Update on large-log resummation in `FeynHiggs`

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- EFT calculations allow to resum large logarithms
  $\rightarrow$ should be accurate for high SUSY scale M_{Susy}
- misses however terms $\propto v/M_{Susy}$
- diagrammatic calculation expected to be more accurate for low M_{Susy} ($\lesssim$ few TeV)

Goal

Combine both approaches to get precise results for both regimes.

`FeynHiggs` already contains full 1-loop and partial 2-loop results

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Double counting has to be avoided:
- $\Rightarrow$ Subtract logarithms from the diagrammatic result
- $\Rightarrow$ Subtract non-logarithmic terms from the EFT result

EFT calculation in $\overline{\text{MS}}/\overline{\text{DR}}$, diagrammatic calculation in OS:
- $\Rightarrow$ Conversion $\overline{\text{MS}}/\overline{\text{DR}} \leftrightarrow$ OS is mandatory

$$\Downarrow$$

$$\begin{aligned} M_h^2 =& (M_h^2)^{\text{FD}} + (\Delta M_h^2)^{\text{EFT}}(X_t^{\overline{\text{DR}}}) \\ & - (\Delta M_h^2)^{\text{EFT,non-log}}(X_t^{OS}) - (\Delta M_h^2)^{\text{FD,Logs}}(X_t^{OS}) \end{aligned}$$

since FeynHiggs 2.10

Resummation of leading/next-to-leading logarithms $\propto \alpha_t, \alpha_s$

- weak gauge couplings are neglected ($g = g' = 0$)

New in FeynHiggs 2.11.3:

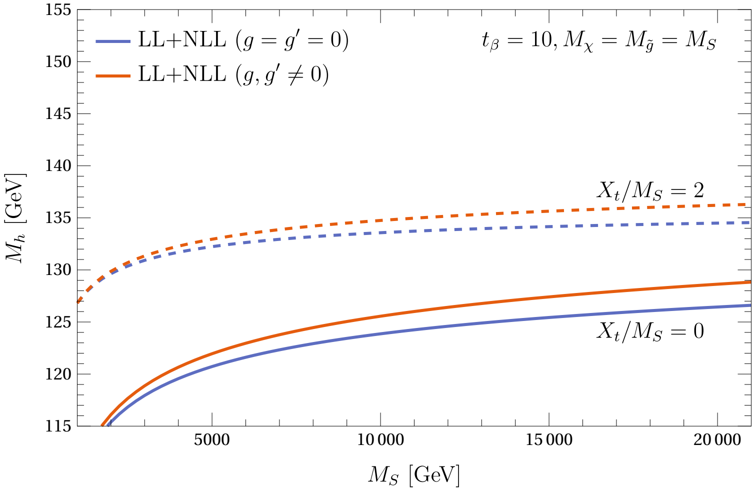
- NNLO formula for calculation of $\overline{\text{MS}}$ top mass
  $\Rightarrow$ Downwards shift of M_h by ~ 1.8 GeV

Improvements for new `FeynHiggs` version:

1. Inclusion of electroweak contributions
2. Introduction of an EWino threshold
3. Introduction of a Gluino threshold

$$X_t^{\overline{\mathrm{DR}}} = X_t^{\mathrm{OS}} \left[1 + \left(\underbrace{\frac{\alpha_s}{\pi}}_{g,\tilde{g}} \underbrace{- \frac{3\alpha_t}{16\pi}\left(1 - \hat{X}_t^2\right)}_{\text{Higgs}} \underbrace{- \frac{\alpha}{96\pi}(1 - 26\, c_w^2)}_{\text{Z,W}^\pm} \right) \ln\left(\frac{M_S^2}{m_t^2}\right) \right]$$

Non-log terms generate NNL logs and non-log terms in ΔM_h^2
$\Rightarrow$ neglected at this level


