Update on large log resummation in `FeynHiggs`

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- EFT calculations allow to resum large logarithms
  $\rightarrow$ should be accurate for high SUSY scale M_{Susy}
- miss however terms $\propto v/M_{\text{Susy}}$
- diagrammatic calculation expected to be more accurate for low M_{Susy} ($\lesssim$ few TeV)

Goal

Combine both approaches to get precise results for both regimes.

Procedure in `FeynHiggs`

1. calculate fixed-order corrections
2. subtract logarithms already contained in fixed-order result
3. resum logarithms using EFT approach
4. add resummed logarithms to fixed-order result

Current status

- fixed-order → full 1L + $\mathcal{O}\left(\alpha_s(\alpha_t + \alpha_b), (\alpha_t + \alpha_b)^2\right)$
- EFT → full LL+NLL, $\mathcal{O}(\alpha_s\alpha_t, \alpha_t^2)$ NNLL,
  intermediary EWino threshold

FeynHiggs 2.14.0

implements changes discussed in [HB Heinemeyer Hollik Weiglein 1706.00346]

- optional $\overline{\text{DR}}$ renormalization of stop sector
- improved calculation of pole masses/Z factors
- small improvements of resummation routines
  - now $v_{\overline{\text{MS}}}$ is used
  - improved 2L subtraction term for `runningMT = 1` ($\overline{\text{MS}}$ top mass)

Optional $\overline{\text{DR}}$ renormalization of stop sector

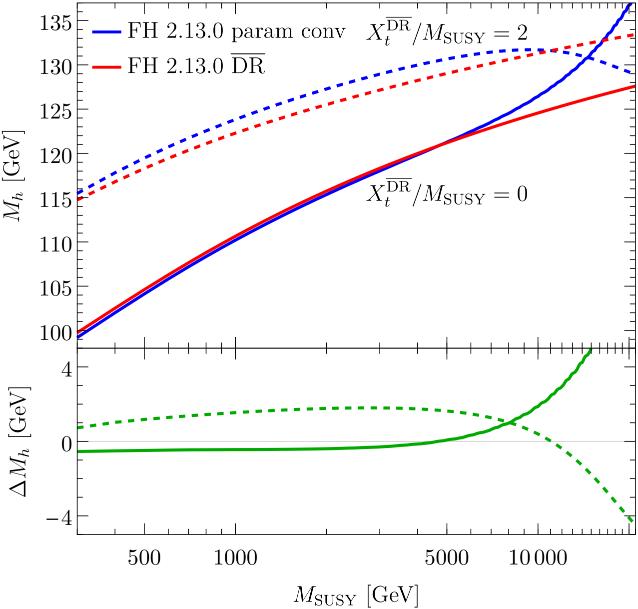
So far

- FH uses OS scheme for renormalization of stop sector
- 1L parameter conversion in case of $\overline{\text{DR}}$ input parameters

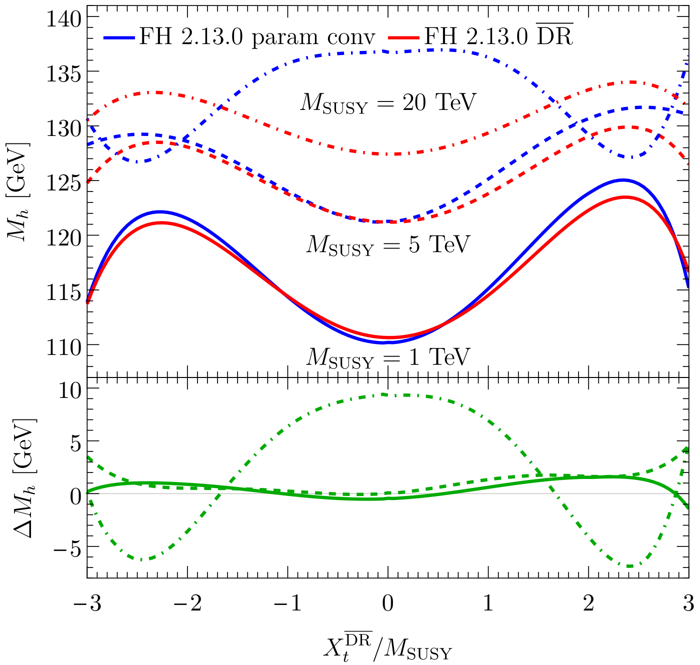
↯ conversion not adequate for result containing resummed logs

