Space Charge in High Current Lower Energy Electron Bunches

DESY Workshop June 20th 2006, S. Becker,
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- Self acceleration: Results from CSRtrack vs. GPT vs. HOMDYN
- Self acceleration: Theoretically
- GPT: The applied calculation method
- Longitudinal phase space
- Spatial distribution
- Considering slices

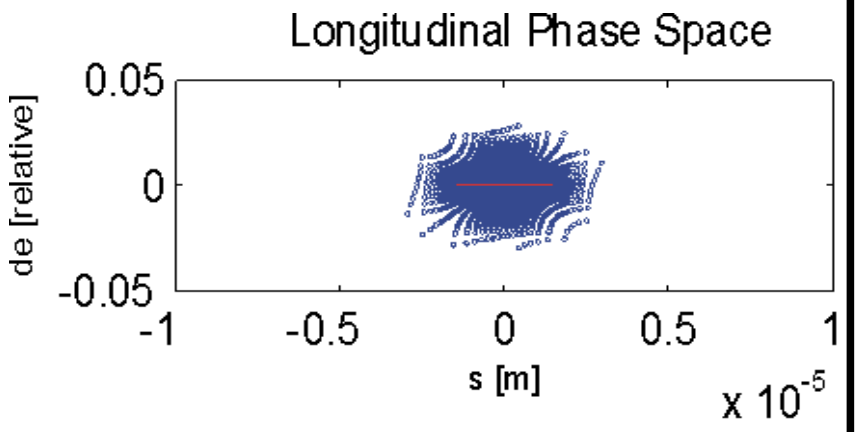
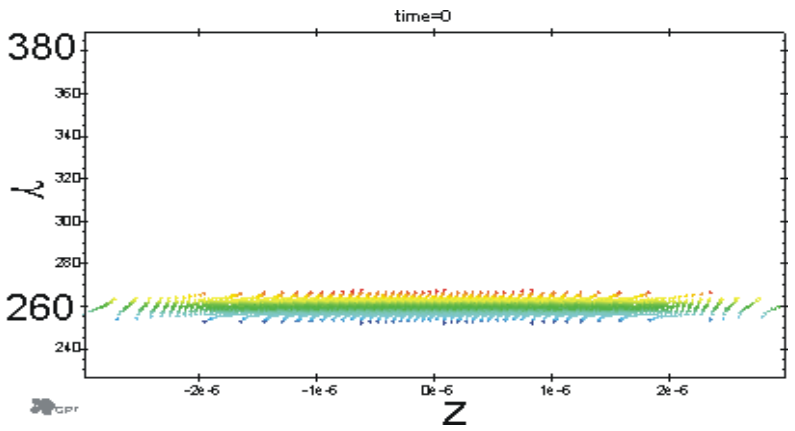
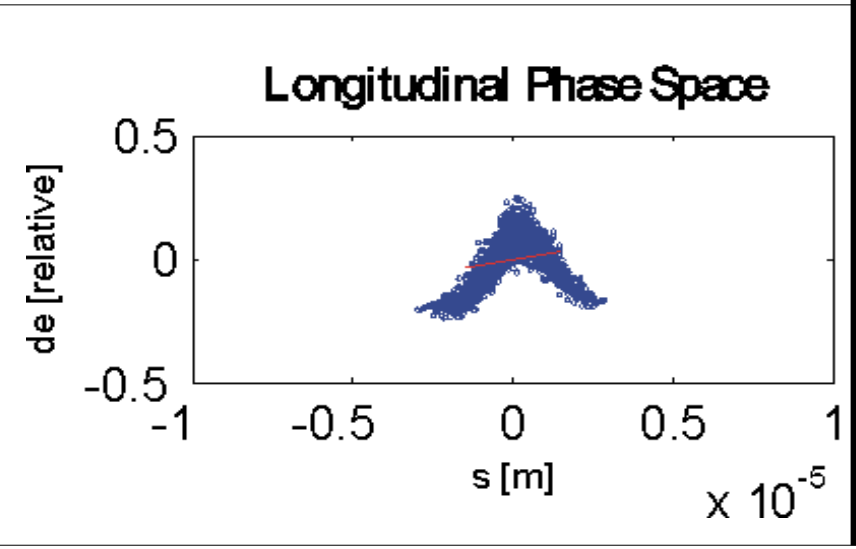
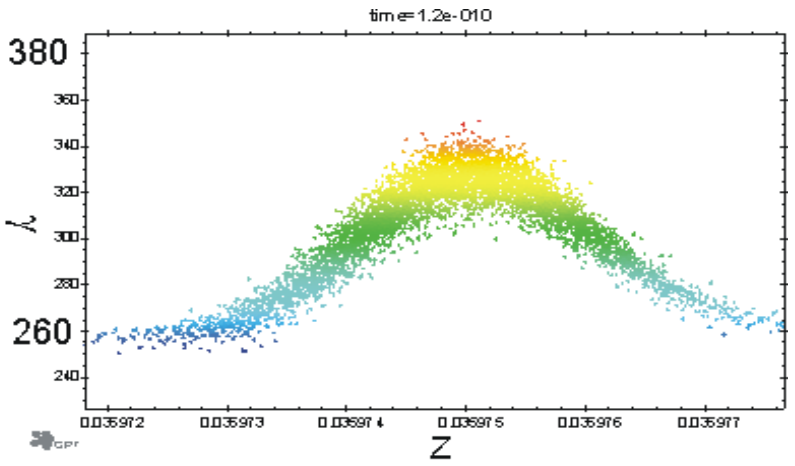
Simulation

“Extreme” case:

- PIC-Simulations -> $\sigma_x = \sigma_y = \sigma_z = 1$ µm
- Charge: 1.25 nC -> I = 150 kA
- Initial distribution: Gaussian

