ASTRA with Wakes

M. Dohlus and C. Henning

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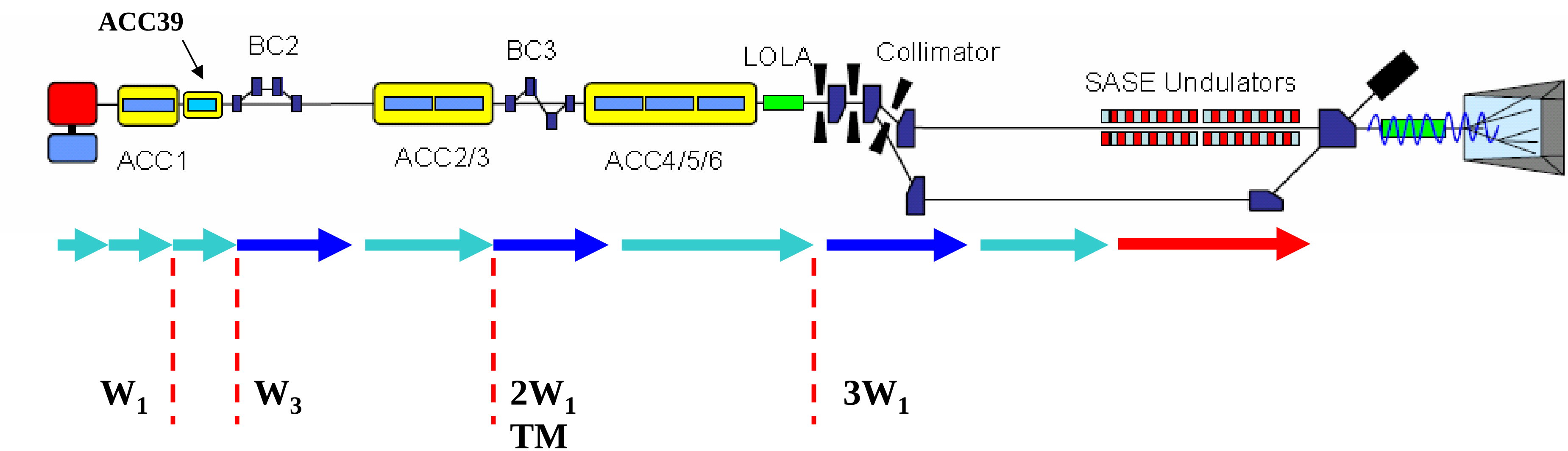
1. Example 1 (Cavity Wakes, Flash)



ASTRA (tracking with space charge, DESY)

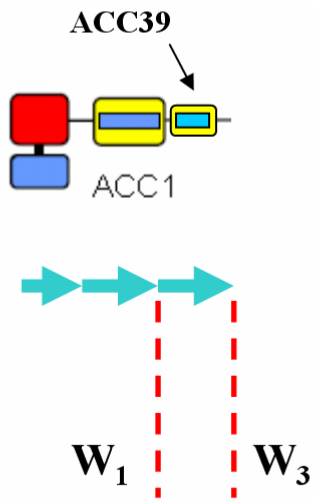
CSRtrack (tracking through dipoles, DESY)

ALICE (3D FEL code, DESY)

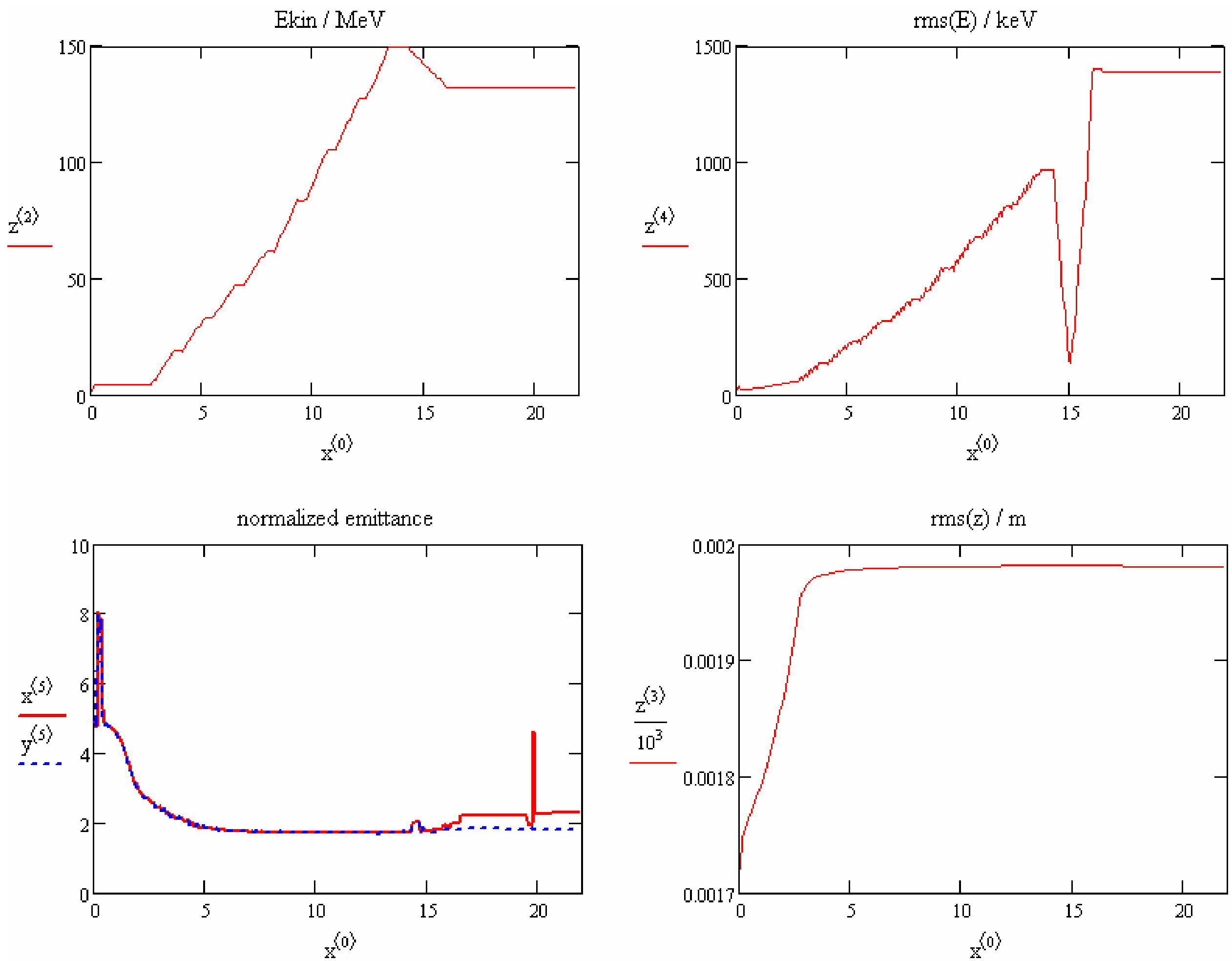
W1 -TESLA cryomodule wake (TESLA Report 2003-19, DESY, 2003)

W3 - ACC39 wake (TESLA Report 2004-01, DESY, 2004)

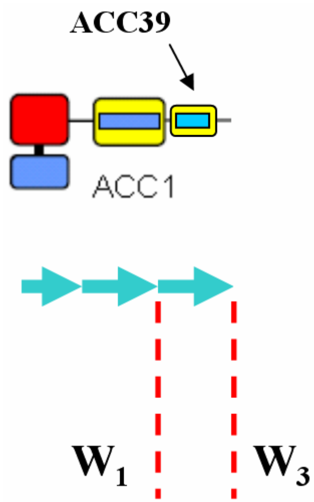
1. Example 1 (Cavity Wakes, Flash)