Therefore

- optional $\overline{\text{DR}}$ renormalization of stop sector
- automatically active if parameter `Qt` $\neq 0$
- for sbottom sector still a parameter conversion is used



FH 2.13.0 param conv
FH 2.13.0 $\overline{\text{DR}}$
$X_t^{\overline{\text{DR}}}/M_{\text{SUSY}} = 2$
$X_t^{\overline{\text{DR}}}/M_{\text{SUSY}} = 0$
M_h [GeV]
ΔM_h [GeV]
M_{SUSY} [GeV]
100
105
110
115
120
125
130
135
4
0
−4
500
1000
5000
10 000


FH 2.13.0 param conv
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$M_{\text{SUSY}} = 20$ TeV
$M_{\text{SUSY}} = 5$ TeV
$M_{\text{SUSY}} = 1$ TeV
M_h [GeV]
ΔM_h [GeV]
$X_t^{\overline{\text{DR}}}/M_{\text{SUSY}}$

Improved calculation of pole masses/Z factors I

For $M_A \gg M_Z$,

$$M_h^2 = m_h^2 - \hat{\Sigma}_{hh}^{(1)}(m_h^2) - \hat{\Sigma}_{hh}^{(2)}(m_h^2) + \hat{\Sigma}_{hh}^{(1)\prime}(m_h^2)\hat{\Sigma}_{hh}^{(1)}(m_h^2) + \ldots$$

- Non-SM contributions to $\hat{\Sigma}_{hh}^{(1)\prime}(m_h^2)\hat{\Sigma}_{hh}^{(1)}(m_h^2)$ are cancelled by subloop-renormalization in $\hat{\Sigma}_{hh}^{(2)}(m_h^2) \rightarrow$ vev-CT
- holds generally at 2L (probably also at higher orders)
- but FH includes $\hat{\Sigma}_{hh}^{(2)}$ only for vanishing electroweak couplings $\rightarrow$ incomplete cancellation

Solution easy for $M_A \gg M_Z$, but what to do for $M_A \sim M_Z$?

Improved calculation of pole masses/Z factors II

Need to determine poles of inverse propagator matrix

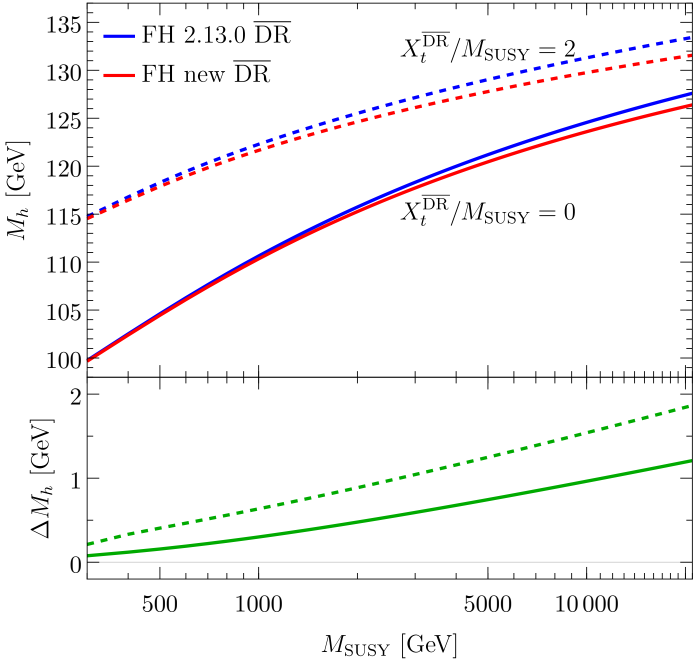
$$\Delta^{-1}(p^2) =$$
$$= \begin{pmatrix} p^2 - m_h^2 + \hat{\Sigma}^{(1)}_{hh}(p^2) + \hat{\Sigma}^{(2)}_{hh}(0) & \hat{\Sigma}^{(1)}_{hH}(p^2) + \hat{\Sigma}^{(2)}_{hH}(0) \\ \hat{\Sigma}^{(1)}_{hH}(p^2) + \hat{\Sigma}^{(2)}_{hH}(0) & p^2 - m_H^2 + \hat{\Sigma}^{(1)}_{HH}(p^2) + \hat{\Sigma}^{(2)}_{HH}(0) \end{pmatrix}$$