Comparison between CSRtrack and GPT

Self Acceleration

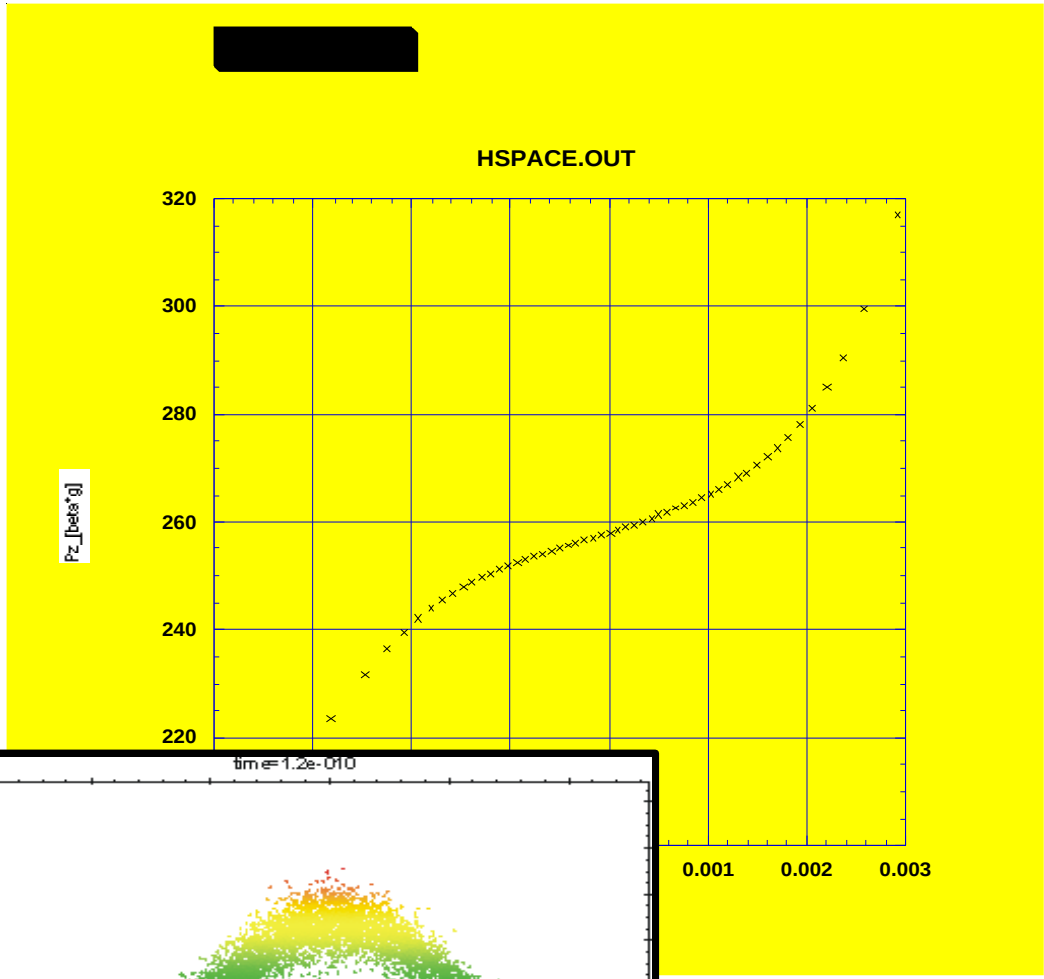
	CSRtrack	GPT
Initial distribution		
After a free drift of approx. 20mm		

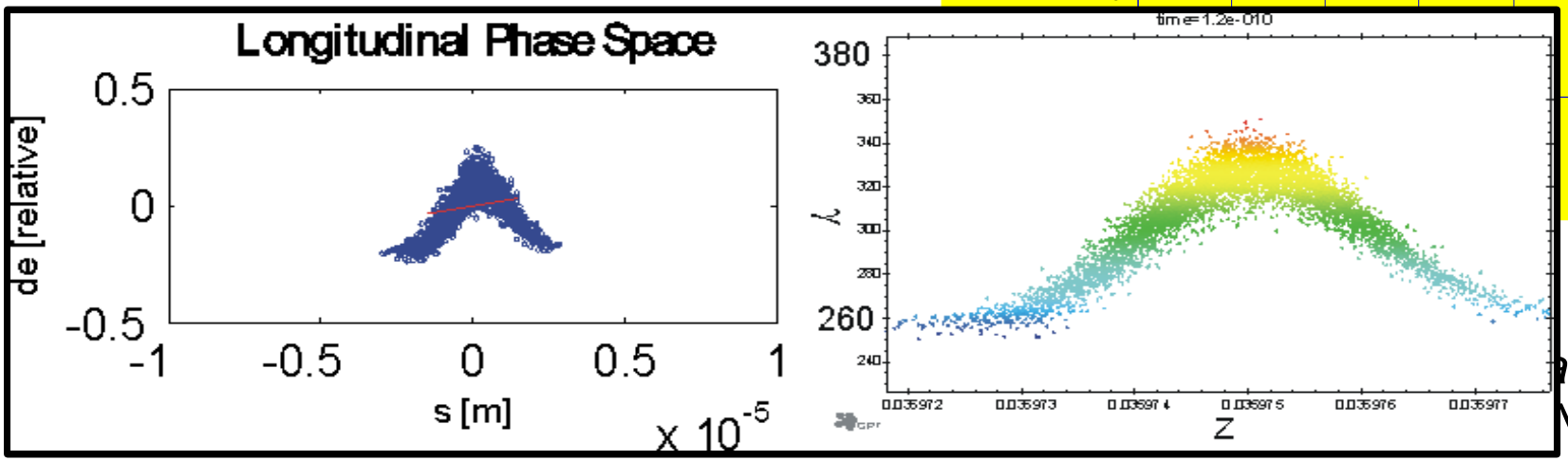
Gain of kinetic energy of approx. 20%

Self Acceleration

HOMDYN results do not comply to CSRtrack and GPT, predicting strong “debunching”.

Locking for the origin of these contradictions is work in progress





y 18th 2006
NFN/MI

Self Acceleration

Electron's rest frame:

Potential energy leads to a mass defect: $E_{pot} = m_d\, c^2$

Total energy in the electrons' mean rest frame:

$$E' = E_0 + E_{pot} = m_0 c^2 + E_{pot} = (m_0 + m_d) c^2$$

$E' = m' c^2 = \gamma' m_0 c^2$ after potential energy "released"

Potential energy in average is estimated to be of the order of **100 keV / electron**

Mean Lorentz factor due to Coulomb explosion:

$$\gamma' = 1 + \frac{m_d}{m_0}$$

Laboratory frame:

Electron bunch is leaves the bubble with γ_0

Total energy:

$$E = \gamma_0 m' c^2 = \gamma_0 m_0 c^2 \cdot \left(1 + \frac{m_d}{m_0}\right)$$

$E = \gamma_0 \gamma' m_0 c^2 = \gamma\, m_0 c^2$ after potential energy "released"

GPT

- GPT adapts time steps dynamically
- Calculation precision is defined by relative momentum changes stepwise
- Space charge is considered point to point using Lorentz transformation. Calculation time t ~ N²

For equation of motion:

$$\mathbf{F}_i = q\left(\mathbf{E}_i + \mathbf{v}_i \times \mathbf{B}_i\right)$$

Interaction in rest frame:

$$\mathbf{E}'_{j\to i} = \frac{Q\mathbf{r}'_{ji}}{4\pi\varepsilon_0 \left|\mathbf{r}'_{ji}\right|^3}$$

