Update on large-log resummation

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2nd International FeynHiggs Meeting
29.11.2016, Munich

- EFT calculations allow to resum large logarithms
  $\rightarrow$ should be accurate for high SUSY scale M_S
- misses however terms $\propto v/M_S$
- diagrammatic calculation expected to be more accurate for low M_S ($\lesssim$ few TeV)

Goal

Combine both approaches to get precise results for both regimes.

If not stated otherwise all plots with parameters

$$\tan\beta = 10, \ \ M_{\text{soft}} = \mu = M_A \equiv M_S, \ \ A_{b,c,s,e,\mu,\tau} = 0$$

Current status

FeynHiggs resummation procedures at the very similar level of accuracy as pure EFT calculations

$\downarrow$

expected to see correspondence for high scales, but so far still large discrepancies could be observed

Discussions mainly about

- $\overline{\text{DR}} \leftrightarrow$ OS conversion
- terms induced by momentum dependence of Higgs self-energy

FeynHiggs uses mixed OS/$\overline{\text{DR}}$ scheme
→ to use $\overline{\text{DR}}$ input parameters conversion necessary

Procedure so far

- $m^{\overline{\text{DR}}}_{\tilde{t}_{1,2}}, X_t^{\overline{\text{DR}}}, X_b^{\overline{\text{DR}}} \overset{\mathcal{O}(\alpha_s,\alpha_t,\alpha_b)}{\longrightarrow} M_{\tilde{t}_{1,2}}, X_t^{\text{OS}}, X_b^{\text{OS}}$
- Forget about $m^{\overline{\text{DR}}}_{\tilde{t}_{1,2}}, X_t^{\overline{\text{DR}}}, X_b^{\overline{\text{DR}}}$, use $M_{\tilde{t}_{1,2}}, X_t^{\text{OS}}, X_b^{\text{OS}}$ as 'new' input parameters
- No conversion of $\mu, M_A, M_{\tilde{b}_{1,2}}, ...$

Two problems with this approach

1. Conversion induces terms beyond 2L level
2. X_t, entering in resummation procedure, is calculated by

$$X_t^{\overline{\text{DR}},\text{EFT}} = X_t^{\text{OS}} \left[1 + \left(\frac{\alpha_s}{\pi} - \frac{3\alpha_t}{16\pi} \left(1 - \hat{X}_t^2 \right) \right) \ln \left(\frac{M_S^2}{m_t^2} \right) \right].$$

$\Rightarrow X_t^{\overline{\text{DR}},\text{EFT}} \neq X_t^{\overline{\text{DR}}}$

Alternative method: fixed-order conversion

Set $X_t^{\overline{\text{DR}}} = X_t^{\text{OS}} = X_t^{\overline{\text{DR}},\text{EFT}}$, add missed 2L terms by hand.

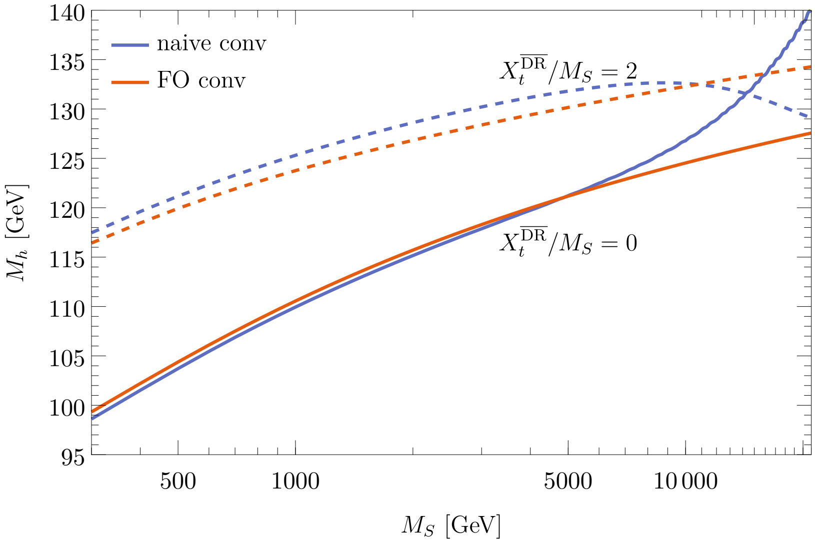
e.g. $(\hat{X}_t = X_t/M_S, \hat{\mu} = \mu/M_S)$

$$\hat{\Sigma}^{1\text{L}}_{\phi_1\phi_1} = 2k\frac{1}{s_\beta^2}\frac{m_t^4}{v^2}\hat{\mu}^2\left(\hat{X}_t^{\text{OS}}\right)^2$$

$$\Big\downarrow X_t^{\text{OS}} = X_t^{\overline{\text{DR}}} + k\Delta X_t, \quad M_S^{\text{OS}} = M_S^{\overline{\text{DR}}} + k\Delta M_S$$

$$2k\frac{1}{s_\beta^2}\frac{m_t^4}{v^2}\hat{\mu}^2\left(\hat{X}_t^{\overline{\text{DR}}}\right)^2 + 4k^2\frac{1}{s_\beta^2}\frac{m_t^4}{v^2}\left[\frac{\Delta X_t}{M_S^{\overline{\text{DR}}}}\hat{X}_t^{\overline{\text{DR}}}\hat{\mu}^2 - 2\frac{\Delta M_S}{M_S^{\overline{\text{DR}}}}\left(\hat{X}_t^{\overline{\text{DR}}}\right)^2\hat{\mu}^2\right]$$