155
150
145
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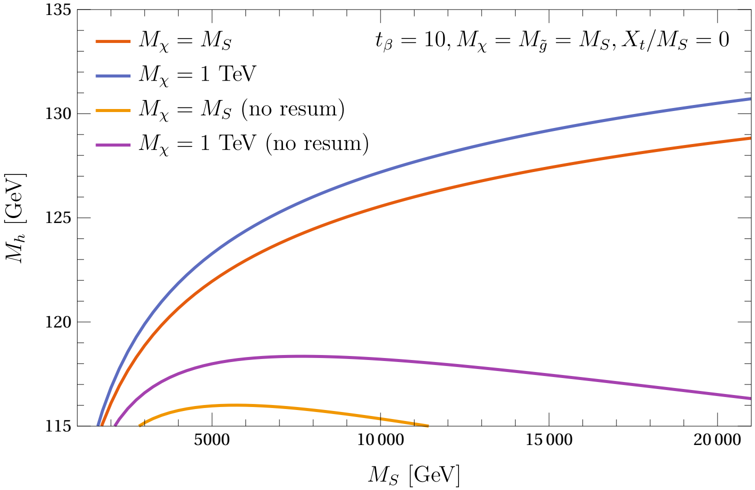
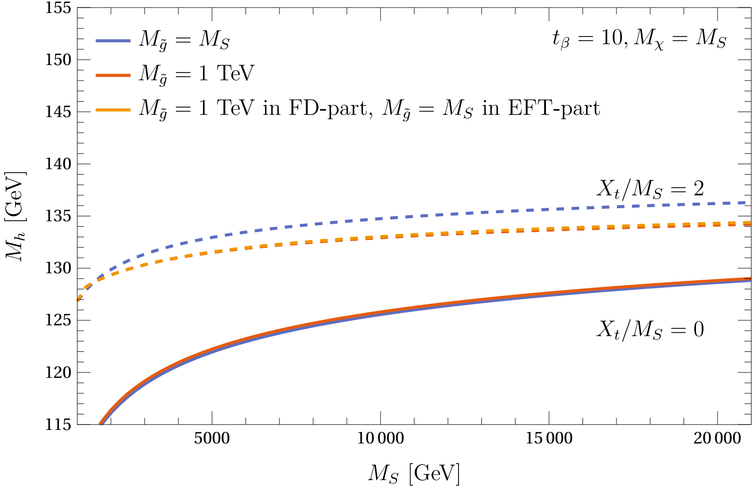
115
M_h [GeV]
5000
10 000
15 000
20 000
M_S [GeV]
LL+NLL ($g = g' = 0$)
LL+NLL ($g, g' \neq 0$)
$t_\beta = 10, M_\chi = M_{\tilde{g}} = M_S$
$X_t/M_S = 2$
$X_t/M_S = 0$


$M_\chi = M_S$
$M_\chi = 1$ TeV
$M_\chi = M_S$ (no resum)
$M_\chi = 1$ TeV (no resum)
$t_\beta = 10, M_\chi = M_{\tilde{g}} = M_S, X_t/M_S = 0$
135
130
125
120
115
M_h [GeV]
5000
10 000
15 000
20 000
M_S [GeV]



$\Rightarrow$ Gluino threshold is neglible!

Soon publicly available in new `FeynHiggs` version!

Resummation of NNL logarithms $\propto \mathcal{O}(\alpha_s, \alpha_t)$

Needed:

- 3-loop RGEs for λ, y_t and g_3 $\rightarrow$ known
  e.g. Buttazzo et al. (2013)
- 2-loop threshold corrections for λ $\rightarrow$ known
  e.g. Vega & Villadoro (2015)

caveat

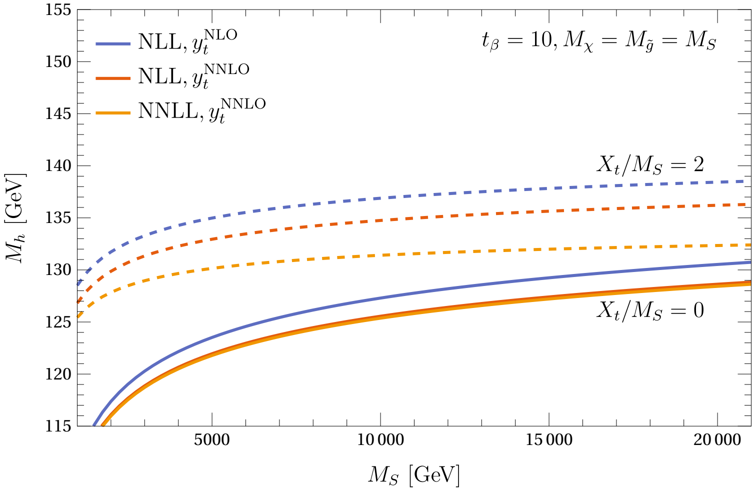
$M_{\tilde{g}} = M_S$ has to be assumed $\rightarrow$ no problem, since Gluino threshold is nevertheless neglible

Conversion:

- X_t has to be converted respecting also non-logarithmic terms ($\propto \alpha_s, \alpha_t$) Espinosa & Zhang (2000)
- now also M_S has to be converted
- conversion of M_χ can be neglected

$\Rightarrow$ New subtraction terms needed:

- subtract non-log terms generated by conversion of 1-loop threshold corrections
- subtract non-log terms originating from 2-loop threshold correction