From Lorentz transformations:

$$\mathbf{r}_{ji} = \mathbf{r}_i - \mathbf{r}_j$$

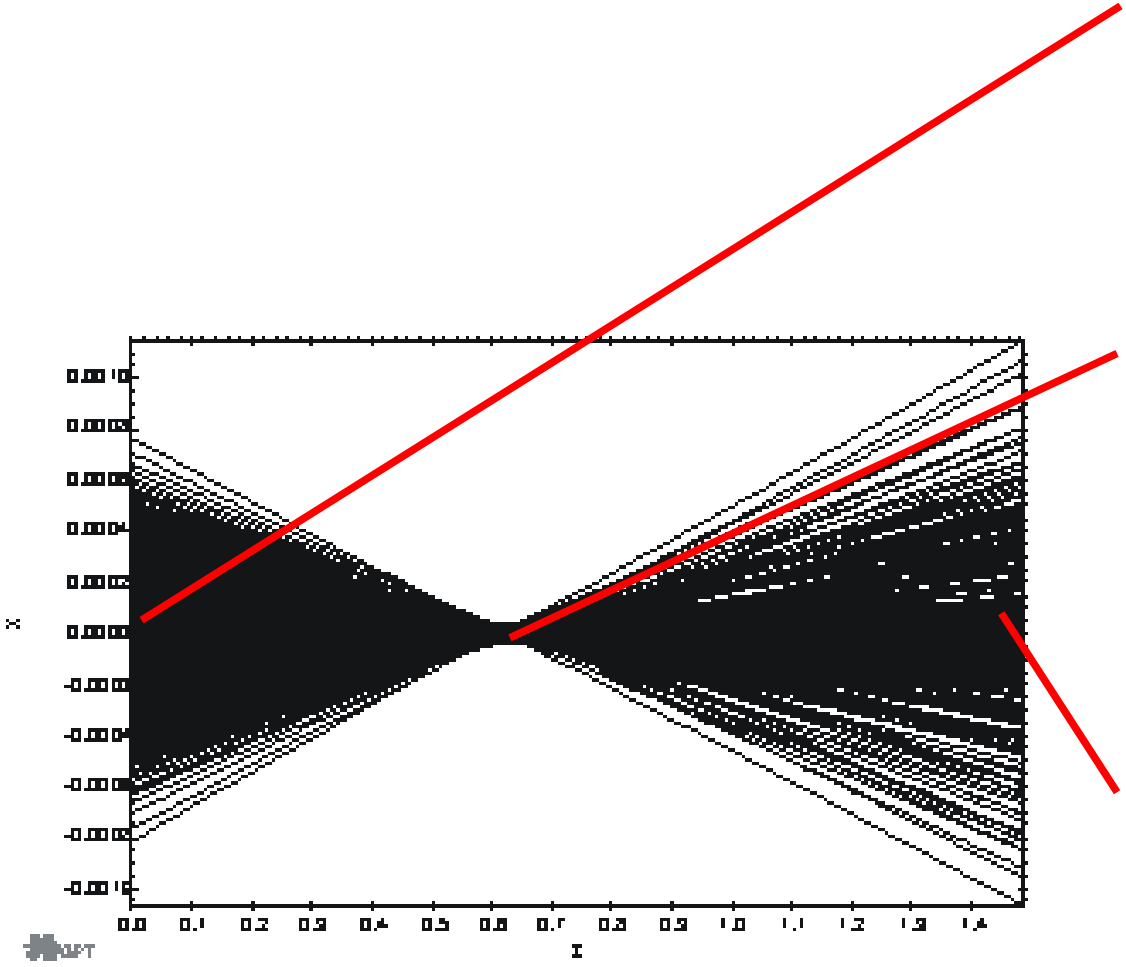
$$\mathbf{r}'_{ji} = \mathbf{r}_{ji} + \frac{\gamma_j^{\,2}}{\gamma_j + 1}\left(\mathbf{r}_{ji}\cdot\boldsymbol{\beta}_j\right)\boldsymbol{\beta}_j$$

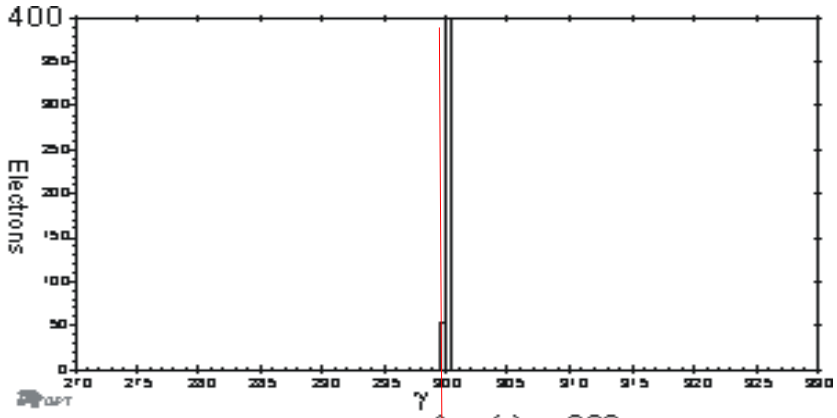
$$\mathbf{E}_i = \sum_{j\neq i}\gamma_j\left[\mathbf{E}'_{j\to i} - \frac{\gamma_j}{\gamma_j + 1}\left(\boldsymbol{\beta}_j\cdot\mathbf{E}'_{j\to i}\right)\boldsymbol{\beta}_j\right]$$

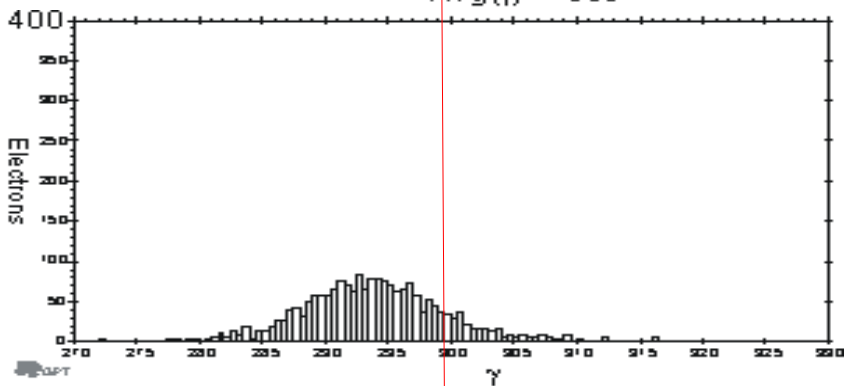
$$\mathbf{B}_i = \sum_{j\neq i}\frac{\gamma_j\boldsymbol{\beta}_j\times\mathbf{E}'_{j\to i}}{c}$$