- solves both problems by construction


naive conv
FO conv
$X_t^{\overline{\text{DR}}}/M_S = 2$
$X_t^{\overline{\text{DR}}}/M_S = 0$
M_h [GeV]
M_S [GeV]
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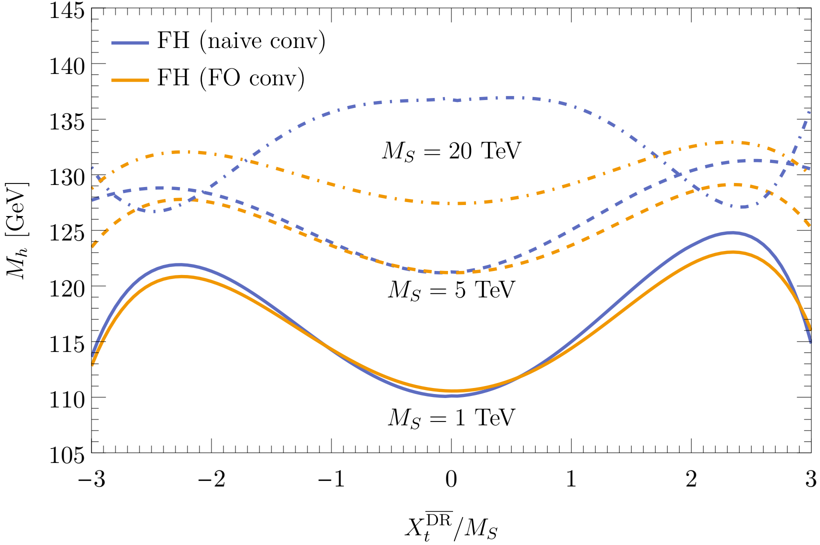
FH (naive conv)
FH (FO conv)
$M_S = 20$ TeV
$M_S = 5$ TeV
$M_S = 1$ TeV
M_h [GeV]
$X_t^{\overline{\text{DR}}}/M_S$
145
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135
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0
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2
3

Diagrammatic calculation

In limit $M_A \gg M_Z$ Higgs pole mass is determined by

$$\begin{aligned}(M_h^2)_{\text{FD}} &= m_h^2 - \hat{\Sigma}_{hh}^{\text{MSSM}}(M_h^2) = \\ &= m_h^2 - \hat{\Sigma}_{hh}^{\text{MSSM}}(m_h^2) + \hat{\Sigma}_{hh}^{\text{MSSM}\prime}(m_h^2)\hat{\Sigma}_{hh}^{\text{MSSM}}(m_h^2) + \ldots\end{aligned}$$

EFT calculation

Calculate $\lambda(M_t)$ by RGE running. Extract pole mass out of $\lambda(M_t)$ via

$$(M_h^2)_{\text{EFT}} = v^2\lambda_{\text{OS}} = v^2\lambda(M_t) - v^2\delta\lambda = \text{ (finite parts only)}$$

$$= v^2\lambda(M_t) - \frac{\delta T}{v} - \delta M_h^2 + \lambda\delta v^2 - \delta\lambda^{(2)}v^2 =$$

$$= v^2_{\overline{\text{MS}}}\lambda(M_t) + \frac{T^{\text{SM}}}{v} - \Sigma_{hh}^{\text{SM}}(M_h^2) - \delta\lambda^{(2)}v^2 =$$

$$= v^2_{\overline{\text{MS}}}\lambda(M_t) + \frac{T^{\text{SM}}}{v} - \Sigma_{hh}^{\text{SM}}(m_h^2) - \delta\lambda^{(2)}v^2 + \Sigma_{hh}^{\text{SM}\prime}(m_h^2)\Big(...\Big) + ...$$

Hybrid approach in FeynHiggs

Calculate $\lambda(M_t)$ by RGE running. Extract pole mass out of $\lambda(M_t)$ via

$$
\begin{aligned}
(M_h^2)_{\text{FH}} =& \\
=& m_h^2 - \underbrace{\hat{\Sigma}_{hh}^{\text{MSSM}}(M_h^2)}_{\text{FO result}} + \underbrace{\left[v^2_{\overline{\text{MS}}}\lambda(M_t)\right]_{\text{logs}}}_{\text{EFT result}} + \underbrace{\left[\hat{\Sigma}_{hh}^{\text{MSSM}}(m_h^2)\right]_{\text{logs}}}_{\text{subtraction term}} = \\
=& m_h^2 + \left[v^2_{\overline{\text{MS}}}\lambda(M_t)\right]_{\text{logs}} - \left[\hat{\Sigma}_{hh}^{\text{MSSM}}(m_h^2)\right]_{\text{nolog}} \\
& - \hat{\Sigma}_{hh}^{\text{MSSM}\prime}(m_h^2)\left(\left[v^2_{\overline{\text{MS}}}\lambda(M_t)\right]_{\text{logs}} - \left[\hat{\Sigma}_{hh}^{\text{MSSM}}(m_h^2)\right]_{\text{nolog}}\right) + \ldots
\end{aligned}
$$