$t_\beta = 10, M_\chi = M_{\tilde{g}} = M_S$
NLL, y_t^{NLO}
NLL, y_t^{NNLO}
NNLL, y_t^{NNLO}
$X_t/M_S = 2$
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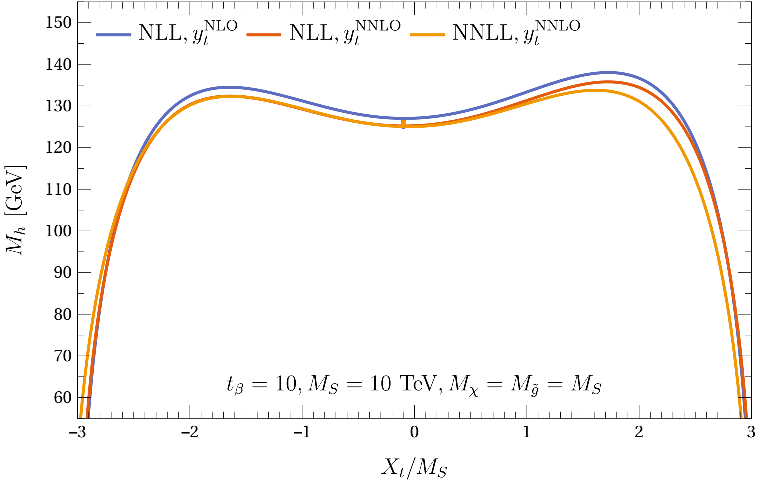
NLL, y_t^{NLO}
NLL, y_t^{NNLO}
NNLL, y_t^{NNLO}
$t_\beta = 10, M_S = 10$ TeV, $M_\chi = M_{\tilde{g}} = M_S$
M_h [GeV]
X_t/M_S
150
140
130
120
110
100
90
80
70
60
−3
−2
−1
0
1
2
3

Contributions included in `SUSYHD` but not in `FeynHiggs`:

- NNNLO value for y_t (same value chosen for comparison)
- running of bottom/tau Yukawa couplings
- 3-loop running of electroweak gauge couplings
- threshold corrections $\propto$ 'small' logarithms (e.g. $\ln \frac{m_{Q_3}}{M_S}$)

Contributions included in `FeynHiggs` but not in `SUSYHD`

- 1- and 2-loop terms $\propto v/M_{Susy}, m_t/M_{Susy}$

Further differences:

- `FeynHiggs` takes $t_\beta(m_t)$ as input, `SUSYHD` takes $t_\beta(M_S)$
- Different definition for X_t in OS-scheme

In normal OS-scheme following Espinosa & Zhang/`FeynHiggs`:

$$X_t^{\mathrm{OS}} = \frac{(m_t X_t)^{\mathrm{OS}}}{m_t^{\mathrm{OS}}} \Rightarrow \delta X_t = \frac{1}{m_t}\delta(m_t X_t)^{\mathrm{OS}} - X_t \frac{\delta m_t^{\mathrm{OS}}}{m_t}$$

In 'modified OS-scheme' following Vega & Villadoro:

$$X_t^{\mathrm{SUSYHD}}(Q) = \frac{(m_t X_t)^{\mathrm{OS}}}{m_t^{\overline{\mathrm{MS}}}(Q)} \Rightarrow \delta X_t = \frac{1}{m_t}\delta(m_t X_t)^{\mathrm{OS}} - X_t \frac{\delta m_t^{\overline{\mathrm{MS}}}}{m_t}$$

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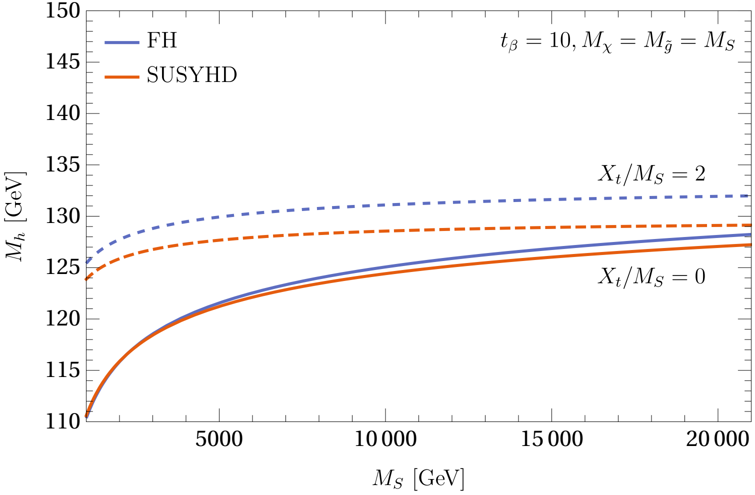
use externally calculated $X_t^{\overline{\mathrm{DR}}}$ as input for `SUSYHD` in $\overline{\mathrm{DR}}$-mode for meaningful comparison

