CSR gain calculations

benchmark BC

500 MeV, gaussian bunch, no modulation (convergence test)

500 MeV, λ = 200 µm, green's & projected

5 GeV, λ = 100 µm, green's & projected

gain curves: 500 MeV, 5 GeV, projected & integral equation

$\delta E/E$ = 2E-4

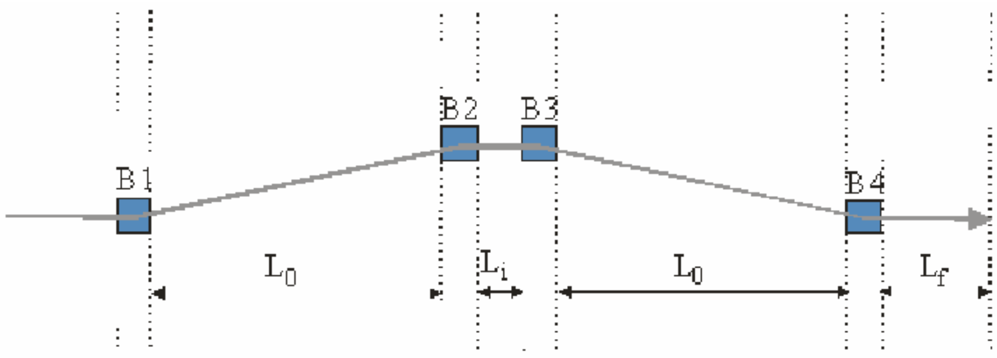
δE = 50 keV @ 500 A (5 keV @ 50 A)

XFEL double BC

E=511 MeV, δE = 5 keV, 50 A $\rightarrow$ 1 kA $\rightarrow$ 5 kA

gain curve

benchmark example



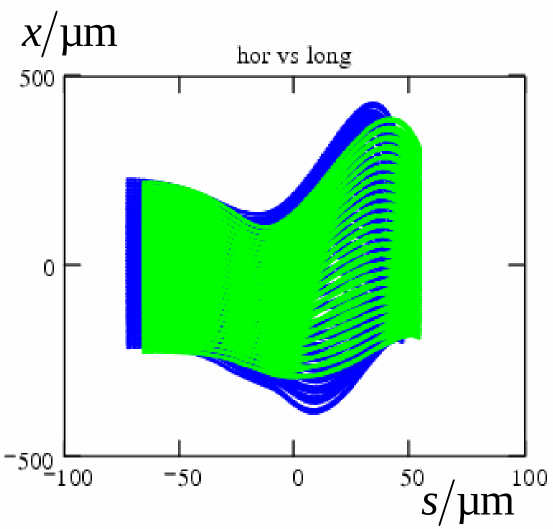
Parameters	*Symbol*	*Value*	*Unit*
Bend magnet length (projected)	L_b	0.5	m
Drift length B1->B2 and B3->B4 (projected)	L_0	5.0	m
Drift length B2->B3	L_i	1.0	m
Post chicane drift	L_f	2.0	m
Bend radius of each dipole magnet	R	10.35	m
Bending Angle	ϕ	2.77	deg
Momentum compaction	R_{56}	-25	mm
2nd order momentum compaction	T_{566}	+37.5	mm
Total projected length of chicane	L_{tot}	13.0	m
Vertical half gap of bends	g	2.5,5	mm

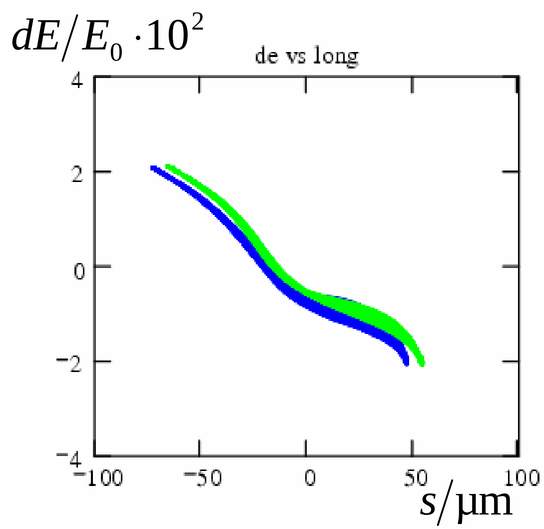
Parameter	*Symbol*	*Value*	*Unit*
Nominal energy	E_0	0.5/5.0	GeV
bunch charge	Q	0.5, 1.0	nC
incoherent rms energy spread	$(\Delta E)_{u\text{-}rms}$	10	keV
linear energy-z correlation	a	+36.0	m^{-1}
total initial rms relative energy spread	$(\Delta E/E_0)$ rms	0.720	%
initial rms bunch length	σ_i	200	µm
final rms bunch length	σ_f	20	µm
initial normalized rms emittance	$\varepsilon_{n,x}$ / $\varepsilon_{n,y}$	1.0 / 1.0	mm-mrad
initial betatron functions at 1st bend entrance	β_x / β_y	40 / 13	m
initial alpha-function at 1st bend entrance	α_x / α_y	+2.6 / + 1.0	

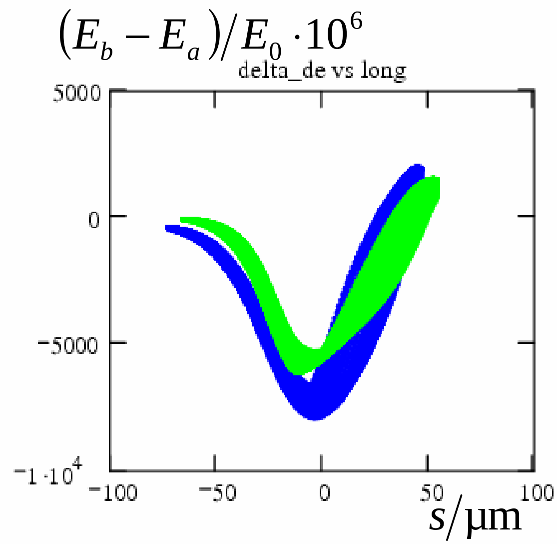
gaussian bunch, q = 1nC, $\varepsilon_{in} = 10^{-9}$ m

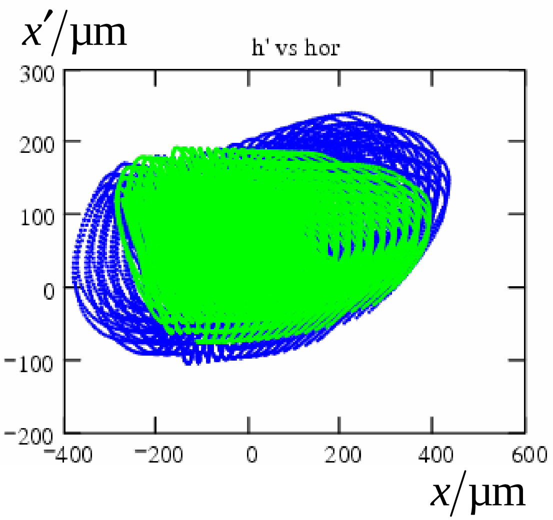
BM500 Example (compression factor = 10)
2m after BC
Green's projected

