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27.06.2005

Transverse space charge forces in the injector area: downstream ACC00

Beam lines (W.Decking, 16.06.2005):

1. Beam line downstream ACC00 up to BC1:
   Quad_ACC00, INJ_DIAG, INJ1_DOG,
   ACC01, ACC02, ACC03, ACC04, ACC3RD.

2. Beam line downstream INJ1_DOG up to BC1:
   ACC01, ACC02, ACC03, ACC04, ACC3RD.

Acceleration in injector linac:

- on-crest
- $E_{acc} = 11.14\,MV/m$

Beam parameters:

- Initial energy: $E_{in} = 130\,MeV$.
- Final energy: $E_{fn} = 501.5\,MeV$.
- ACC00_BC1:
  - Current: $I = 60\,A$.
  - Normalized emittance: 0.2 - 2.0 $mm \cdot mrad$.
- INJ1_DOG_BC1:
  - Current: $I = 60 - 200\,A$ (step 20 A).
  - Normalized emittance: 1.0 $mm \cdot mrad$.
- Energy spread in slice: $P_z = 1.5 \cdot 10^{-4}$ (20 keV at 130 MeV).

Beam model:

- Transverse plane: Gaussian beam (truncated at $3\,\sigma$), matched to upstream optics.
- Energy spread in slice: Gaussian distribution.

Setup for simulations:

- Number of particles in slice: $N_p = 10^5$.
- Number of space charge kicks per each element: 5.
- Grid size: $\Delta x, y = 5\,\mu m$.

Statistical values:

- Statistical emittance:

$$\epsilon_x \;=\; \sqrt{\langle x^2\rangle \langle p_x^2\rangle \;-\; \langle xp_x\rangle^2}$$

- Statistical β-function:

$$\beta_x \;=\; \frac{\langle x^2\rangle}{\epsilon_x}$$

- Moment invariant of coupled 2D linear motion (includes linear space charge when treated in the Vlasov approximation) (first discovered by W.Lysenko):

$$I_{xy}^2 \;=\; \frac{\epsilon_x^2 \;+\; \epsilon_y^2 \;+\; 2\cdot(\langle xy\rangle \langle p_x p_y\rangle \;-\; \langle xp_y\rangle \langle yp_x\rangle)}{2}$$

- Statistical normalized emittance:

$$\epsilon_{x,n} \;=\; \beta_0\gamma_0\epsilon_x$$

where p_x, p_y are particle momentum divided by design momentum.

Mismatch:

$$\lambda_x \;=\; M_x \;+\; \sqrt{M_x^2 \;-\; 1}$$

$$M_x \;=\; \frac{\beta_x\gamma_{x,sc} \;-\; 2\alpha_x\alpha_{x,sc} \;+\; \gamma_x\beta_{x,sc}}{2}$$

$$\frac{1}{\lambda_x} \;\leq\; \frac{\beta_{x,sc}}{\beta_x} \;\leq\; \lambda_x$$

where $x \;=\; x, y$ and $\beta_x, \beta_{x,sc}$ are $\beta-$function without and with space charge effect, respectively.

