Gaussian Laser Mode (TEM00)

electrical field:

$$\mathbf{E}(r,z) = E_{x0}\mathbf{e}_x \frac{\exp\left(-\left(\frac{r}{\hat{\sigma}}\right)^2 \frac{1}{1+i\,z/z_R}\right)}{1+i\,z/z_R}\exp\left(i\left(kz-\omega t\right)\right)$$ with $$z_R = \frac{k\hat{\sigma}^2}{2}$$

Rayleigh length

RMS intensity:

$$\left\|\mathbf{E}(r,0)\right\|^2 = E_{x0}^2 \exp\left(-2\left(\frac{r}{\hat{\sigma}}\right)^2\right) = E_{x0}^2 \exp\left(-\frac{1}{2}\left(\frac{r}{\sigma_I}\right)^2\right)$$

therefore: $\sigma_I = \hat{\sigma}/2$

$z_R = 2k\sigma_I^2$

Interaction in Undulator (1 electron)

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \approx \begin{pmatrix} x_0 \\ y_0 \\ 0 \end{pmatrix} + \begin{pmatrix} x_0' \\ y_0' \\ 1 \end{pmatrix} \bar{v}(t-t_0) + \begin{pmatrix} \hat{x}\sin\left(k_u \bar{v}(t-t_0)\right) \\ 0 \\ -\hat{z}\sin\left(2k_u \bar{v}(t-t_0)\right) \end{pmatrix}$$

with $z_a \le z \le z_a + L$

$$z_a = \frac{\lambda_u}{4} \qquad L = N\lambda_u$$

$$z_m = (z_a + z_b)/2$$

$$\delta \mathrm{E} = e\int_{t_a}^{t_b} E_x(x,y,z,t)v_x(t)dt \approx e\frac{\hat{K}}{2\gamma}\int_{z_a}^{z_b} E_x(x_0 + x_0'z, y_0 + y_0' - z, z)dz \cdot \cos(\omega t_0)$$

with $E_x(x,y,z) = E_{x0}\dfrac{\exp\left(-\dfrac{x^2+y^2}{\hat{\sigma}^2}\dfrac{1}{1+i(z-z_m)/z_R}\right)}{1+i(z-z_m)/z_R}$

K undulator parameter

$$\hat{K} = K\left[J_0\left(\frac{K^2}{4+2K^2}\right) - J_1\left(\frac{K^2}{4+2K^2}\right)\right]$$

Interaction in Undulator (1 electron)

$L << z_R$

$$E_x(x,y,z) \approx E_{x0}\exp\left(-\frac{(x_0+\delta x)^2+(y_0+\delta y)^2}{\hat{\sigma}^2}\right)$$

$$\approx E_{x0}\exp\left(-\frac{x_0^2+y_0^2}{\hat{\sigma}^2}\right)\exp\left(-\frac{2(x_0\delta x+y_0\delta y)}{\hat{\sigma}^2}\right)$$

$$= E_{x0}\exp\left(-\frac{x_0^2+y_0^2}{\hat{\sigma}^2}\right)\exp\left(-\frac{2(x_0x_0'+y_0y_0')}{\hat{\sigma}^2}(z-z_m)\right)$$

$$\int_{z_a}^{z_b} E_x(x,y,z)dz = LE_{x0}\exp\left(-\frac{x_0^2+y_0^2}{\hat{\sigma}^2}\right)\underbrace{\frac{\sinh(X)}{X}}_{1+O(X^2)}$$

$$X = \frac{(x_0x_0'+y_0y_0')L}{\hat{\sigma}^2} << 1$$

$$\delta\mathrm{E}(x_0,x_0',y_0,y_0',t_0) \approx e\frac{\hat{K}}{2\gamma}E_{x0}L\exp\left(-\frac{x_0^2+y_0^2}{4\sigma_I^2}\right)\cdot\cos(\omega t_0)$$