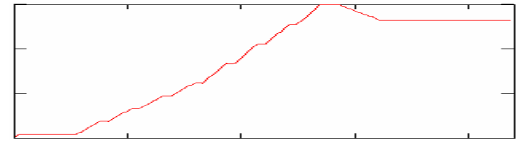


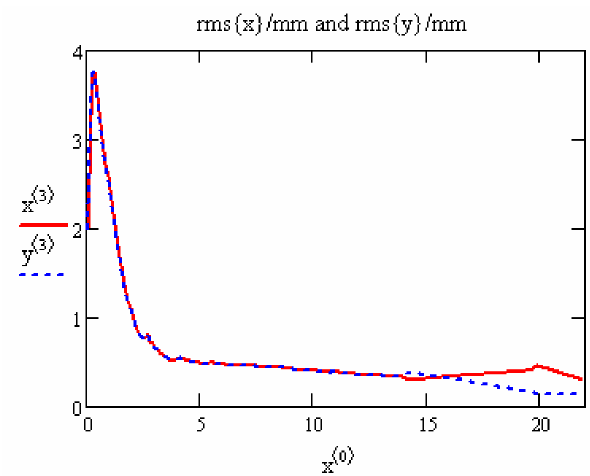
calculation with monopole- und dipole wakes
with 3D cavity fields (including coupler assymmetries)

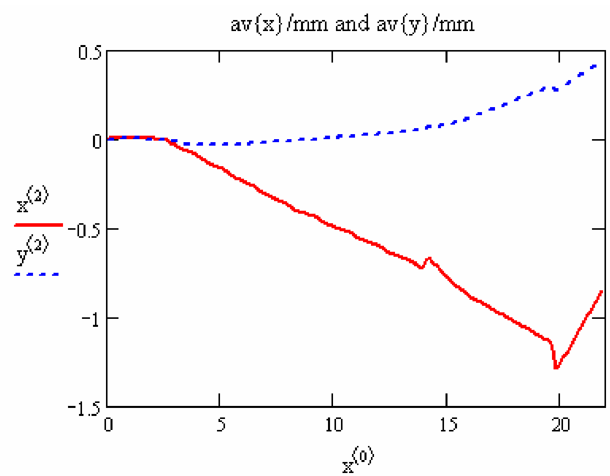


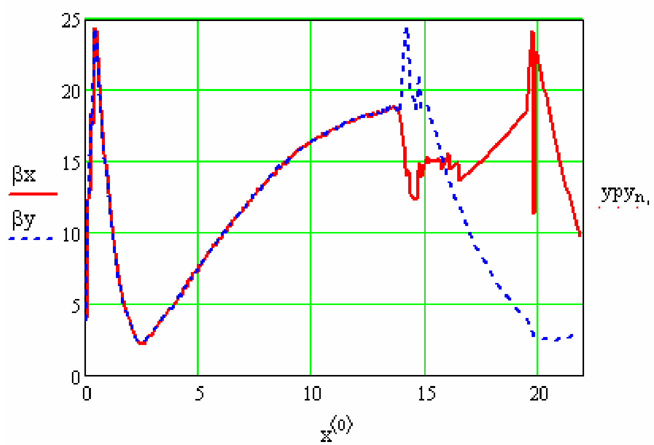
1. Example 1 (Cavity Wakes, Flash)

