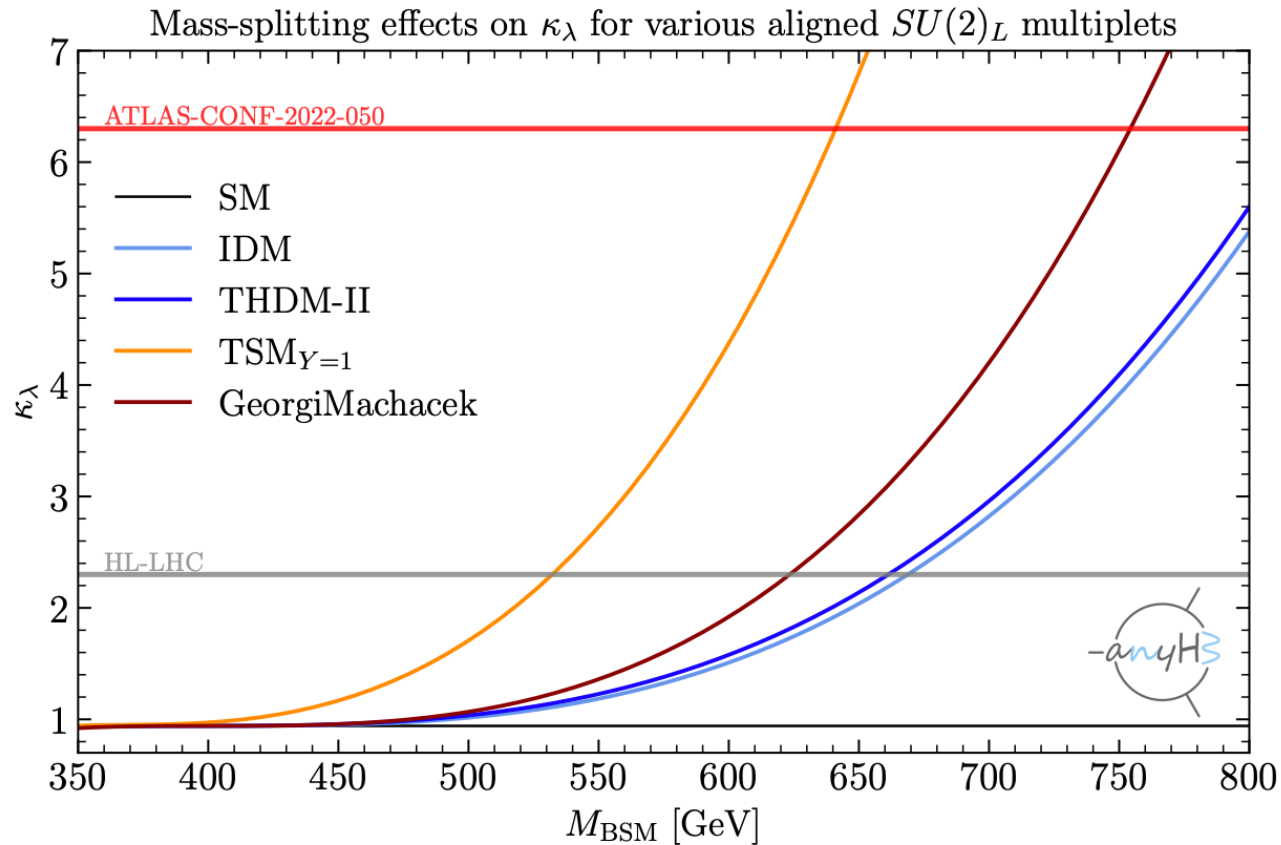
# Constraints on $\kappa_\lambda$ — benchmark plane

[HB,Braathen,Weiglein, 2202.03453]



# Other extension of SM Higgs sector

[HB,Braathen,Gabelmann,Weiglein,2305.03015]



- Large loop corrections to  $\kappa_\lambda$  possible in various models.
- $\kappa_\lambda$  very sensitive to BSM scalar couplings.
- Automatized calculation of  $\kappa_\lambda$  available in Python package anyH3.
- Uses UFO models as input.
- See also [1704.01953,1902.05936,2209.00666] for non-Higgs models, EFT discussion, etc.

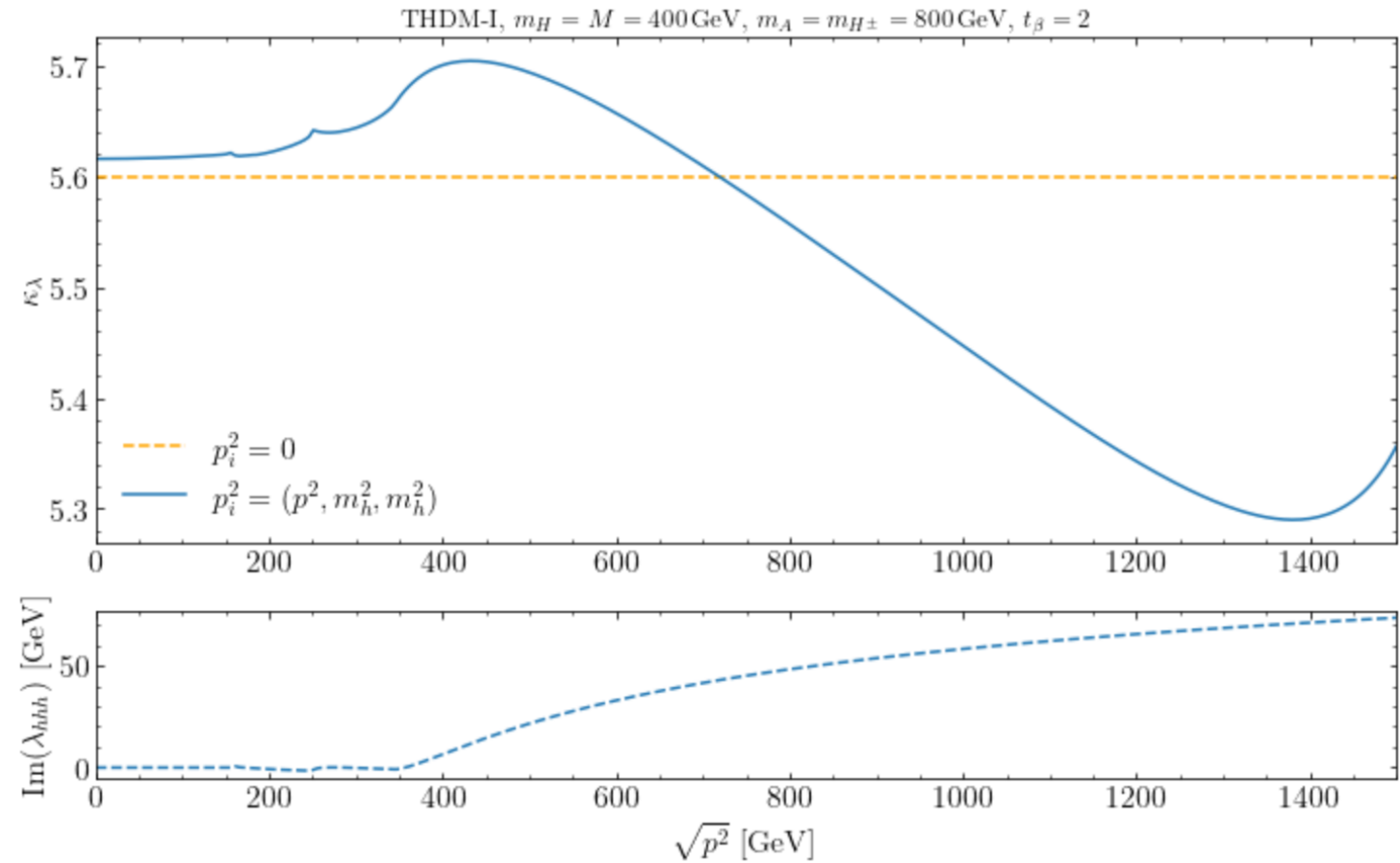
# Momentum dependence

[HB,Braathen,Gabelmann,Weiglein,2305.03015]