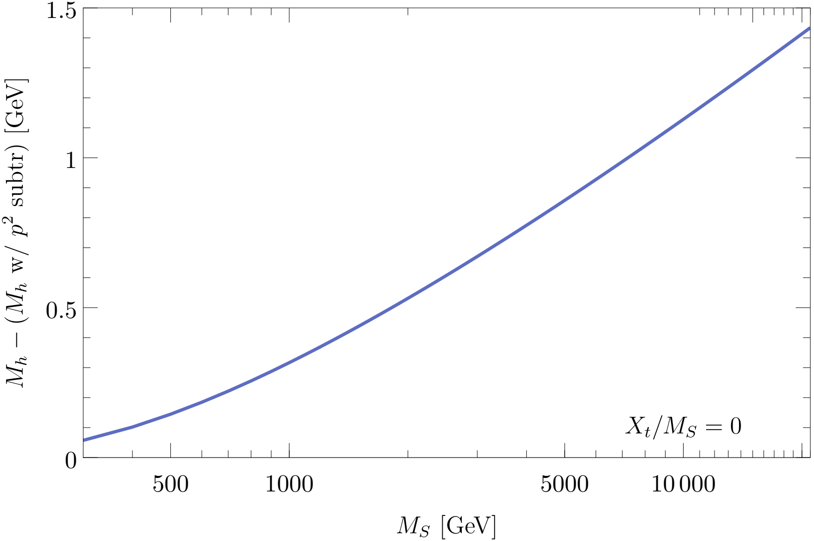
Comparison of logarithmic terms

$$(M_h^2)^{\text{logs}}_{\text{EFT}} = \left[v^2_{\overline{\text{MS}}}\lambda(M_t)\right]_{\text{logs}} - \hat{\Sigma}^{\text{SM}\prime}_{hh}(m_h^2)\left[v^2_{\overline{\text{MS}}}\lambda(M_t)\right]_{\text{logs}} + \ldots$$

$$\begin{aligned}(M_h^2)^{\text{logs}}_{\text{FH}} = &\left[v^2_{\overline{\text{MS}}}\lambda(M_t)\right]_{\text{logs}} + \left[\hat{\Sigma}^{\text{MSSM}\prime}_{hh}(m_h^2)\right]_{\text{logs}}\left[\hat{\Sigma}^{\text{MSSM}}_{hh}(m_h^2)\right]_{\text{nolog}}\\ &- \left[\hat{\Sigma}^{\text{MSSM}\prime}_{hh}(m_h^2)\right]_{\text{logs+nologs}}\left[v^2_{\overline{\text{MS}}}\lambda(M_t)\right]_{\text{logs}} + \ldots\end{aligned}$$

In heavy SUSY limit $\hat{\Sigma}^{\text{MSSM}}_{hh} \simeq \hat{\Sigma}^{\text{SM}}_{hh} + \hat{\Sigma}^{\text{SUSY}}_{hh} \rightarrow$ difference is

$$\begin{aligned}\Delta M_h^2 = &\left[\hat{\Sigma}^{\text{SUSY}\prime}_{hh}(m_h^2)\right]_{\text{logs}}\left[\hat{\Sigma}^{\text{MSSM}}_{hh}(m_h^2)\right]_{\text{nolog}}\\ &- \hat{\Sigma}^{\text{SUSY}\prime}_{hh}(m_h^2)\left[v^2_{\overline{\text{MS}}}\lambda(M_t)\right]_{\text{logs}} + \ldots\end{aligned}$$


1.5
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$M_h - (M_h$ w/ p^2 subtr) [GeV]
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M_S [GeV]
$X_t/M_S = 0$

Differences to SUSYHD

- different renormalization schemes
- different extraction of pole mass
- small differences in EFT calculations
- different renormalization of $\tan\beta$

- $\mathcal{O}(v/M_S)$ terms
- non-logarithmic terms
- ...?

SUSYHD by default uses NNNLO for $y_t(M_t)$ $\rightarrow$ sizable impact
$\rightarrow$ deactivated for all comparison plots

Differences to SUSYHD

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Differences to SUSYHD

- different renormalization schemes → under control ✓
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- different renormalization schemes → under control ✓
- different extraction of pole mass → effect isolated ✓
- small differences in EFT calculations → negligible ✓
- different renormalization of $\tan\beta$
  → negligible for $\tan\beta \gtrsim 5$ ✓
- $\mathcal{O}(v/M_S)$ terms
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- ...?