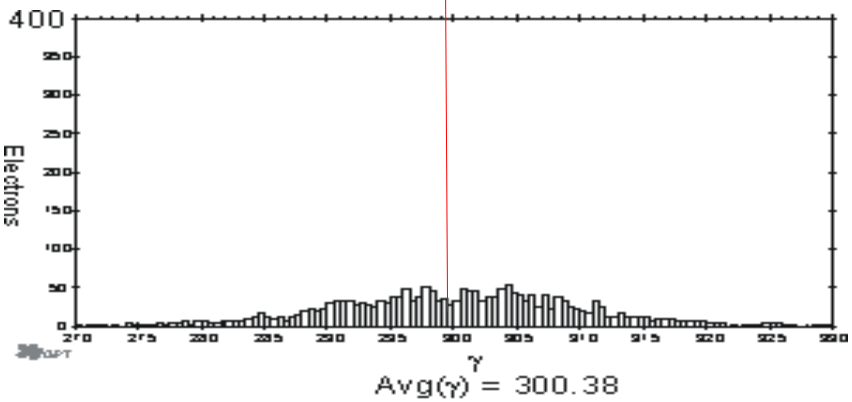
GPT- Energy Conservation

Our personal “Lackmustest”
for the evaluation of GPT results

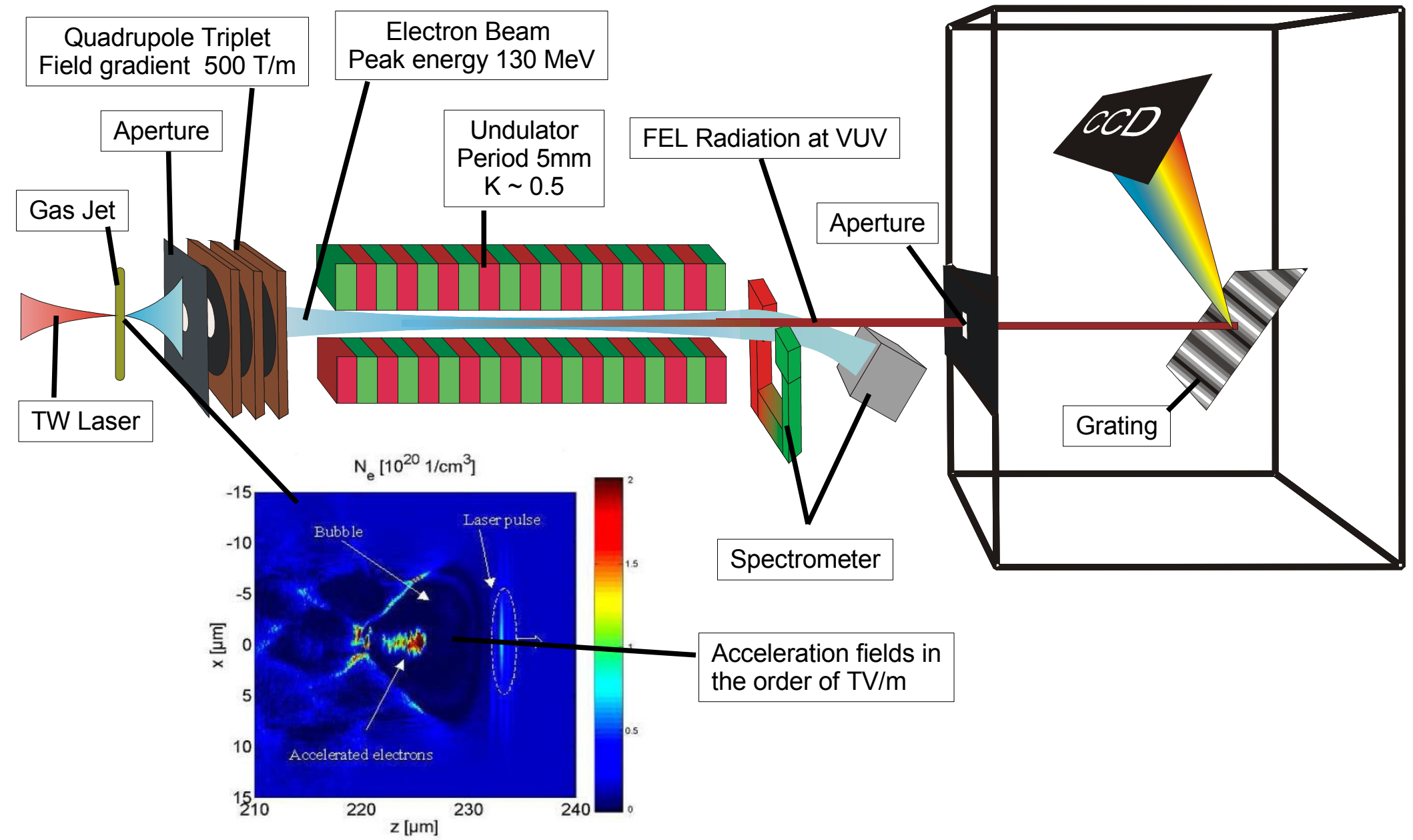




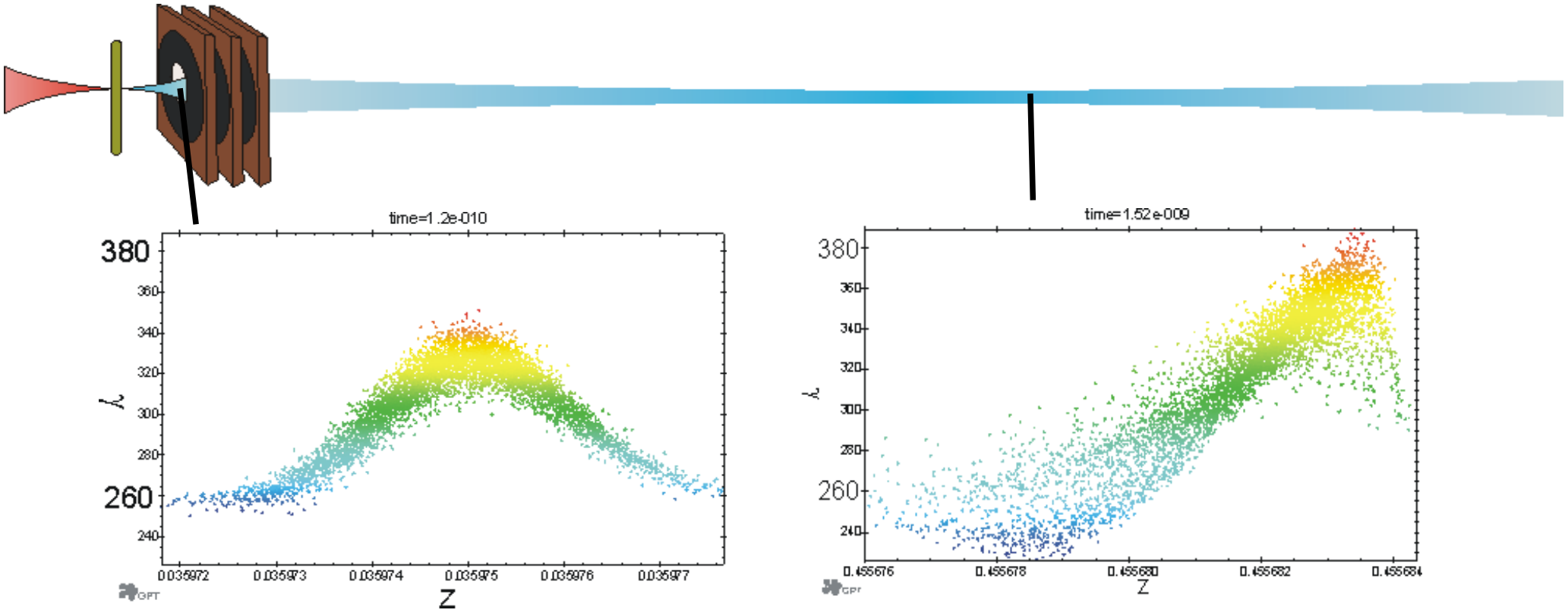




Experimental Overview

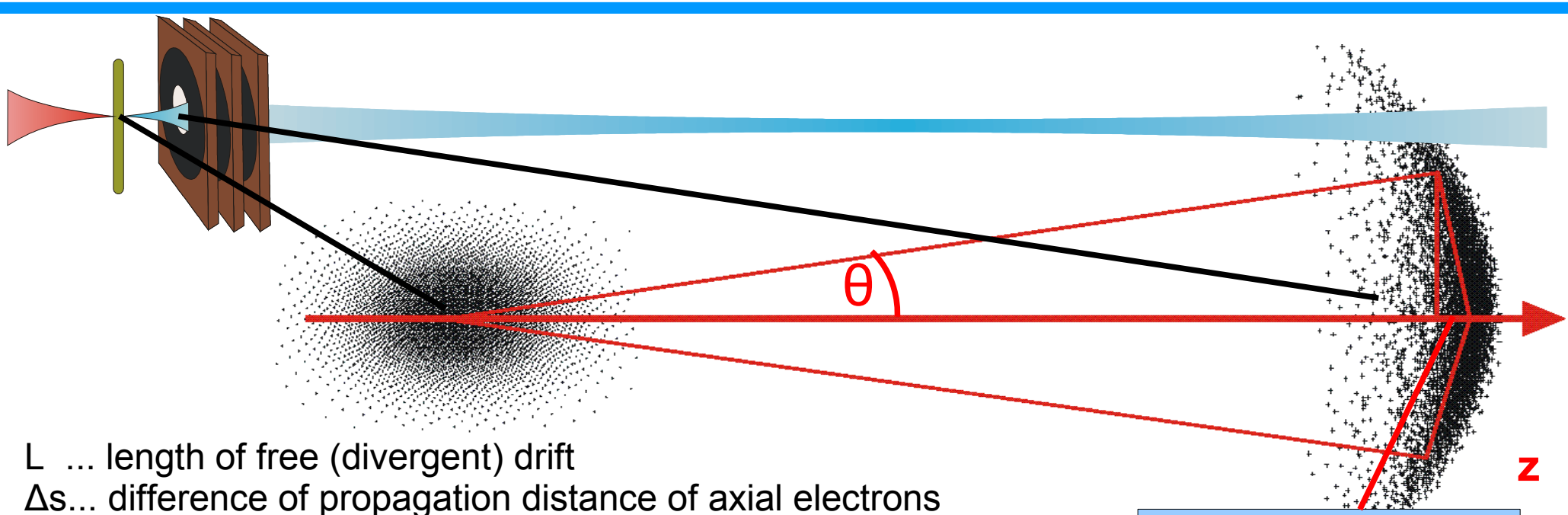


Longitudinal Phase Space



Evolution from “hat” to an almost linear energy chirp

Longitudinal Phase Space



L ... length of free (divergent) drift
Δs... difference of propagation distance of axial electrons compared to electrons of angle θ

$$\Delta s \sim (1-\cos^2\theta)\cdot L$$

Criteria for the phase space to become a linear energy shift:

-Axial electron propagates with γ_0
-Electron at angle θ propagates with $(\alpha+1)\cdot\gamma_0$

$$\Delta s \approx L\cdot\left(\frac{\theta^2}{2}-\frac{\alpha}{\gamma_0^2}\right) < 0 \quad \rightarrow$$

phase space becomes “linear” chirp for $\theta < \frac{\sqrt{2\alpha}}{\gamma_0}$