Spectrum

$$P_{\delta\mathcal{E}}(\delta\mathcal{E}) = \int dx_0 dx_0' dy_0 dy_0' dt_0 \times P(x_0, x_0', y_0, y_0', t_0)\,\delta(\delta\mathcal{E} - \delta\mathcal{E}(x_0, x_0', y_0, y_0', t_0))$$

$$P_{\delta\mathcal{E}}(\delta\mathcal{E}) = T^{-1}\int dx_0 dy_0 dt_0 \times P_{x_0 y_0}(x_0, y_0)\,\delta(\delta\mathcal{E} - \delta\mathcal{E}(x_0, y_0, t_0))$$

$$P_{\delta\mathcal{E}}(\delta\mathcal{E}) = \hat{\mathcal{E}}^{-1}\int dr \times Q\left(\delta\mathcal{E}/\hat{\mathcal{E}}, R(r)\right) P_r(r)$$

with $P_r(r) = \int d\varphi \times r P_{x_0 y_0}(r\cos\varphi, r\sin\varphi)$

$$\hat{\mathcal{E}} = e\frac{\hat{K}}{2\gamma} E_{x0} L$$

$$R(r) = \exp\left(-(r/2\sigma_I)^2\right)$$

$$Q(x, \hat{y}) = \mathrm{Re}\left\{\frac{1}{\pi\sqrt{\hat{y}^2 - x^2}}\right\}$$

Spatial Probability of Particles

origin = origin of laser beam

(in plane of focus)

rms width: $\sigma_x = \sqrt{\beta_x \varepsilon}$ $\qquad$ $\sigma_y = \sqrt{\beta_y \varepsilon}$

offset: x_c $\quad$ y_c

$$P_{x_0 y_0}(x_0, y_0) = \frac{1}{2\pi\sigma_x\sigma_y} \exp\left[-\frac{1}{2}\left(\frac{(x_0 - x_c)^2}{\sigma_x^2} + \frac{(y_0 - y_c)^2}{\sigma_y^2}\right)\right]$$

$$\downarrow$$

$$P_r(r) = \int d\varphi \times r P_{x_0 y_0}(r\cos\varphi, r\sin\varphi)$$

$$\downarrow$$

$$P_{\delta E}(\delta E) = \hat{E}^{-1} \int dr \times Q\left(\delta E / \hat{E}, R(r)\right) P_r(r)$$

Example 1: 0.1nC, perfect match

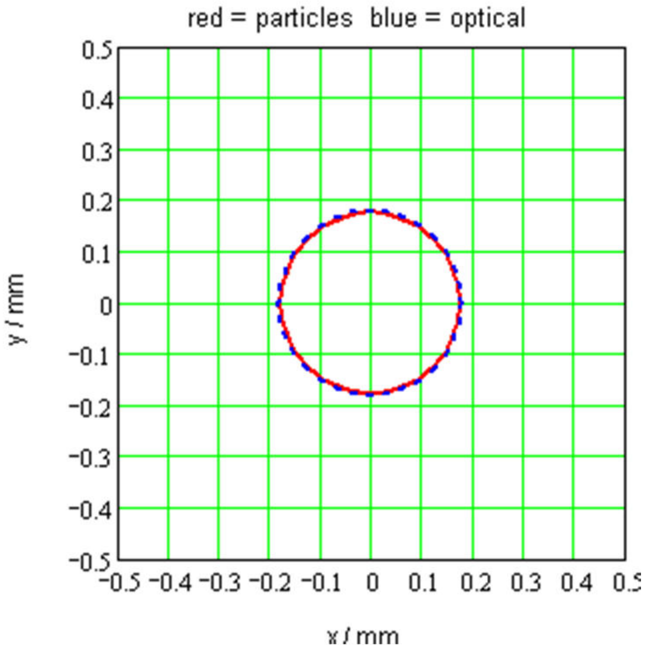
working point for $q = 0.1\,\mathrm{nC}$