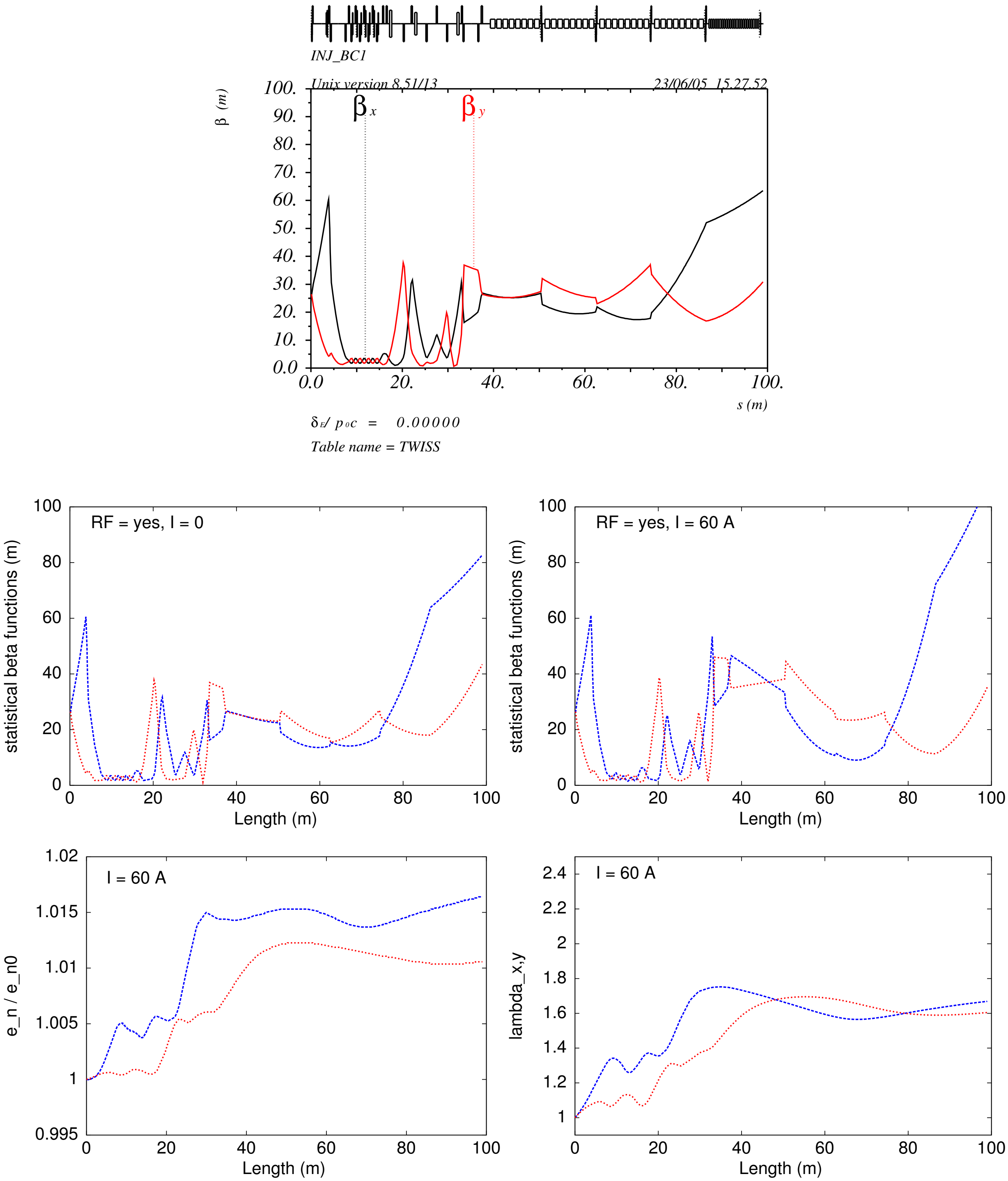

Figure 1: Beam line between ACC09 and BC1. Current: $I = 0\,A$ and $I = 60\,A$. $E_{init} = 130\,MeV$. $E_{acc} \approx 11.14 MeV/m$, on-crest. $E_{fin} = 501.5\,MeV$. $\varepsilon_{nx,ny} = 1\,mm \cdot mrad$. Blue (black on top picture) and red colours represent the horizontal and vertical planes, respectively. Dogleg: L = 9.75 m (from 22.69 to 32.44 m).