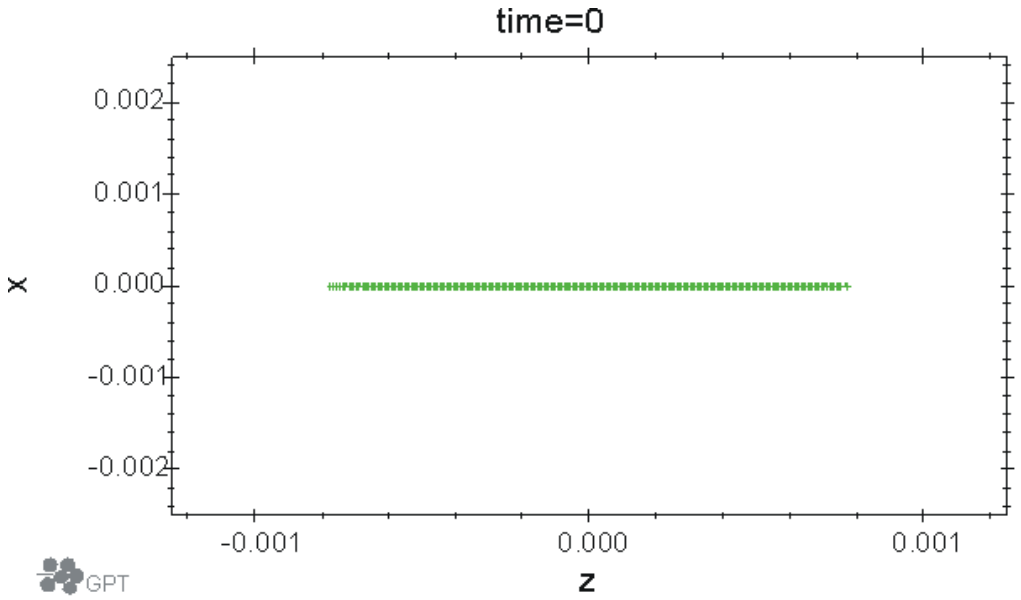
Longitudinal Phase Space

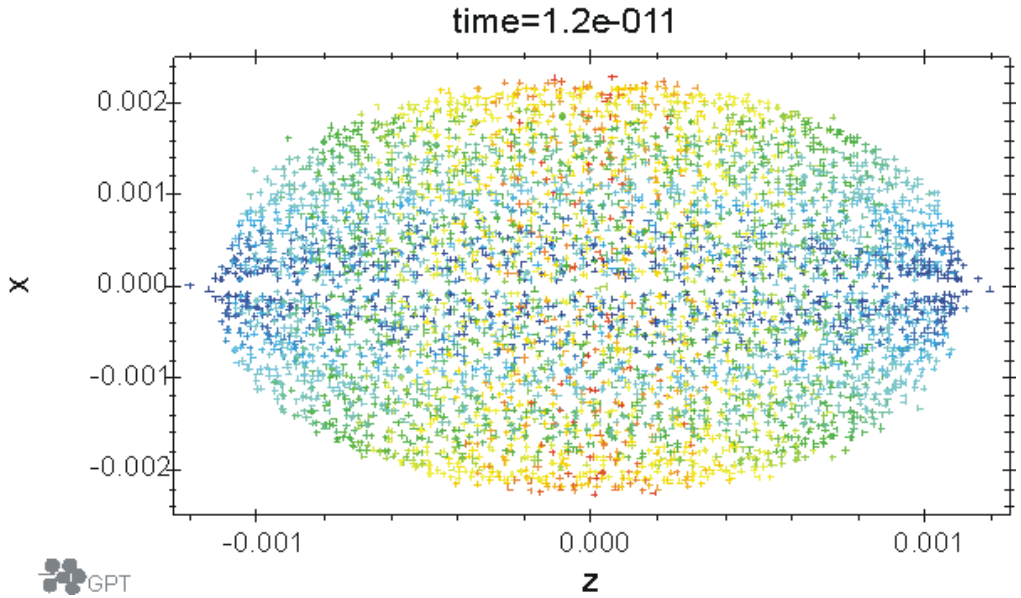
Coulomb explosion of resting electrons of a respective bunch yields:

$$\beta' = 0.7 \;\Rightarrow\; \alpha = \frac{\beta'}{\gamma_0} \approx 0.25$$

$$\Rightarrow \frac{\sqrt{2\alpha}}{\gamma_0} \approx 3\,mrad \quad > \theta$$

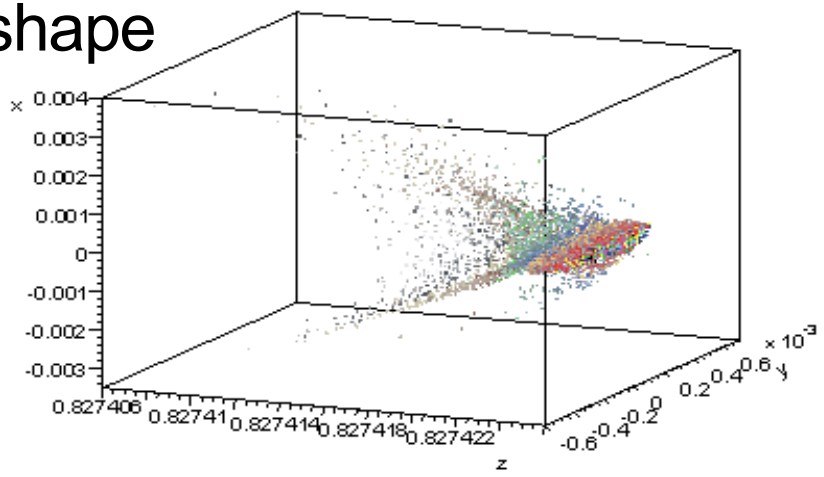
→ This θ is smaller than expected, especially after collimating the beam



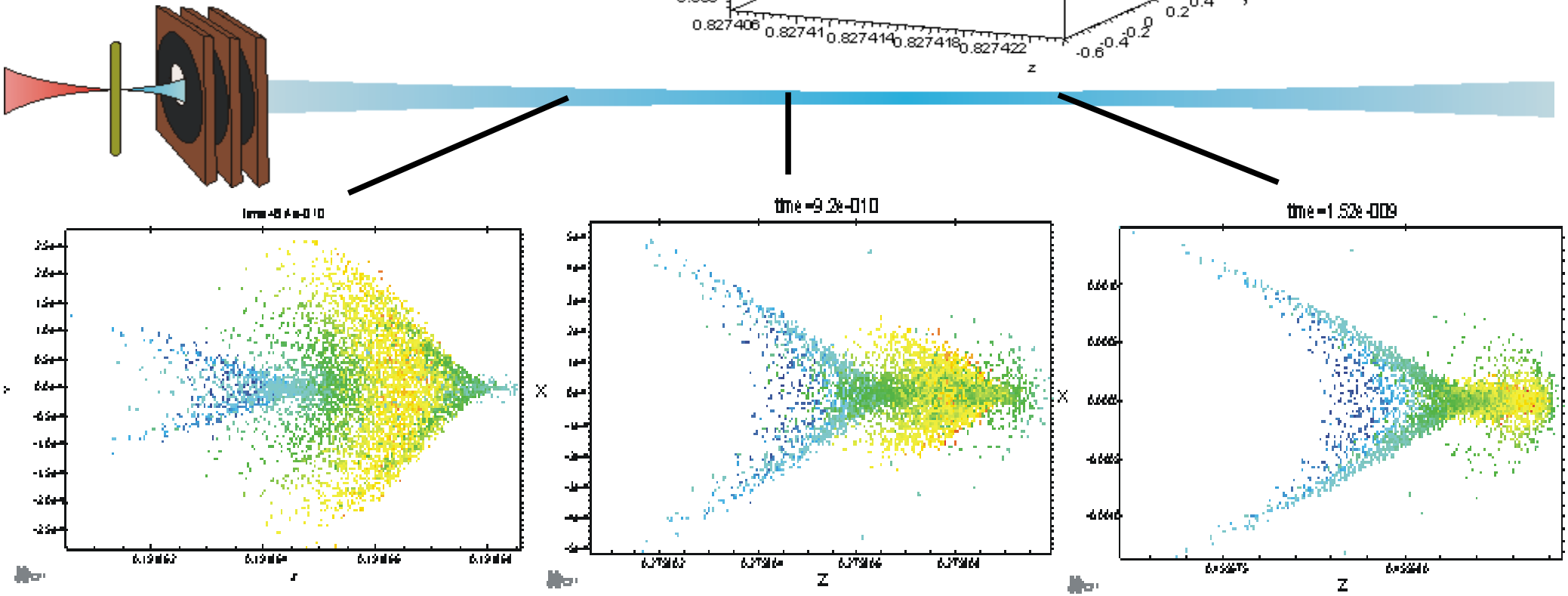


Spatial Distribution

After refocusing, the spacial shape of the bunch reminds of...



a fish...