- Typical assumption: zero momentum ( $p_1^2 = p_2^2 = p_3^2 = 0$ )
- Using anyH3, check validity



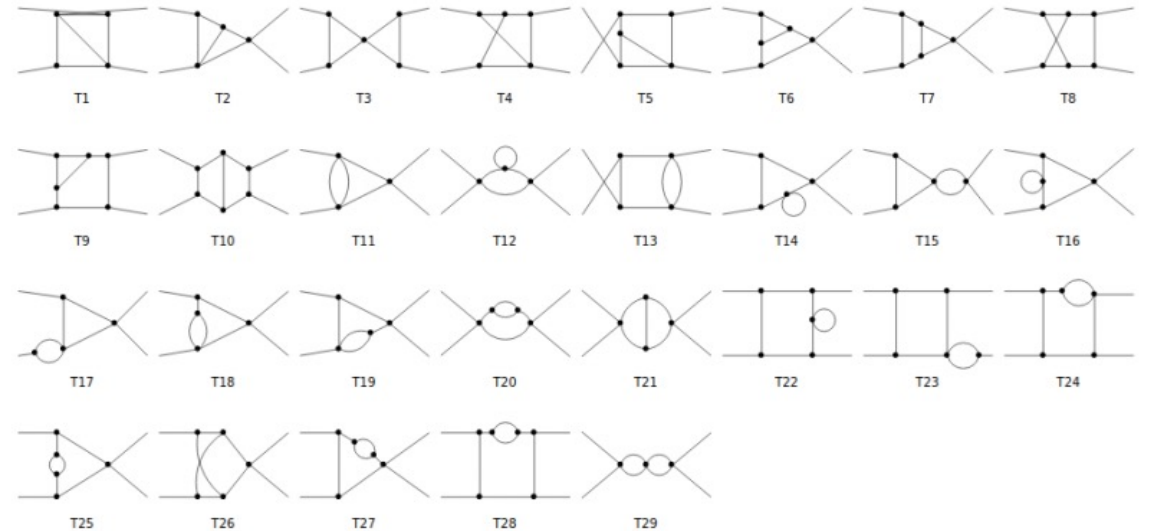
Zero-momentum correction captures bulk of correction



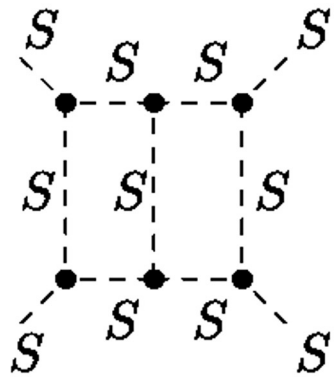
# Generic 2L results for $\lambda_{hhh}$

[HB,Braathen,Gabelmann,Paßehr,2503.15645]

- 1L predictions full automated: anyH3 and BSMPT [Basler et al.,1803.02846,...]
- next step: automation for 2L
- building blocks: scalar 0,1,2,3,4-point functions
- work in zero-momentum approximation
- write down all possible Feynman diagrams with generic fields/couplings
- reduce number of diagrams using known symmetries



# Exemplary result



T-integral

[Weiglein,Scharf,Böhm,hep-ph/9310358]

```
{
  {1, 1, 1},
  {
    edge[iv[1], v[5], S[i3]], edge[iv[2], v[6], S[i4]],
    edge[iv[3], v[7], S[i1]], edge[iv[4], v[8], S[i2]],
    edge[v[5], v[6], -S[i9]], edge[v[5], v[7], -S[i6]],
    edge[v[5], v[8], -S[i8]], edge[v[6], v[7], -S[i5]],
    edge[v[6], v[8], -S[i7]]
  },
  -((lfac^2*SSS[i1, -i5, -i6, 1]*SSS[i2, -i7, -i8, 1]*
    SSSS[i3, i6, i8, i9, 1]*SSSS[i4, i5, i7, -i9, 1]*
    T[Df[k1, MS[i7]]*Df[k1, MS[i8]]*Df[k3, MS[i5]]*
    Df[k3, MS[i6]]*Df[k4, MS[i9]]])
  )/SF[1, 1/6, 1])
},
```

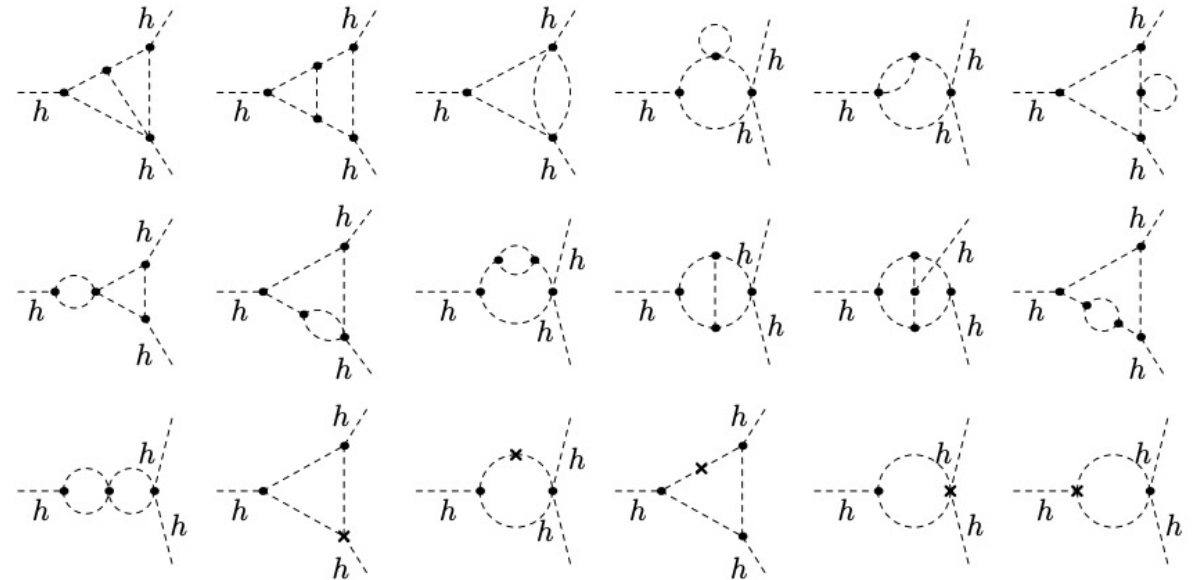
Unique edge list  
defining diagram

Generic couplings

Symmetry factor

# Application to concrete model

- map generic results to concrete model using FeynArts [Hahn et al., hep-ph/0012260]
- numerical evaluation of loop integrals: on-the-fly reduction using new Python package Tintegrals
- cross-checked against many known results in the literature