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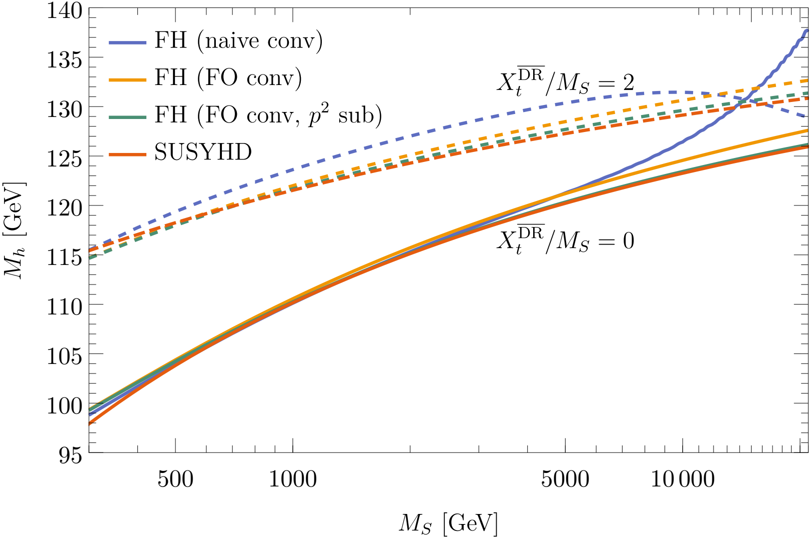
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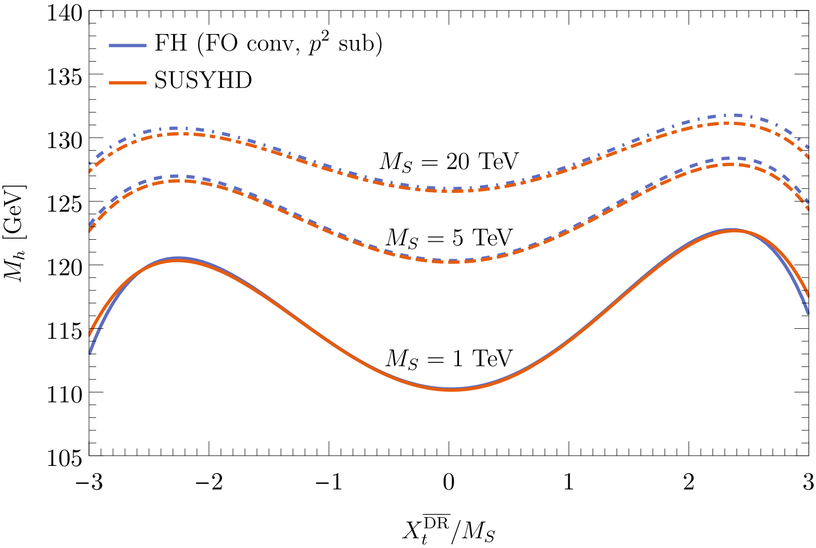
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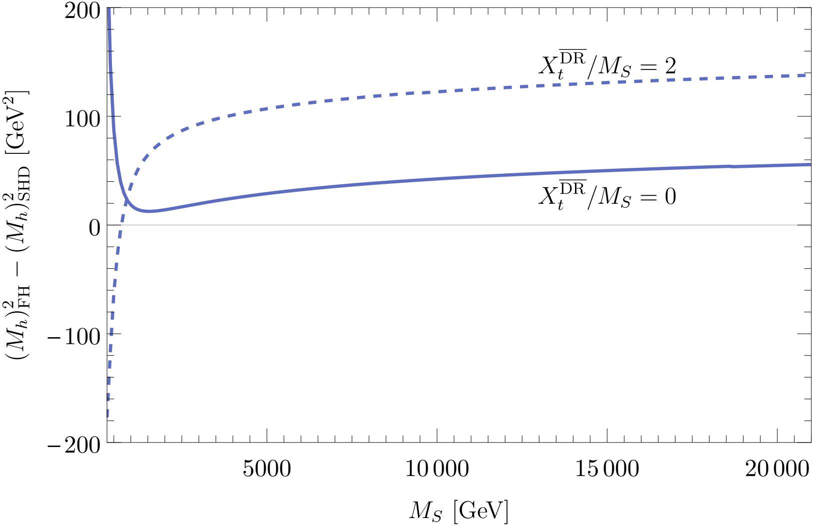
$\longrightarrow$ **overall very good agreement**

- different renormalization schemes → under control ✓
- different extraction of pole mass → effect isolated ✓
- small differences in EFT calculations → negligible ✓
- different renormalization of $\tan\beta$
  → negligible for $\tan\beta \gtrsim 5$ ✓
- $\mathcal{O}(v/M_S)$ terms → for $M_S \gtrsim 1$ TeV negligible ✓
- non-logarithmic terms
- ...?



$\longrightarrow$ nearly constant difference for high scales

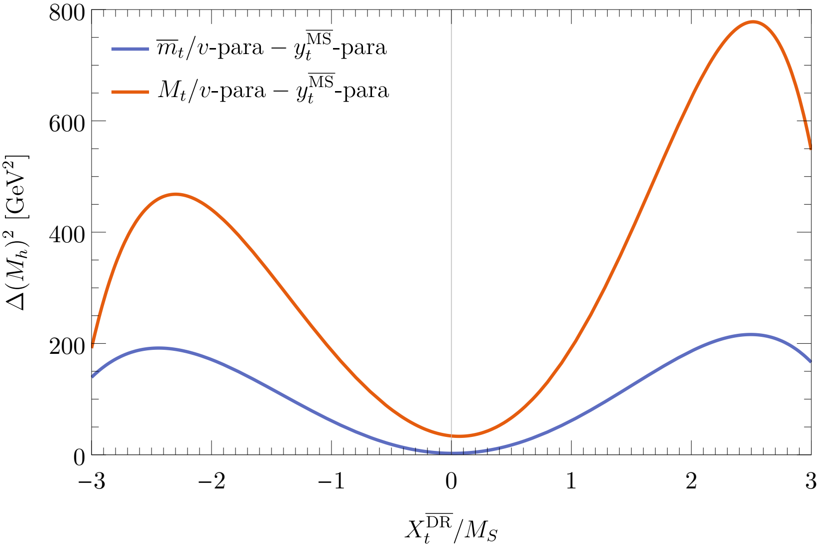
Origin

Different parametrization of non-logarithmic terms

Three ways to parametrize top Yukawa coupling in FO result

- $M_t/v \to$ `FeynHiggs` with `runningMT = 0`
- $\overline{m}_t/v \to$ `FeynHiggs` with `runningMT = 1`
- $y_t^{\overline{\text{MS}}} = \overline{m}_t/v_{\overline{\text{MS}}} \to$ `SUSYHD`