no uncorrelated energy spread @ entrance







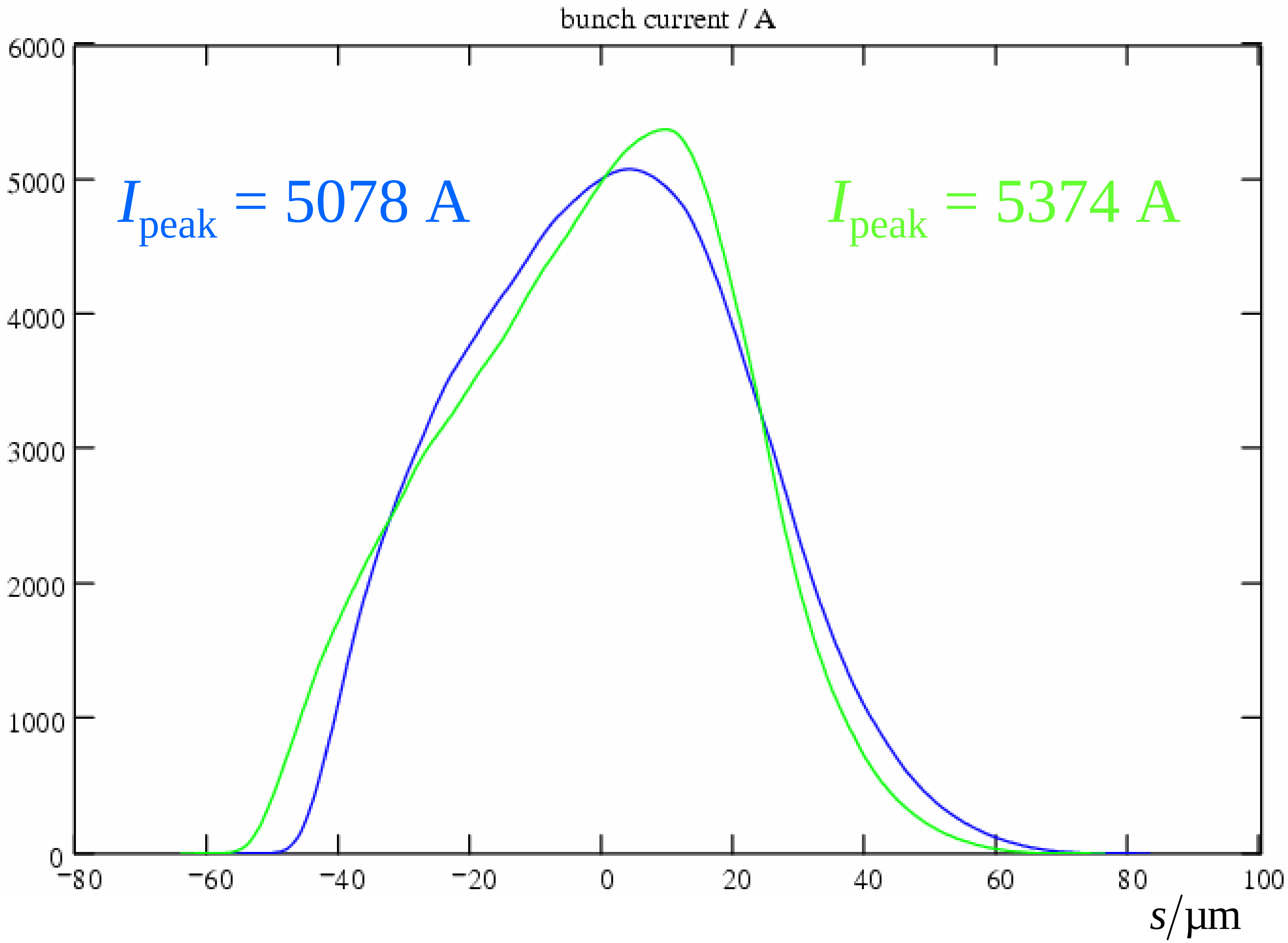


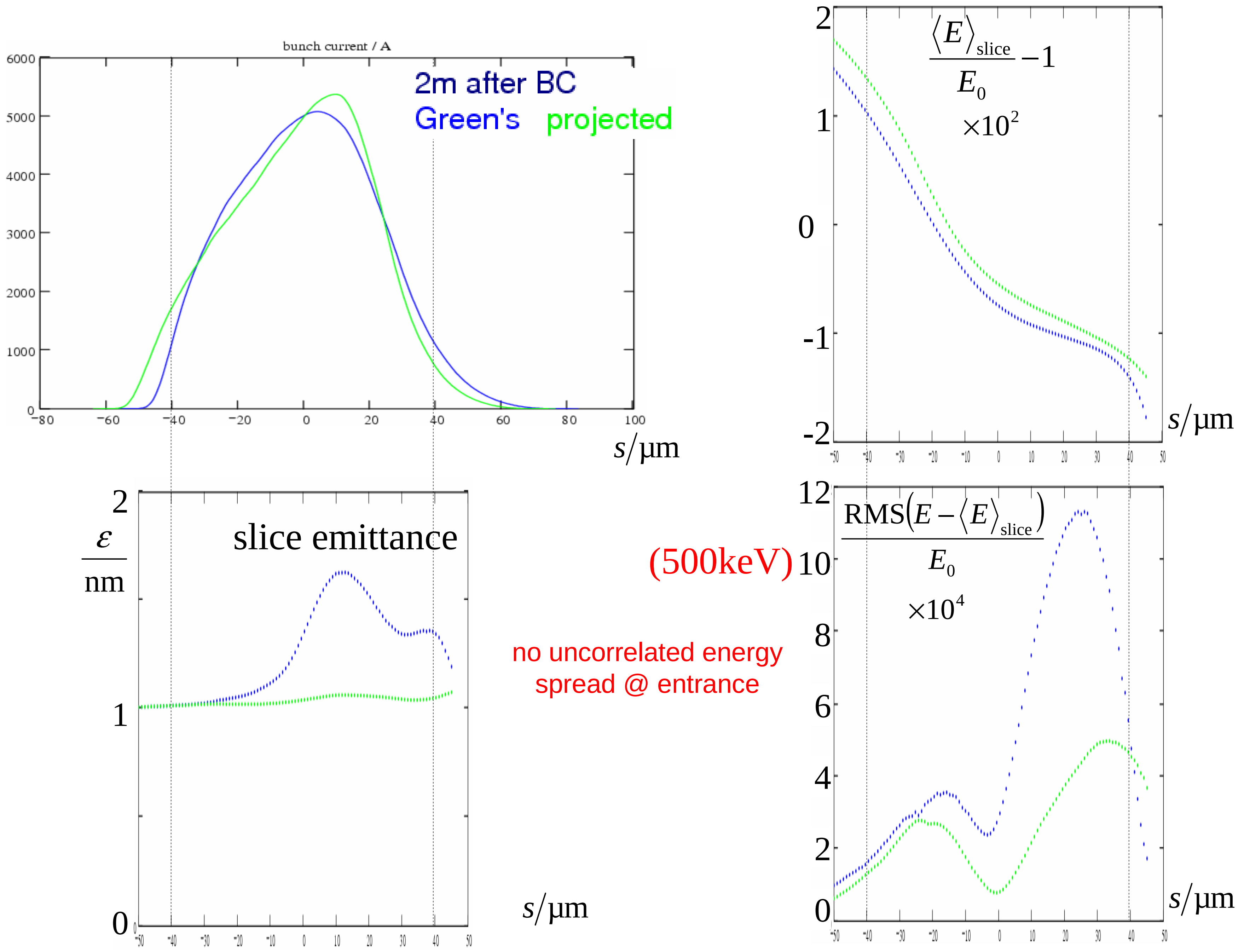
total bunch:

$\varepsilon_{out} = 7.35 \cdot 10^{-9}$ m
$\Delta E/E_0 = -5.2 \cdot 10^{-3}$

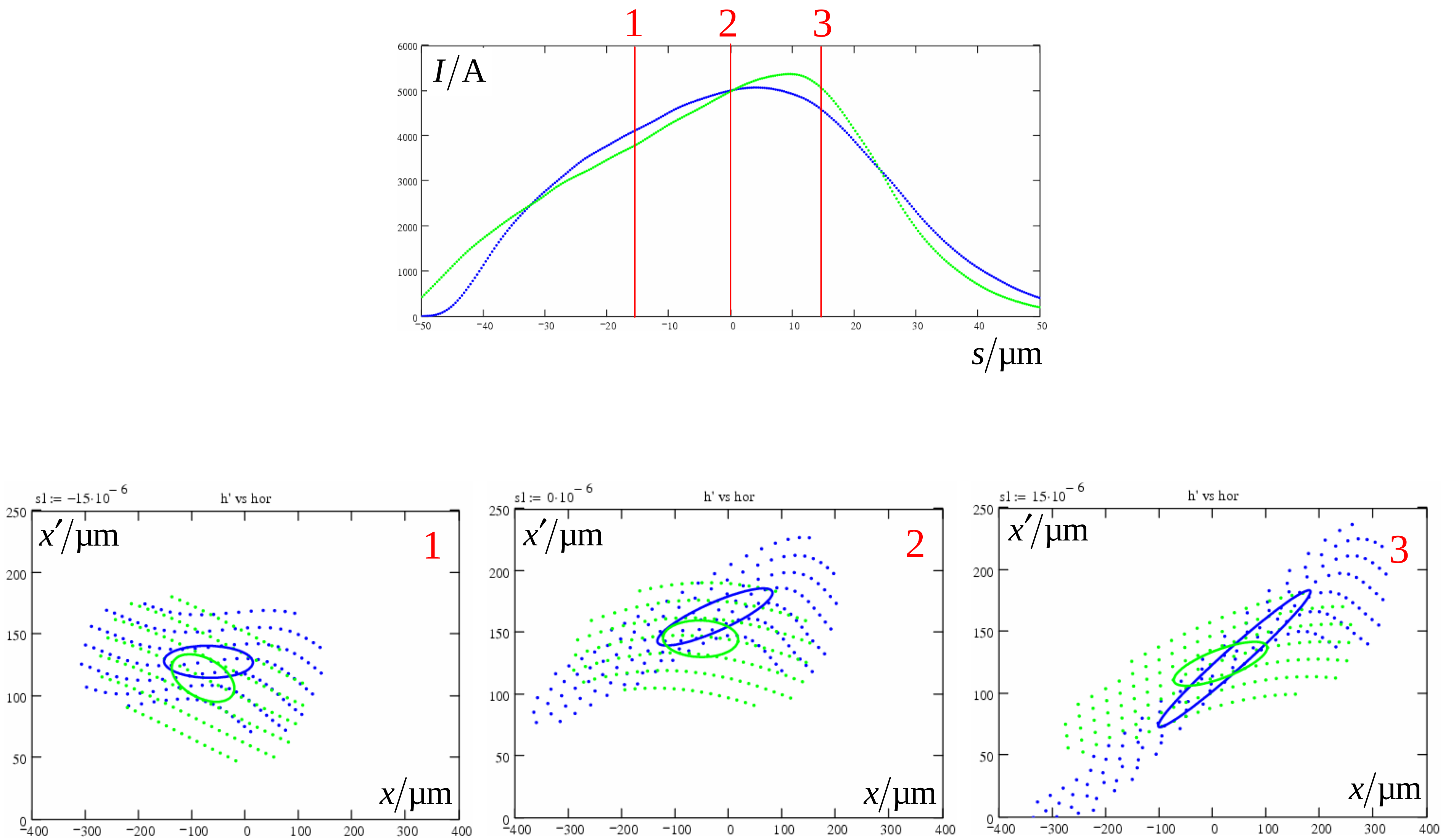
$\varepsilon_{out} = 4.25 \cdot 10^{-9}$ m
$\Delta E/E_0 = -3.9 \cdot 10^{-3}$