→ all parameters in following plots are OS-parameters


FH
SUSYHD
$t_\beta = 10, M_\chi = M_{\tilde{g}} = M_S$
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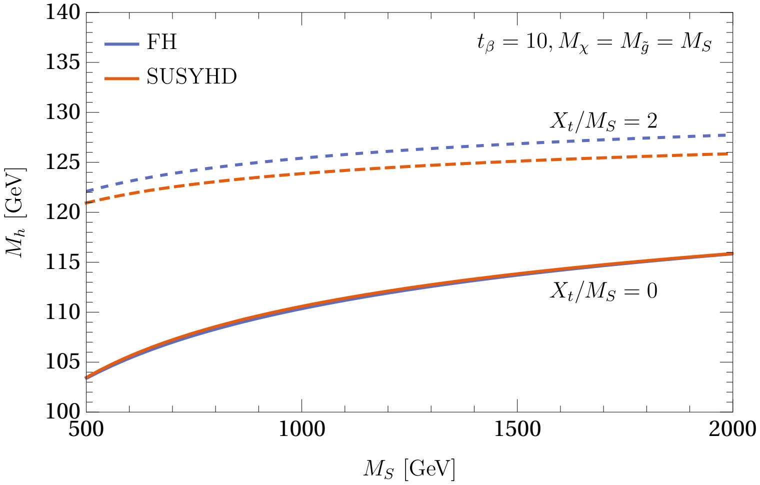
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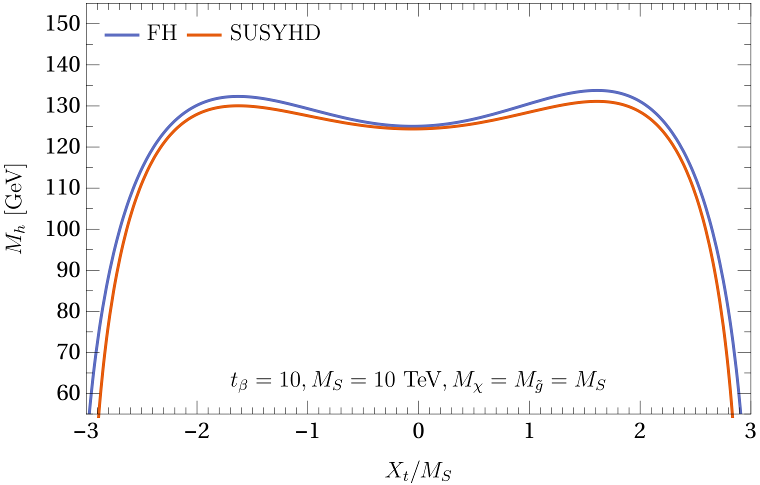
FH
SUSYHD
$t_\beta = 10, M_S = 10$ TeV, $M_\chi = M_{\tilde{g}} = M_S$
M_h [GeV]
X_t/M_S
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−1
0
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3

- Results for $\lambda(m_t)v^2$ coincide within $\mathcal{O}(0.1\text{ GeV})$ (at least for $t_\beta = 10$)

$$\Downarrow$$

differences not due to contributions included in `SUSYHD` but not in `FeynHiggs`

Possible explanations:

- terms $\propto v/M_{Susy}$
- ...

Stop-sector vertex corrections ($h_t = y_t/s_\beta$)

$$\Delta_{\tilde{t}}^{\rm Ver}\lambda_1 = -\frac{1}{2}kh_t^4\hat{\mu}^4 + \frac{3}{4}k\left(g^2+g'^2\right)h_t^2\hat{\mu}^2$$

$$\Delta_{\tilde{t}}^{\rm Ver}\lambda_2 = 6kh_t^4\hat{A}_t^2\left(1-\frac{1}{12}\hat{A}_t^2\right) - \frac{3}{4}k\left(g^2+g'^2\right)h_t^2\hat{A}_t^2$$

$$\Delta_{\tilde{t}}^{\rm Ver}\lambda_3 = \frac{1}{2}kh_t^4\hat{\mu}^2\left(3-\hat{A}_t^2\right) - \frac{3}{8}k\left(g^2-g'^2\right)h_t^2\left(\hat{A}_t^2-\hat{\mu}^2\right)$$

$$\Delta_{\tilde{t}}^{\rm Ver}\lambda_4 = \frac{1}{2}kh_t^4\hat{\mu}^2\left(3-\hat{A}_t^2\right) + \frac{3}{4}kg^2h_t^2\left(\hat{A}_t^2-\hat{\mu}^2\right)$$

$$\Delta_{\tilde{t}}^{\rm Ver}\lambda_5 = -\frac{1}{2}kh_t^4\hat{\mu}^2\hat{A}_t^2$$

$$\Delta_{\tilde{t}}^{\rm Ver}\lambda_6 = \frac{1}{2}kh_t^4\hat{\mu}^3\hat{A}_t - \frac{3}{8}k(g^2+g'^2)h_t^2\hat{\mu}\hat{A}_t$$

$$\Delta_{\tilde{t}}^{\rm Ver}\lambda_7 = \frac{1}{2}kh_t^4\hat{\mu}\hat{A}_t\left(\hat{A}_t^2-6\right) + \frac{3}{8}k(g^2+g'^2)h_t^2\hat{\mu}\hat{A}_t$$

Terms in red not present in Cheung et. al. (2015) and Lee & Wagner (2015), but in Haber & Hempfling (1993)

Lightest Higgs 4-point self-coupling:

$$\lambda_{\text{THDM}} = \lambda_1 c_\beta^4 + \lambda_2 s_\beta^4 + 2(\lambda_3 + \lambda_4 + \lambda_5) c_\beta^2 s_\beta^2 + 4\lambda_6 c_\beta^3 s_\beta + 4\lambda_7 c_\beta s_\beta^3$$

$$\Rightarrow \Delta_{\tilde{t}}^{\text{Ver}} \lambda_{\text{THDM}} = \Delta_{\tilde{t}}^{\text{Ver}} \lambda_1 c_\beta^4 + \Delta_{\tilde{t}}^{\text{Ver}} \lambda_2 s_\beta^4 + 2\Delta_{\tilde{t}}^{\text{Ver}} (\lambda_3 + \lambda_4 + \lambda_5) c_\beta^2 s_\beta^2 + 4\Delta_{\tilde{t}}^{\text{Ver}} \lambda_6 c_\beta^3 s_\beta + 4\Delta_{\tilde{t}}^{\text{Ver}} \lambda_7 c_\beta s_\beta^3$$