Equivalent at 2L order, but induces differences at higher order



→ explains constant difference almost completely

- different renormalization schemes → under control ✓
- different extraction of pole mass → effect isolated ✓
- small differences in EFT calculations → negligible ✓
- different renormalization of $\tan\beta$
  → negligible for $\tan\beta \gtrsim 5$ ✓
- $\mathcal{O}(v/M_S)$ terms → for $M_S \gtrsim 1$ TeV negligible ✓
- non-logarithmic terms → sizeable differences due to different parametrization of top Yukawa coupling ✓
- ...? → nothing significant ✓

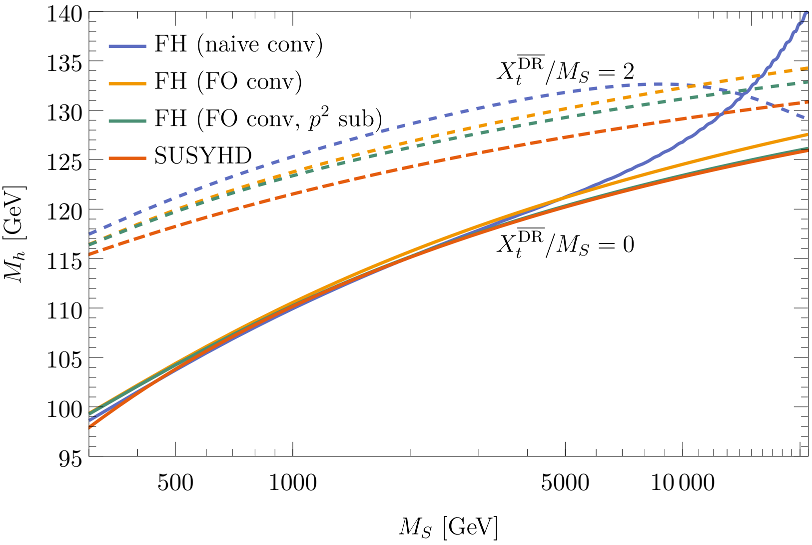
↓

Differences between FeynHiggs and SUSYHD (completely) understood ?!

Uncertainty estimate of SUSYHD

1. EFT uncertainty
   - $\mathcal{O}(v/M_S)$ terms
   - estimated by $v/M_S\cdot$(1L correction)
   - ok for $M_S \gtrsim 1$ TeV ✓
2. SM uncertainty:
   - higher order corrections to pole mass extraction
   - estimated by (de)activating higher order corrections to y_t and $\delta\lambda$
   - probably also ok (haven't checked) (✓)
3. SUSY uncertainty:
   - higher order threshold corrections
   - estimated by variation of matching scale $1/2 < Q/M_S < 2$
   - does not capture non-log terms ✗

Comparison for `runningMT=0` (OS top mass)



Rough estimate

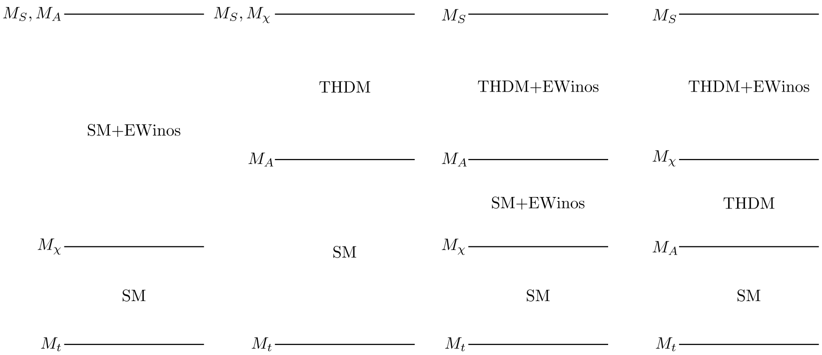
Uncertainty of ~ 2 GeV seems to be realistic for high scales and nearly maximal stop mixing

↓

Needed

More complete 2L calculation and explicit 3L calculation

Low M_A EFT



- Full dependence on effective couplings (thresholds and 2L RGEs)

Effective Lagrangians

$$\mathcal{L}_{\text{THDM}} = \ldots - V_{\text{THDM}}(H_u, H_d) - h_t \epsilon_{ij} \bar{t}_R Q_L^i H_u^j - h_t' \bar{t}_R Q_L H_d$$

$\to$ 9 effective couplings

$$\begin{aligned}\mathcal{L}_{\text{THDM+EWinos}} = \ldots &- \frac{1}{\sqrt{2}} H_u^\dagger \left(\hat{g}_{2uu} \sigma^a \tilde{W}^a + \hat{g}_{1uu} \tilde{B}\right) \tilde{\mathcal{H}}_u \\ &- \frac{1}{\sqrt{2}} H_d^\dagger \left(\hat{g}_{2dd} \sigma^a \tilde{W}^a - \hat{g}_{1dd} \tilde{B}\right) \tilde{\mathcal{H}}_d \\ &- \frac{1}{\sqrt{2}} (-i H_d^T \sigma_2) \left(\hat{g}_{2du} \sigma^a \tilde{W}^a + \hat{g}_{1du} \tilde{B}\right) \tilde{\mathcal{H}}_u \\ &- \frac{1}{\sqrt{2}} (-i H_u^T \sigma_2) \left(\hat{g}_{2ud} \sigma^a \tilde{W}^a - \hat{g}_{1ud} \tilde{B}\right) \tilde{\mathcal{H}}_d \\ &+ h.c. - V_{\text{THDM}}(H_u, H_d),\end{aligned}$$