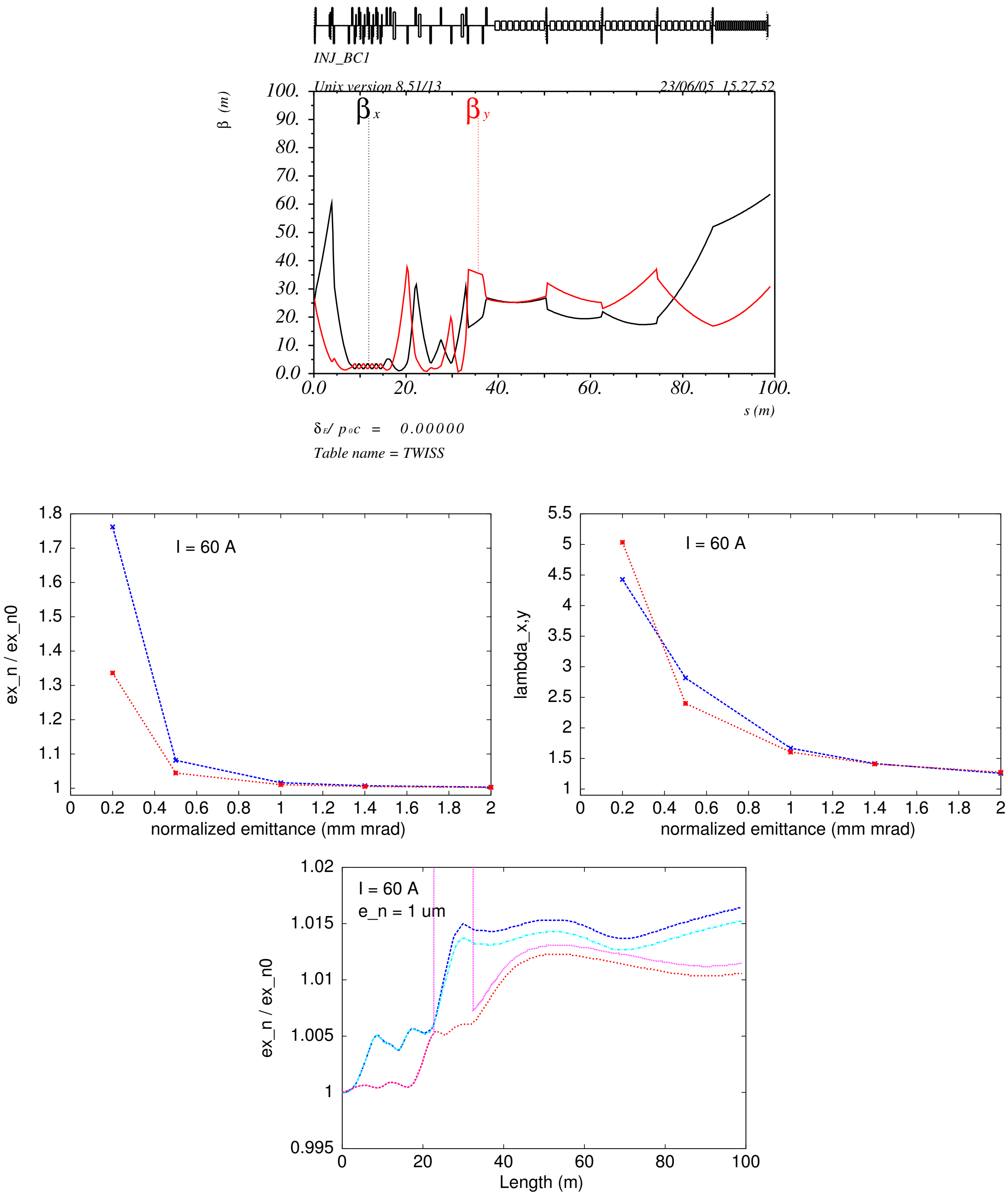

Figure 2: Exit of Beam line between ACC00 and BC1. Current: $I = 60\,A$. $E_{init} = 130\,MeV$. $E_{acc} \approx 11.14 MeV/m$, on-crest. $E_{fin} = 501.5\,MeV$. $\varepsilon_{nx,ny} = 0.2$ - $2\ mm \cdot mrad$. Blue (black on top picture) and red colours represent the horizontal and vertical planes, respectively. Bottom: light-blue and magenta: with energy spread. Dogleg: L = 9.75 m (from 22.69 to 32.44 m).