In the case $M_A = M_S$ ($X_t = A_t - \mu/t_\beta$):

$$\Delta_{\tilde{t}}^{\text{Ver}} \lambda_{\text{SM}} = 6k y_t^2 \left\{ \left(y_t^2 + \frac{1}{8}\left(g^2 + g'^2\right) c_{2\beta} \right) \hat{X}_t^2 - \frac{1}{12} y_t^2 \hat{X}_t^4 \right\}$$

$M_A \to M_S$ provides test of threshold corrections

$$\Delta_{\tilde{t}}^{\text{Ver}} \lambda_{\text{THDM}} \stackrel{!}{=} \Delta_{\tilde{t}}^{\text{Ver}} \lambda_{\text{SM}}$$

$\rightarrow$ only fullfilled if red terms are included

Higgs wavefunction renormalization:

- again Cheung et. al. (2015), Lee & Wagner (2015) $\neq$ Haber & Hempfling (1993)
- $\Delta_{\tilde{t}}^{\mathrm{WFR}}\lambda_{\mathrm{SM}} = -\frac{1}{4} y_t^2 (g^2 + g'^2) c_{2\beta}^2 \hat{X}_t^2$
- none of the results yields SM correction
- also $\Delta_{\tilde{t}}^{\mathrm{Ver}}\lambda_{\mathrm{SM}} + \Delta_{\tilde{t}}^{\mathrm{WFR}}\lambda_{\mathrm{SM}} \stackrel{!}{=} \Delta_{\tilde{t}}^{\mathrm{Ver}}\lambda_{\mathrm{THDM}} + \Delta_{\tilde{t}}^{\mathrm{WFR}}\lambda_{\mathrm{THDM}}$ (for $M_A \to M_S$) not fullfilled

Extensions already implemented:

- NNLO conversion of $\overline{\text{MS}}$ top mass
- Electroweak contributions
- EWino and Gluino threshold

Soon available:

- NNLL resummation for logarithms of $\mathcal{O}(\alpha_t, \alpha_s)$

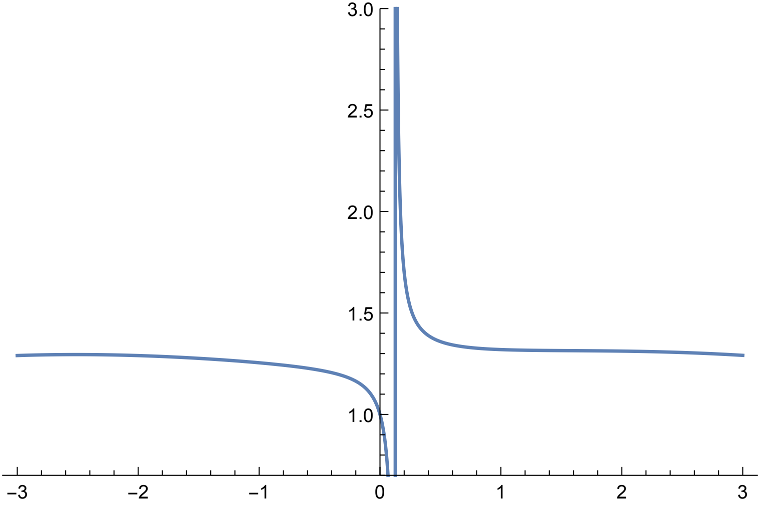
Next step:

- Low M_A scenario with full dependence on effective couplings

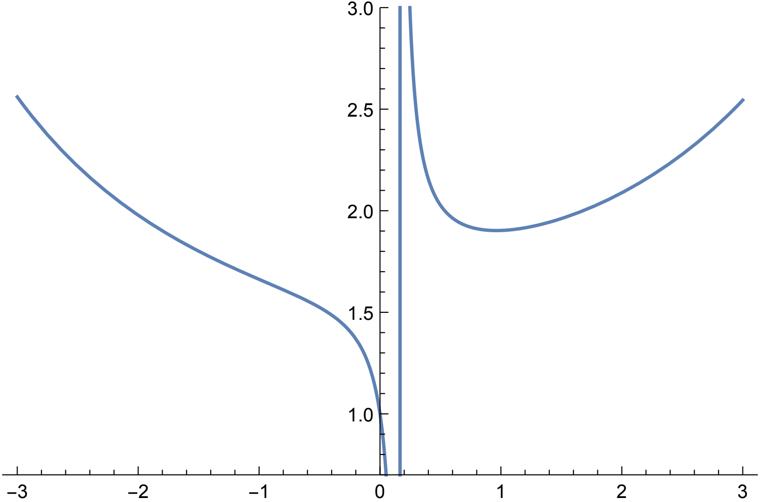
For $\mu = M_S$:

$$\begin{aligned}
X_t^{\overline{\text{DR}}}(M_S) =& X_t^{\text{OS}} \\
&+ \frac{32}{3} k g_3^2 M_S \\
&+ \frac{3}{s_\beta^2} k y_t^2 \left[2(X_t s_\beta^2 + Y_t c_\beta^2) - \frac{\pi}{\sqrt{2}} Y_t c_\beta^2 + X_t + \left(-\frac{1}{2} + \frac{\pi}{3\sqrt{3}} \right) c_\beta^2 \hat{Y}_t^2 \; X_t \right. \\
&\qquad \left. - \frac{1}{2} s_\beta^2 \hat{X}_t^2 X_t \ln \frac{m_t X_t}{M_S^2} \right] \\
&+ \frac{4}{3} k g_3^2 X_t \left(5 + 3 \ln \frac{M_S^2}{m_t^2} - \hat{X}_t \right) - \frac{3}{4 s_\beta^2} k y_t^2 X_t \left[\frac{1}{2}(1 + c_\beta^2) + \left(\frac{8}{3} + \ln \frac{M_S^2}{m_t^2} \right) s_\beta^2 - \frac{1}{2} \right] \\
X_t^{\overline{\text{DR}}}(M_S) =& X_t^{\text{SUSYHD}}(M_S) \\
&+ \frac{32}{3} k g_3^2 M_S \\
&+ \frac{3}{s_\beta^2} k y_t^2 \left[2(X_t s_\beta^2 + Y_t c_\beta^2) - \frac{\pi}{\sqrt{2}} Y_t c_\beta^2 + X_t \right] \\
&+ \frac{4}{3} k g_3^2 X_t \left(1 - \hat{X}_t \right) - \frac{3}{4 s_\beta^2} k y_t^2 X_t \left[\frac{1}{2}(1 + c_\beta^2) - \frac{1}{2} \right]
\end{aligned}$$

$X_t = A_t - \mu / t_\beta, \; Y_t = A_t + \mu t_\beta, \; \hat{X}_t = X_t / M_S, \; \hat{Y}_t = Y_t / M_S$



$X_t^{\text{SUSYHD}}(M_S)/X_t^{\text{OS}}$ in dependence of $\hat{X}_t^{\overline{\text{DR}}}$ for $M_S = 1$ TeV



$X_t^{\text{SUSYHD}}(M_S)/X_t^{\text{OS}}$ in dependence of $\hat{X}_t^{\overline{\text{DR}}}$ for $M_S = 10$ TeV

Standard Model RGEs (1 loop):

$$\frac{d\lambda}{dt} = k\left[6\left(\lambda^2 + \lambda h_t^2 - h_t^4\right) - \lambda\left(\frac{9}{2}g^2 + \frac{3}{2}g'^2\right) + \frac{9}{8}g^4 + \frac{3}{8}g'^4 + \frac{3}{4}g^2 g'^2\right]$$

$$\frac{dh_t}{dt} = \frac{1}{2}h_t k\left(\frac{9}{2}h_t^2 - 8g_3^2 - \frac{9}{4}g^2 - \frac{17}{12}g'^2\right)$$

$$\frac{dg_3}{dt} = -\langle\frac{7}{2};\frac{5}{2}\rangle k g_3^3$$

$$\frac{dg'}{dt} = \frac{41}{12}g'^3$$

$$\frac{dg}{dt} = -\frac{19}{12}g^3$$

Split Model Langrangian:

$$\mathcal{L}_{\rm split} = \langle 0; -\frac{1}{2} M_3 \tilde{g}^a \tilde{g}^a \rangle - \frac{1}{2} M_\chi \tilde{W}^a \tilde{W}^a - \frac{1}{2} M_\chi \tilde{B}^a \tilde{B}^a - M_\chi \tilde{H}_u^T \epsilon \tilde{H}_d$$
$$- \frac{1}{\sqrt{2}} H^\dagger \left(\tilde{g}_{2u} \sigma^a \tilde{W}^a + \tilde{g}_{1u} \tilde{B} \right) \tilde{H}_u$$
$$- \frac{1}{\sqrt{2}} H^T \epsilon \left(-\tilde{g}_{2d} \sigma^a \tilde{W}^a + \tilde{g}_{1d} \tilde{B} \right) \tilde{H}_d$$
$$+ h.c. + ...$$