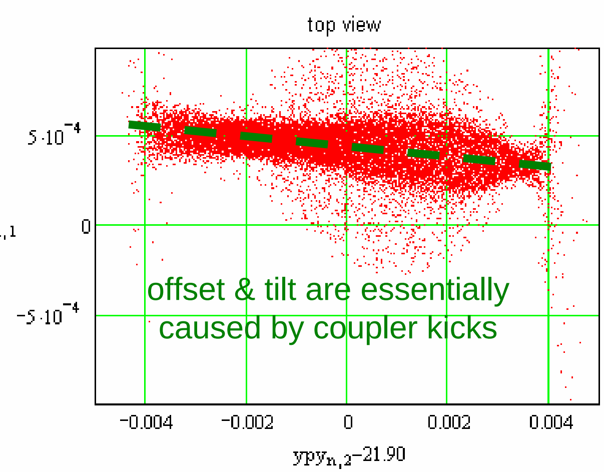




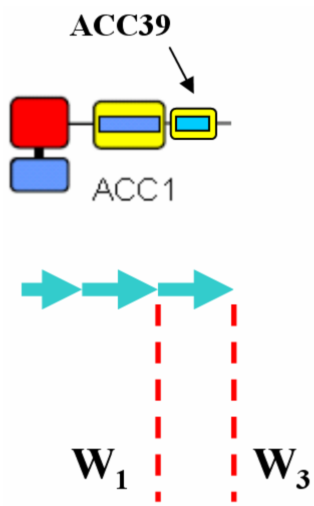


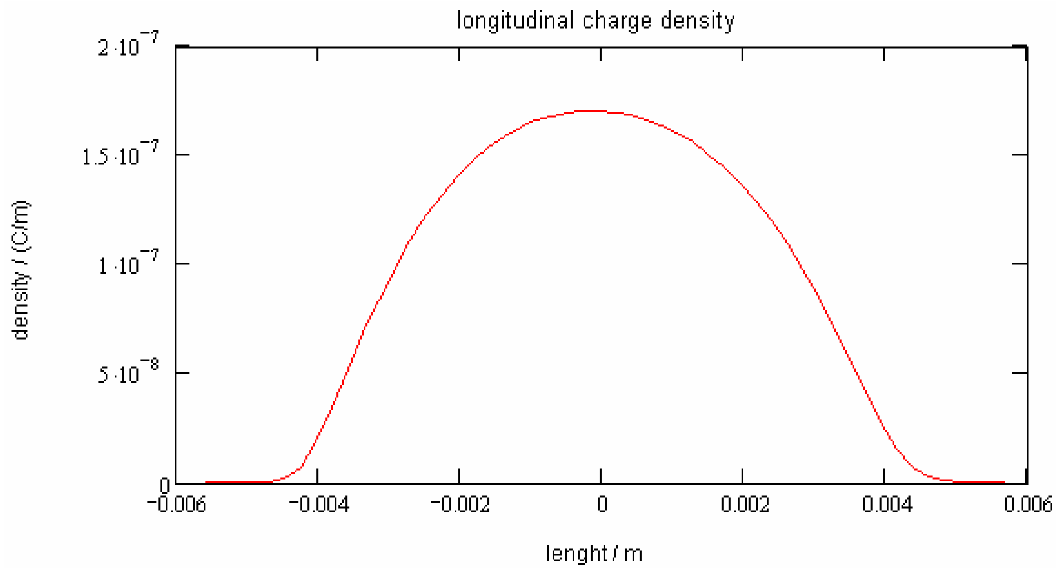


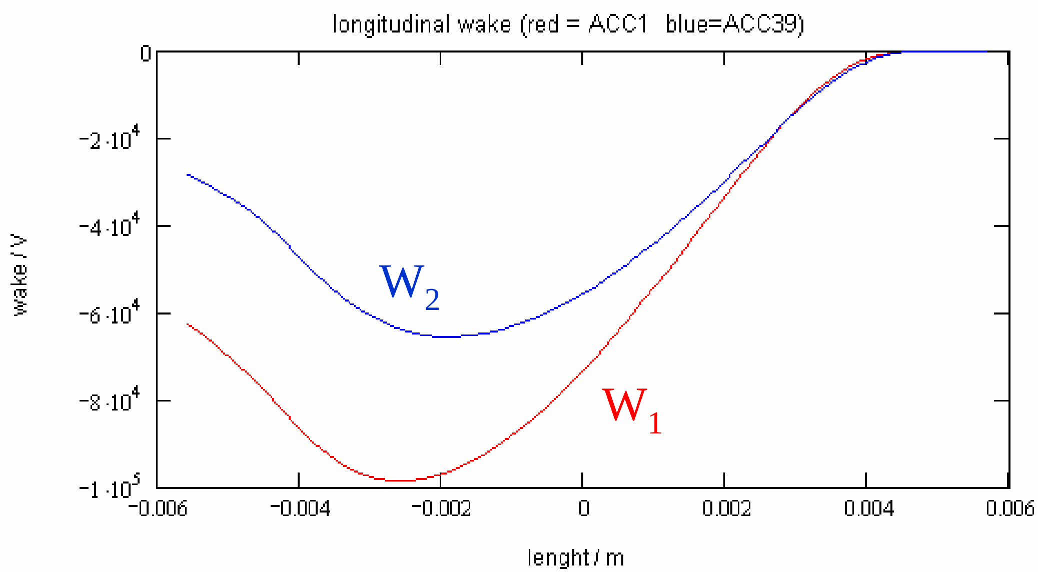




1. Example 1 (Cavity Wakes, Flash)





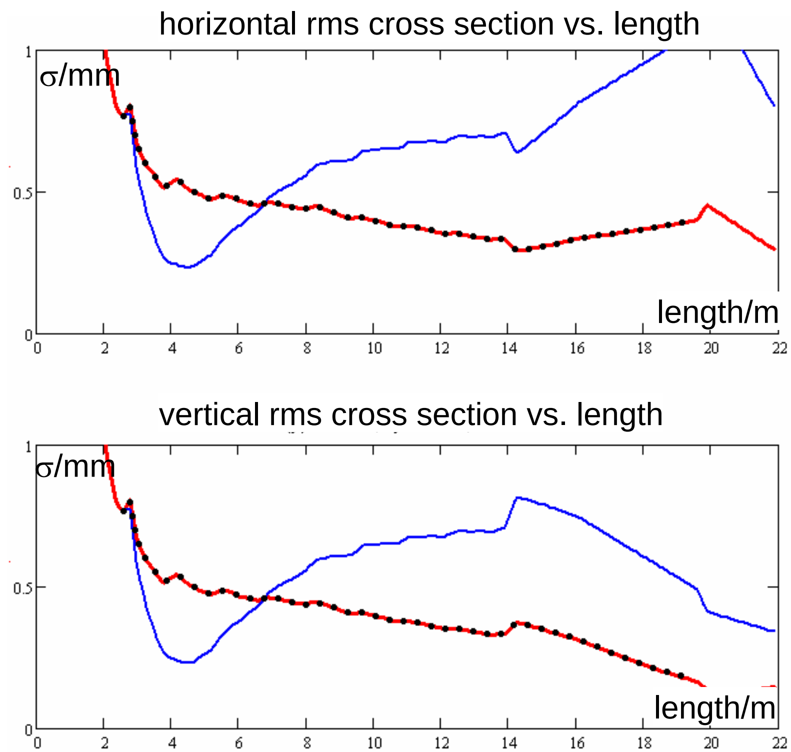


1. Example 1 (Cavity Wakes, Flash)

… but:

3d effects need 3d simulation
more particles needed
mirror charges in rz implemented

to be investigated:



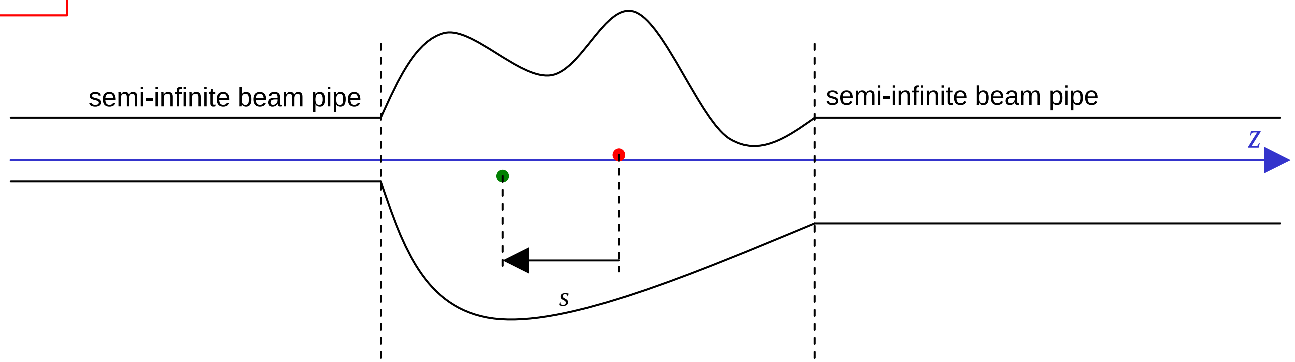
rz run
. . . rz run restarted at z=2.6m
xyz run started at z=2.6m

2. About Wakes

definition of wake function: 2 point particles, "C7 convention"

$$\mathbf{w}_f(u_s, v_s, u_t, v_t, s) = -\frac{c}{q_t q_s} \Delta\mathbf{p}(u_t, v_t, -s) \leftarrow \int_{-\infty}^{\infty} dz (\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

!!!



change of momentum: many particles, in principle

$$\Delta\mathbf{p}(u_t, v_t, w_t) = -\frac{q_t}{c} \sum_n q^{(n)} \mathbf{w}_f(u_n, v_n, u_n, v_t, w_s - w_t)$$

change of momentum – continuous distribution

$$\Delta\mathbf{p}(u_t, v_t, w_t) = -\frac{q_t}{c} \int \rho(u, v, r) \mathbf{w}_f(u, v, u_t, v_t, w_t - r) dudvdr$$

2. About Wakes

representation of wake function:

monopole

$$\mathbf{w}_f(u,v,u_t,v_t,s)=\mathbf{e}_w w_{m,w}(s)$$

dipole

$$\mathbf{w}_f(u,v,u_t,v_t,s)=(u\mathbf{e}_u+v\mathbf{e}_v)w_i(s)+(uu_t+vv_t)\mathbf{e}_w w_{d,w}(s)$$

with $w_i(s)=-\int_{-\infty}^{s} w_{d,w}(s)dx$

Taylor expansion

$$\mathbf{w}_f(u,v,u_t,v_t,s)=\cdots+\mathbf{e}_w w_{t,w}(u,v,u_t,v_t,s)$$

with

$$w_{t,w}(u,v,u_t,v_t,s)\approx w_0(s)+\begin{bmatrix} w_1(s) \\ w_2(s) \\ w_3(s) \\ w_4(s) \end{bmatrix}^t \begin{bmatrix} u \\ v \\ u_t \\ v_t \end{bmatrix}+\begin{bmatrix} u \\ v \\ u_t \\ v_t \end{bmatrix}^t \begin{bmatrix} w_{11}(s) & w_{12}(s) & w_{13}(s) & w_{14}(s) \\ w_{12}(s) & w_{22}(s) & w_{23}(s) & w_{24}(s) \\ w_{13}(s) & w_{23}(s) & w_{33}(s) & w_{34}(s) \\ w_{14}(s) & w_{24}(s) & w_{34}(s) & -w_{33}(s) \end{bmatrix} \begin{bmatrix} u \\ v \\ u_t \\ v_t \end{bmatrix}$$

fulfills longitudinal theorem, transverse components follow from Panofsky Wenzel theorem

special case (monopole + dipole wake): $w_0(s)=w_{m,w}(s)$

$w_{13}(s)=w_{24}(s)=0{,}5w_{d,w}(s)$

all other components vanish

2. About Wakes

transverse theorem (Panofsky Wenzel):

$$\frac{\partial}{\partial s} w_u(u, v, x_t, v_t, s) = -\frac{\partial}{\partial u_t} w_w(u, v, u_t, v_t, s)$$

$$\frac{\partial}{\partial s} w_v(u, v, u_t, v_t, s) = -\frac{\partial}{\partial v_t} w_w(u, v, u_t, v_t, s)$$

longitudinal theorem:

$$\left(\frac{\partial^2}{\partial u_t^2} + \frac{\partial^2}{\partial v_t^2} \right) w_w(u, v, u_t, v_t, s) = 0$$