At 1L level $M_h^2 = m_h^2 - \hat{\Sigma}^{(1)}_{hh}(m_h^2) \rightarrow$ expand around 1L solution

$\Rightarrow$ determine poles of

$$\Delta^{-1}_{hh}(p^2) = p^2 - m_h^2 \quad + \hat{\Sigma}^{(1)}_{hh}(m_h^2) \quad + \hat{\Sigma}^{(2)}_{hh}(0) \quad - \left[\hat{\Sigma}^{(1)\prime}_{hh}(m_h^2)\hat{\Sigma}^{(1)}_{hh}(m_h^2)\right]_{g=g_Y=0}$$
$$\Delta^{-1}_{hH}(p^2) = \quad + \hat{\Sigma}^{(1)}_{hH}(m_h^2) \quad + \hat{\Sigma}^{(2)}_{hH}(0) \quad - \left[\hat{\Sigma}^{(1)\prime}_{hH}(m_h^2)\hat{\Sigma}^{(1)}_{hh}(m_h^2)\right]_{g=g_Y=0}$$
$$\Delta^{-1}_{HH}(p^2) = p^2 - m_H^2 \quad + \hat{\Sigma}^{(1)}_{HH}(m_h^2) \quad + \hat{\Sigma}^{(2)}_{HH}(0) \quad - \left[\hat{\Sigma}^{(1)\prime}_{HH}(m_h^2)\hat{\Sigma}^{(1)}_{hh}(m_h^2)\right]_{g=g_Y=0}$$

For determination of M_H expand around $M_H^2 = m_H^2 - \hat{\Sigma}^{(1)}_{HH}(m_H^2)$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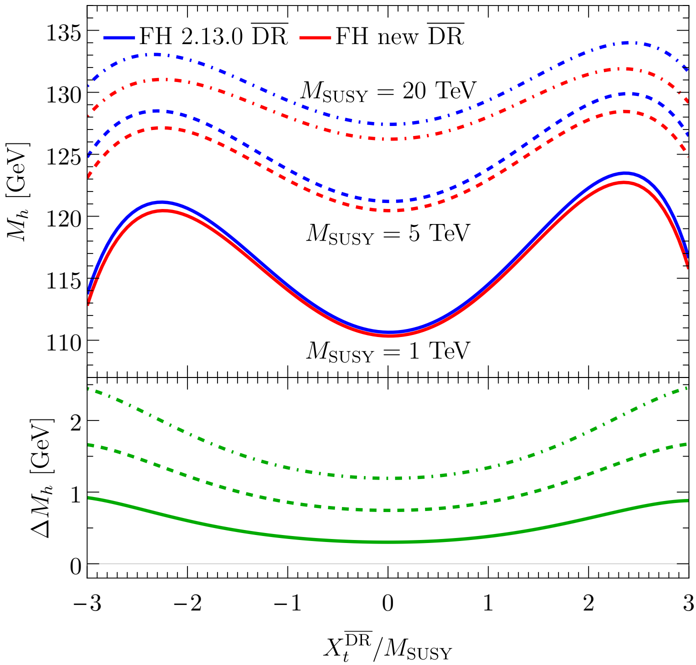
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115
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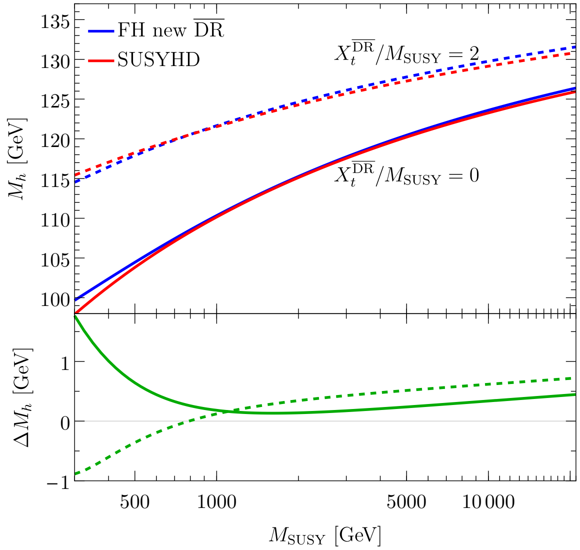
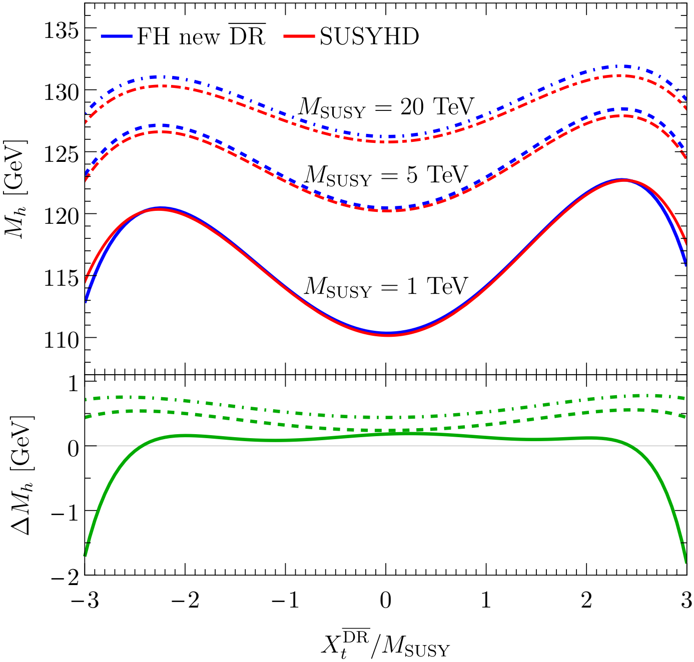
2
1
0
−3
−2
−1
0
1
2
3