$\to$ 17 effective couplings

- Running from M_S to $M_A \to \Delta\hat{\Sigma}_{11}, \Delta\hat{\Sigma}_{12}, \Delta\hat{\Sigma}_{22}$, e.g.

$$\begin{aligned}\Delta\hat{\Sigma}_{11} =& v^2\left(3\lambda_1 c_\beta^2 + (\lambda_3+\lambda_4+\lambda_5)s_\beta^2 + 6\lambda_6 s_\beta c_\beta\right)(Q = M_A)\\ &- \text{1L,2L subtraction terms}\end{aligned}$$

- Running from M_A to $m_t \to \Delta\hat{\Sigma}_{22} = \lambda(m_t)v^2/s_\beta^2$ (as in high M_A case)
- still issue with definition of t_β:

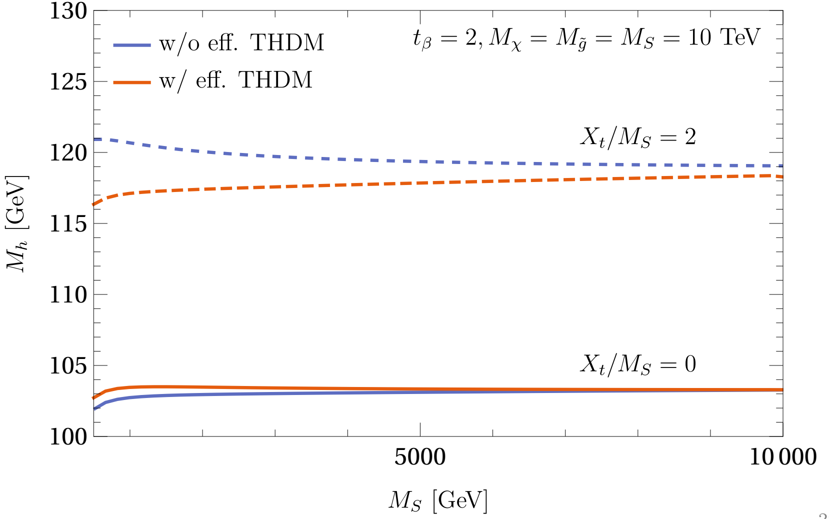
$$\underbrace{t_\beta^{\text{MSSM}}(m_t)}_{\text{FH}} \leftrightarrow \underbrace{t_\beta^{\text{MSSM}}(M_S) \leftrightarrow t_\beta^{\text{THDM}}(M_A)}_{\text{EFT}}$$

- idea: change renormalization scale of t_β in diagrammatic result

$$t_\beta(M_t) = t_\beta(M_S) + \Delta t_\beta$$

Current status

Running → result not yet trustworthy



Conclusion

- Newly implemented fixed-order $\overline{\mathrm{DR}} \rightarrow$ OS conversion $\rightarrow$ more reliable results
- Momentum dependence of SUSY contributions to Higgs self-energy induces terms not present in pure EFT calculation
- Parametrization of non-logarithmic terms has sizeable effect for large stop-mixing
- Apart of these effects excellent agreement of `FeynHiggs` with `SUSYHD` found

How to proceed

- Reach conclusion about uncertainty estimates
- Finish effective THDM calculation

The OS vev-counterterm is given by

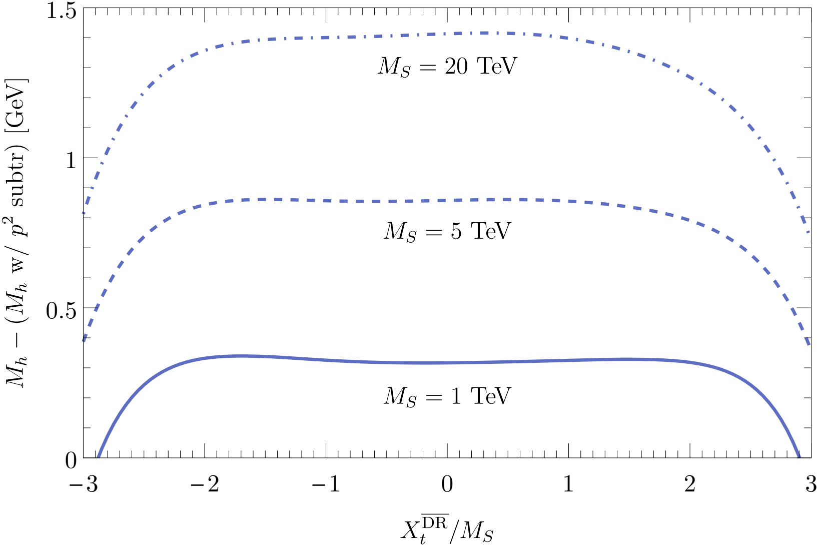
$$\delta v^2 = v^2 \left[\frac{\delta M_W^2}{M_W^2} + \frac{c_w^2}{s_w^2} \left(\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2} \right) - \frac{\delta e^2}{e^2} \right] \overset{\mathcal{O}(\alpha_s,\alpha_t)}{=}$$
$$= v^2 \left(-\hat{\Sigma}_{hh}^{(1)\prime}(m_h^2) + \text{ SM corrections} \right).$$

The Higgs pole mass is calculated via

$$M_h^2 = m_h^2 - \hat{\Sigma}_{hh}^{(1)}(m_h^2) - \hat{\Sigma}_{hh}^{(2)}(0) + \Sigma_{hh}^{(1)\prime}(m_h^2)\Sigma_{hh}^{(1)}(m_h^2) + ...$$