2. About Wakes

$$\Delta \mathbf{p}(u_t, v_t, w_t) = -\frac{q_t}{c}\int \rho(u,v,r)\,\mathbf{w}_f(u,v,u_t,v_t,w_t - r)\,du\,dv\,dr$$

change of momentum - Taylor expansion

$$\Delta p_x(x_t,y_t,z_t) = -\frac{q_t}{c}\begin{bmatrix} w_{3i}\otimes\lambda + 2w_{13i}\otimes\lambda_x + 2w_{23i}\otimes\lambda_y \\ 2w_{33i}\otimes\lambda \\ 2w_{34i}\otimes\lambda \end{bmatrix}^t \begin{bmatrix} 1 \\ x_t \\ y_t \end{bmatrix} \qquad \Delta p_y(x_t,y_t,z_t) = \cdots$$

$$\Delta p_z(x_t,y_t,z_t) = -\frac{q_t}{c}\begin{bmatrix} w_0\otimes\lambda + w_1\otimes\lambda_x + w_2\otimes\lambda_y + w_{11}\otimes\lambda_{xx} + 2w_{12}\otimes\lambda_{xy} + w_{22}\otimes\lambda_{yy} \\ w_3\otimes\lambda + 2w_{13}\otimes\lambda_x + 2w_{23}\otimes\lambda_y \\ w_4\otimes\lambda + 2w_{14}\otimes\lambda_x + 2w_{24}\otimes\lambda_y \\ 2w_{34}\otimes\lambda \\ w_{33}\otimes\lambda \end{bmatrix}^t \begin{bmatrix} 1 \\ x_t \\ y_t \\ x_t y_t \\ x_t x_t - y_t y_t \end{bmatrix}$$

with $a\otimes b = \int a(z_t)\,b(r+z_t)\,dr$

Taylor coefficients $w_0(s)$, w_1, w_2, w_3, w_4, w_{11}, w_{12}, w_{22}, w_{13}, w_{14}, w_{23}, w_{24}, w_{33}, w_{34}

and 1D distribution functions:

$$\lambda(w) = \sum_n q^{(n)}\delta(w - w_n)$$
$$\lambda_u(w) = \sum_n u_n q^{(n)}\delta(w - w_n)$$
$$\ldots$$

in continuous representation:

$$\lambda(w) = \int \rho(u,v,w)\,du\,dv$$
$$\lambda_u(w) = \int u\,\rho(u,v,w)\,du\,dv$$
$$\ldots$$

→ binning & smoothing

3. ASTRA Input

logical & control

LCSR	[F]	F/T = use_not/use discrete wake kick
wk_screen()	[F]	F/T = write_not/write particle file (after kick)

location and directions

wk_x()	[0]	pointer to origin of wake
wk_y()	[0]	unit = meter
wk_z()	[1]	
wk_ex()	[0]	vector of longitudinal direction
wk_ey()	[0]	(will be normalized internally)
wk_ez()	[1]	
wk_hx()	[1]	vector of horizontal direction
wk_hy()	[0]	(will be normalized internally)
wk_hz()	[0]	

binning and smoothing

wk_equi_grid()	[T]	T/F = binning to equi grid/charge
wk_N_bin()	[10]	number of bins
wk_ip_method()	[2]	interpolation method 0/1/2 = rectangular/triagular/gaussian
wk_smooth()	[0.5]	smoothing parameter (for gaussina interpolation)

wake functions and scaling

wk_type()	[‘undefined’]	type or method of wake calculation (character)
wk_filename()	[‘undefined’]	file name with required information
wk_testfile()	[‘undefined’]	file name for test output; writes test output if filename is defined
wk_scaling()	[1]	scaling factor for wake kick

3. ASTRA Input

logical & control

LCSR	[F]
wk_screen()	[F]

location and directions

wk_x()	[0]
wk_y()	[0]
wk_z()	[1]
wk_ex()	[0]
wk_ey()	[0]
wk_ez()	[1]
wk_hx()	[1]
wk_hy()	[0]
wk_hz()	[0]

defines plane of where discrete wake kick is applied and coordinate transformation

$$\begin{pmatrix}\mathbf{e}_x & \mathbf{e}_y & \mathbf{e}_z\end{pmatrix}\begin{pmatrix}x_n\\ y_n\\ z_n\end{pmatrix} = \mathbf{r}_p + \begin{pmatrix}\mathbf{e}_u & \mathbf{e}_v & \mathbf{e}_w\end{pmatrix}\begin{pmatrix}u_n\\ v_n\\ w_n\end{pmatrix}$$

$$\begin{pmatrix}\mathbf{e}_x & \mathbf{e}_y & \mathbf{e}_z\end{pmatrix}\begin{pmatrix}p_{x,n}\\ p_{y,n}\\ p_{z,n}\end{pmatrix} = \begin{pmatrix}\mathbf{e}_u & \mathbf{e}_v & \mathbf{e}_w\end{pmatrix}\begin{pmatrix}p_{u,n}\\ p_{v,n}\\ p_{w,n}\end{pmatrix}$$

(wake is calculated with respect to uvw origin)

binning and smoothing

wk_equi_grid()	[T]
wk_N_bin()	[10]
wk_ip_method()	[2]
wk_smooth()	[0.5]

binning and smoothing parameters

wake functions and scaling

wk_type()	['undefined']
wk_filename()	['undefined']
wk_testfile()	['undefined']
wk_scaling()	[1]

type or method of wake calculation (monopole/dipole/taylor) and file with wake coefficient functions

3. ASTRA Input

coefficient functions:

only longitudinal coefficients (w-component) have to be specified,
transverse coefficients follow from Panofsky Wenzel theorem

We follow the proposal of Igor Zagorodnov to describe each coefficient function by an expression of the following type:

$$w_i(s) = w_i^{(0)}(s) + \frac{\Phi(s)}{C_i} + R_i c\delta(s) - c\frac{\partial}{\partial s}\left[L_i c\delta(s) + w_i^{(-1)}(s)\right]$$

or:

$$w_{ij}(s) = w_{ij}^{(0)}(s) + \cdots$$

Each coefficient function is defined by three network parameters R, L, C and by two functions $w^{(0)}(s)$ and $w^{(-1)}(s)$ that are all together described in a single table:

table$_i$ or table$_{ij}$ =

N_0	N_1
R	L
$\tilde{C}$	i or $i+10j$
$s_1^{(0)}$	$w^{(0)}\left(s_1^{(0)}\right)$
$s_2^{(0)}$	$w^{(0)}\left(s_2^{(0)}\right)$
$\vdots$	$\vdots$
$s_{N_0}^{(0)}$	$w^{(0)}\left(s_{N_0}^{(0)}\right)$
$s_1^{(-1)}$	$w^{(-1)}\left(s_1^{(-1)}\right)$
$s_2^{(-1)}$	$w^{(-1)}\left(s_2^{(-1)}\right)$
$\vdots$	$\vdots$
$s_{N_1}^{(-1)}$	$w^{(-1)}\left(s_{N_1}^{(-1)}\right)$