Comparison to SUSYHD for single-scale scenario

$$\tan\beta = 10,$$
$$M_{\text{soft}} = M_{\text{Susy}},$$
$$\mu = M_A = M_{\text{Susy}},$$
$$A_{b,c,s,e,\mu,\tau} = 0$$

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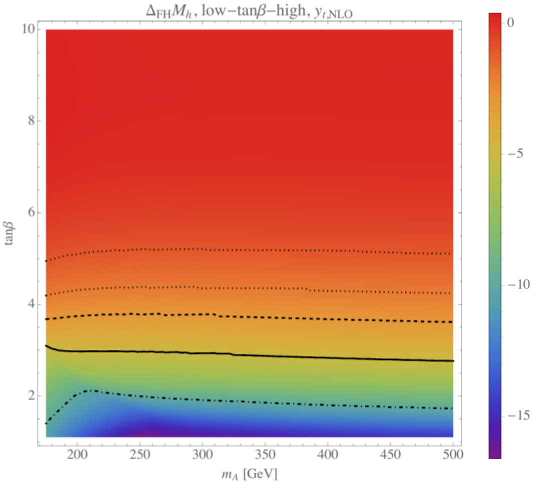
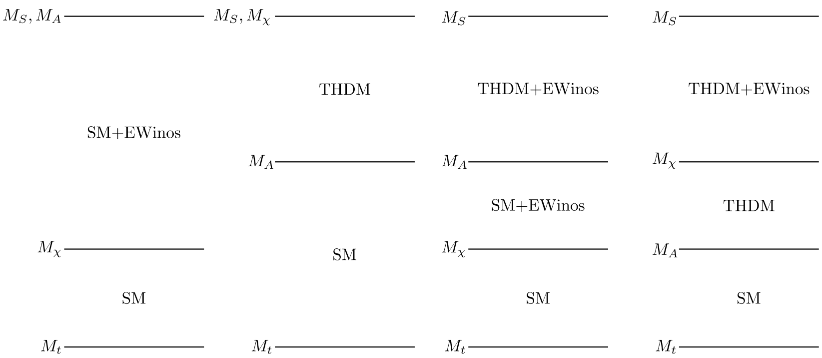
3

Current status / motivation

- MhEFT [Lee Wagner 1508.00576]
- FlexibleSUSY [Athron Park Steudtner Stöckinger Voigt 1609.00371]



EFTs for low M_A



Effective Lagrangians

$$\mathcal{L}_{\text{THDM}} = \ldots - V_{\text{THDM}}(H_u, H_d) - h_t \epsilon_{ij} \bar{t}_R Q_L^i H_u^j - h_t' \bar{t}_R Q_L H_d$$