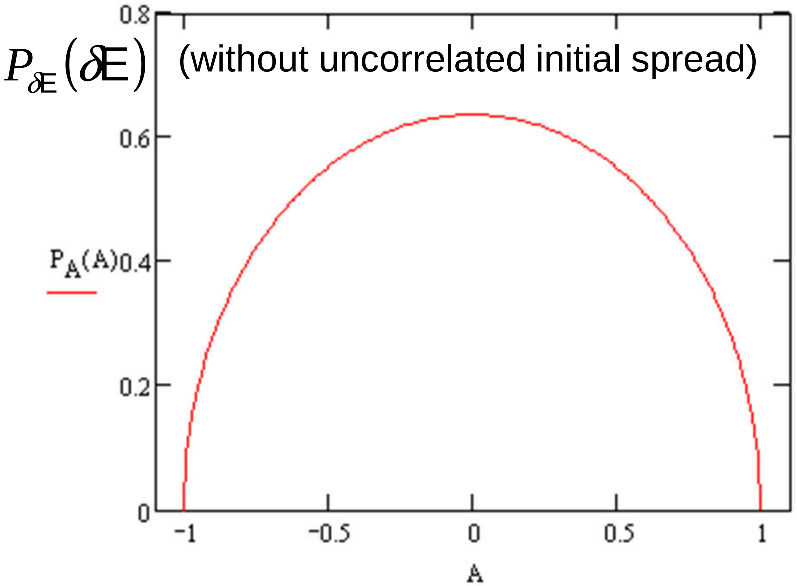
$\varepsilon\gamma = 0.34\,\mu\mathrm{m}$