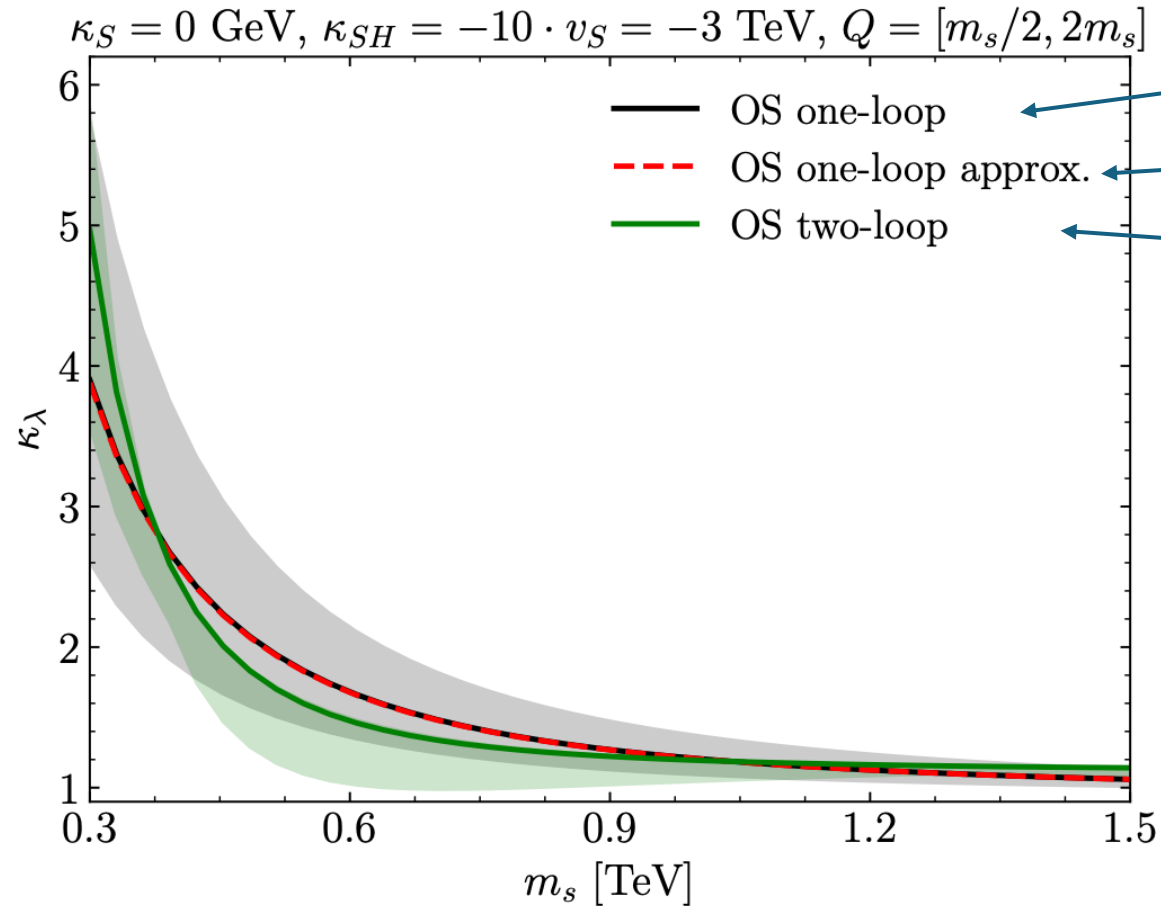


- **new 2L result:** general singlet extension in alignment limit

$$V_{SSM} = \mu^2 |\Phi|^2 + \frac{\lambda_H}{2} |\Phi|^4 + \frac{m_S^2}{2} S^2 + \frac{\lambda_S}{2} S^4 + \frac{\lambda_{SH}}{2} S^2 |\Phi|^2 + \kappa_{SH} S |\Phi|^2 + \frac{\kappa_S}{3} S^3$$

- All parameters OS renormalized apart from  $\overline{\text{MS}}$   $v_S, \kappa_S, \kappa_{SH}$

# 2L corrections in SSM



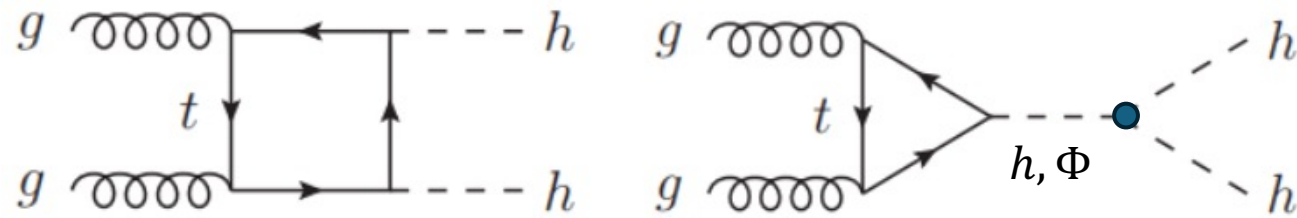
full 1L result

1L result with  $m_h = 0$

2L result with  $m_h = 0$

all building blocks for  
automated 2L prediction  
now available!

# Large trilinear couplings and di-HH production



# Di-Higgs production in a nutshell

- $\lambda_{hhh}$  is not a physical observable but  $\sigma_{hh}$  is

external Higgs

trilinear Higgs  
couplings

top-coupling  
modifiers

di-HH formfactors

$$\frac{d\sigma_{h_i h_j}}{d\hat{t}} = \frac{\gamma}{256\pi} \frac{\alpha_s^2}{(4\pi)^2} \left\{ \left| \sum_k \frac{-c_{t,h_k} \lambda_{h_i h_j h_k} v_{EW}}{s - m_{h_k}^2 + i m_{h_k} \Gamma_{h_k}} F_{\Delta}^e + c_{t,h_i} c_{t,h_j} F_{\square}^{ee} + \tilde{c}_{t,h_i} \tilde{c}_{t,h_j} F_{\square}^{oo} \right|^2 \right. \\ \left. + \left| \sum_k \frac{-\tilde{c}_{t,h_k} \lambda_{h_i h_j h_k} v_{EW}}{s - m_{h_k}^2 + i m_{h_k} \Gamma_{h_k}} F_{\Delta}^o + c_{t,h_i} \tilde{c}_{t,h_j} F_{\square}^{eo} \right|^2 \right. \\ \left. + |c_{t,h_i} c_{t,h_j} G_{\square}^{ee} + \tilde{c}_{t,h_i} \tilde{c}_{t,h_j} G_{\square}^{oo}|^2 + |c_{t,h_i} \tilde{c}_{t,h_j} G_{\square}^{eo}|^2 \right\}$$