2m after BC
Green's projected





bunch current / A
2m after BC
Green's
projected
s/µm
⟨E⟩slice / E0 − 1
×10^2
s/µm
ε / nm
slice emittance
s/µm
(500keV)
no uncorrelated energy spread @ entrance
RMS(E − ⟨E⟩slice) / E0
×10^4
s/µm



1
2
3
I/A
s/µm
s1 := −15·10−6
h' vs hor
s1 := 0·10−6
h' vs hor
s1 := 15·10−6
h' vs hor
x′/µm
x/µm
x′/µm
x/µm
x′/µm
x/µm

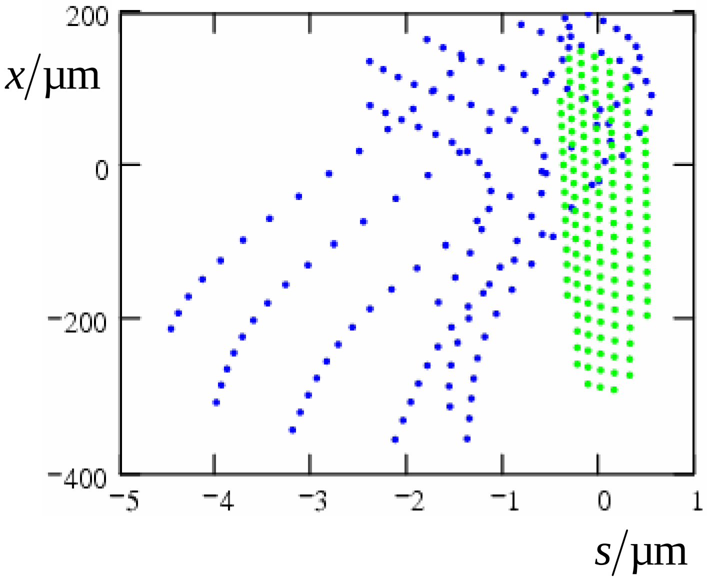
slice at entrance → @ exit

2m after BC

Green's projected

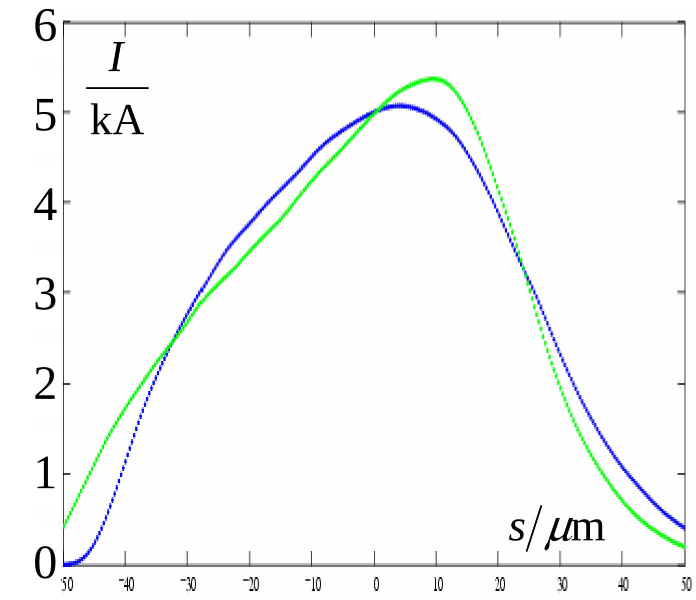
no uncorrelated energy
spread @ entrance

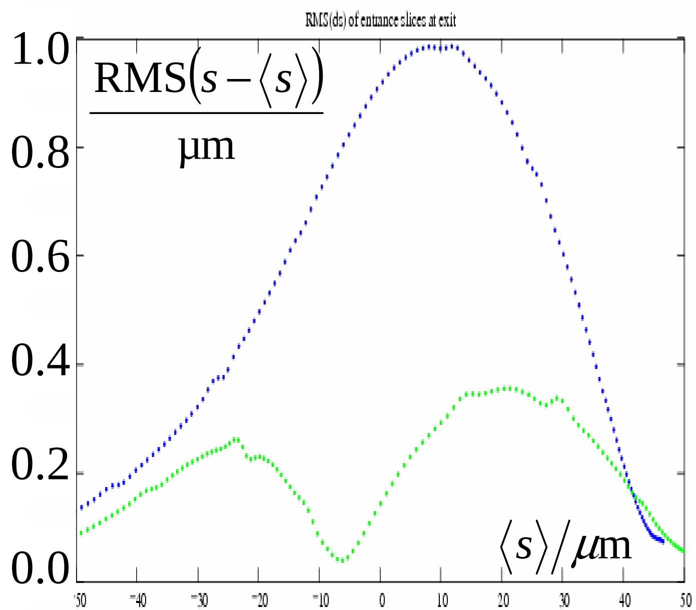
middle of bunch @ entrance



$\mathrm{RMS}(s-\langle s\rangle)=0.9\ \mu\mathrm{m}$

$\mathrm{RMS}(s-\langle s\rangle)=0.14\ \mu\mathrm{m}$



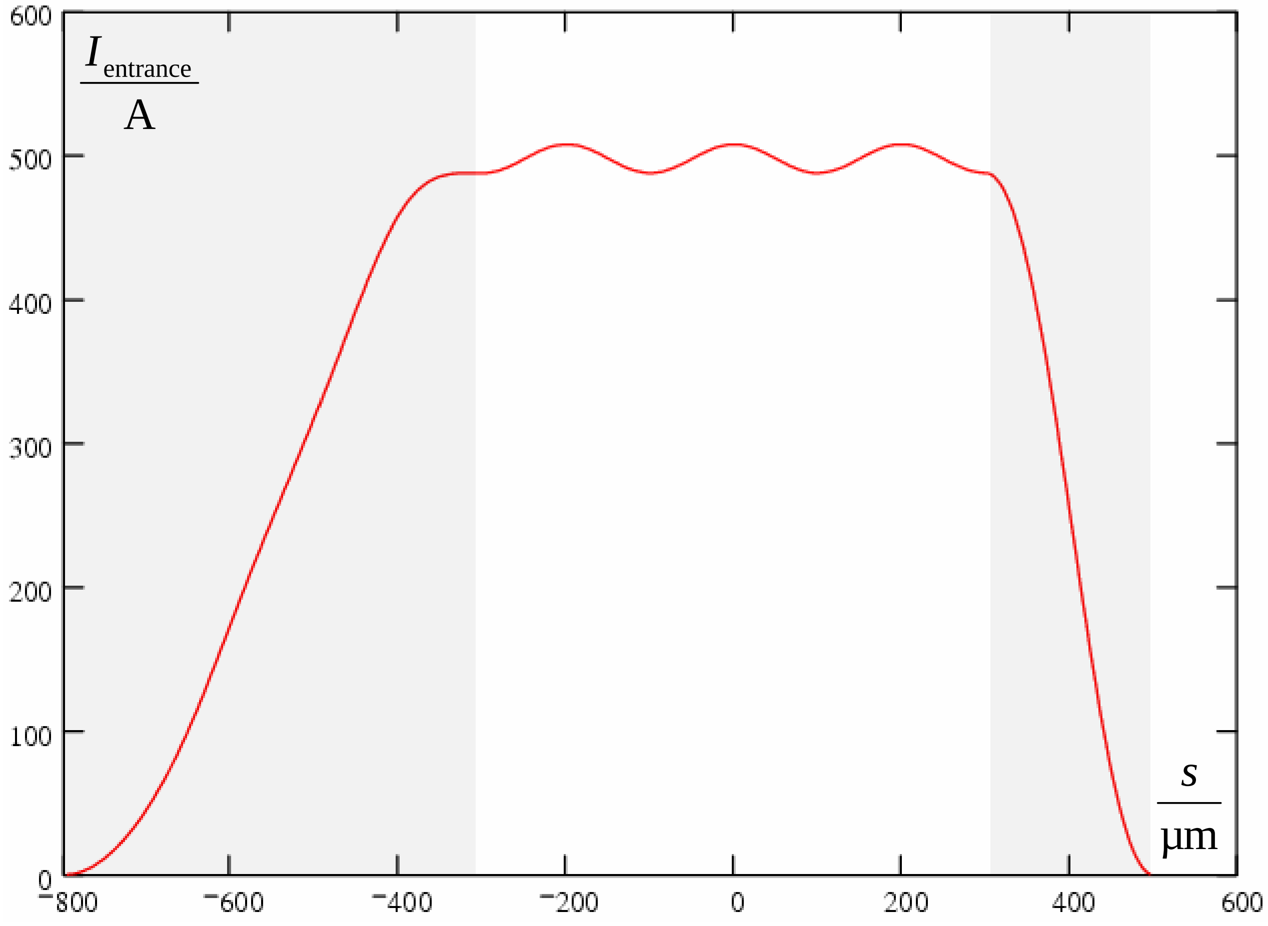


BM 500 MeV, μ-bunching

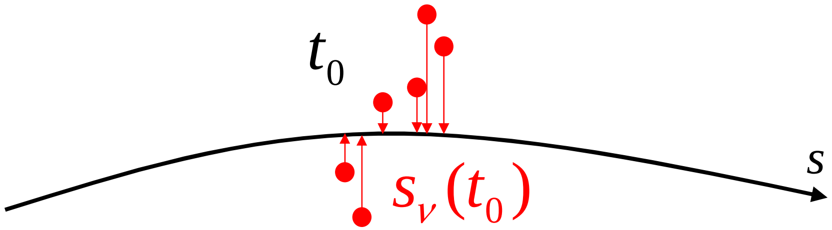
$I_{\text{entrance}} = 500\text{A} \rightarrow I_{\text{exit}} = 5\text{kA}$

$\lambda_{\text{entrance}} = 200\,\mu\text{m}$

50000 sub-bunches   flat top, δE(s) : linear compression (×10)



$$\dot{\mathbf{p}}_\nu = q\left(\mathbf{e}_\parallel E^{(\lambda)}(s_\nu, t) + \mathbf{v}_\nu \times \mathbf{B}^{(\text{ext})}\right)$$



$$\rightarrow \lambda(s)$$

projected method:

$$\lambda(s) \rightarrow E^{(\lambda)}(s,t)$$

filtered projected method:
(µ-bunch instab.)