$\mathrm{E} = 130\,\mathrm{MeV}$

$\beta_x = \beta_y = 25\,\mathrm{m}$

$\sigma_I = 175\,\mu\mathrm{m}$

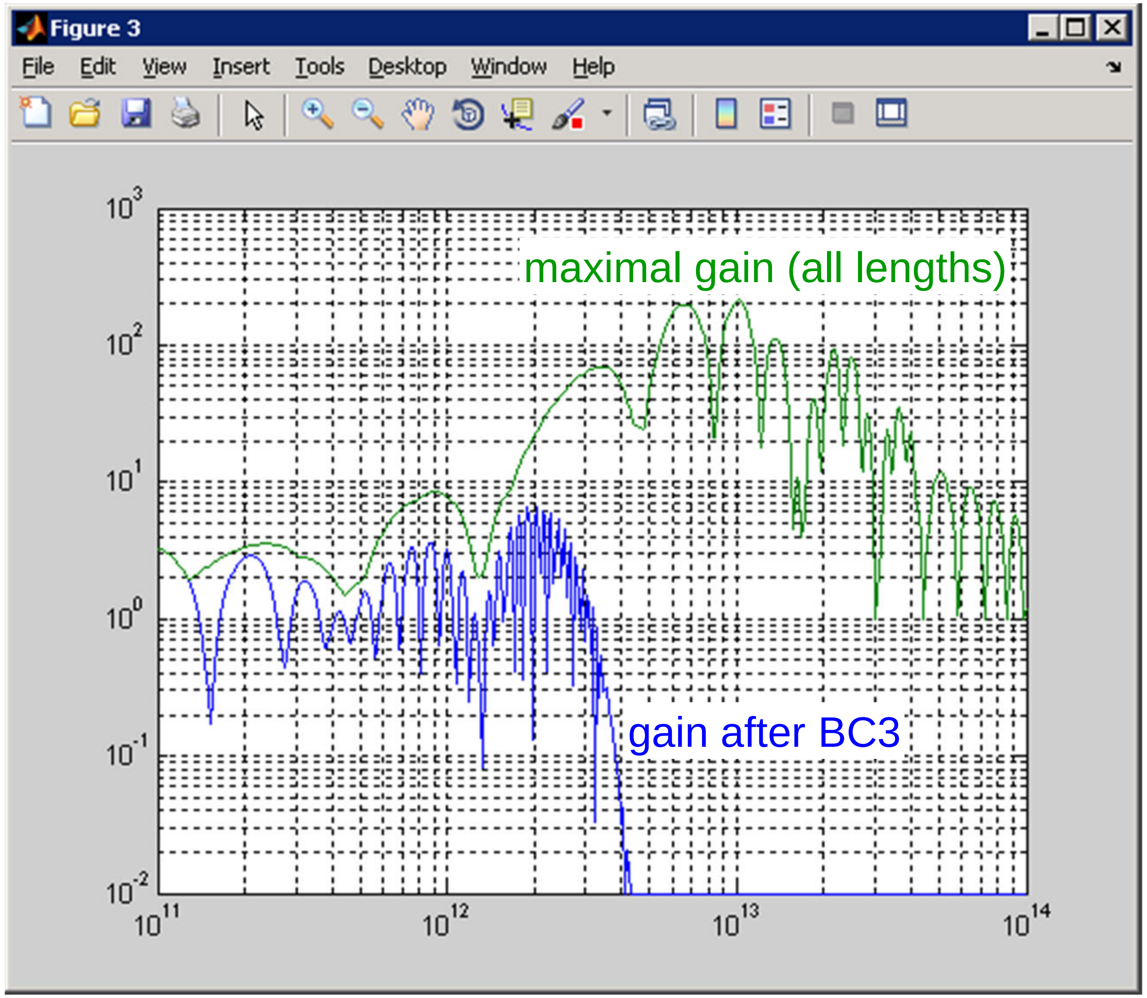




Example 1:   0.1nC, perfect match

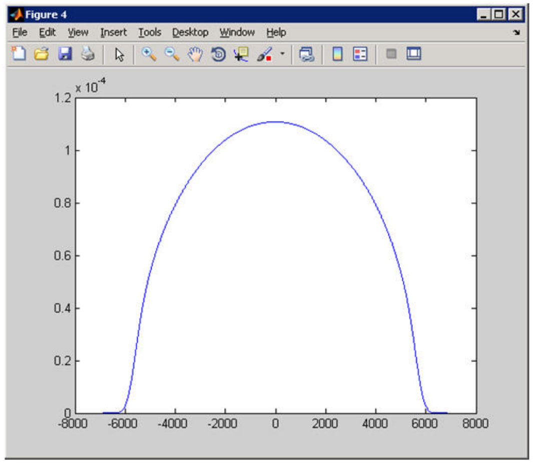
µb amplification for Igor's working point