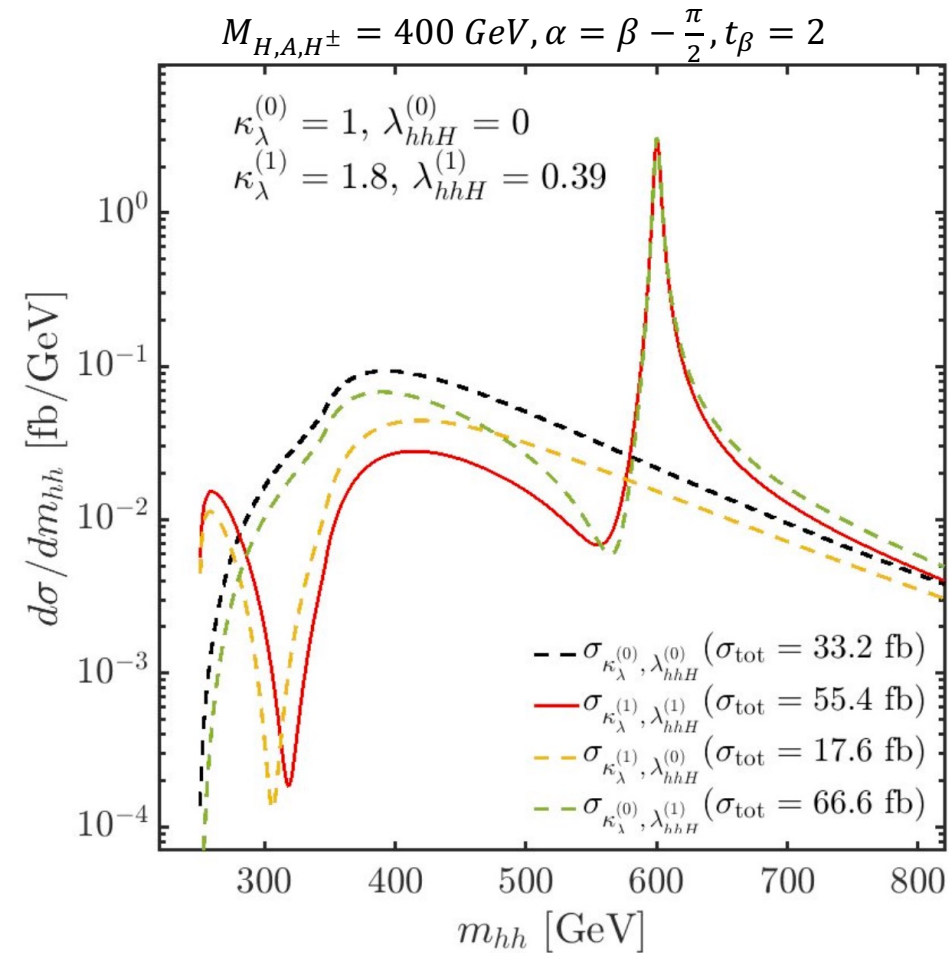
- implement generic LO QCD differential XS into anyH3 → new anyHH subpackage

[Plehn,Spira,Zerwas,hep-ph/9603205]

- also available in HPAIR (+ NLO QCD) [Mühlleitner et al.,2112.12515,...]

# Effect on invariant mass distribution in 2HDM

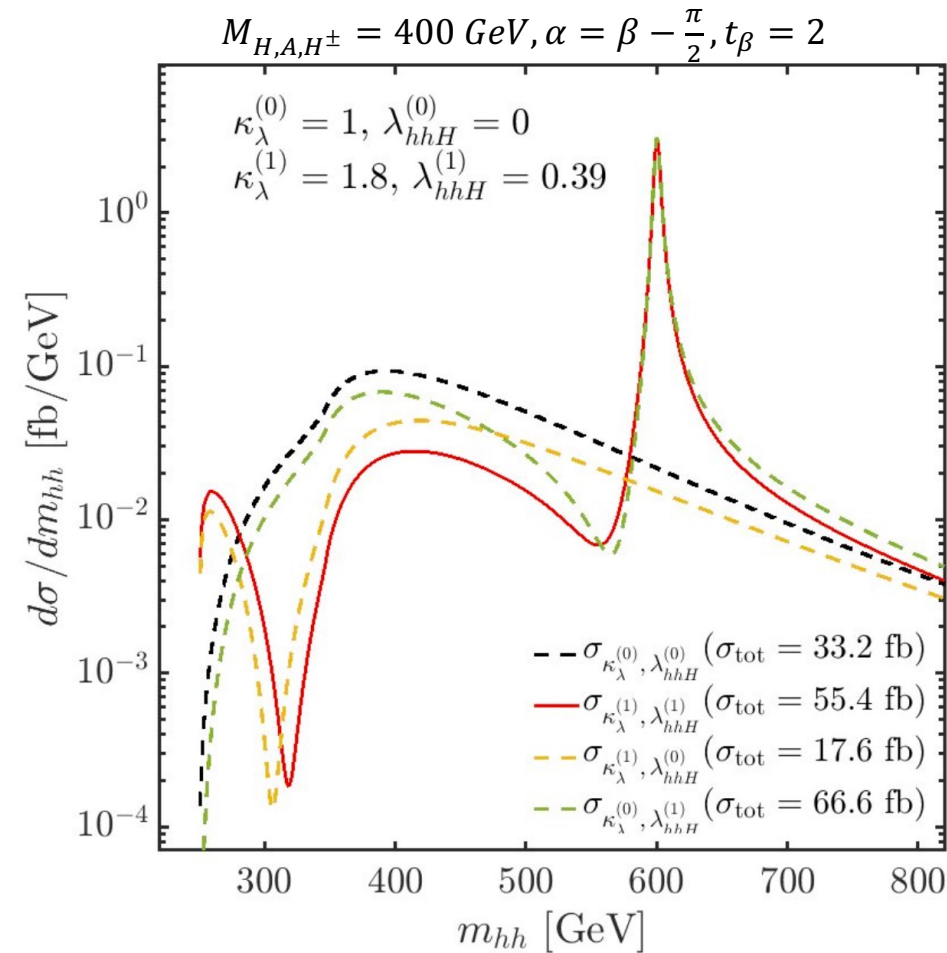
[Heinemeyer,Mühlleitner,Radchenko,Weiglein,2403.14776]



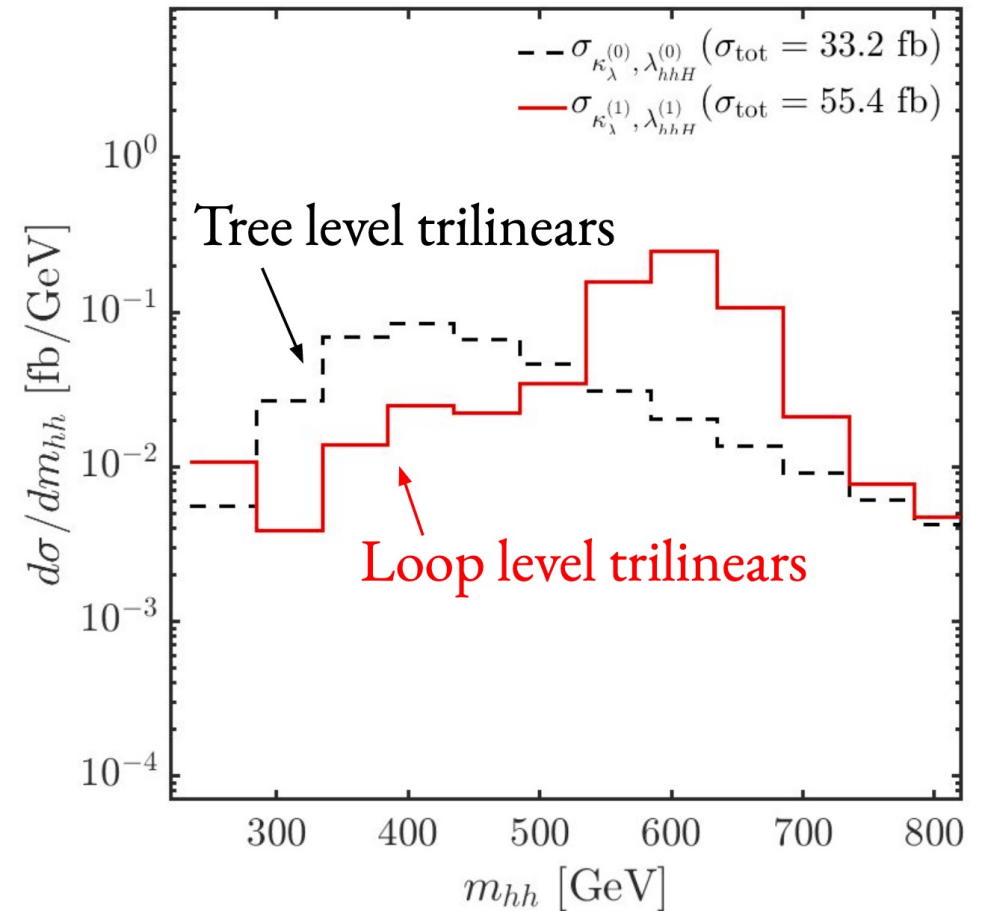
Significant distortion of distribution; loop corrections to BSM trilinears important.