Split Model RGEs (1 loop):

$$
\begin{aligned}
\frac{dg'}{dt} =& \frac{15}{4}kg'^3 \\
\frac{dg}{dt} =& -\frac{7}{12}kg^3 \\
\frac{dg_3}{dt} =& -\langle\frac{7}{2};\frac{5}{2}\rangle kg_3^3 \\
\frac{dh_t}{dt} =& \frac{1}{2}kh_t\left[-\frac{9}{4}g^2 - 8g_3^2 - \frac{17}{12}g'^2 + \frac{9}{2}h_t^2 + \frac{1}{2}(\tilde{g}_{1d}^2 + \tilde{g}_{1u}^2 + 3\tilde{g}_{2d}^2 + 3\tilde{g}_{2u}^2)\right] \\
\frac{d\lambda}{dt} =& \frac{1}{2}k\left[-\tilde{g}_{1d}^4 - \tilde{g}_{1u}^4 + \frac{9}{4}g^4 - 5\tilde{g}_{2d}^4 - 4\tilde{g}_{1d}\tilde{g}_{1u}\tilde{g}_{2d}\tilde{g}_{2u} - 5\tilde{g}_{2u}^4\right. \\
& -2(\tilde{g}_{1u}^2 + \tilde{g}_{2d}^2)(\tilde{g}_{1d}^2 + \tilde{g}_{2u}^2) \\
& +\frac{3}{2}g^2g'^2 + \frac{3}{4}g'^4 - 12h_t^4 + 2(\tilde{g}_{1d}^2 + \tilde{g}_{1u}^2 + 3\tilde{g}_{2d}^2 + 3\tilde{g}_{2u}^2)\lambda \\
& \left.-9\left(g^2 + \frac{1}{3}g'^2\right)\lambda + 12h_t^2\lambda + 12\lambda^2\right]
\end{aligned}
$$

Split Model RGEs (1 loop), cont.:

$$
\begin{aligned}
\frac{d\tilde{g}_{1u}}{dt} =& \frac{1}{2}k\left[3\tilde{g}_{1d}\tilde{g}_{2d}\tilde{g}_{2u} + \tilde{g}_{1u}\left(\frac{5}{4}\tilde{g}_{1u}^2 + 2\tilde{g}_{1d}^2 + \frac{9}{4}\tilde{g}_{2u}^2 + \frac{3}{2}\tilde{g}_{2d}^2 - \frac{9}{4}g^2 - \frac{3}{4}g'^2 + 3h_t^2\right)\right] \\
\frac{d\tilde{g}_{1d}}{dt} =& \frac{1}{2}k\left[3\tilde{g}_{1u}\tilde{g}_{2d}\tilde{g}_{2u} + \tilde{g}_{1d}\left(\frac{5}{4}\tilde{g}_{1d}^2 + 2\tilde{g}_{1u}^2 + \frac{9}{4}\tilde{g}_{2d}^2 + \frac{3}{2}\tilde{g}_{2u}^2 - \frac{9}{4}g^2 - \frac{3}{4}g'^2 + 3h_t^2\right)\right] \\
\frac{d\tilde{g}_{2u}}{dt} =& \frac{1}{2}k\left[\tilde{g}_{1d}\tilde{g}_{1u}\tilde{g}_{2d} + \tilde{g}_{2u}\left(\frac{1}{2}\tilde{g}_{1d}^2 + \frac{3}{4}\tilde{g}_{1u}^2 + \tilde{g}_{2d}^2 + \frac{11}{4}\tilde{g}_{2u}^2 - \frac{33}{4}g^2 - \frac{3}{4}g'^2 + 3h_t^2\right)\right] \\
\frac{d\tilde{g}_{2d}}{dt} =& \frac{1}{2}k\left[\tilde{g}_{1d}\tilde{g}_{1u}\tilde{g}_{2u} + \tilde{g}_{2d}\left(\frac{1}{2}\tilde{g}_{1u}^2 + \frac{3}{4}\tilde{g}_{1d}^2 + \tilde{g}_{2u}^2 + \frac{11}{4}\tilde{g}_{2d}^2 - \frac{33}{4}g^2 - \frac{3}{4}g'^2 + 3h_t^2\right)\right]
\end{aligned}
$$

$$\lambda(M_S) = \underbrace{\frac{1}{4}\cos^2(2\beta)\left(g^2+g'^2\right)}_{\text{tree level}}$$
$$\underbrace{-k\left[\left(\frac{3}{4}-\frac{1}{6}c_{2\beta}^2\right)g^4+\frac{1}{2}g^2g'^2+\frac{1}{4}g'^4\right]}_{\text{tree-level term }\overline{\mathrm{DR}}\to\overline{\mathrm{MS}}}$$
$$\underbrace{+6h_t^2k\left\{\left[h_t^2+\frac{1}{8}\left(g^2+g'^2\right)c_{2\beta}\right]\hat{X}_t^2-\frac{1}{12}h_t^2\hat{X}_t^4\right\}}_{\text{stop-threshold corr.}}$$
$$\underbrace{-\frac{1}{4}kh_t^2(g^2+g'^2)c_{2\beta}^2\hat{X}_t^2}_{\text{stop-threshold corr.}}$$
$$\underbrace{-\frac{3}{16}k(g'^2+g^2)^2s_{4\beta}^2}_{\text{heavy Higgs threshold corr.}}$$