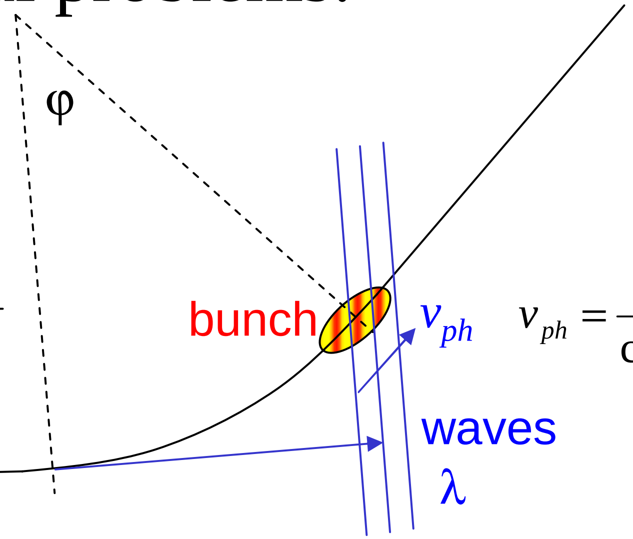
$$\lambda(s) \rightarrow \lambda_F(s) \rightarrow E^{(\lambda_F)}(s,t)$$

band pass filter for µ-modulation

some numerical problems:

a) correct integration of transients:

$$(v_{ph} - c)\Delta t << \lambda \quad \rightarrow \quad \frac{c\Delta t}{\lambda} << \frac{\cos\varphi}{1-\cos\varphi} \approx \frac{2}{\varphi^2}$$



$$v_{ph} = \frac{c}{\cos\varphi}$$

e.g. $\varphi = 0.05\,\text{rad} \quad \lambda = 1\,\mu\text{m} \quad \rightarrow \quad c\Delta t << 800\,\mu\text{m}$ very short time steps

b) sub-bunch length: $\lambda^{(0)} = C(s)\lambda(s) \quad \sigma_{\|} << \dfrac{\lambda^{(0)}}{2\pi C(s)}$

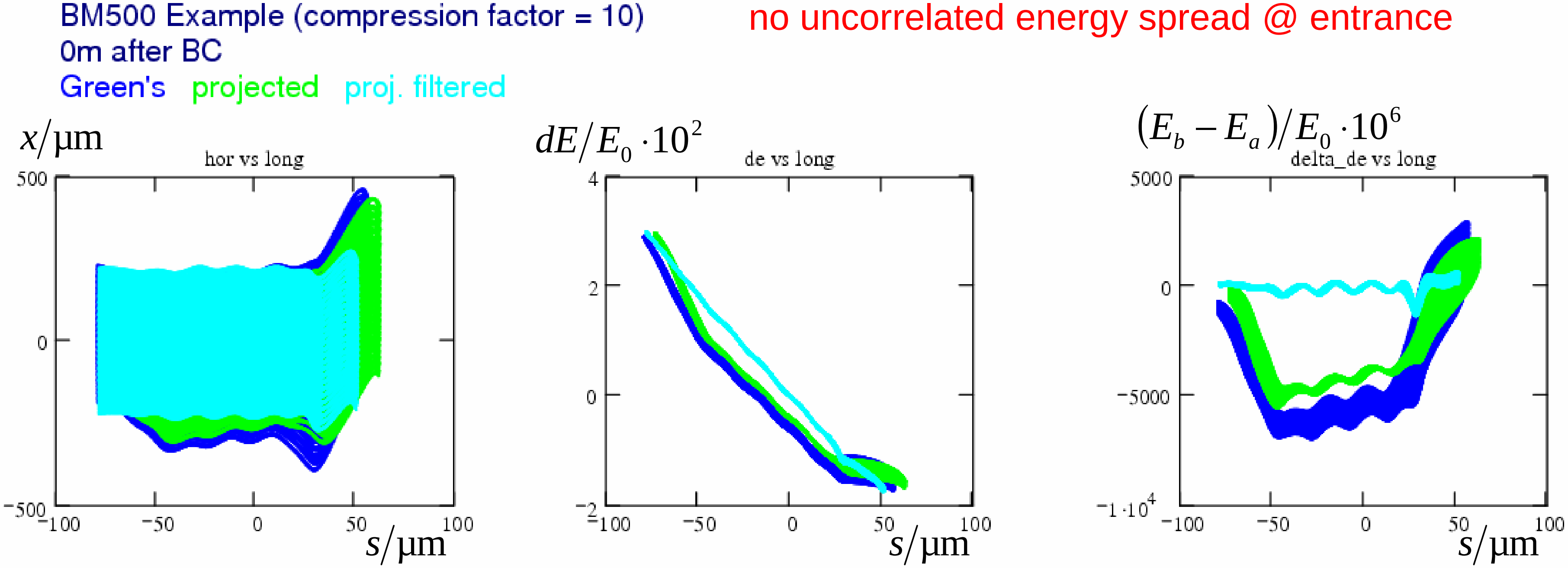
c) uncorrelated energy spread:

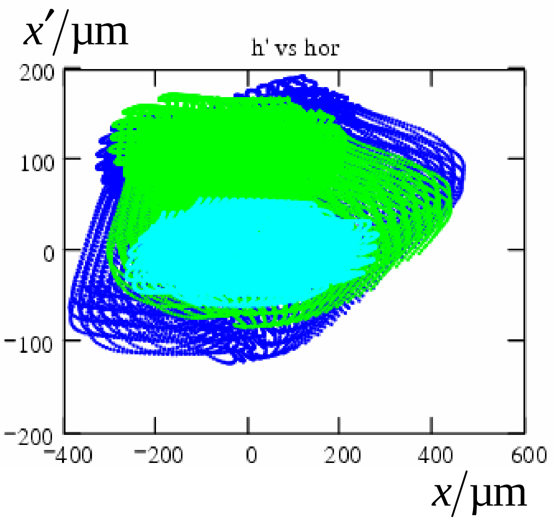
initial distribution without $\delta E_{\text{unc.}}$ but: $\tilde{\sigma}(s) << \dfrac{\lambda^{(0)}}{2\pi C(s)}$

$$\sigma_{\|}(s) = \max\left(\tilde{\sigma}(s), R_{56}\delta E_{\text{unc.}}\right)$$

d) sub-bunch width: $\sigma_r << \sqrt[3]{R\sigma_{\|}^2} \quad \rightarrow \sigma_r <\approx 10\sigma_{\|}$ (convolution method)

BM 500 MeV, μ-bunching



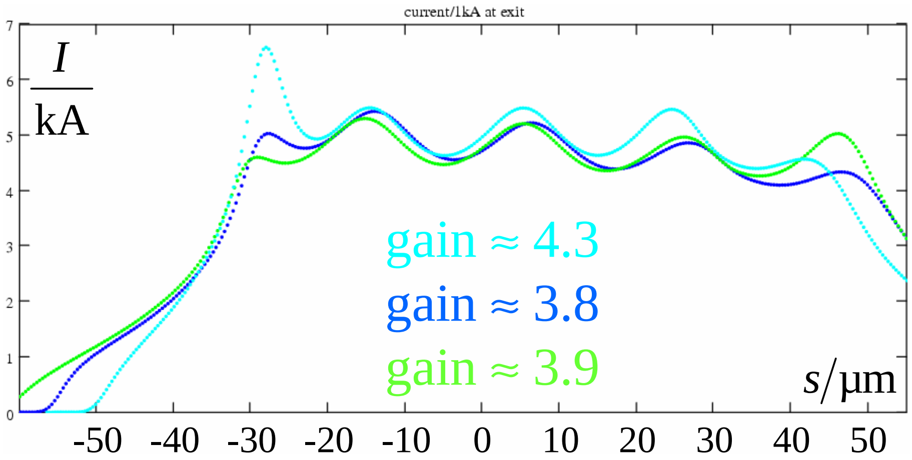


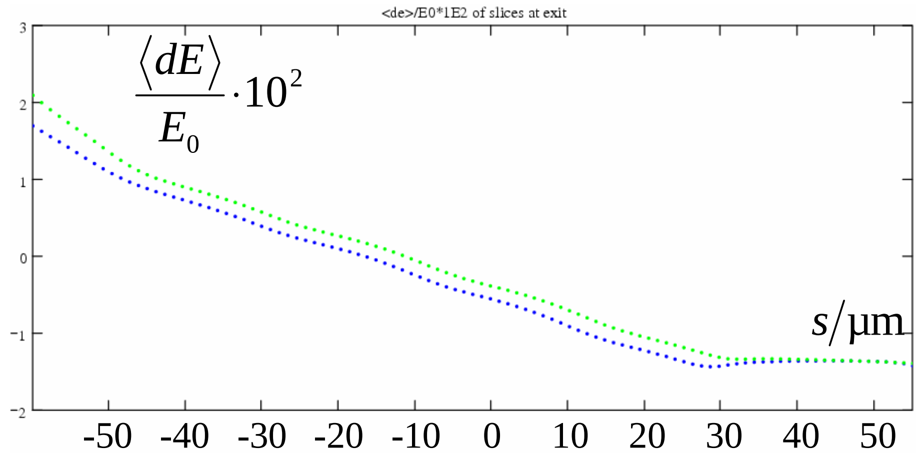
$$I_{entrance} = 500\text{A} \rightarrow I_{exit} = 5\text{kA}$$