# Effect on invariant mass distribution in 2HDM

[Heinemeyer, Mühlleitner, Radchenko, Weiglein, 2403.14776]



smearing +  
binning



Significant distortion of distribution; loop corrections to BSM trilinears important.

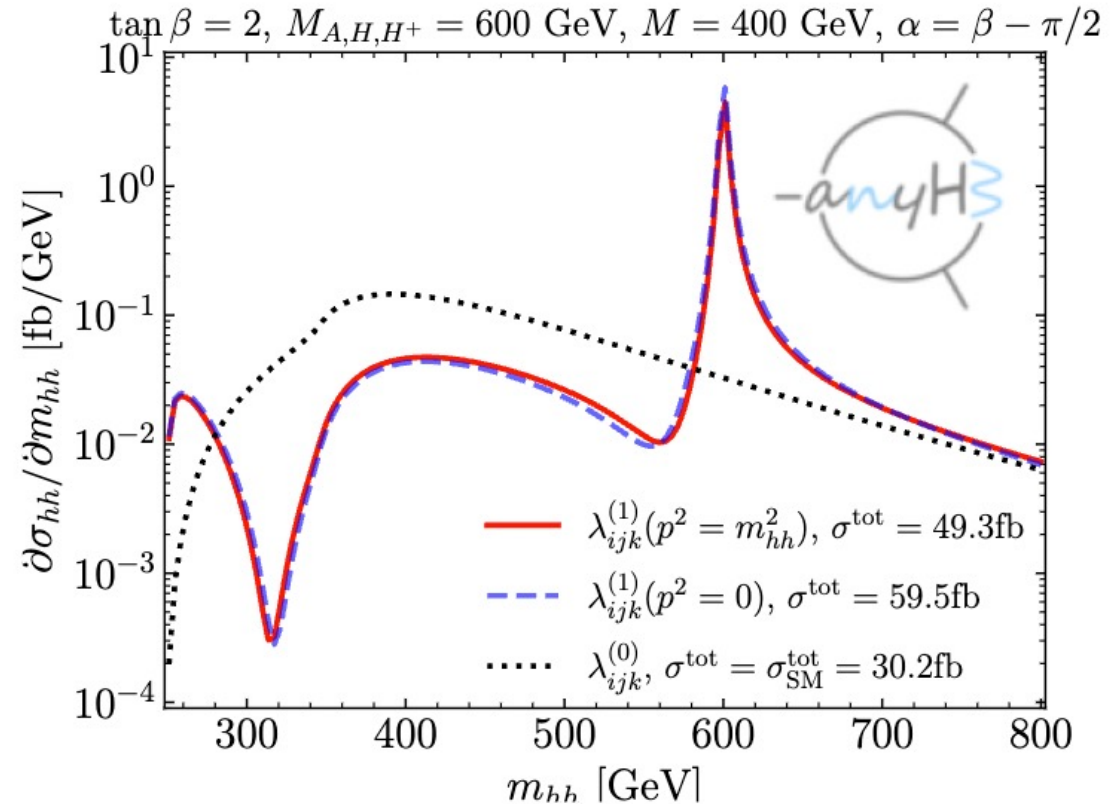
# Momentum effects

[HB,Braathen,Gabelmann,Radchenko,Weiglein,WIP]

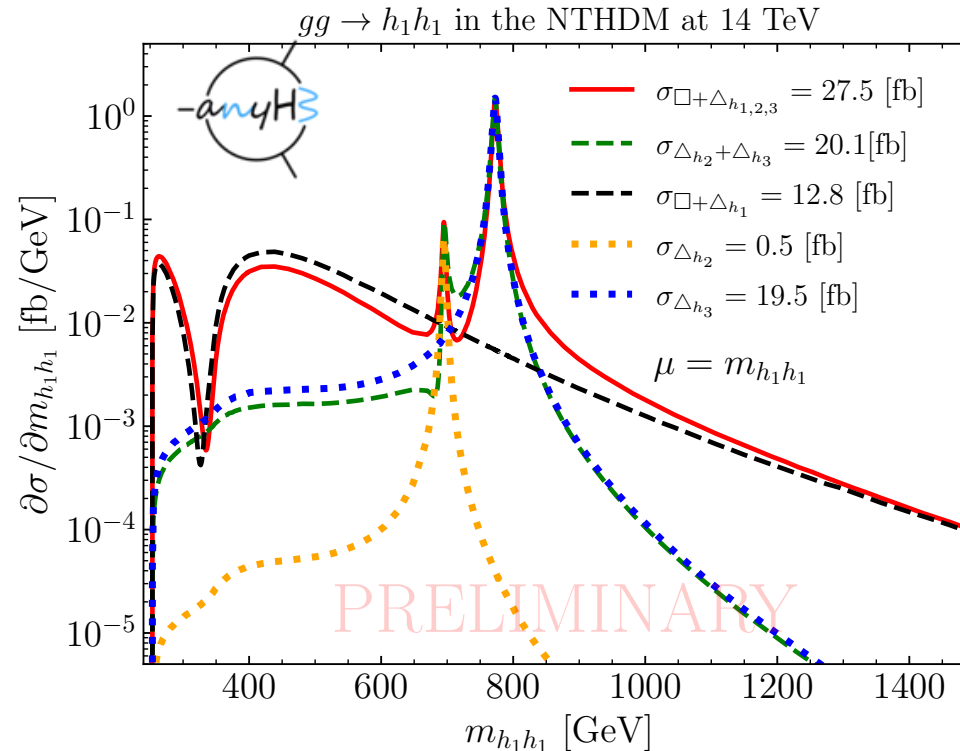
- momentum effects seemingly small
- but large impact on total XS due to  $p^2$ -dependence of  $\lambda_{hhH}$

Future improvements:

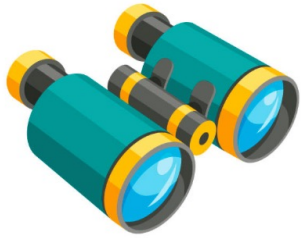
- link to MC generators
- subleading corrections



# Going beyond the 2HDM – multiple resonances



**anyHH:** automated di-HH results for models without colored BSM states including loop corrected trilinears.



# Can we discover BSM first in di-Higgs?

Or are the other Higgs coupling/BSM searches always more sensitive?