The renormalized two-loop self-energy reads

$$\hat{\Sigma}_{hh}^{(2)}(0) = \Sigma_{hh}^{(2)}(0) + \frac{\partial}{\partial v^2}\hat{\Sigma}_{hh}^{(1)}(m_h^2) \cdot \delta v^2 + ... =$$
$$= \Sigma_{hh}^{(2)}(0) - \hat{\Sigma}_{hh}^{(1)}(m_h^2)\frac{\delta v^2}{v^2} + ... =$$
$$= \Sigma_{hh}^{(2)}(0) + \hat{\Sigma}_{hh}^{(1)}(m_h^2)\hat{\Sigma}_{hh}^{(1)\prime}(m_h^2) + ...$$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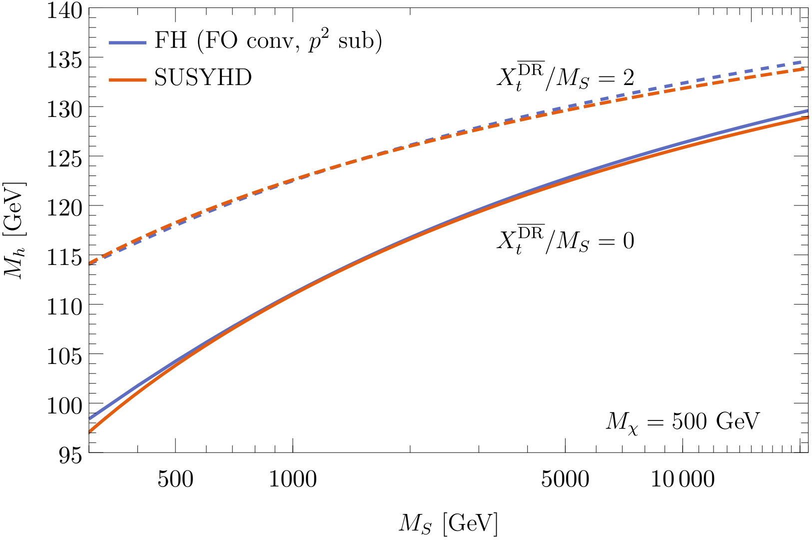
$X_t^{\overline{\mathrm{DR}}}/M_S$



FH (FO conv, p^2 sub)
SUSYHD
$X_t^{\overline{\mathrm{DR}}}/M_S = 2$
$X_t^{\overline{\mathrm{DR}}}/M_S = 0$
$M_\chi = 500$ GeV
M_h [GeV]
M_S [GeV]
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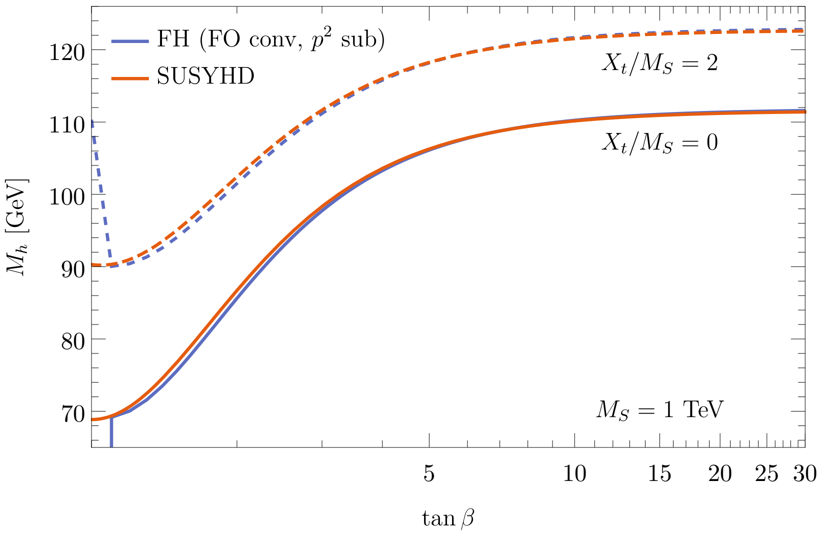
FH (FO conv, p^2 sub)
SUSYHD
$X_t/M_S = 2$
$X_t/M_S = 0$
$M_S = 1$ TeV
120
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M_h [GeV]
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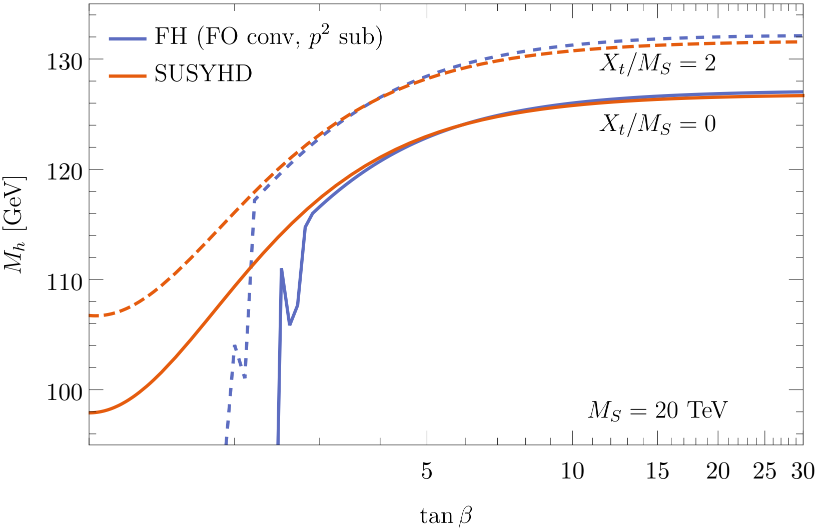
$\tan\beta$