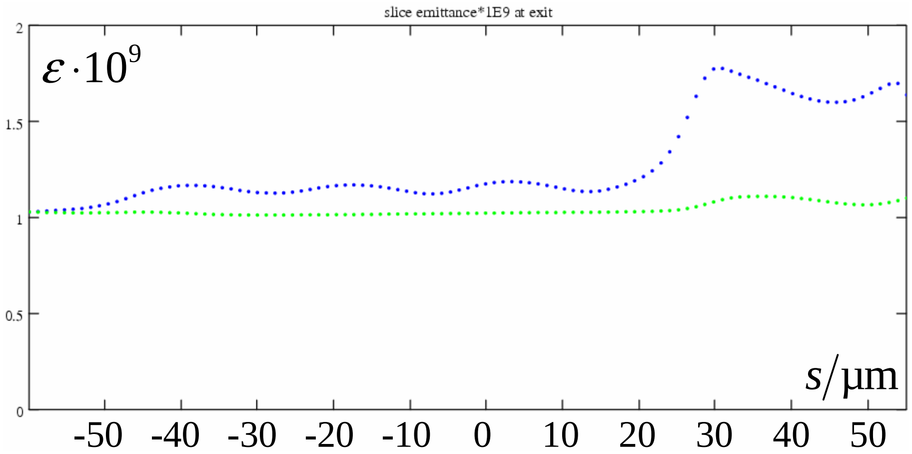
$$\lambda_{entrance} = 200\ \mu\text{m}$$

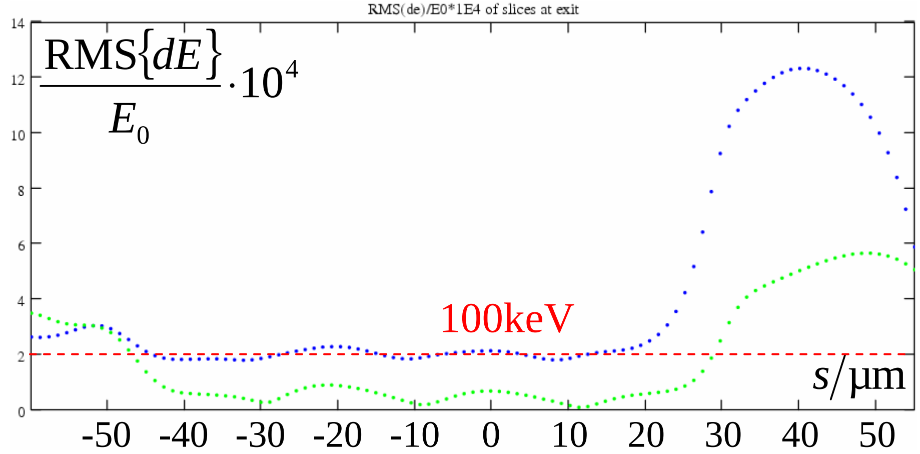
$$m_{entrance} = 2\% \rightarrow m_{exit} \approx 8\%$$

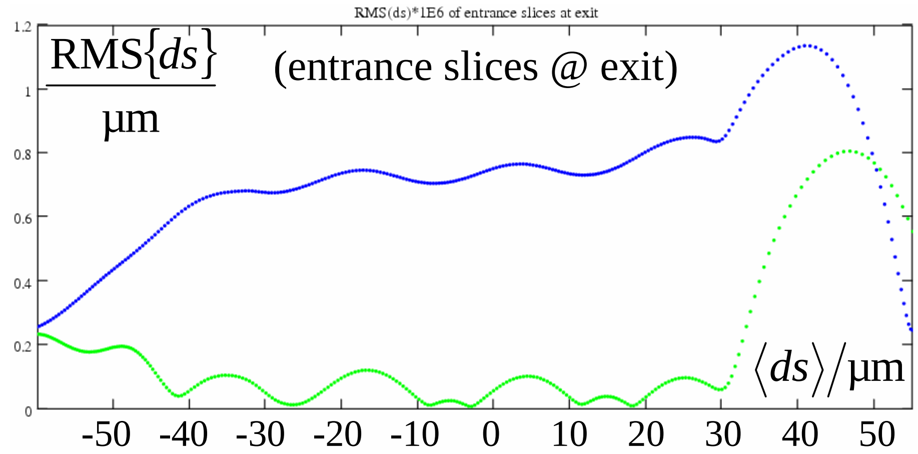
$$\text{gain} \approx 4$$









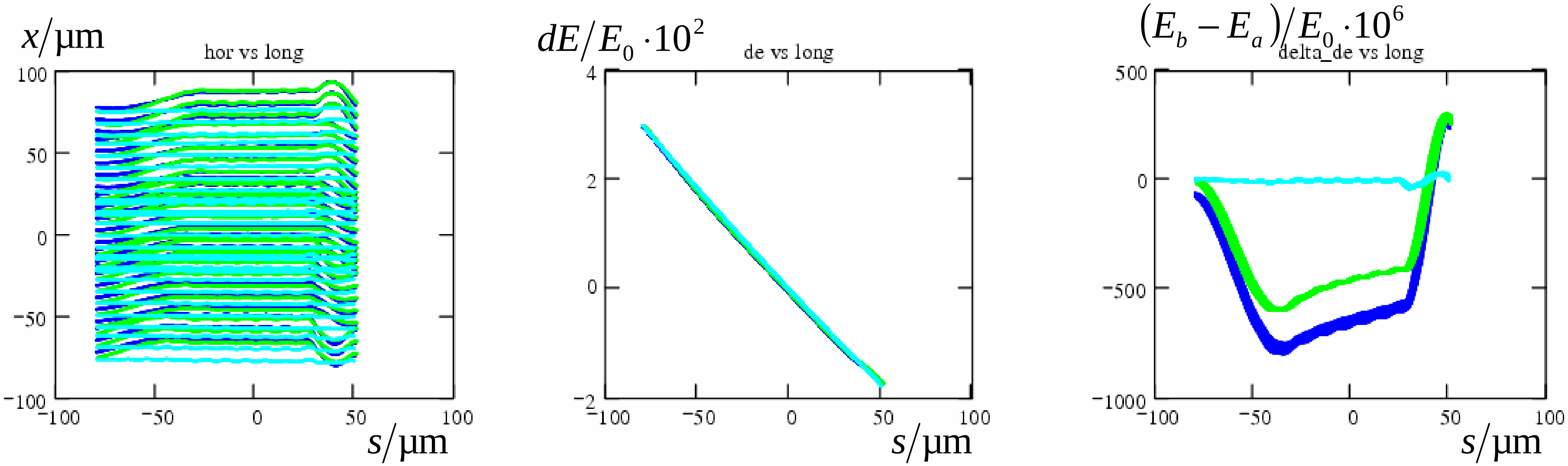


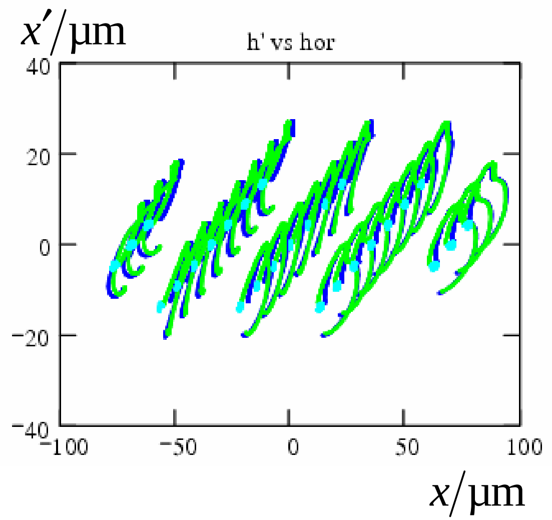
BM500 Example (compression factor = 10)
2m after BC
Green's projected
no uncorrelated energy spread @ entrance