$$\tilde{g}_{1u}(M_S) = g s_\beta \left\{ 1 + k \left[\frac{3}{16} g^2 \left(-2 + 7 c_\beta^2\right) + \frac{1}{20} g'^2 \left(-44 + 7 c_\beta^2\right) + \frac{9}{4 s_\beta^2} h_t^2 \right] \right\}$$

$$\tilde{g}_{1d}(M_S) = g c_\beta \left\{ 1 + k \left[\frac{3}{16} g^2 \left(-2 + 7 s_\beta^2\right) + \frac{1}{20} g'^2 \left(-44 + 7 s_\beta^2\right) \right] \right\}$$

$$\tilde{g}_{2u}(M_S) = g' s_\beta \left\{ 1 + k \left[-g^2 \left(\frac{2}{3} + \frac{11}{16} c_\beta^2 \right) + \frac{1}{20} g'^2 \left(-2 + 7 c_\beta^2\right) + \frac{9}{4 s_\beta^2} h_t^2 \right] \right\}$$

$$\tilde{g}_{2u}(M_S) = g' c_\beta \left\{ 1 + k \left[-g^2 \left(\frac{2}{3} + \frac{11}{16} s_\beta^2 \right) + \frac{1}{20} g'^2 \left(-2 + 7 s_\beta^2\right) \right] \right\}$$

$$
\begin{aligned}
\lambda_{\rm SM}(M_\chi) &= \lambda_\chi(M_\chi) \\
&\quad + k\Bigg\{-\frac{7}{12}(\tilde{g}_{1d}^4+\tilde{g}_{1d}^4)-\frac{9}{4}(\tilde{g}_{2d}^4+\tilde{g}_{1u}^4)-\frac{3}{2}\tilde{g}_{1d}^2 g_{1u}^2-\frac{7}{2}g_{2d}^2 g_{2u}^2 \\
&\quad -\frac{8}{3}\tilde{g}_{1d}\tilde{g}_{1u}\tilde{g}_{2d}\tilde{g}_{2u}-\frac{7}{6}(\tilde{g}_{1d}^2\tilde{g}_{2d}^2+\tilde{g}_{1u}^2\tilde{g}_{2u}^2)-\frac{1}{6}(\tilde{g}_{1d}^2\tilde{g}_{2u}^2+\tilde{g}_{1u}^2\tilde{g}_{2d}^2) \\
&\quad -\frac{4}{3}(\tilde{g}_{1d}\tilde{g}_{2u}^2+\tilde{g}_{1u}\tilde{g}_{2d})(\tilde{g}_{1d}\tilde{g}_{2d}+\tilde{g}_{1u}\tilde{g}_{2u}) \\
&\quad +\frac{2}{3}\tilde{g}_{1d}\tilde{g}_{1u}(\lambda_\chi-2\tilde{g}_{1d}^2-2\tilde{g}_{1u}^2)+2\tilde{g}_{2d}\tilde{g}_{2u}(\lambda_\chi-2\tilde{g}_{2d}^2-2\tilde{g}_{2u}^2) \\
&\quad +\frac{1}{3}\lambda_\chi(\tilde{g}_{1d}^2+\tilde{g}_{1u}^2)+\lambda_\chi(\tilde{g}_{2d}^2+\tilde{g}_{2u}^2)\Bigg\}
\end{aligned}
$$

$$
\begin{aligned}
g_\chi(M_\chi) &= g_{\rm SM}(M_\chi) \\
g'_\chi(M_\chi) &= g'_{\rm SM}(M_\chi) \\
g_{3,\chi}(M_\chi) &= g_{3,{\rm SM}}(M_\chi) \\
h_{t,\chi}(M_\chi) &= h_{t,{\rm SM}}(M_\chi)\left\{1 - k\left[\frac{1}{6}\tilde{g}_{1u}\tilde{g}_{1d} + \frac{1}{12}(\tilde{g}_{1u}^2 + \tilde{g}_{1d}^2)\right.\right. \\
&\qquad \left.\left. + \ \frac{1}{2}\tilde{g}_{2u}\tilde{g}_{2d} + \frac{1}{4}(\tilde{g}_{2u}^2 + \tilde{g}_{2d}^2)\right]\right\}
\end{aligned}
$$

one-loop logarithms ($M_\chi = M_S$)

$$M_h^{\rm LL} = -\frac{\alpha}{192\pi M_W^2 s_w^2}\Big\{288 M_t^4 + 144 m_t^2 M_Z^2 c_{2\beta}$$
$$+4c_{4\beta}\left(22 M_W^4 - 64 M_W^2 M_Z^2 + 41 M_Z^4\right) - 9 M_Z^4 c_{8\beta}$$
$$-56 M_W^4 - 256 M_W^2 M_Z^2 + 101 M_Z^4\Big\} \ln\left(\frac{M_S^2}{m_t^2}\right)$$

chargino/neutralino contribution

$$\frac{\alpha M_Z^2}{24\pi c_w^4}\Big[(10 c_w^2(2c_w^2-1)-1)c_{4\beta} + 2c_w^2(22c_w^2-5)+11\Big]\ln\left(\frac{M_\chi^2}{m_t^2}\right)$$