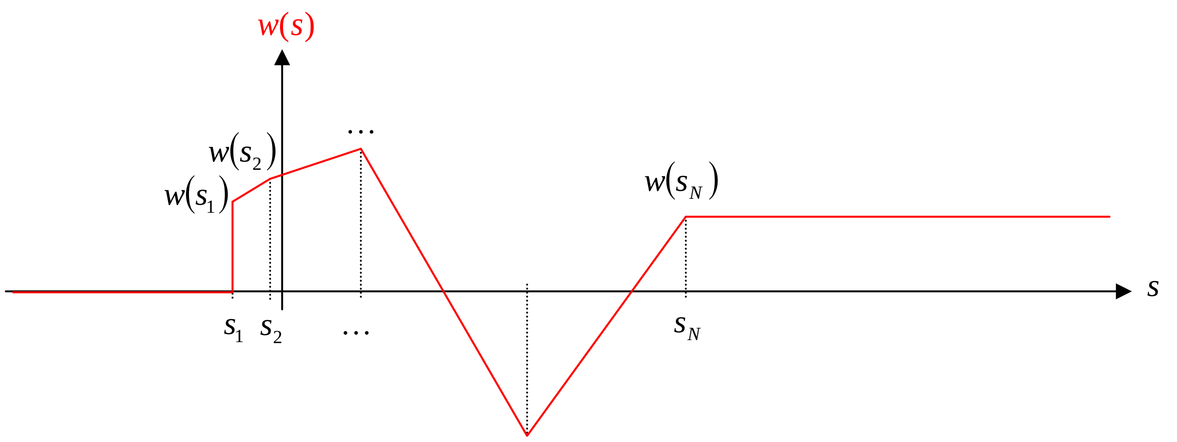
N_0 or N_1 or both may be zero

3. ASTRA Input

description of wake functions, “C7 convention”

capacitive coefficient $\frac{1}{C} = \begin{cases} 1/\tilde{C} & \text{if } \tilde{C} > 0 \\ 0 & \text{otherwise} \end{cases}$

table description of $w^{(0)}(s)$ and $w^{(-1)}(s)$:



with $s_1 < s_2 < \cdots < s_N$

monopole wake function

$\mathbf{w}_f(u, v, u_t, v_t, s) = \mathbf{e}_w w_{m,w}(s)$

wk_type() = **monopole_method_f**
wk_filename() = **filename**

3. ASTRA Input

dipole wake function

$$\mathbf{w}_f(u,v,u_t,v_t,s) = (u\mathbf{e}_u + v\mathbf{e}_v)w_i(s) + (uu_t + vv_t)\mathbf{e}_w w_{d,w}(s)$$

wk_type() = **dipole_method_f**
wk_filename() = **filename**

filename describes $w_{d,w}(s)$
the transverse wake is calculated with the Panofsky-Wenzel theorem

Taylor expansion of wake function

wk_type() = **taylor_method_f**
wk_filename() = **filename**

file describes a “multi-table” with up to 14 coefficient functions:

w_0, w_1, w_2, w_3, w_4
$w_{11}, w_{12}, w_{22}, w_{13}, w_{14}, w_{23}, w_{24}, w_{33}, w_{34}$

format:

K	0
table 1	
table 2	
…	
table K	

K is the number of non vanishing coefficient functions
(vanishing coefficients need no sub-tables)
the order of sub-tables is arbitrary

3. ASTRA Input

other formats:

monopole wake potential

wk_type() = **monopole_method_p**
wk_filename() = **filename**

dipole wake potential

wk_type() = **dipole_method_p**
wk_filename() = **filename**

4. Some Wake Files

(A) Cavities

TESLA 2004-01 (659KB)

Wake Fields Generated by the LOLA-IV Structure and the 3rd Harmonic Section in TTF-II

Igor Zagorodnov, Thomas Weiland - TU Darmstadt;
Martin Dohlus - DESY

TESLA 2003-19 (374KB)

The Short-Range Transverse Wake Function for TESLA Accelerating Structure

Thomas Weiland, I. Zagorodnov - TEMF, TU Darmstadt

The TESLA cavity is a main element of the LINAC and it is reasonable to compare the obtained wakes to ones of the TESLA cryomodule [8]: $L_a = 8.288[m/module]$

$$w_{\parallel}^{cryo}(s) = -\theta(s)\cdot 41.5 e^{-\sqrt{\frac{s}{1.74\cdot 10^{-3}}}} \quad \left[\frac{V}{pC\cdot m}\right],$$

$$w_{\perp}^{cryo}(s) = \theta(s)\left[121\left(1-\left(1+\sqrt{\frac{s}{0.92\cdot 10^{-3}}}\right)e^{-\sqrt{\frac{s}{0.92\cdot 10^{-3}}}}\right)\right] \quad \left[\frac{V}{pC\cdot m\cdot m}\right].$$

The active length of the LOLA structure is $L_{total} = 3.64\ m$. And the normalized short range wake functions of the LOLA read

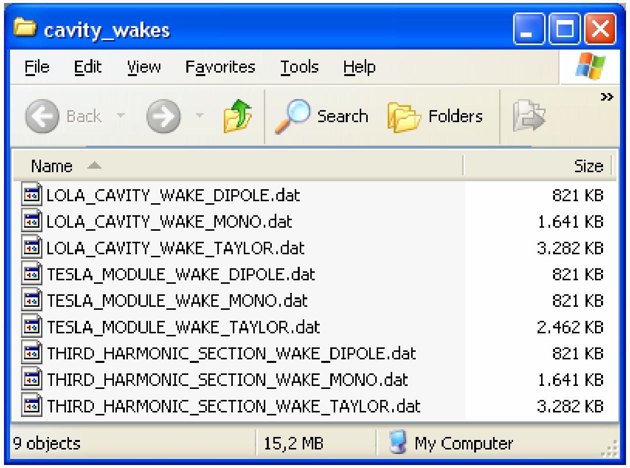
$$w_{\parallel}^{LOLA}(s) = -\theta(s)\left[70.8 e^{-\sqrt{\frac{s}{3.96\cdot 10^{-3}}}} + 0.32\frac{\cos(1760 s^{0.72})}{\sqrt{s}+1600 s^{1.23}}\right] \quad \left[\frac{V}{pC\cdot m}\right],$$

$$w_{\perp}^{LOLA}(s) = \theta(s)\left[2804\left(1-\left(1+\sqrt{\frac{s}{11.7\cdot 10^{-3}}}\right)e^{-\sqrt{\frac{s}{11.7\cdot 10^{-3}}}}\right)+2530\sqrt{s}\right] \quad \left[\frac{V}{pC\cdot m\cdot m}\right].$$

The active length of the 3rd harmonic section is $L_{total} = 36\cdot 0.03844 = 1.3838\ m$. And the normalized short range wake functions of the section read

$$w_{\parallel}^{3rd}(s) = -\theta(s)\left[230 e^{-\sqrt{\frac{s}{8.4\cdot 10^{-4}}}} + 0.65\frac{\cos(5830 s^{0.83})}{\sqrt{s}+195 s} + 0.026\delta(s)\right]\left[\frac{V}{pC\cdot m}\right],$$

$$w_{\perp}^{3rd}(s) = \theta(s)\left[1612\left(1-\left(1+\sqrt{\frac{s}{0.56\cdot 10^{-3}}}\right)e^{-\sqrt{\frac{s}{0.56\cdot 10^{-3}}}}\right)+3932\sqrt{s}+64\right] \quad \left[\frac{V}{pC\cdot m\cdot m}\right].$$