BM 5 GeV, μ-bunching

BM5000 Example (compression factor = 10)
100um mod., 2m after BC
Green's projected pr.&filtered

no uncorrelated energy spread @ entrance

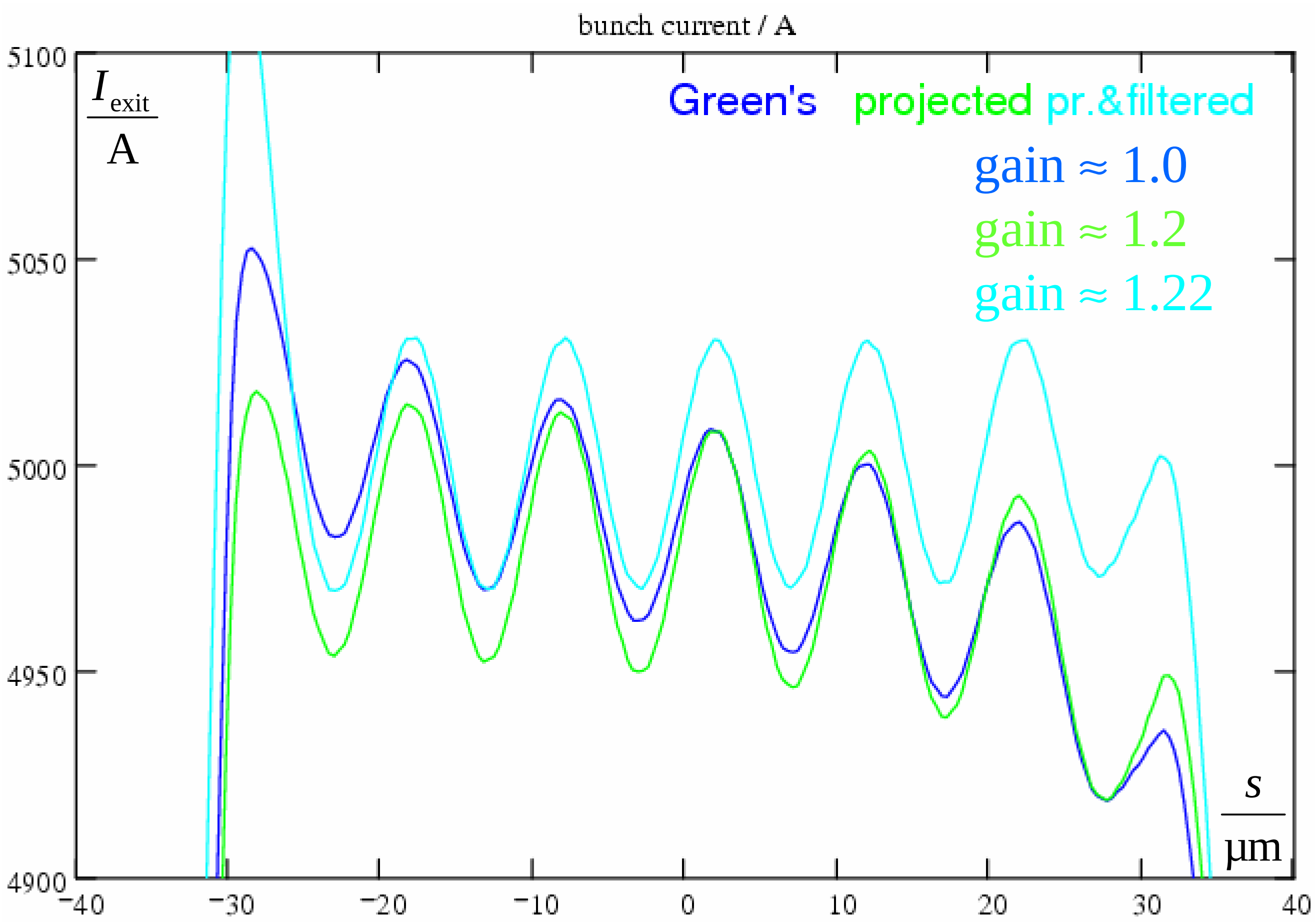




$$I_{\text{entrance}} = 500\text{A} \rightarrow I_{\text{exit}} = 5\text{kA}$$

$$\lambda_{\text{entrance}} = 100\ \mu\text{m}$$

$$\text{gain} \approx 1$$



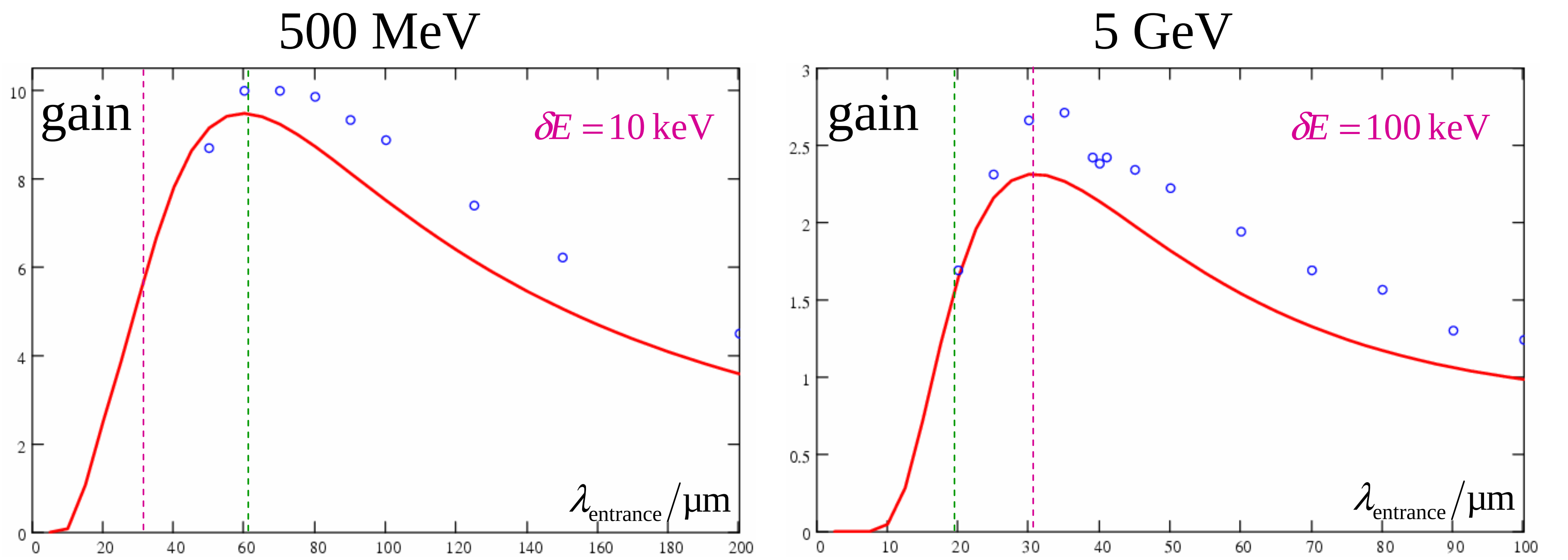
bunch current / A
5100
5050
5000
4950
4900
$\frac{I_{exit}}{A}$
Green's
projected
pr.&filtered
gain ≈ 1.0
gain ≈ 1.2
gain ≈ 1.22
$\frac{s}{\mu m}$
−40
−30
−20
−10
0
10
20
30
40

BM 500 MeV/5 GeV, μ-bunching

projected method, integral equation

$I_{\text{exit}} = 5\,\text{kA}$

$\delta E/E_0 = 2\cdot 10^{-5}$ (uncorrelated energy spread at entrance)



$2\pi\varphi\sqrt{\varepsilon\beta}$

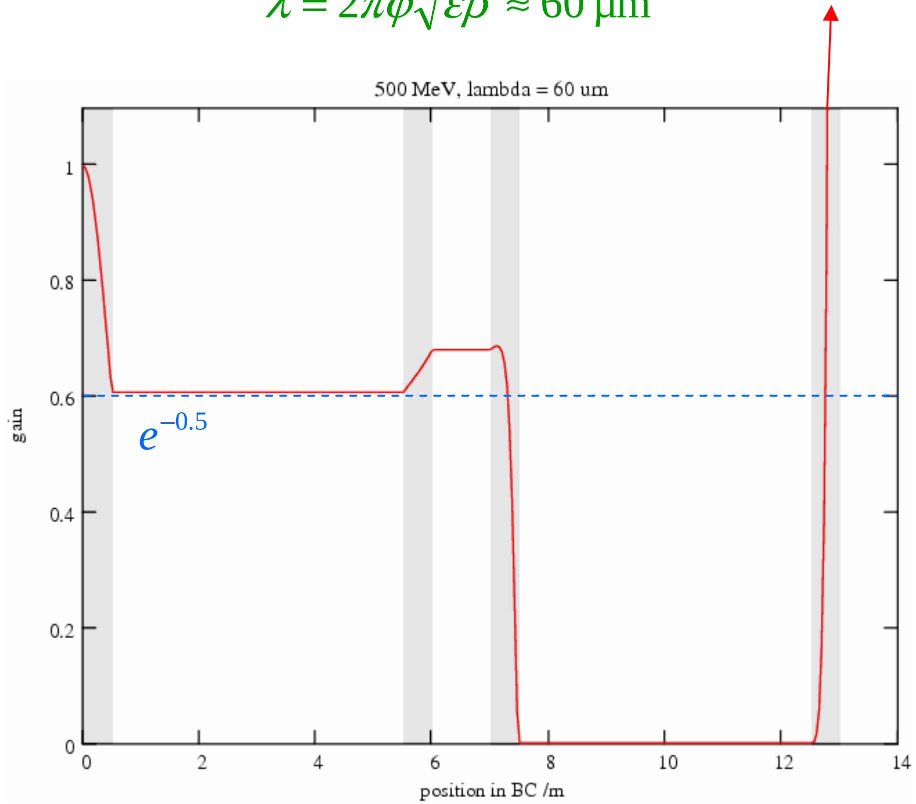
$\frac{\delta E}{E_0}\cdot R_{56}\cdot C\cdot 2\pi = 31\,\mu\text{m}$