$\rightarrow$ 9 effective couplings $(\lambda_{1..7}, h_t, h_t')$

$$\begin{aligned}
\mathcal{L}_{\text{THDM+EWinos}} = \ldots &- \frac{1}{\sqrt{2}} H_u^\dagger \left(\hat{g}_{2uu} \sigma^a \tilde{W}^a + \hat{g}_{1uu} \tilde{B} \right) \tilde{\mathcal{H}}_u \\
&- \frac{1}{\sqrt{2}} H_d^\dagger \left(\hat{g}_{2dd} \sigma^a \tilde{W}^a - \hat{g}_{1dd} \tilde{B} \right) \tilde{\mathcal{H}}_d \\
&- \frac{1}{\sqrt{2}} (-i H_d^T \sigma_2) \left(\hat{g}_{2du} \sigma^a \tilde{W}^a + \hat{g}_{1du} \tilde{B} \right) \tilde{\mathcal{H}}_u \\
&- \frac{1}{\sqrt{2}} (-i H_u^T \sigma_2) \left(\hat{g}_{2ud} \sigma^a \tilde{W}^a - \hat{g}_{1ud} \tilde{B} \right) \tilde{\mathcal{H}}_d \\
&+ h.c. - V_{\text{THDM}}(H_u, H_d),
\end{aligned}$$

$\rightarrow$ 17 effective couplings

Status of EFT calculation

- all possible hierarchies taken into account.
- full 2L running for all effective couplings (RGEs via `SARAH`)
- full 1L threshold corrections for all effective couplings
- $\mathcal{O}(\alpha_s \alpha_t)$ threshold corrections for λ_i's

$\rightarrow$ finished

Preparation of fixed-order calculation I

Problem

Renormalization scale of fixed-order calculation is by default chosen to be $M_t \to t_\beta = t_\beta^{\text{MSSM}}(M_t)$

- we however need $t_\beta^{\text{MSSM}}(M_S)$ as input for EFT calculation
- solved so far by including 1L running of t_β
- this procedure however fails for $M_A < M_S$ since it misses threshold corrections relating $t_\beta^{\text{THDM}}(M_S) \leftrightarrow t_\beta^{\text{MSSM}}(M_S)$,

$$t_\beta^{\text{THDM}}(M_S) = t_\beta^{\text{MSSM}}(M_S)\left[1 + \frac{1}{4} k h_t^2 (\hat{A}_t - \hat{\mu}/t_\beta)(\hat{A}_t + \hat{\mu} t_\beta)\right]$$

Preparation of fixed-order calculation II

Solution

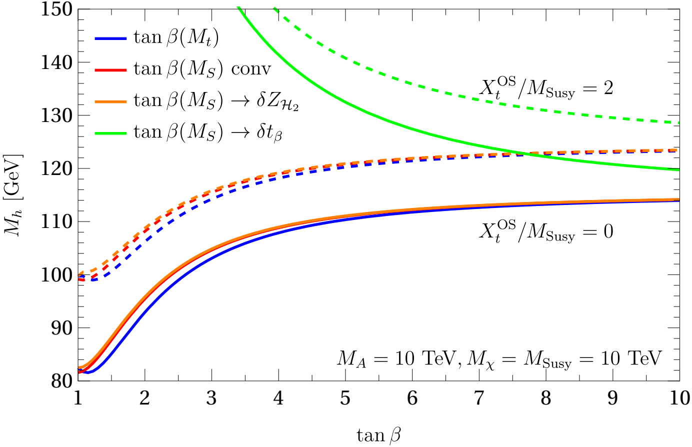
Change renormalization scale of t_β in fixed-order calculation

Idea: Introduce finite CT for t_β adapting its scale to M_S

$$t_\beta^{\text{MSSM}}(M_S) = t_\beta^{\text{MSSM}}(M_t)\left(1 - \frac{3}{2}k\frac{y_t^2}{s_\beta^2}\ln\frac{M_S^2}{M_t^2}\right) \Rightarrow \delta t_\beta^{\text{fin}} = \frac{3}{2}k\frac{y_t^2}{s_\beta c_\beta}\ln\frac{M_S^2}{M_t^2}$$

Two different methods:

1. regard $\delta t_\beta^{\text{fin}}$ as independent finite CT
2. employ relation $\delta t_\beta = \frac{1}{2}t_\beta\left(\delta Z_{\mathcal{H}_2} - \delta Z_{\mathcal{H}_1}\right)$,
   i.e. $\delta Z_{\mathcal{H}_2}\big|_{\text{fin}} = 2\delta t_\beta|_{\text{fin}}/t_\beta$