FH (FO conv, p^2 sub)
SUSYHD
$X_t/M_S = 2$
$X_t/M_S = 0$
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M_h [GeV]
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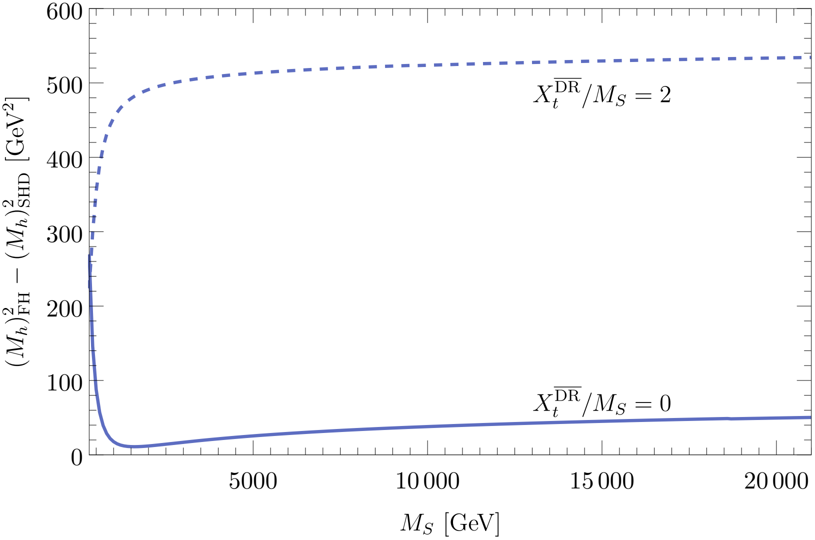
$\tan\beta$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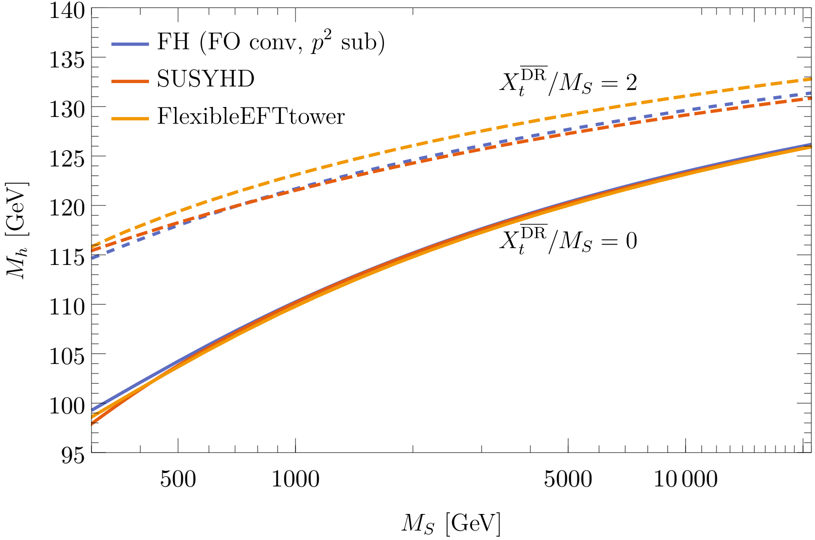
M_S [GeV]
$(M_h)^2_{\text{FH}} - (M_h)^2_{\text{SHD}}$ [GeV^2]
$X_t^{\overline{\text{DR}}}/M_S = 2$
$X_t^{\overline{\text{DR}}}/M_S = 0$



FH (FO conv, p^2 sub)
SUSYHD
FlexibleEFTtower
$X_t^{\overline{\mathrm{DR}}}/M_S = 2$
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140
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95
M_h [GeV]
500
1000
5000
10 000
M_S [GeV]

$$t_\beta^{\rm THDM}(M_S) = t_\beta^{\rm MSSM}\left\{1 + \frac{1}{4}k h_t^2(\hat{A}_t - \hat{\mu}/t_\beta)(\hat{A}_t + \hat{\mu}t_\beta)\right\}$$

$$h_t^{\rm THDM}(M_S) = h_t^{\rm MSSM}\left\{1 + k\left[\frac{4}{3}g_3^2(1 - \hat{A}_t) - \frac{1}{4}h_t^2\hat{A}_t^2\right]\right\}$$

$$(h_t')^{\rm THDM}(M_S) = h_t^{\rm MSSM}k\left[\frac{4}{3}g_3^2\hat{\mu} + \frac{1}{4}h_t^2\hat{A}_t\hat{\mu}\right]$$

$$y_t^{\rm SM}(M_A) = \left(h_t^{\rm THDM}s_{\beta^{\rm THDM}} + (h_t')^{\rm THDM}c_{\beta^{\rm THDM}}\right)\cdot \left\{1 - \frac{3}{8}k\left(h_t c_\beta - h_t' s_\beta\right)^2\right\}$$

$$X_t^{\overline{\rm DR}}(M_S) = X_t^{\rm OS}\left\{1 + \left[\frac{\alpha_s}{\pi} - \frac{3\alpha_t}{16\pi}\left(1 - \hat{X}_t^2\right)\right]\ln\frac{M_S^2}{m_t^2} - \frac{3}{16\pi}\frac{\alpha_t}{t_\beta^2}\left(1 - \hat{Y}_t^2\right)\ln\frac{M_S^2}{M_A^2}\right\}$$