C = compression factor (=10), φ = bending angle

modulation gain along BC

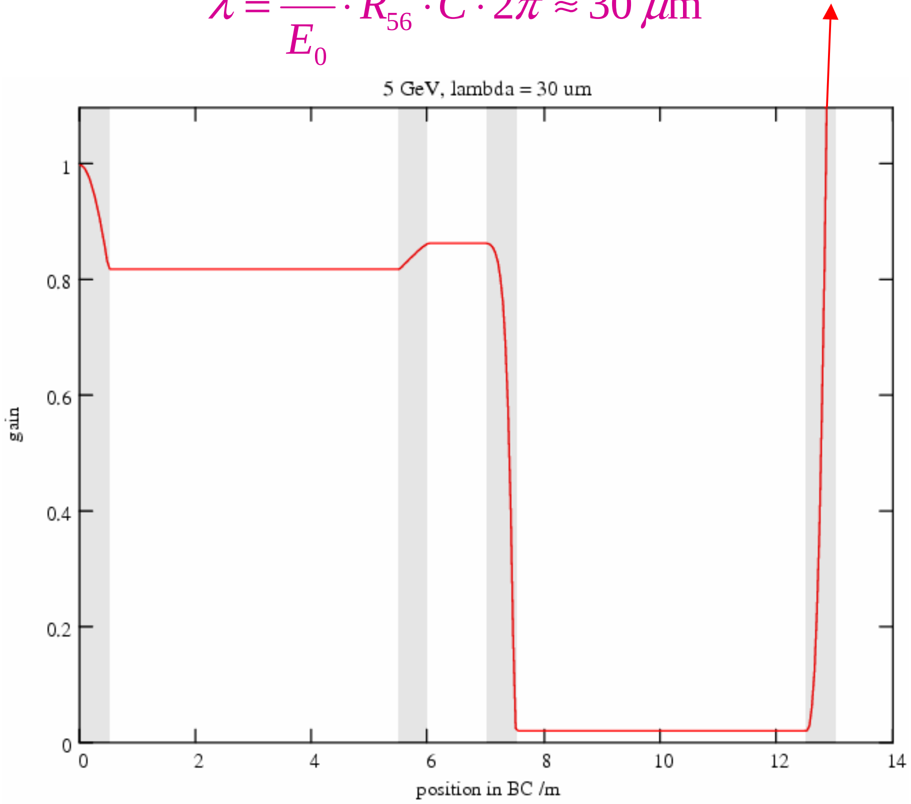
500 MeV

$$\lambda = 2\pi\varphi\sqrt{\varepsilon\beta} \approx 60\ \mu\text{m}$$



5 GeV

$$\lambda = \frac{\delta E}{E_0} \cdot R_{56} \cdot C \cdot 2\pi \approx 30\ \mu\text{m}$$



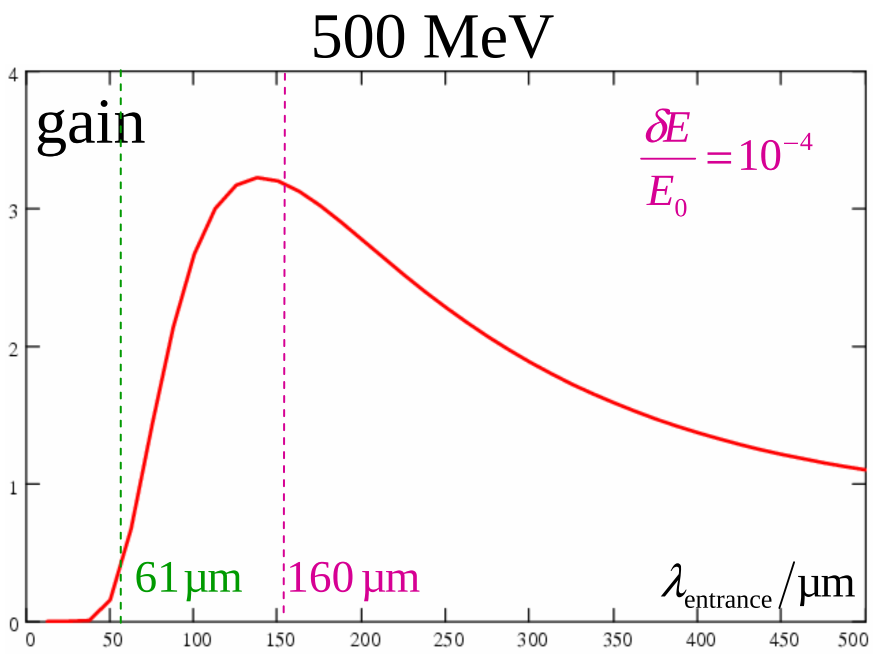
$$\delta E / E_0 = 2 \cdot 10^{-5}$$

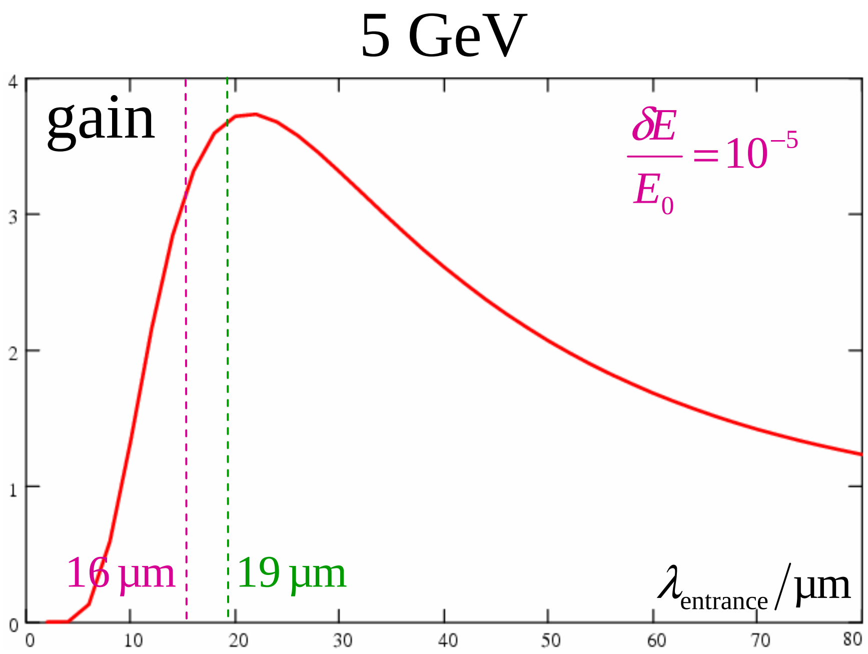
BM 500 MeV/5 GeV, μ-bunching

integral equation

$$I_{\text{exit}} = 5\,\text{kA}$$

$\delta E_{50\,\mathbf{A}} \approx 5\,\text{keV} \rightarrow \delta E_{500\,\mathbf{A}} \approx 50\,\text{keV}$ (uncorrelated energy spread at entrance)



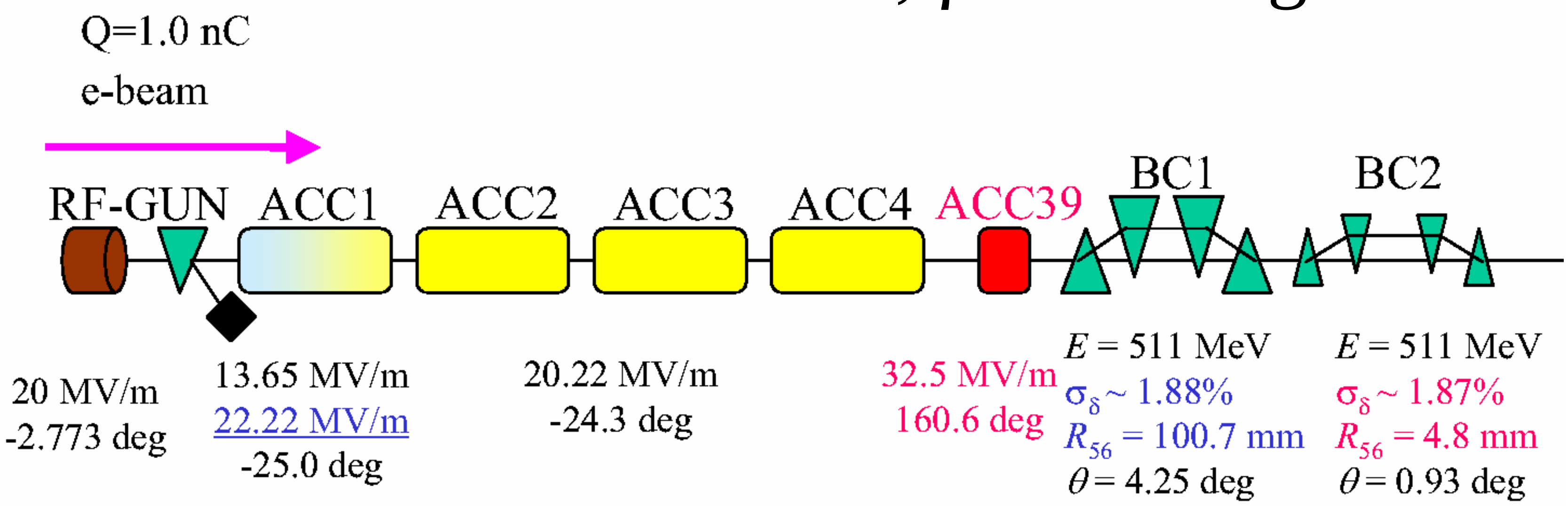


$$2\pi\varphi\sqrt{\varepsilon\beta}$$

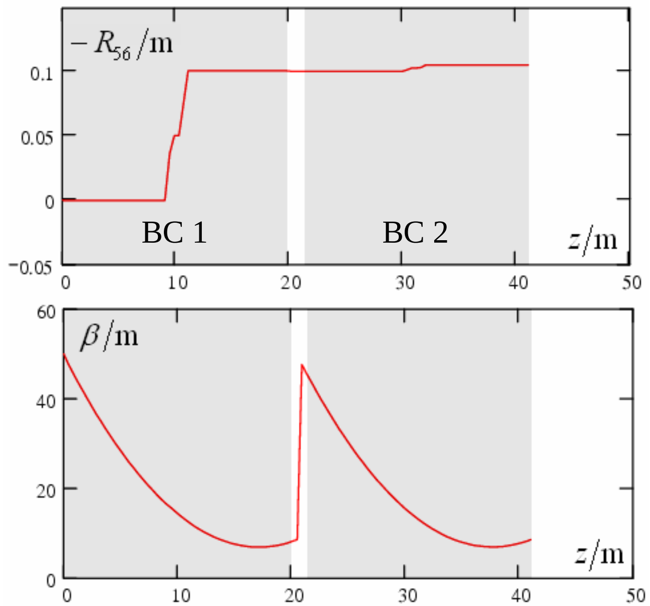
$$\frac{\delta E}{E_0} \cdot R_{56} \cdot C \cdot 2\pi$$