# Exemplary scenario: SSM ”nightmare” scenario

$$V_{\text{SSM}}(\Phi, S) = V_{\text{SM}}(\Phi) + \frac{1}{2} \mu_S^2 S^2 + \frac{1}{4!} \lambda_S S^4 + \lambda_{S\Phi} S^2 \Phi^\dagger \Phi$$

If there is an exact  $S \rightarrow -S$  symmetry,  $S$  does not get a vev.

- No mixing with SM Higgs.
- All Higgs couplings are SM-like at the tree level.
- Also searches very difficult, since  $S$  has to be pair produced via the 125 GeV Higgs.

What about loop-level effects on the Higgs couplings?

# Case study: $Z_2$ SSM

$$V(\Phi, S) = V_{\text{SM}}(\Phi) + \frac{1}{2} \mu_S^2 S^2 + \frac{1}{4!} \lambda_S S^4 + \lambda_{S\Phi} S^2 \Phi^\dagger \Phi$$

If  $S$  does not get a vev,  $\lambda_{hhh} = \lambda_{hhh}^{\text{SM}}$  at the tree-level ( $m_S^2 = \mu_S^2 + \lambda_{S\Phi} v^2$ ).

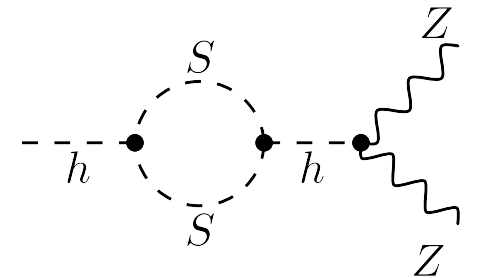
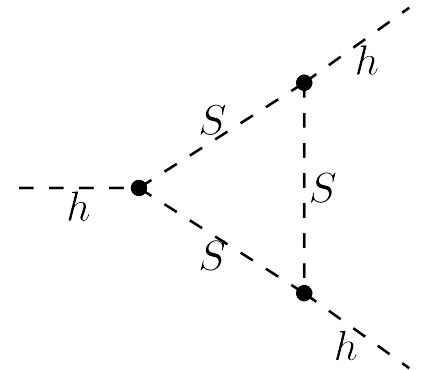
The 1L correction to  $\lambda_{hhh}$  scales like

$$\lambda_{hhh}^{1L} \propto \frac{g_{hSS}^3}{(4\pi)^2} C_0(\dots) \propto \frac{g_{hSS}^3}{(4\pi)^2} \frac{1}{m_S^2} \propto \frac{1}{(4\pi)^2} \frac{m_S^4}{v^3} \left(1 - \frac{\mu_S^2}{m_S^2}\right)^3 \Rightarrow \kappa_\lambda \equiv \frac{\lambda_{hhh}}{\lambda_{hhh}^{\text{SM}}} = 1 + \frac{1}{(4\pi)^2} \frac{m_S^4}{v^4 \lambda_\Phi^{\text{SM}}} \left(1 - \frac{\mu_S^2}{m_S^2}\right)^3$$

whereas the dominant correction to other Higgs couplings scale like

$$g^{1L} \propto \frac{g_{hSS}^2}{(4\pi)^2} B'_0(\dots) \cdot g_{\text{tree}} \propto \frac{1}{(4\pi)^2} \frac{m_S^2}{v^2} \left(1 - \frac{\mu_S^2}{m_S^2}\right)^2 \Rightarrow c_{\text{eff}} \equiv \frac{g}{g^{\text{SM}}} = 1 + \frac{1}{(4\pi)^2} \frac{m_S^2}{v^2} \left(1 - \frac{\mu_S^2}{m_S^2}\right)^2$$

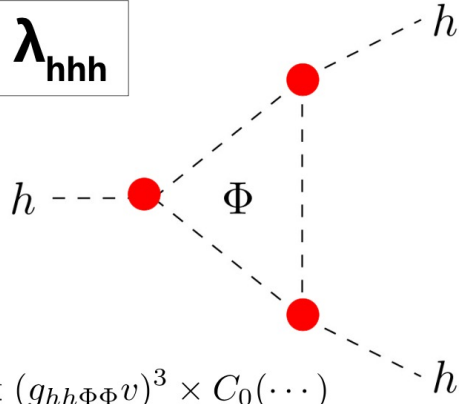
➡ Deviation in  $\lambda_{hhh}$  enhanced by a factor  $\frac{m_S^2}{v^2 \lambda_\Phi^{\text{SM}}} \left(1 - \frac{\mu_S^2}{m_S^2}\right)$  w.r.t. to other Higgs couplings!



# Higgs couplings: power counting

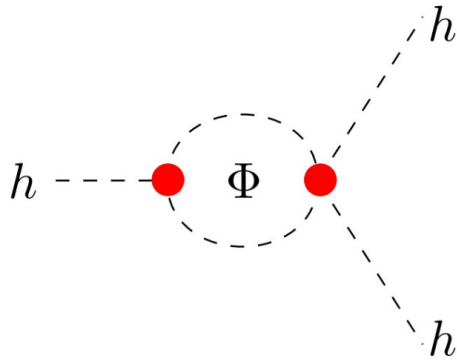
$$m_\Phi^2 = M^2 + \frac{1}{2} g_{hh\Phi\Phi} v^2$$

$\lambda_{hhh}$



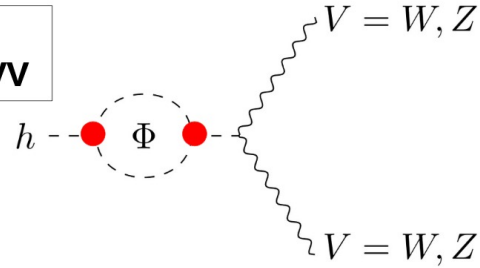
$$\propto (g_{hh\Phi\Phi} v)^3 \times C_0(\dots)$$

$$\sim \frac{(g_{hh\Phi\Phi} v)^3}{m_\Phi^2} \xrightarrow{g_{hh\Phi\Phi} v^2 \gg M^2} \mathcal{O}(g_{hh\Phi\Phi}^2)$$



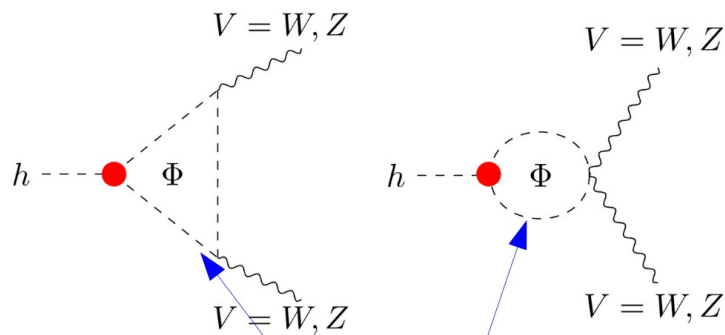
$$\propto g_{hh\Phi\Phi}^2 v \times \underbrace{B_0(\dots)}_{\text{mass. dim. 0}} \sim \mathcal{O}(g_{hh\Phi\Phi}^2)$$