Independent $\delta t_\beta^{\mathrm{fin}}$ yields stable results only at strict 2L level,

$$M_h^2 = m_h^2 - \hat{\Sigma}_{hh}^{(1)}(m_h^2) - \hat{\Sigma}_{hh}^{(2)}(m_h^2) + \hat{\Sigma}_{hh}^{(1)\prime}(m_h^2)\hat{\Sigma}_{hh}^{(1)}(m_h^2) + \frac{\left(\hat{\Sigma}_{hH}^{(1)}(m_h^2)\right)^2}{m_h^2 - m_H^2}$$

Matching to fixed order calculation

- Running from M_S to $M_A \to \Delta\hat{\Sigma}_{11}, \Delta\hat{\Sigma}_{12}, \Delta\hat{\Sigma}_{22}$, e.g.

$$\Delta\hat{\Sigma}_{11} =$$
$$= \left[M_A^2 s_\beta^2 + v^2 \left(3\lambda_1 c_\beta^2 + (\lambda_3 + \lambda_4 + \lambda_5) s_\beta^2 + 6\lambda_6 s_\beta c_\beta\right)\right]_{Q=M_A}$$
$$- \text{subtraction terms}$$

- Running from M_A to $M_t \to \Delta\hat{\Sigma}_{22} = \lambda(m_t) v^2 / s_\beta^2$ (as in high M_A case)

Matching to fixed order calculation II

Normalization of Higgs doublets

MSSM and THDM Higgs doublets have not the same normalization

- LSZ theorem yields

$$\begin{pmatrix} \Phi_1^{\text{THDM}} \\ \Phi_2^{\text{THDM}} \end{pmatrix} = \underbrace{\begin{pmatrix} 1 + \frac{1}{2}\Delta\Sigma'_{11} & \frac{1}{2}\Delta\Sigma'_{12} \\ \frac{1}{2}\Delta\Sigma'_{12} & 1 + \frac{1}{2}\Delta\Sigma'_{22} \end{pmatrix}}_{=U_{\Delta\Phi}} \begin{pmatrix} \Phi_1^{\text{MSSM}} \\ \Phi_2^{\text{MSSM}} \end{pmatrix}$$

with $\Delta\Sigma'_{ij} = \Sigma_{ij}^{\text{MSSM}} - \Sigma_{ij}^{\text{THDM}}$

$$\Rightarrow \Delta^{-1}_{\text{MSSM}}(p^2) = U^T_{\Delta\Phi}\Delta^{-1}_{\text{THDM}}(p^2)U_{\Delta\Phi}$$

- pole masses do not depend on absolute field normalization
$\rightarrow$ not important for pure EFT calculation

Matching to fixed order calculation III

Hybrid calculation in `FeynHiggs`:

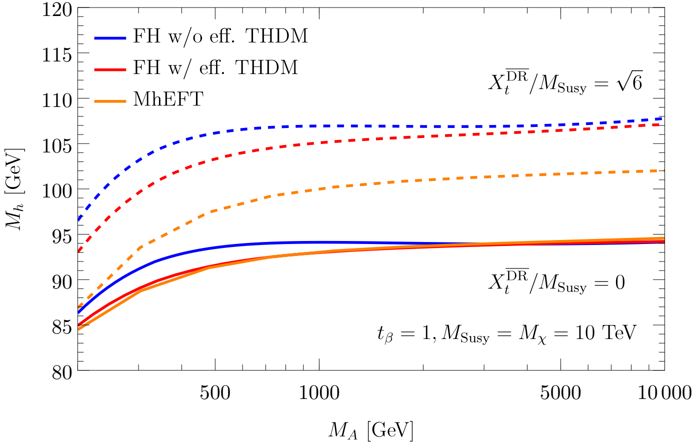
$$\Delta_{\texttt{FH}}^{-1}(p^2) =$$
$$= \begin{pmatrix} p^2 - m_h^2 + \hat{\Sigma}_{hh}^{\text{FO}}(p^2) + \Delta\Sigma_{hh}^{\text{logs}} & \hat{\Sigma}_{hH}^{\text{FO}}(p^2) + \Delta\Sigma_{hH}^{\text{logs}} \\ \hat{\Sigma}_{hH}^{\text{FO}}(p^2) + \Delta\Sigma_{hH}^{\text{logs}} & p^2 - m_H^2 + \hat{\Sigma}_{HH}^{\text{FO}}(p^2) + \Delta\Sigma_{HH}^{\text{logs}} \end{pmatrix}$$

with $\Delta\Sigma_{ij}^{\text{logs}} = \Sigma_{ij}^{\text{EFT}} - \Sigma_{ij}^{\text{sub}}$

