Pole mass determination in the presence of heavy particles

[based on 1812.06452]

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Pole determination for mixing scalars

- Mixing appears in the SM and many extensions

General problem

How to determine the pole masses?

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Use MSSM Higgs pole mass determination as example, but **arguments/methods are generally applicable**:

- Tree-level mass eigenstates: $\mathcal{CP}$-even h and H, $\mathcal{CP}$-odd A, $H^{\pm}$,
- loop corrections lead to mixing between h and H and A in case of $\mathcal{CP}$-violation (and Goldstone boson G^0)

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2. construct inverse Higgs propagator matrix,
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3. find poles of inverse propagator matrix.
   $\rightarrow$ straightforward??

1. Calculate Higgs self-energies

Hybrid approach of FeynHiggs:

$$\hat{\Sigma}_{ij}(p^2) = \hat{\Sigma}^{(1)}_{ij}(p^2) + \hat{\Sigma}^{(2)}_{ij}(0)\big|_{g=g'=0} + \text{higher-order logs}$$

- 1L and 2L self-energies obtained in diagrammatic fixed-order approach,
- approximation of vanishing electroweak gauge couplings and external momentum @ 2L,

  ($p^2 \neq 0$ can be included for QCD corrections)
- large logarithms resummed in EFT approach.

  (full LL+NLL, $\mathcal{O}(\alpha_s, \alpha_t)$ NNLL)

2. Construct inverse Higgs propagator matrix

$$i\Delta_{hH}^{-1}(p^2) = \begin{pmatrix} p^2 - m_h^2 + \hat{\Sigma}_{hh}(p^2) & \hat{\Sigma}_{hH}(p^2) \\ \hat{\Sigma}_{hH}(p^2) & p^2 - m_H^2 + \hat{\Sigma}_{HH}(p^2) \end{pmatrix}$$

General remarks:

- Discussion here restricted to 2×2 mixing between $\mathcal{CP}$ even states h and H
  (but also applies for 3×3 mixing),
- pole masses labelled by $M_{h_1} \leq M_{h_2} (\leq M_{h_3})$,
- $M_h \to h$-like state, $M_H \to H$-like state.

3. Find poles of inverse propagator matrix

Have to solve

$$\det\left(\Delta_{hH}^{-1}(p^2)\right) = 0.$$

How to solve this equation?

1. Fixed-order determination,
2. numerical determination,
3. improved fixed-order determination,
4. numerical determination with heavy-OS field renormalization.

Fixed-order pole determination

- Conceptionally very easy,
- truncate at the 2L level.

Solutions:

$$M_{h_1}^2 = m_h^2 - \hat{\Sigma}_{hh}^{(1)}(m_h^2) - \hat{\Sigma}_{hh}^{(2)}(0)\Big|_{\text{g=g'=0}} + \left[\hat{\Sigma}_{hh}^{(1)\prime}(m_h^2)\hat{\Sigma}_{hh}^{(1)}(m_h^2) + \frac{\left(\hat{\Sigma}_{hH}^{(1)}(m_h^2)\right)^2}{m_h^2 - m_H^2}\right]_{\text{g=g'=0}}$$

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- Problematic term: Gets very large if masses degenerate $\rightarrow$ break down of perturbative expansion.

Numerical pole determination

- Conceptionally very easy,
- solve for pole numerically.

Solutions:

$$M_{h_1}^2 = m_h^2 - \hat{\Sigma}_{hh}^{(1)}(m_h^2) - \hat{\Sigma}_{hh}^{(2)}(0)\Big|_{g=g'=0} + \hat{\Sigma}_{hh}^{(1)\prime}(m_h^2)\hat{\Sigma}_{hh}^{(1)}(m_h^2) + \frac{\left(\hat{\Sigma}_{hH}^{(1)}(m_h^2)\right)^2}{m_h^2 - m_H^2} + \ldots$$

- Including higher order terms proportional to $1/(m_h^2 - m_H^2)$ cures perturbative expansion.

Problems of numerical pole determination

For $M_{\text{SUSY}} \gg M_t$, we have

$$\hat{\Sigma}^{(1)} = \underbrace{\hat{\Sigma}^{(1),\text{heavy}}}_{\text{SUSY contr.}} + \underbrace{\hat{\Sigma}^{(1),\text{light}}}_{\text{SM contr.}} =$$

Comparison between EFT and hybrid approach showed:

- $\hat{\Sigma}_{hh}^{(1),\text{heavy}\prime}(m_h^2)\hat{\Sigma}_{hh}^{(1)}(m_h^2)$ is cancelled by parts of subloop renormalization contained in $\hat{\Sigma}_{hh}^{(2)}(0)\big|_{g=g'=0}$,
- cancellation incomplete, since terms are included at different orders of accuracy,
- similar incomplete cancellation at higher orders.