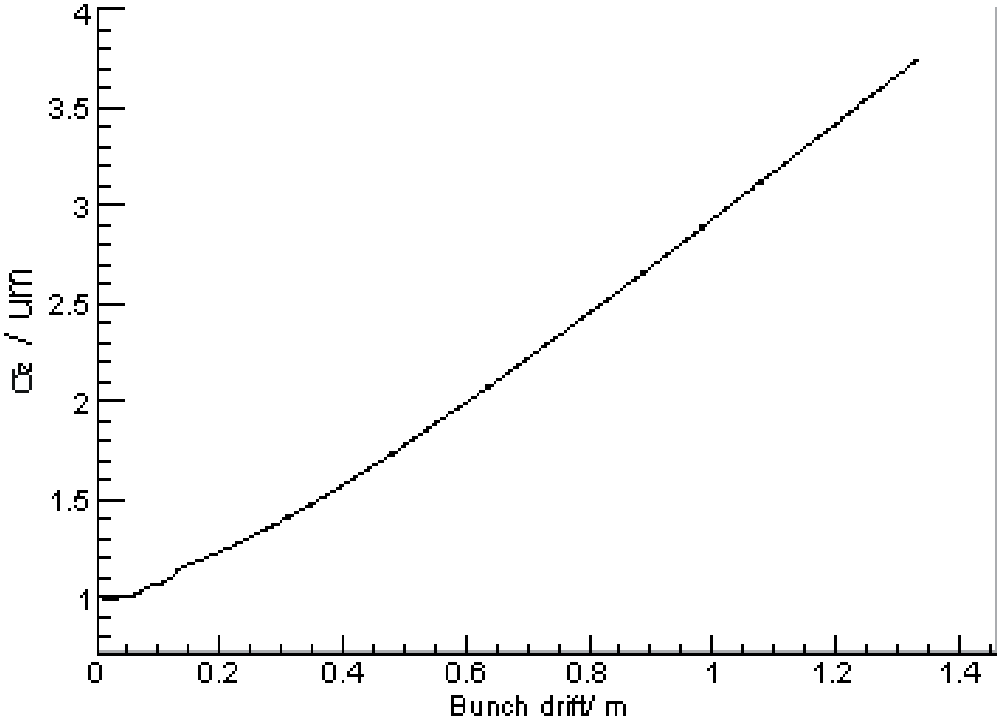


Spatial Distribution

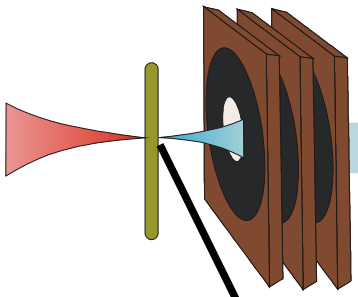
- The presented calculation is far from being optimized.
  The “fish” in this case is not “slim” but rather looks like a “hammerhead shark”.
- Optimization using linear beam optics is not possible, as the beam keeps gaining kinetic energy while propagating though the lens
- Optimization hence is work in progress as being very time consuming