…_MONO = monopole wake
…_DIPOLE = dipole wake
…_TAYLOR = monopole & dipole wake (together)

4. Some Wake Files

(B) resistive walls (round pipes)

monopole- and dipole impedance functions **(per length)** and wakes **(per length)**

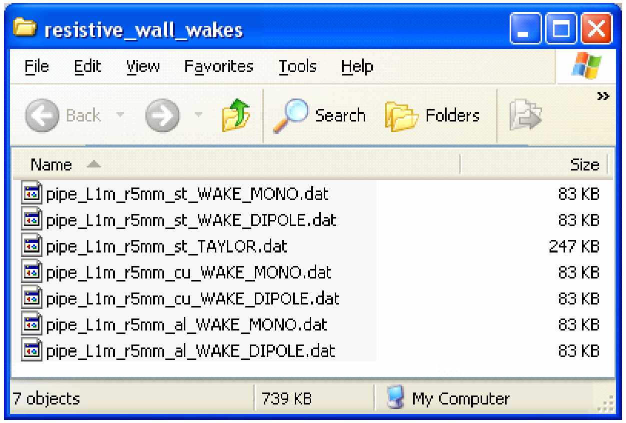
$$\mathbf{Z}^{(m)}(x,y,x_t,y_t,\omega)=\frac{Z_s}{2\pi R}\frac{1}{1+jk\frac{R}{2}\frac{Z_s}{Z_0}}\begin{pmatrix}0\\0\\1\end{pmatrix}$$

$$\mathbf{w}^{(m)}(x,y,x_t,y_t,s)=\begin{pmatrix}0\\0\\w_r(s)\end{pmatrix}$$

$$\mathbf{Z}^{(d)}(x,y,x_t,y_t,\omega)\approx\frac{Z_s}{2\pi R}\frac{2}{R^2}\frac{1}{1+jk\frac{R}{2}\frac{Z_s}{Z_0}}\begin{pmatrix}jx/k\\jy/k\\xx_t+yy_t\end{pmatrix}$$

$$\mathbf{w}^{(d)}(x,y,x_t,y_t,s)\approx\begin{pmatrix}\cdots\\\cdots\\\frac{2w_r(s)}{R^2}(xx_t+yy_t)\end{pmatrix}$$

for usual beam parameters (not ultra-long bunches)



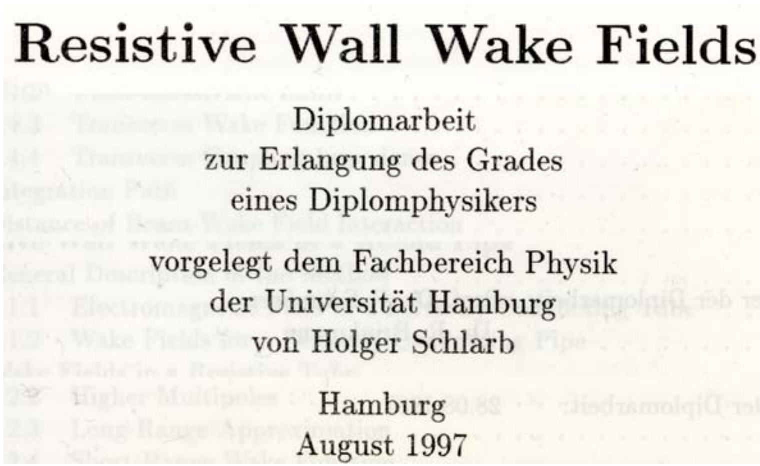
wakes for 1m beam pipe with 5mm radius for Al, Cu and steel (for frequency dependant conductivity)

use wk_scaling() for different length

5. Example 2 (Resistive Wakes {per length}, Undulator)

emittance growth in a FLASH-like undulator due to resistive wall wakes:

Holger's DA:

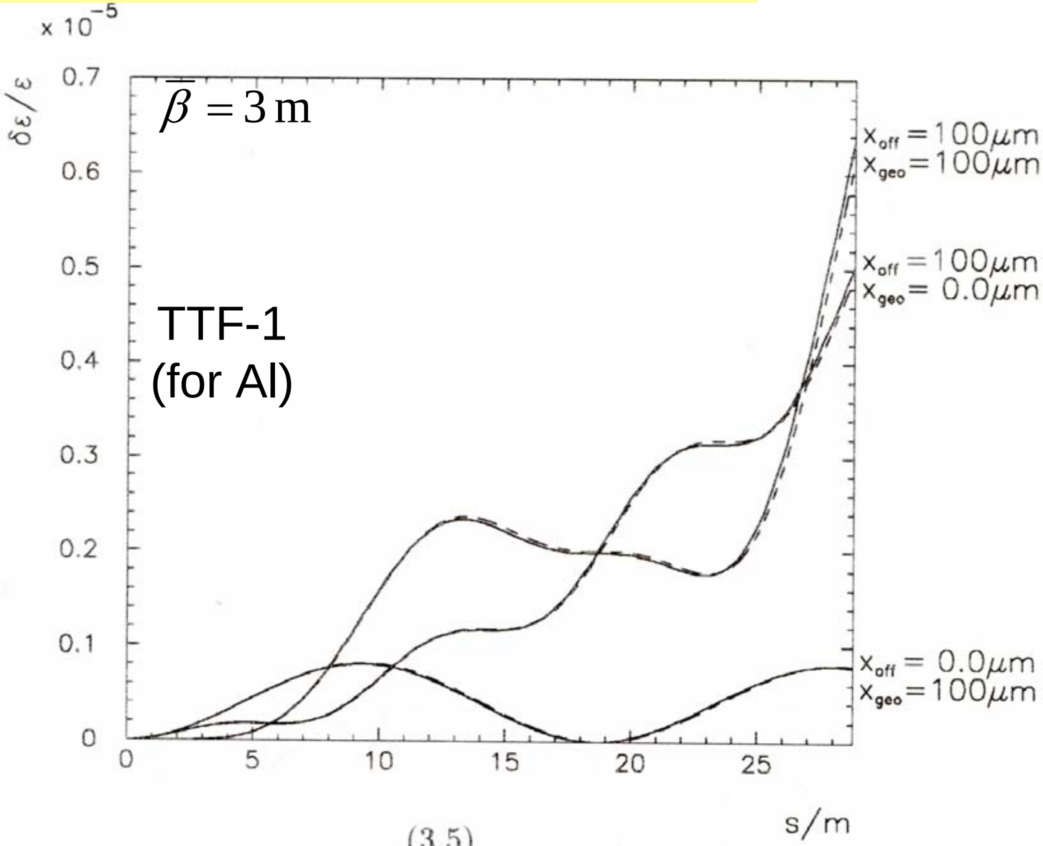


Resistive Wall Wake Fields

Diplomarbeit
zur Erlangung des Grades
eines Diplomphysikers

vorgelegt dem Fachbereich Physik
der Universität Hamburg
von Holger Schlarb

Hamburg
August 1997



analytic estimation:

$$\frac{\delta\epsilon_x(s)}{\epsilon_x} \approx \frac{1}{2\epsilon_x}\cdot\left(\frac{Ne^2}{E_0}\right)^2\cdot\left(<W_\perp^{\lambda 2}>_\lambda - k_\perp^2\right) \tag{3.5}$$

$$\times \left\{\left(\frac{x_{off}\sqrt{\overline{\beta}}}{2}\, s\,\sin\left(k_{\overline{\beta}}\, s\right) - x_{geo}\,\overline{\beta}^{3/2}\left[1 - \cos\left(k_{\overline{\beta}}\, s\right)\right]\right)^2\right.$$

$$\left. + \left(\frac{x_{off}\sqrt{\overline{\beta}}}{2}\left[s\,\cos\left(k_{\overline{\beta}}\, s\right) + \overline{\beta}\sin\left(k_{\overline{\beta}}\, s\right)\right] - x_{geo}\,\overline{\beta}^{3/2}\sin\left(k_{\overline{\beta}}\, s\right)\right)^2\right\}.$$