C = compression factor (=10), φ = bending angle

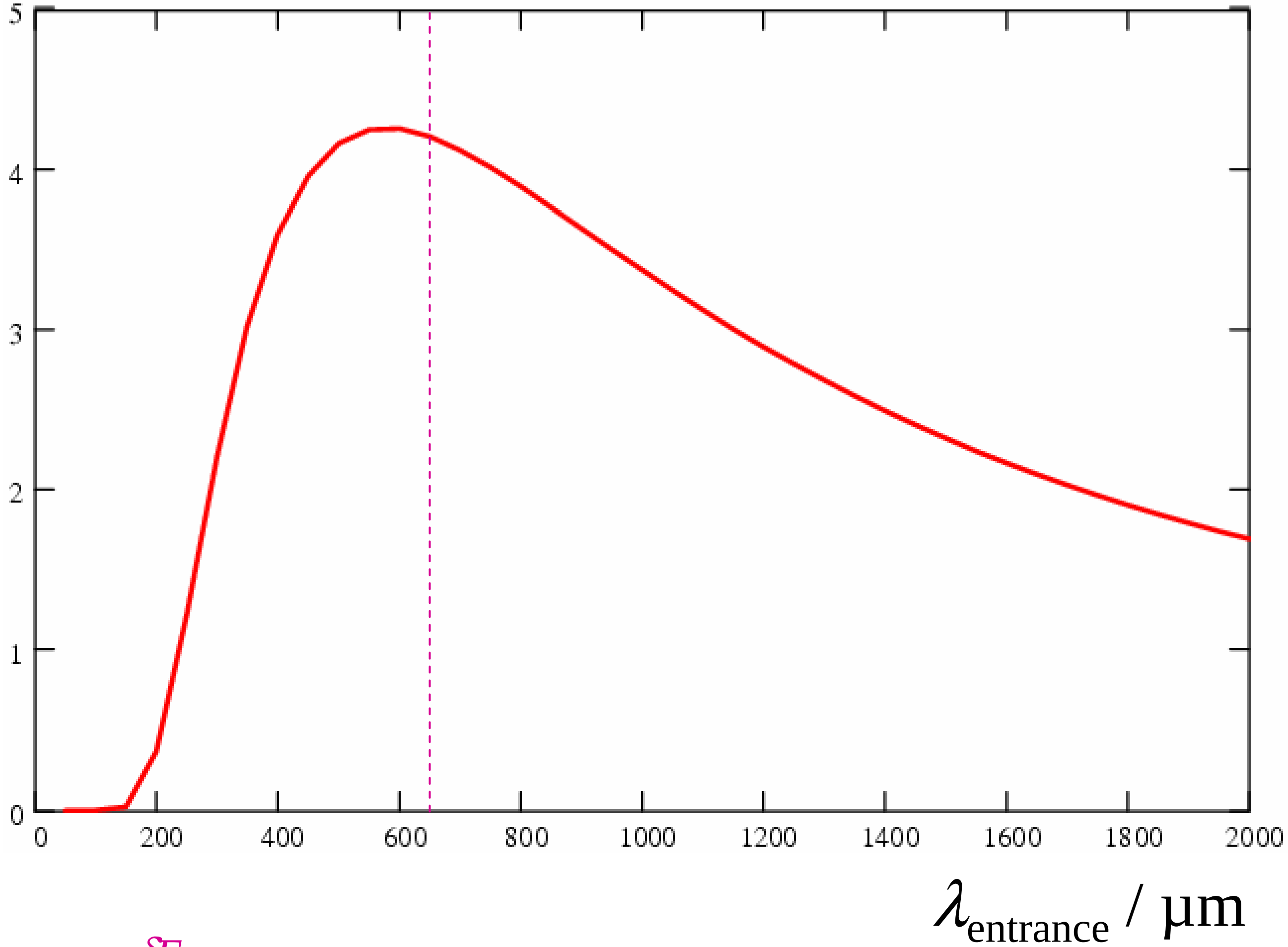
XFEL double BC, μ-bunching



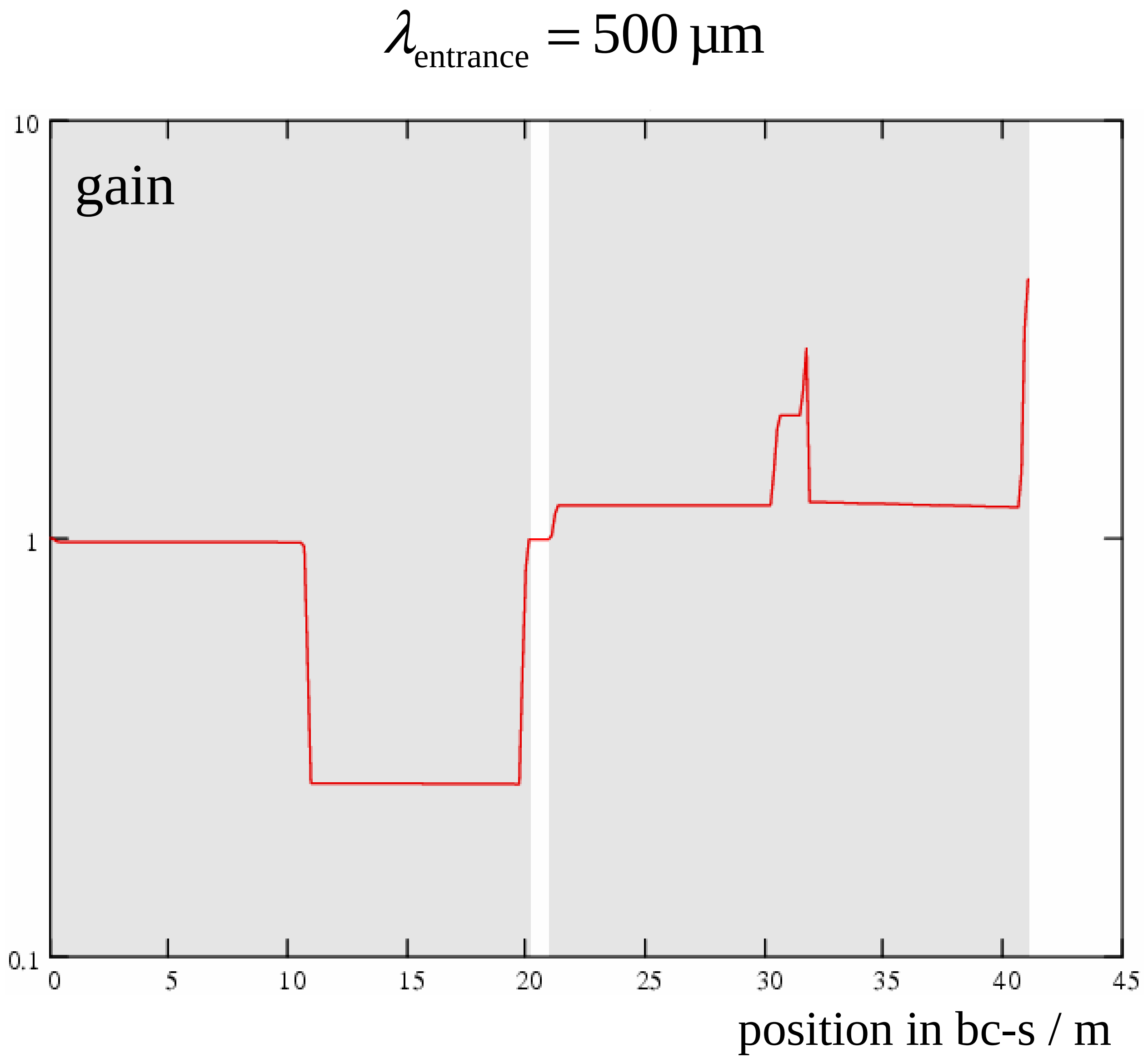
compression: $C = 100 = 20 \times 5$
current: 50 A $\rightarrow$ 5 kA
emittance: 10^{-9} m
uncorrelated energy spread: δe = 5 keV



gain in double bc



$$\frac{\delta E}{E_0} \cdot R_{56} \cdot C \cdot 2\pi = 650\ \mu\mathrm{m}$$



$\lambda_{entrance} = 500\ \mu m$
gain
10
1
0.1
0
5
10
15
20
25
30
35
40
45
position in bc-s / m

[redacted]