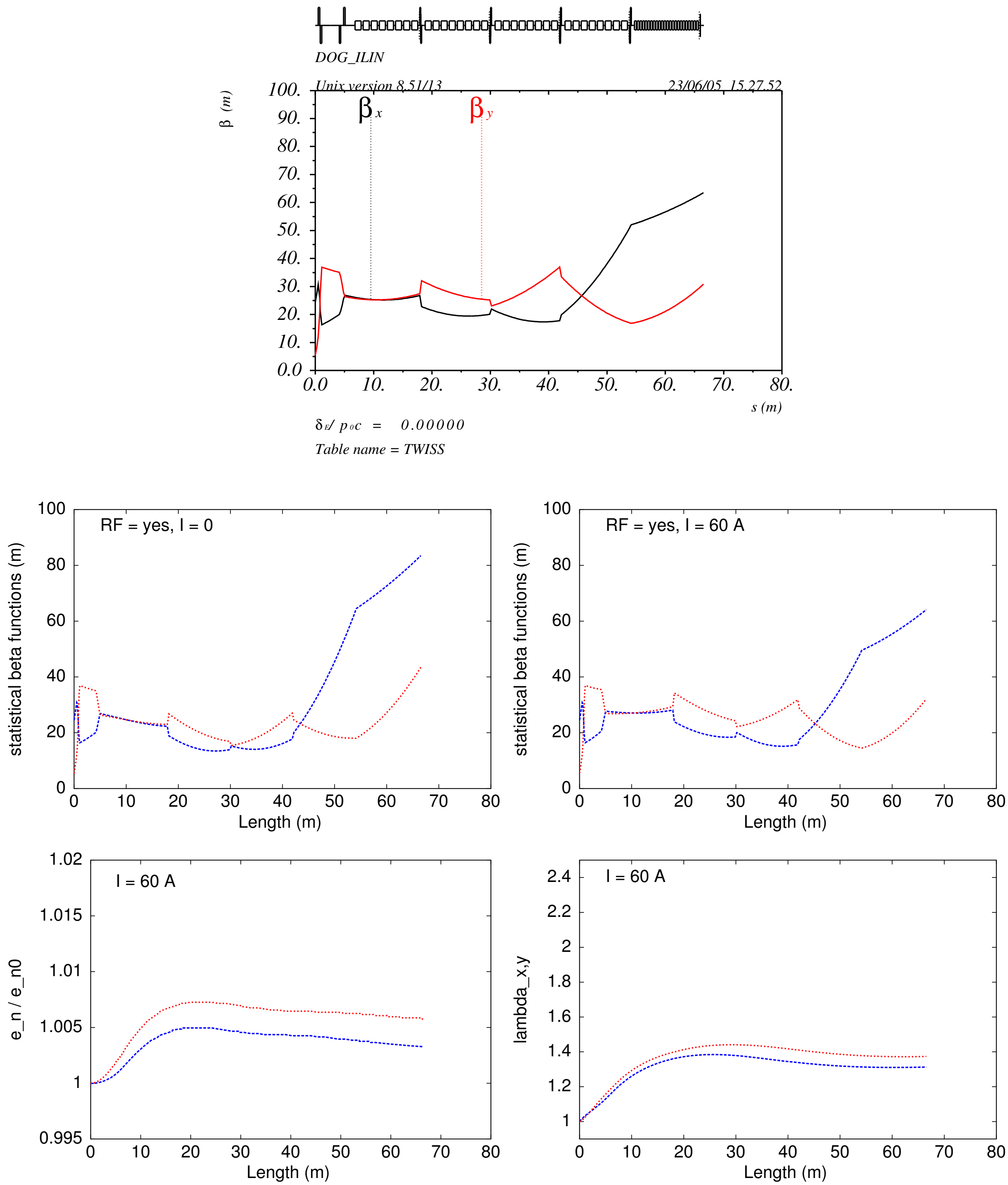

Figure 3: Beam line downstream Injector Dogleg up to BC1. Currents: $I = 0\,A$, $I = 60\,A$. $E_{init} = 130\,MeV$. $E_{acc} \approx 11.14 MeV/m$, on-crest. $E_{fin} = 501.5\,MeV$. $\varepsilon_{nx,ny} = 1\,mm \cdot mrad$. Blue (black on top pictute) and red colours represent the horizontal and vertical planes, respectively.