“Relative” normalization important for

- correct merging of EFT result (THDM normalization) with fixed order result (MSSM renormalization)
- calculation of 2L subtraction terms

Current status:
Works fine at 1L, derivatives of 2L self-energies missing



- preliminary (subtraction terms modified by hand)
- `MhEFT` employs $\overline{\text{MS}}$ renormalization of X_t, no conv. taken into account

Conclusion

Next version: `FeynHiggs 2.14.0`

- optional $\overline{\mathrm{DR}}$ renormalization of stop sector
- improved calculation of pole masses/Z factors
- small improvements of resummation routines

Single-scale SUSY:

- good agreement between various codes
- time to look at scenarios with more mass scales

Low M_A scenario:

- Upcoming extension of `FH` with effective THDM
- needs careful defintion of $\tan\beta$
- important to take different normalizations of Higgs doublets into account

The OS vev-counterterm is given by

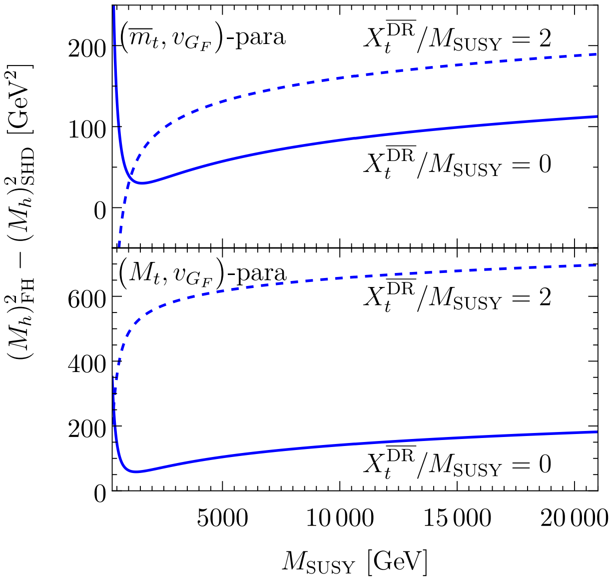
$$\delta v^2 = v^2 \left[\frac{\delta M_W^2}{M_W^2} + \frac{c_w^2}{s_w^2} \left(\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2} \right) - \frac{\delta e^2}{e^2} \right] \overset{\mathcal{O}(\alpha_s,\alpha_t)}{=}$$
$$= v^2 \left(-\hat{\Sigma}_{hh}^{(1)\prime}(m_h^2) + \text{ SM corrections} \right).$$

The Higgs pole mass is calculated via

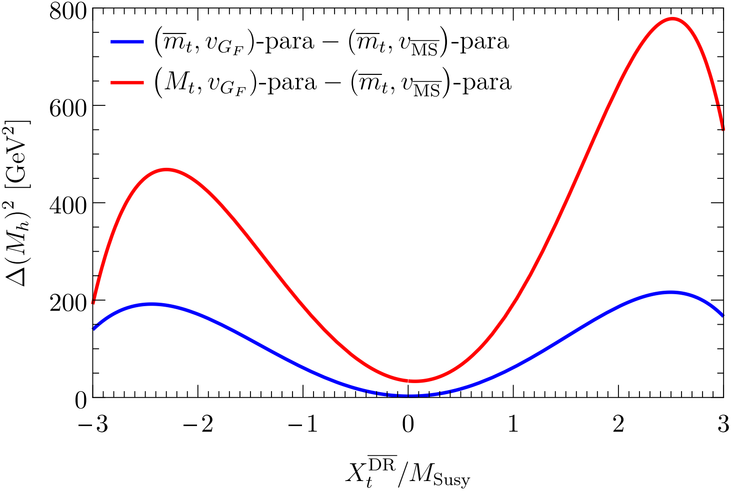
$$M_h^2 = m_h^2 - \hat{\Sigma}_{hh}^{(1)}(m_h^2) - \hat{\Sigma}_{hh}^{(2)}(0) + \Sigma_{hh}^{(1)\prime}(m_h^2)\Sigma_{hh}^{(1)}(m_h^2) + ...$$