5. Example 2 (Resistive Wakes {per length}, Undulator)

ASTRA input:

```
&NEWRUN
Version=2
Head='FODO LP=0.96m beta_av=3m'
Distribution=particles.ini
...
/

&OUTPUT
ZSTART          = 0.000
ZSTOP           = 30.000
...
/
```

30 m undulator

```
&SCAN
/

&MODULES
/

&ERROR
/

&CHARGE
LSPCH           =F
...
/
```

no space charge forces

5. Example 2 (Resistive Wakes {per length}, Undulator)

ASTRA input:

```
&CSR
LCSR=T
WK_X        (1)=0.001
WK_Z        (1)=0.47
WK_EQUI_GRID(1)=T
WK_N_BIN    (1)=50
WK_TYPE     (1)='taylor_method_f'
WK_IP_METHOD(1)=2
WK_SMOOTH   (1)=1.0
WK_FILENAME (1)='pipe_L1m_r5mm_st_TAYLOR.dat'
WK_SCALING  (1)=0.48
WK_SCREEN   (1)=T

 • • •

WK_X        (62)=0.001
WK_Z        (62)=29.75
WK_EQUI_GRID(62)=T
WK_N_BIN    (62)=50
WK_TYPE     (62)='taylor_method_f'
WK_IP_METHOD(62)=2
WK_SMOOTH   (62)=1.0
WK_FILENAME (62)='pipe_L1m_r5mm_st_TAYLOR.dat'
WK_SCALING  (62)=0.48
/
```

name list “CSR” for wakes!

62 x wake elements in half-fodo cells

1 mm offset

R = 5 mm beam pipe from steel

5. Example 2 (Resistive Wakes {per length}, Undulator)

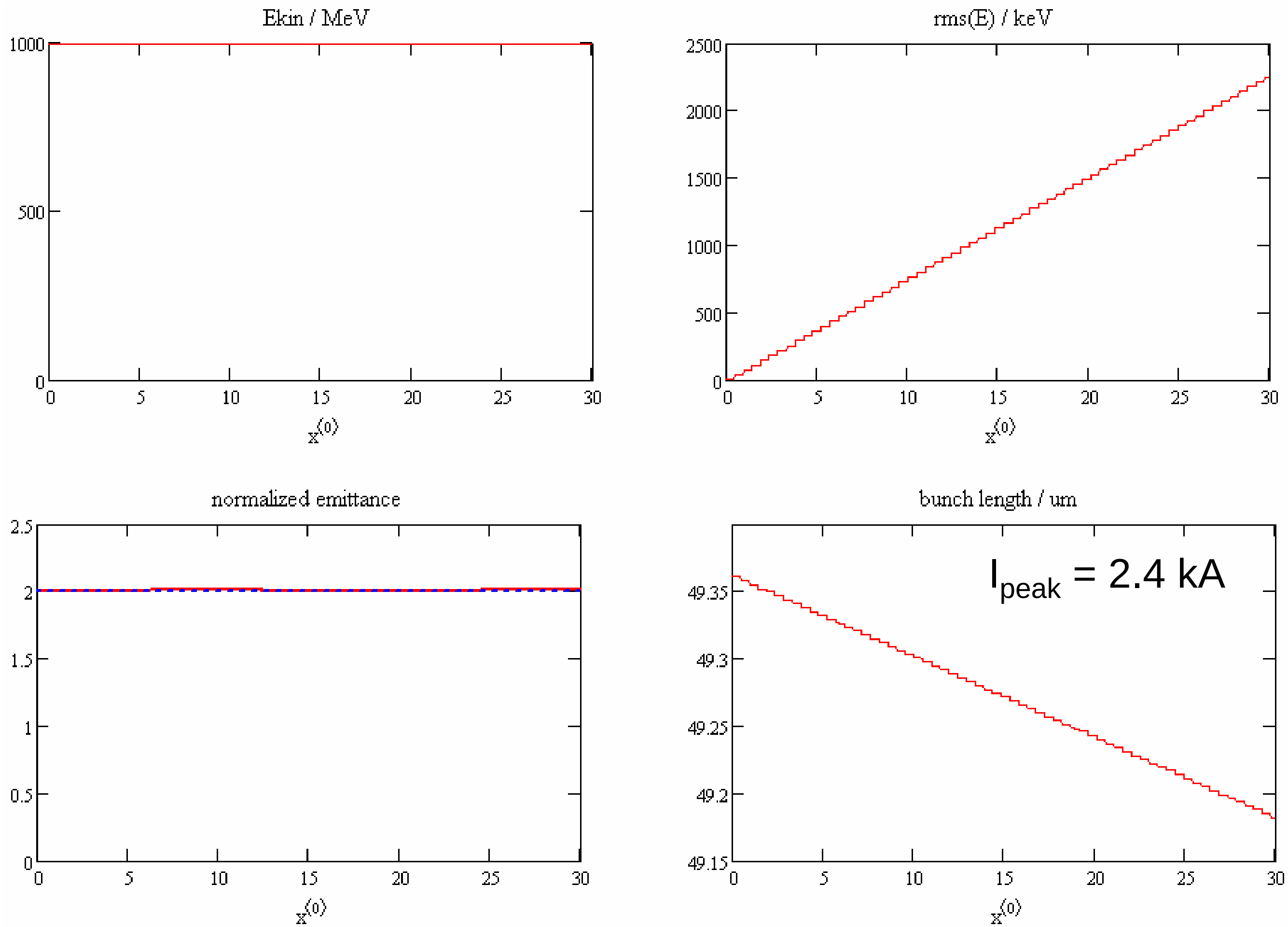
ASTRA input:

```
&APERTURE
/

&CAVITY
/

&SOLENOID
/

&QUADRUPOLE
LQUAD          =T

Q_length(1)=0.1365
Q_K(1)     =5.625472
! Q_Bore(1)  =0.0000001
Q_pos(1)   =0.24

Q_length(2)=0.1365
Q_K(2)     =-5.625472
! Q_Bore(2)  =0.0000001
Q_pos(2)   =0.72

• • •

/

&DIPOLE
/
```

fodo lattice with
Lp=0.9600m, Lq=0.1365m, kq=5.625472m^{-2}
(av. beta function is $\approx$ 3.0m)

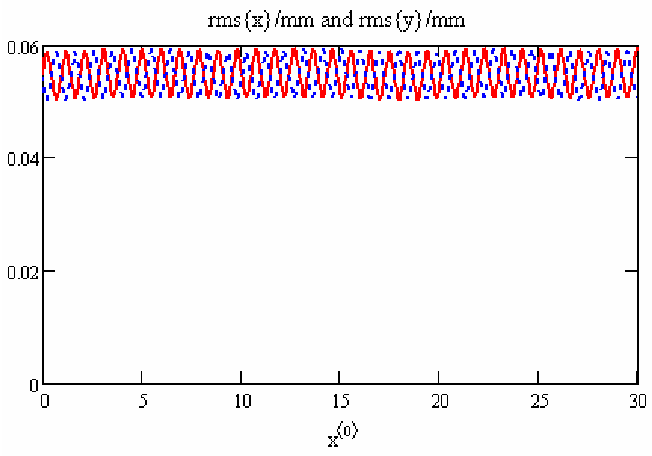
5. Example 2 (Resistive Wakes {per length}, Undulator)