$\rightarrow$ Easy to solve in decoupling limit, but what's for low M_A?

Improved fixed-order pole determination

Determination of h-like state:

1. Expand $i\Delta_{hH}^{-1}$ around 1L solution $\left(M_h^{(1)}\right)^2 = m_h^2 - \hat{\Sigma}_{hh}^{(1)}(m_h^2)$,
2. get eigenvalues of expanded matrix

$$\left(i\Delta_{hH}^{-1,h-\exp}(p^2)\right)_{jk} = (p^2 - m_j^2)\delta_{jk} + \hat{\Sigma}_{jk}^{(1)}(m_h^2) + \hat{\Sigma}_{jk}^{(2)}(0)\Big|_{\text{g=g'=0}} - \left[\hat{\Sigma}_{jk}^{(1)\prime}(m_h^2)\hat{\Sigma}_{hh}^{(1)}(m_h^2)\right]_{\text{g=g'=0}} + \Delta_{jk}^{\text{logs}},$$

3. pick h-like eigenvalue corresponding to $M_h^2 = m_h^2 - \hat{\Sigma}_{hh}^{(1)}(m_h^2) + \ldots$,

   other eigenvalue would be $m_H^2 - \hat{\Sigma}_{HH}^{(1)}(m_h^2) + \ldots$

Determination of H-like state analogously, just have to $h \leftrightarrow H$.

Assement of the numerical pole determination

- Includes higher order $1/(m_h^2 - m_H^2)$ terms,
- cancellation between different terms of same order ensured,
- faster since no numerical pole search is required.

$\rightarrow$ Everything fine?

But: M_H^{125} scenario [Bagnaschi,HB,Fuchs,Hahn,Heinemeyer,Liebler,Patel,Slavich,Stefaniak,Wagner,Weiglein,1808.07542]

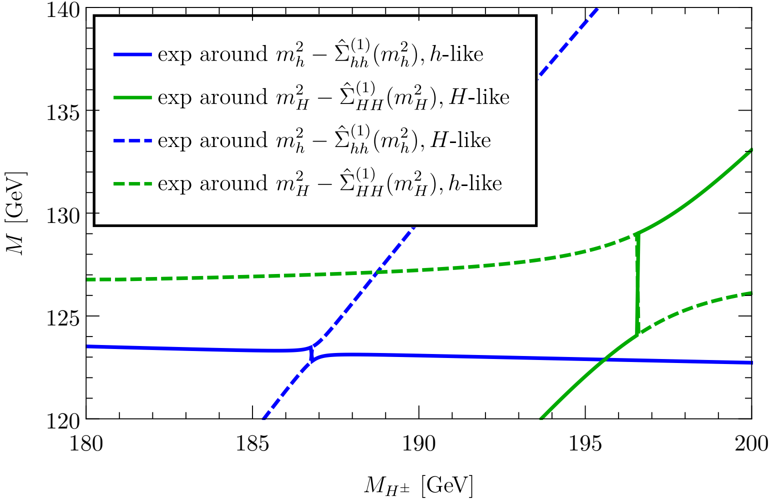
- MSSM Higgs benchmark scenario,
- parameters:

$$M_{\text{SUSY}} = 2\ \text{TeV},\ M_{\tilde{Q}_3} = M_{\tilde{U}_3} = 700\ \text{GeV},$$
$$\mu = 6\ \text{TeV},\ M_1 = 675\ \text{GeV},\ M_2 = 1\ \text{TeV},\ M_3 = 2.5\ \text{TeV},$$
$$A_t = 450\ \text{GeV},\ A_{b,c,s,u,d} = 0,$$

- scan over $M_{H^\pm}$ and $\tan\beta$.

M_{h_2} is suppossed to play role of SM-like Higgs boson

But: M_H^{125} scenario



Solid lines: "right" solutions; dashed lines: "wrong" solutions.

Assessment of improved fixed-order pole determination

- Algorithm works as wanted,
- 2L truncation, however, introduces "unphysical" jumps close to crossing points,
- reason: found solutions are not poles of every element of inverse propagator matrix.

$\rightarrow$ Still unsatisfying, can we find better method?

What is the origin of the observed cancellation?

- Uncancelled terms originate from p^2 dependence of **heavy** (non-SM) contributions to 1L self-energies,

$$\begin{aligned}\hat{\Sigma}(p^2) &= \Sigma(p^2) + \delta Z\,(p^2 - m^2) - \delta m^2 = \\ &= \Sigma^{\text{heavy}}(p^2) + \Sigma^{\text{light}}(p^2) \\ &\quad + \delta Z\,(p^2 - m^2) - \delta m^2 + \mathcal{O}(v/M_{\text{SUSY}}) = \\ &= \Sigma^{\text{heavy}}(m^2) + \left(\Sigma^{\text{heavy}\prime}(m^2) + \delta Z\right)\,(p^2 - m^2) \\ &\quad + \Sigma^{\text{light}}(p^2) - \delta m^2 + \mathcal{O}(v/M_{\text{SUSY}}),\end{aligned}$$

- higher derivatives of $\hat{\Sigma}^{\text{heavy}}$ suppressed, $\mathcal{O}(v/M_{\text{SUSY}})$

↓

p^2 dep. of “heavy” contributions can be absorbed into Higgs field renormalization

Heavy-OS field normalization

Field (re)normalization

Should drop out if calculating physical observables order by order!