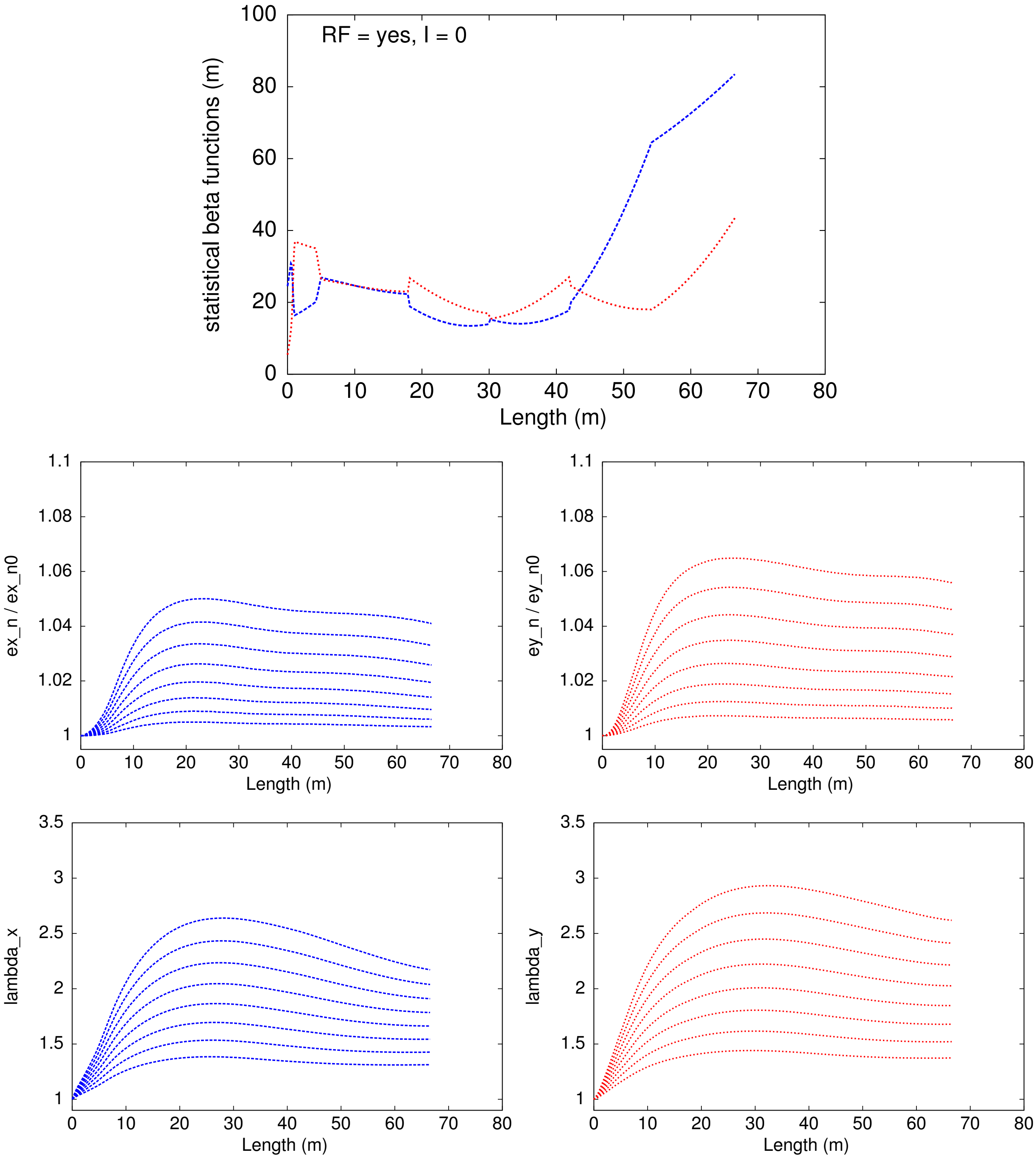

Figure 4: Beam line downstream Injector Dogleg up to BC1. Currents: $I = 60 - 200\,A$ (step 20 A). $E_{init} = 130\,MeV$. $E_{acc} \approx 11.14 MeV/m$, on-crest. $E_{fin} = 501.5\,MeV$. $\varepsilon_{nx,ny} = 1\,mm \cdot mrad$. Blue and red colours represent the horizontal and vertical planes, respectively.