$g_{hVV}$



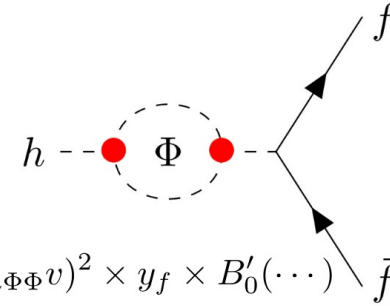
$$\propto (g_{hh\Phi\Phi} v)^2 \times g_{EW} \times B'_0(\dots)$$

$$\propto \frac{(g_{hh\Phi\Phi} v)^2}{m_\Phi^2} \xrightarrow{g_{hh\Phi\Phi} v^2 \gg M^2} \mathcal{O}(g_{hh\Phi\Phi})$$



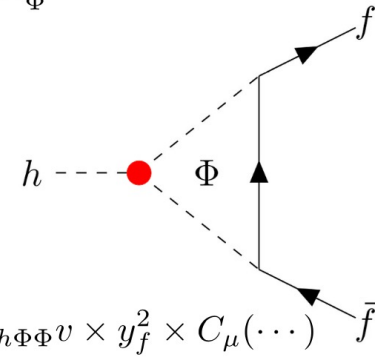
$$\propto g_{hh\Phi\Phi} v \times g_{EW}^2 \times \underbrace{\{C_{\mu\nu}(\dots) \text{ or } B_0(\dots)\}}_{\text{mass. dim. 0}} \sim \mathcal{O}(g_{hh\Phi\Phi})$$

$g_{hff}$



$$\propto (g_{hh\Phi\Phi} v)^2 \times y_f \times B'_0(\dots)$$

$$\propto \frac{(g_{hh\Phi\Phi} v)^2}{m_\Phi^2} \xrightarrow{g_{hh\Phi\Phi} v^2 \gg M^2} \mathcal{O}(g_{hh\Phi\Phi})$$



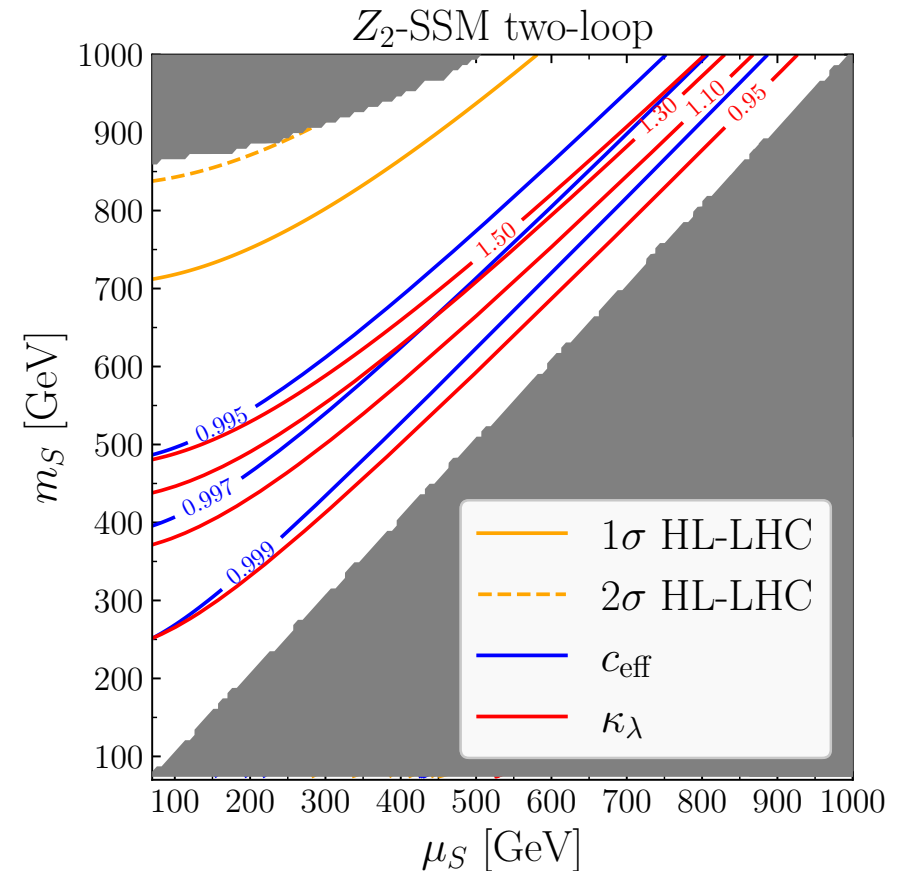
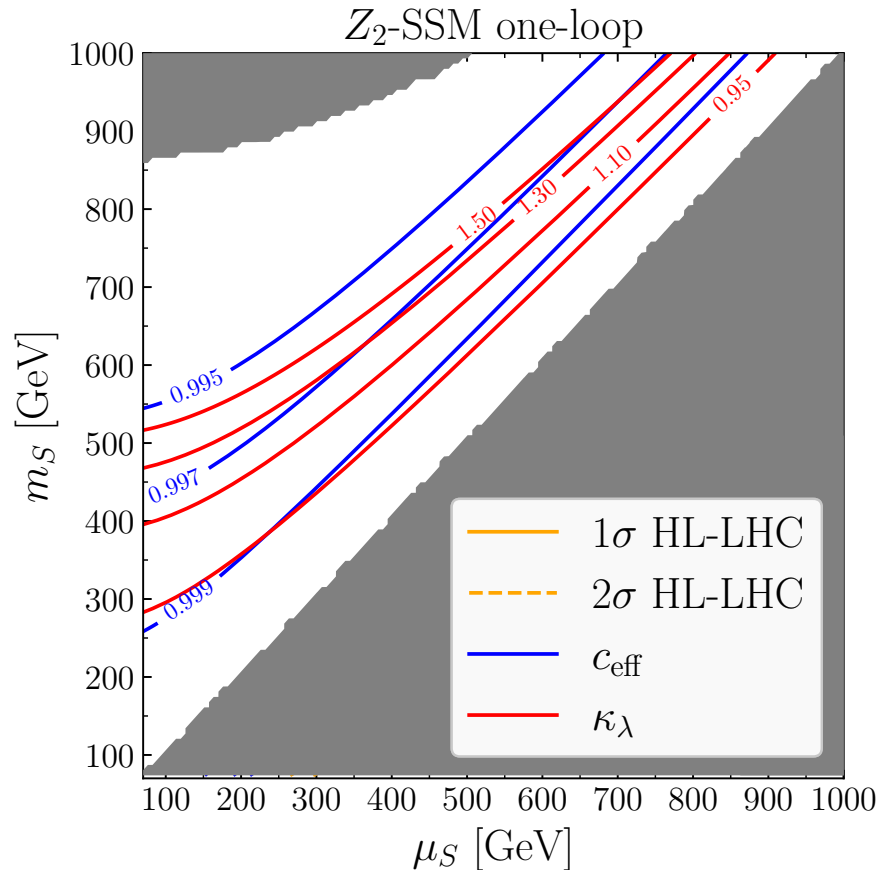
$$\propto g_{hh\Phi\Phi} v \times y_f^2 \times C_\mu(\dots)$$

$$\sim \frac{g_{hh\Phi\Phi} v}{m_\Phi} \xrightarrow{g_{hh\Phi\Phi} v^2 \gg M^2} \mathcal{O}(g_{hh\Phi\Phi}^{1/2})$$

[overview by J. Braathen]