$\rightarrow$ Reinstates decoupling theorem.

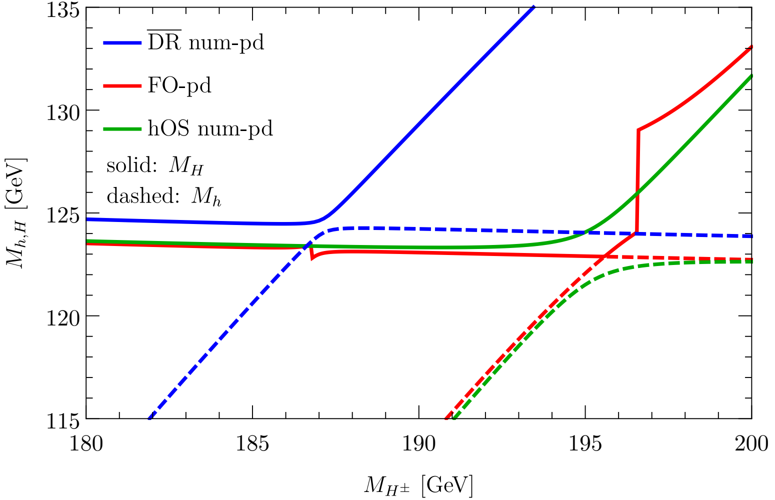
Prevent numerical det. from inducing terms $\propto \hat{\Sigma}^{\text{heavy}\prime}(m^2)$:

- Choose $\delta Z = -\hat{\Sigma}^{\text{heavy}\prime}(m^2)$, i.e.:

$$\delta^{(1)} Z_{hh} = -\hat{\Sigma}^{\text{heavy}\prime}_{hh}(0), \delta^{(1)} Z_{hH} = -\hat{\Sigma}^{\text{heavy}\prime}_{hH}(0), \ldots,$$

- can be evaluated at arbitrary $p^2 \ll M_{\text{SUSY}}$,
- evaluating at zero convenient
  $\rightarrow$ no unphysical thresholds are introduced.

M_H^{125} scenario



Further implications

Definition of $\tan\beta$:

- Heavy-OS field renormalization affects definition of $\tan\beta$:

$$\tan\beta^{\text{MSSM}}(\mu_R) \to \tan\beta^{\text{THDM}}(M_t),$$

- can be prevented introducing independent finite $\tan\beta$ counterterm.

Z matrix connecting physical and tree-level mass states:

- Depends on scheme for Higgs field renormalization,
- can be transformed back to normal $\overline{\text{DR}}$ scheme,
- using heavy-OS scheme helps to improve decoupling behaviour of Higgs decay and production calculations.

Conclusions

1. Fixed-order pole determination:
   - breakdown of perturbative expansion close to crossing points, ✗
   - proper decoupling of heavy particles, ✓
2. numerical pole determination:
   - smooth behaviour close to crossing points, ✓
   - incomplete decoupling of heavy particles, ✗
3. improved fixed-order pole determination:
   - unphysical jumps close to crossing points, ✗
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Thanks for your attention!

Comparison to other methods

Compare in the limit $M_A \gg M_t$:

- numerical pole determination:

$$M_{h_1}^2 = \ldots + \hat{\Sigma}_{hh}^{(1)\prime}(m_h^2)\hat{\Sigma}_{hh}(m_h^2) + \ldots$$

- fixed-order determination:

$$M_{h_1}^2 = \ldots + \left[\hat{\Sigma}_{hh}^{\mathrm{SM},(1)\prime}(m_h^2)\hat{\Sigma}_{hh}^{(1)}(m_h^2)\right]_{\mathrm{g}=\mathrm{g}'=0} + \ldots$$

- numerical pole determination with finite field renormalization:

$$M_{h_1}^2 = \ldots + \hat{\Sigma}_{hh}^{\mathrm{SM},(1)\prime}(m_h^2)\hat{\Sigma}_{hh}(m_h^2) + \ldots$$

Implications for high-scale scenario

all SUSY particles at common scale M_{SUSY}, $\tan\beta = 10$. Solid: $X_t^{\overline{\text{DR}}} = 0$; dashed: $X_t^{\overline{\text{DR}}} = \sqrt{6}$