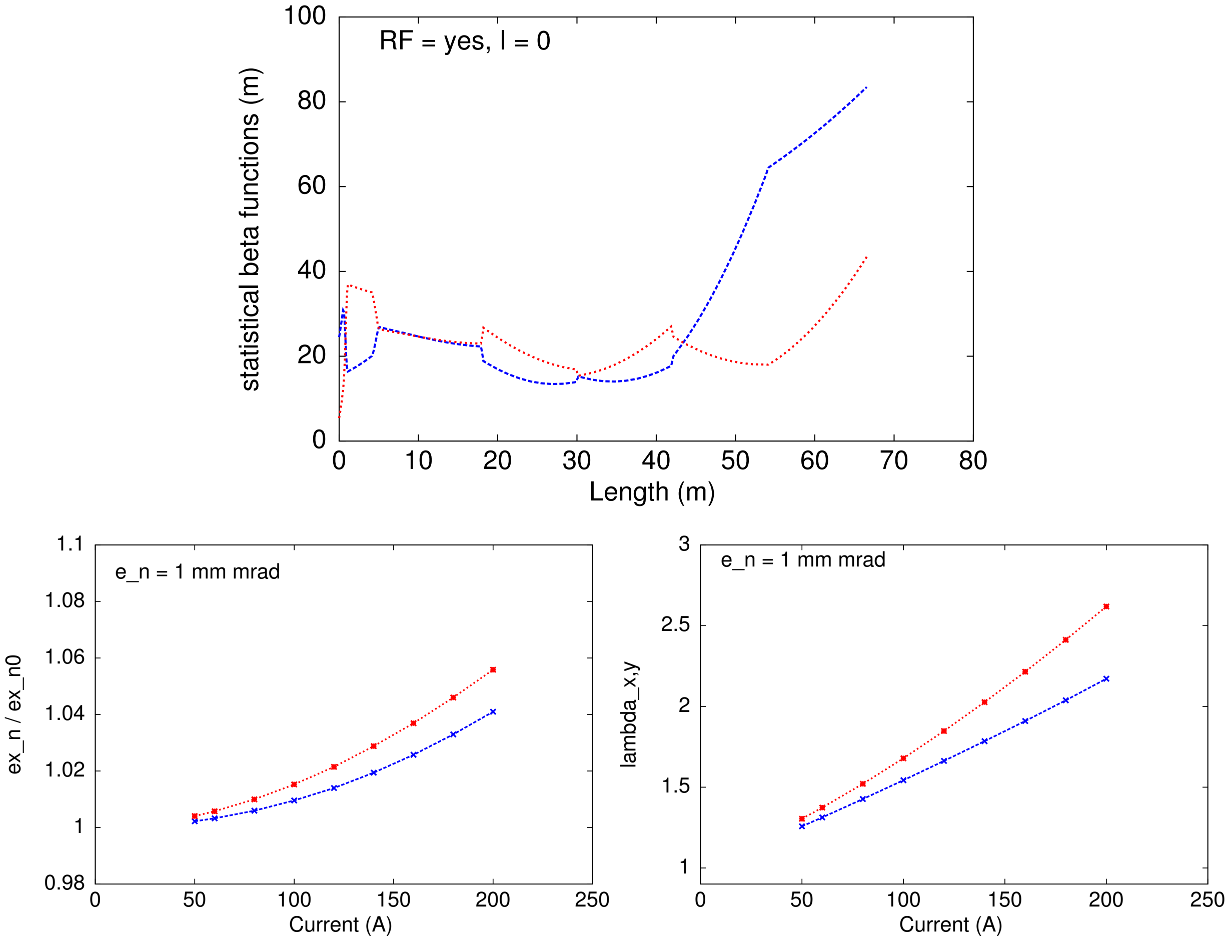

Figure 5: Beam line downstream Injector Dogleg up to BC1. Currents: $I = 50, 60 - 200\,A$ (step 20 A). $E_{init} = 130\,MeV$. $E_{acc} \approx 11.14 MeV/m$, on-crest. $E_{fin} = 501.5\,MeV$. $\varepsilon_{nx,ny} = 1\,mm \cdot mrad$. Blue and red colours represent the horizontal and vertical planes, respectively.