The renormalized two-loop self-energy reads

$$\hat{\Sigma}_{hh}^{(2)}(0) = \Sigma_{hh}^{(2)}(0) + \frac{\partial}{\partial v^2}\hat{\Sigma}_{hh}^{(1)}(m_h^2) \cdot \delta v^2 + ... =$$
$$= \Sigma_{hh}^{(2)}(0) - \hat{\Sigma}_{hh}^{(1)}(m_h^2)\frac{\delta v^2}{v^2} + ... =$$
$$= \Sigma_{hh}^{(2)}(0) + \hat{\Sigma}_{hh}^{(1)}(m_h^2)\hat{\Sigma}_{hh}^{(1)\prime}(m_h^2) + ...$$



$\longrightarrow$ nearly constant difference for high scales


800
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1
2
3
$(\overline{m}_t, v_{G_F})$-para − $(\overline{m}_t, v_{\overline{\rm MS}})$-para
(M_t, v_{G_F})-para − $(\overline{m}_t, v_{\overline{\rm MS}})$-para
$\Delta(M_h)^2$ [GeV^2]
$X_t^{\overline{\rm DR}}/M_{\rm Susy}$

Origin

Different parametrization of non-logarithmic terms

Three ways to parametrize top Yukawa coupling in FO result

- $M_t/v \rightarrow$ `FeynHiggs` with `runningMT = 0`
- $\overline{m}_t/v \rightarrow$ `FeynHiggs` with `runningMT = 1`
- $y_t^{\overline{\text{MS}}} = \overline{m}_t/v_{\overline{\text{MS}}} \rightarrow$ `SUSYHD`

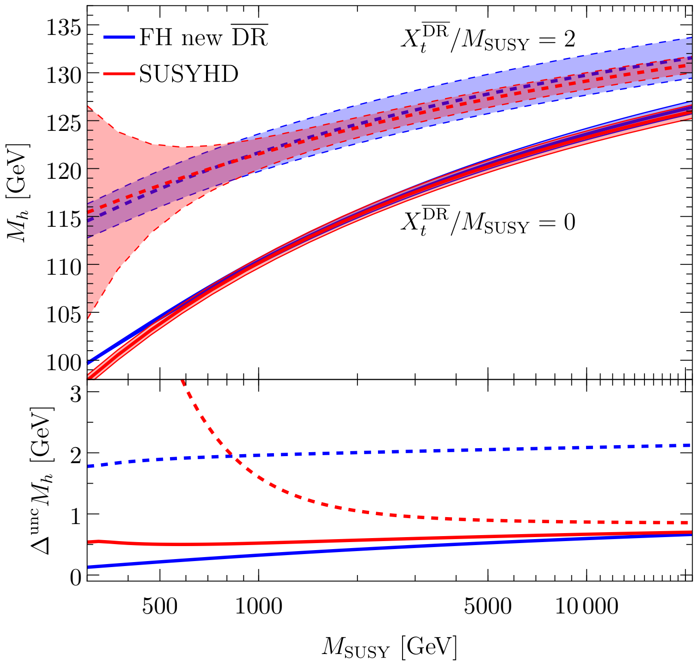
Equivalent at 2L order, but induces differences at higher order

Uncertainty estimate of SUSYHD

1. EFT uncertainty
   - $\mathcal{O}(v/M_S)$ terms
   - estimated by $v/M_S \cdot$(1L correction)
2. SM uncertainty:
   - higher order corrections to pole mass extraction
   - estimated by (de)activating higher order corrections to y_t and $\delta\lambda$
3. SUSY uncertainty:
   - higher order threshold corrections
   - estimated by variation of matching scale $1/2 < Q/M_S < 2$

Uncertainty estimate of FeynHiggs

1. Scale variation:
    - variation of renormalization scale between $1/2M_t$ and $2M_t$
2. Renormalization scheme dependence:
    - switching between OS top mass and $\overline{\text{MS}}$ top mass
3. $\tan\beta$ enhanced correction
    - (de)activating resummation of bottom Yukawa coupling



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135
130
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120
115
110
105
100
3
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0
500
1000
5000
10 000

$$\overline{X}_t(M_S) = X_t^{\mathrm{OS}} \left\{ 1 + \left[\frac{\alpha_s}{\pi} - \frac{3\alpha_t}{16\pi} (1 - \hat{X}_t^2) \right] L + \frac{3}{16\pi} \frac{\alpha_t}{t_\beta^2} \hat{Y}_t^2 L_A \right\}$$