# $Z_2$ SSM benchmark plane

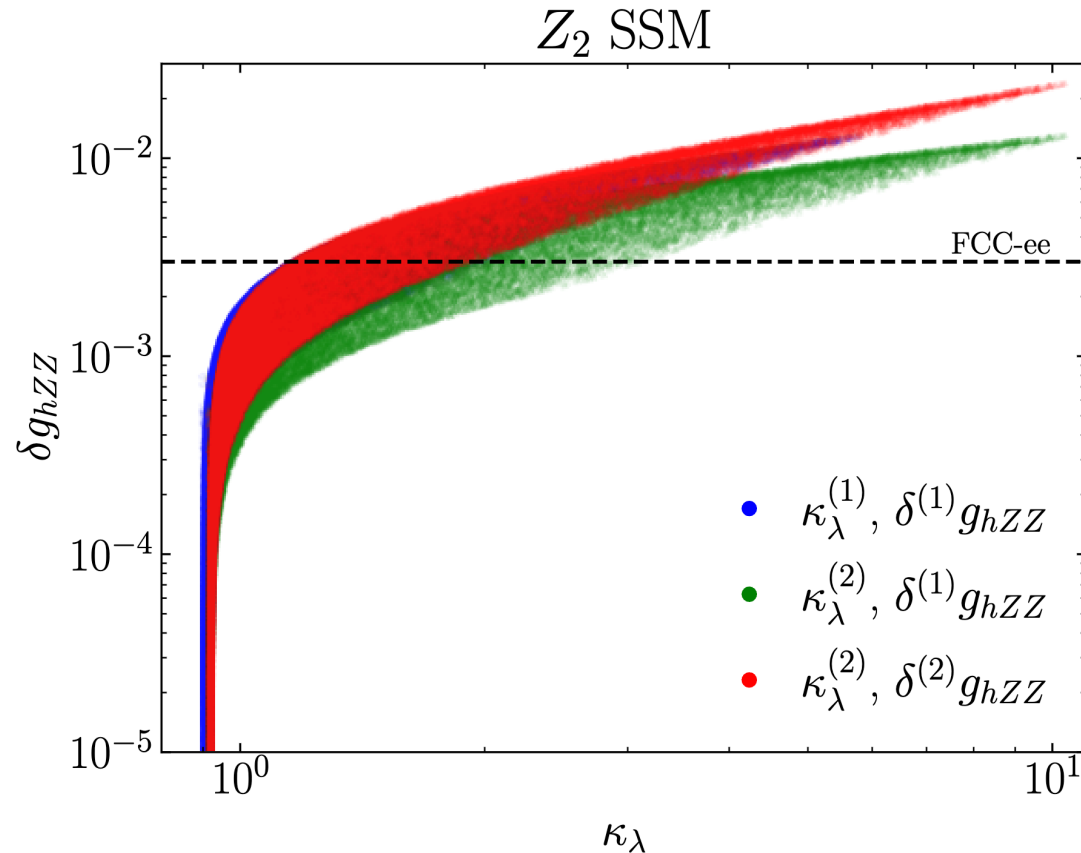
[HB et al., WIP]



Large deviations in  $\kappa_\lambda$  possible while other Higgs couplings very SM-like

# Parameter scan

[HB et al.,WIP]



- all points pass experimental and theoretical bounds
- $\kappa_\lambda \lesssim 2$  feasible within FCC-ee bounds on  $\delta g_{hZZ}$
- Projected future collider sensitivity on  $\kappa_\lambda$ :  $\mathcal{O}(10)\%$
- similar results in the other models (i.e., IDM, 2HDM)



Trilinear Higgs coupling  
can be discovery tool!

# Conclusions



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- precise determination of  $\lambda_{hhh}$  crucial to probe
    - shape of the Higgs potential
    - nature of EW phase transition
    - search for BSM physics
  - radiative corrections in extended Higgs sectors can significantly enhance  $\lambda_{hhh}$  within theoretical and experimental bounds
- ➡  $\lambda_{hhh}$  as a new precision probe of BSM parameter space
- loop-corrected trilinear can significantly distort invariant mass spectrum in di-HH
  - BSM physics could be found first in  $\lambda_{hhh}$ !

# Conclusions

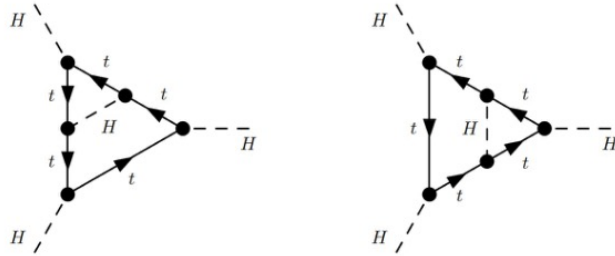
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**Thanks for your attention!**

# Appendix

# Reduction algorithm

## Automating the reduction



```
edgelist1 = {edge[v[1], v[4], S[1]], edge[v[2], v[5], S[1]],
             edge[v[3], v[6], S[1]], edge[v[4], v[7], -F[3]],
             edge[v[4], v[8], F[3]], edge[v[5], v[6], F[3]],
             edge[v[5], v[8], -F[3]], edge[v[6], v[7], F[3]],
             edge[v[7], v[8], S[1]]}
edgelist2 = {edge[v[1], v[4], S[1]], edge[v[2], v[5], S[1]],
             edge[v[3], v[6], S[1]], edge[v[4], v[5], F[3]],
             edge[v[4], v[7], -F[3]], edge[v[5], v[8], F[3]],
             edge[v[6], v[7], F[3]], edge[v[6], v[8], -F[3]],
             edge[v[7], v[8], S[1]]}
```

### Unique representation of diagram

→ "**canonical edges**"

- list of "edges" (= lines in diagram)
- identical diagrams ↔ permutations of edges
- canonical form = one particular choice of ordering

### Reduction algorithm with "canonical edges" in pseudo code:

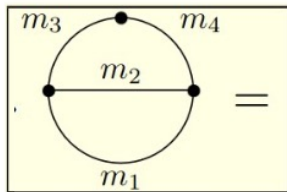
- identify internal and external indices
- generate permutations of external indices
- generate permutations of internal indices
- combine permutations of internal and external indices
- permute edge list following the combined list of permutations
- sort list of permuted edge lists
- return first element of sorted list of "edge lists"

# T-integral reduction

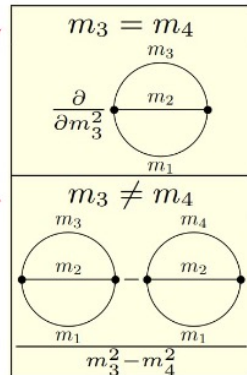
## Aparté 2: Reduction of T-integrals



E.g. reduction of  $T_{1134}$  integral



Analytically  
or  
on-the-fly  
during  
numerical  
evaluation



- Reduction of T-integrals to 2L sunrise ( $T_{134}$ ) and products of 1L integrals
- Can be done in with our Mathematica routines to handle and use the generic results
- Or can be done on-the-fly during numerical evaluation in our Python routines (similar to what is implemented in FeynHiggs now [Paßehr '22])

[Figure adapted from S. Paßehr]

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New: release of "Tintegrals"

```
pip install Tintegrals
```

```
import Tintegrals as T
```

```
x,y=125.**2,250.**2
```

```
T.setMudim(x)
```

```
T.T1134fin(x,y,x,y)
```

```
T.T1134fin(x,x,y,y)
```