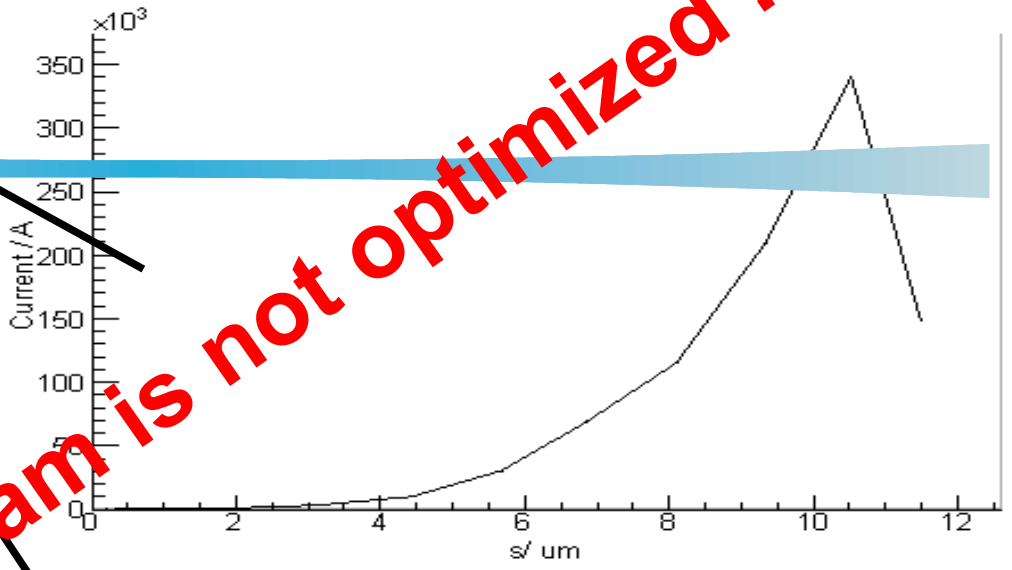
Longitudinal beam prolongation

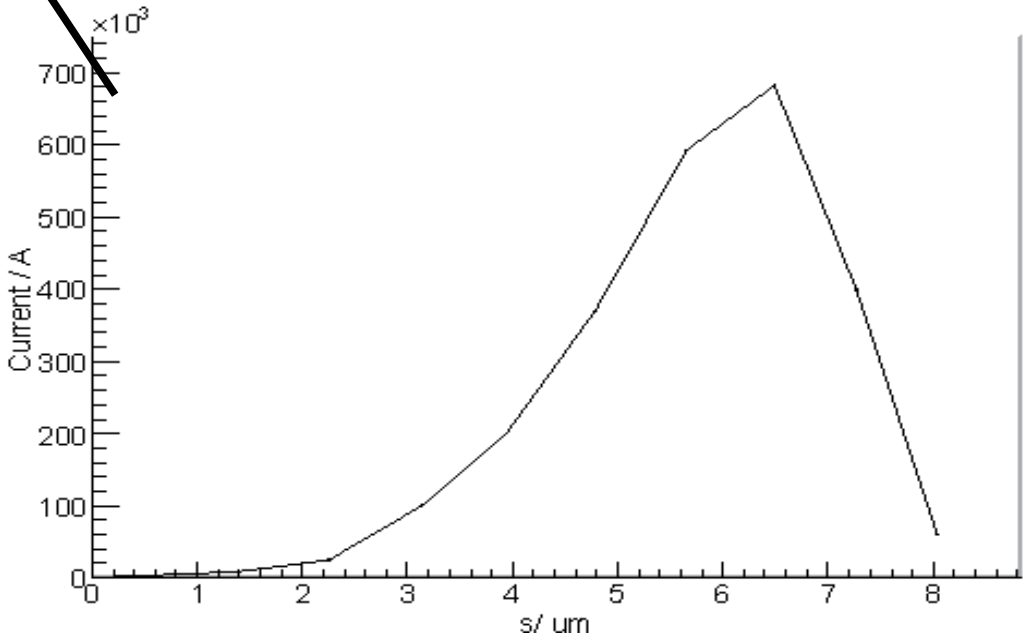
→ **slicing**



Slices

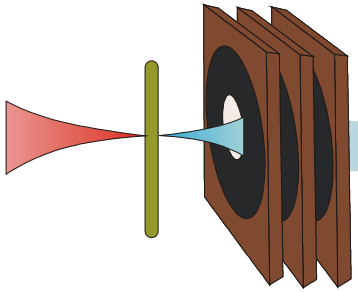


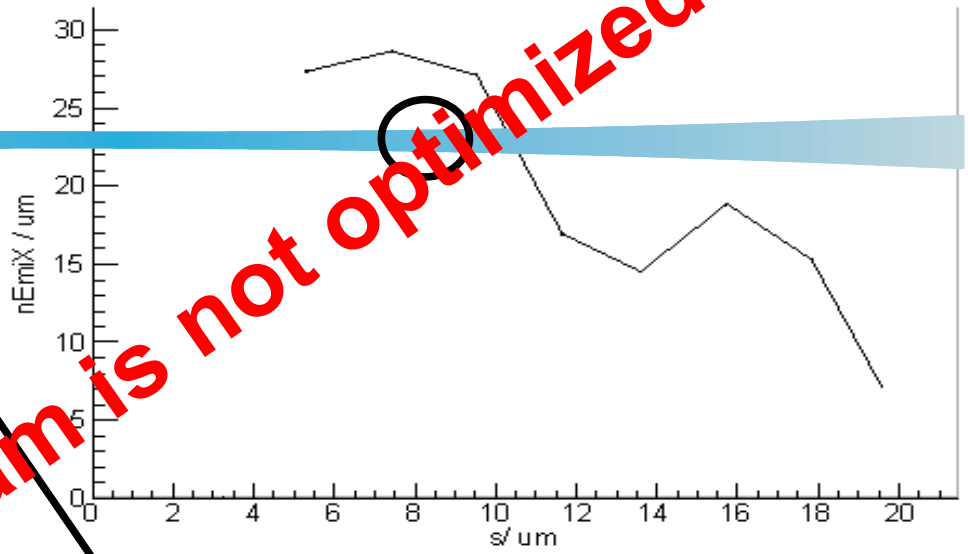


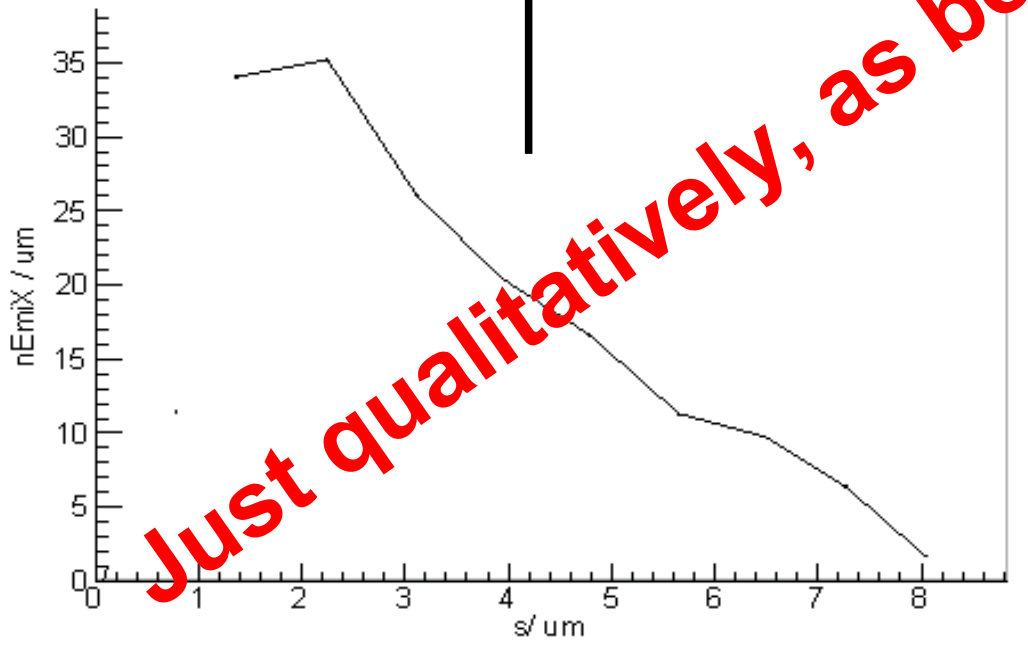


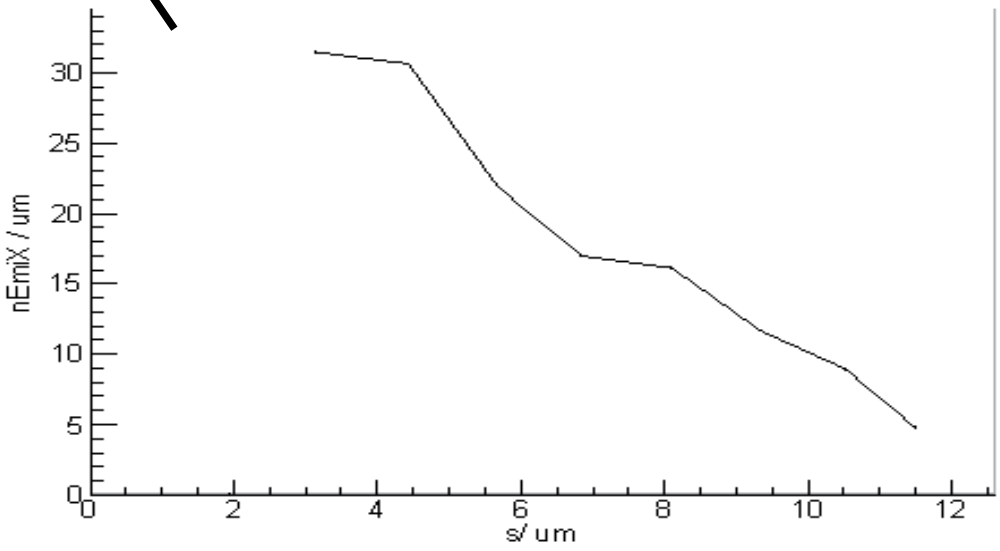
Just qualitatively, as beam is not optimized !!!

Slices









Just qualitatively, as beam is not optimized !!!

Summary

- Self acceleration: Results from CSRtrack vs. GPT vs. HOMDYN
- Self acceleration: Theoretically
- GPT: The applied calculation method
- Longitudinal phase space
- Spatial distribution
- Considering slices - qualitatively