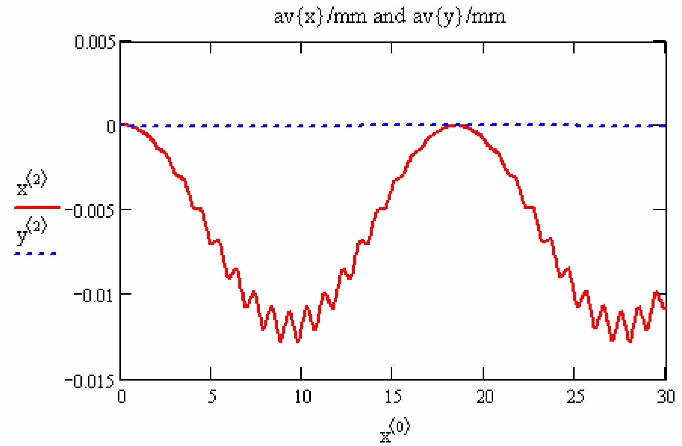
some results:

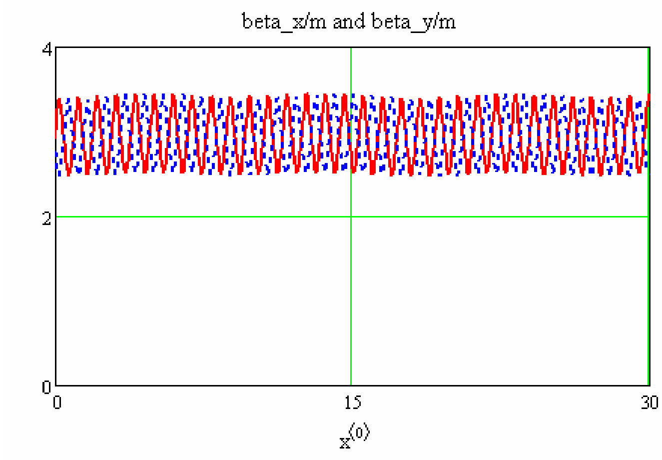


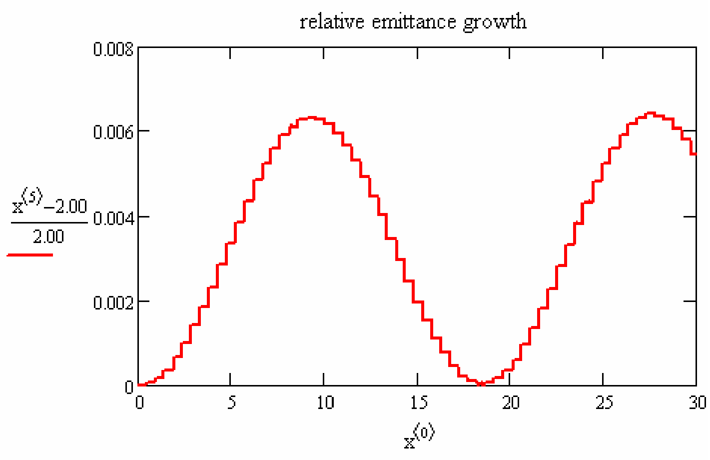
5. Example 2 (Resistive Wakes {per length}, Undulator)

some results:









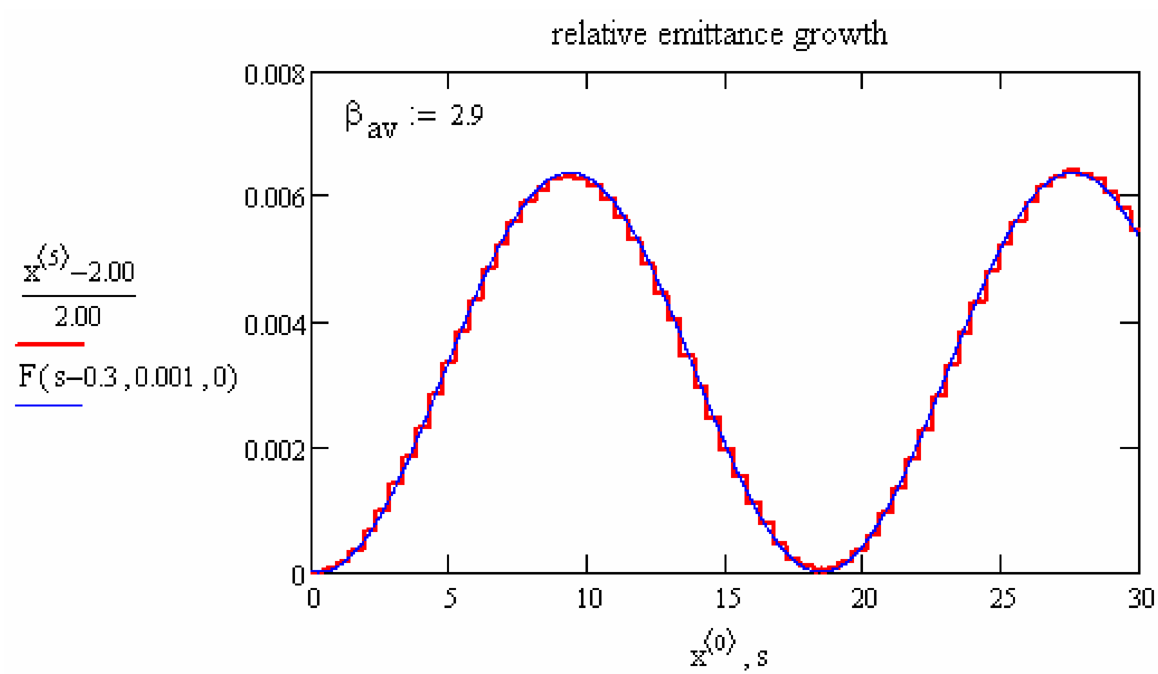
5. Example 2 (Resistive Wakes {per length}, Undulator)

comparison with analytic estimation for averaged beta function:

$$fact := \frac{1}{2\cdot\varepsilon x}\cdot\left(\frac{q_b}{E0}\right)^2\cdot\left(\frac{1}{q}\cdot rms(s,\lambda,TWake\_st\_tr)\right)^2$$

$$fact = 65.371$$

$$F(s,x_{geo},x_{off}) := fact\cdot\left[\left[\frac{x_{off}\cdot\sqrt{\beta_{av}}}{2}\cdot s\cdot\sin\left(\frac{s}{\beta_{av}}\right) - x_{geo}\cdot\beta_{av}^{\frac{3}{2}}\cdot\left(1-\cos\left(\frac{s}{\beta_{av}}\right)\right)\right]^2 + \left[\frac{x_{off}\cdot\sqrt{\beta_{av}}}{2}\cdot\left(s\cdot\cos\left(\frac{s}{\beta_{av}}\right)+\beta_{av}\cdot\sin\left(\frac{s}{\beta_{av}}\right)\right) - x_{geo}\cdot\beta_{av}^{\frac{3}{2}}\cdot\sin\left(\frac{s}{\beta_{av}}\right)\right]^2\right]$$



6. More ?!

to be done: wakes per length

projected CSR in ASTRA – a second attempt?

MATLAB tool in preparation:

test binning and smoothing

use (and test) wake files without ASTRA