rms spread of compressed beam = 2.5 MeV

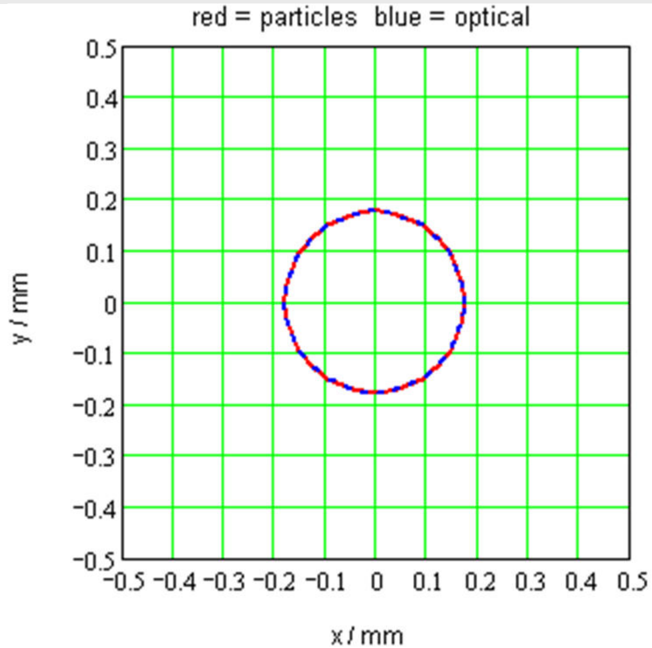


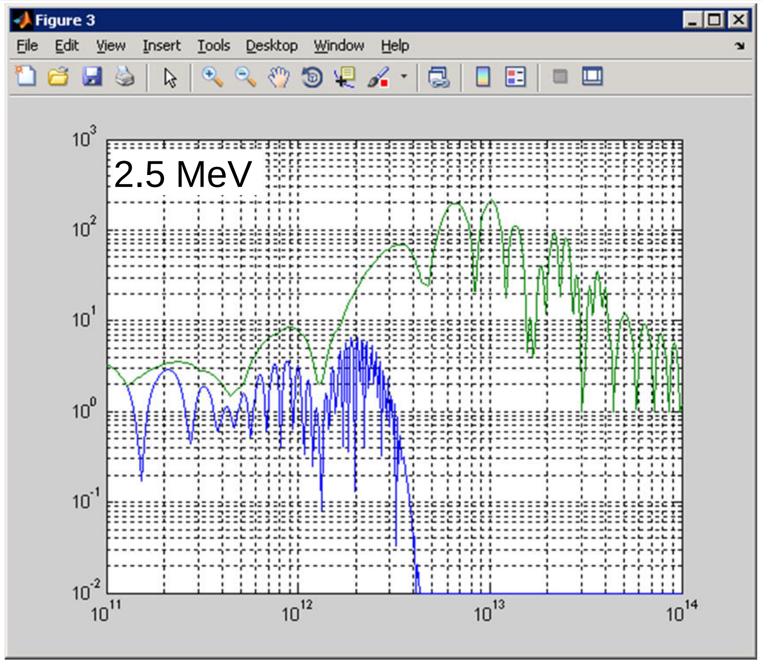
$P_{\delta E}(\delta E)$

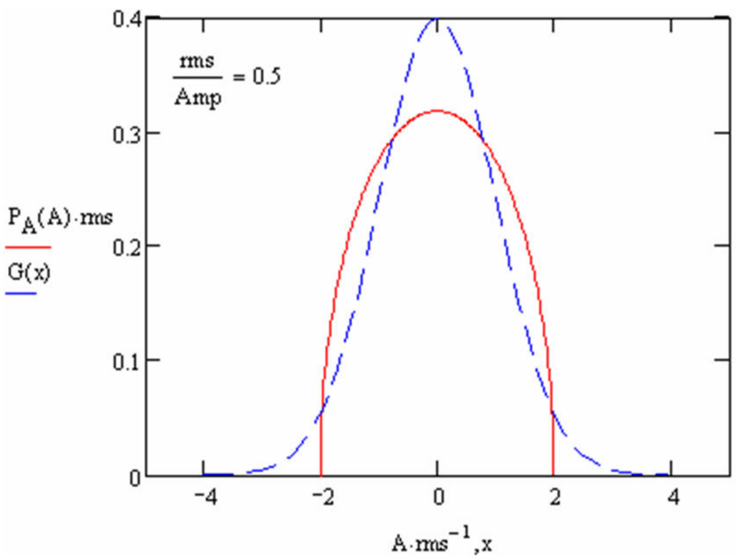
with uncorrelated initial spread

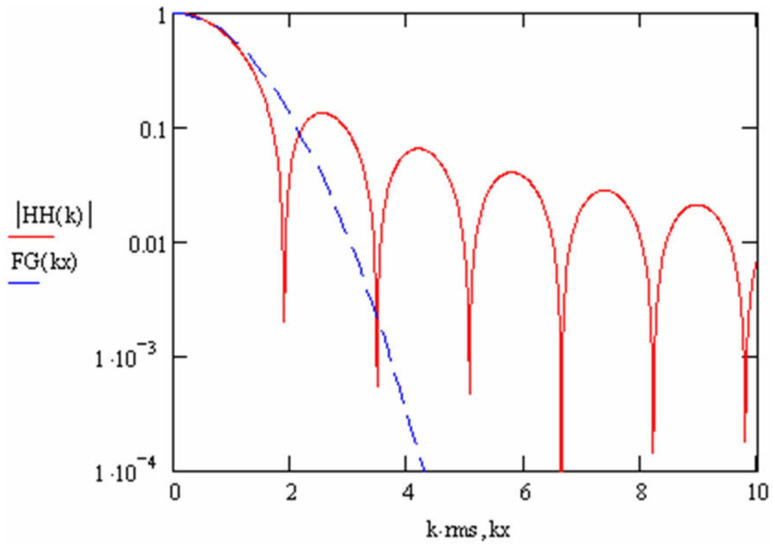


Example 1: 0.1nC, perfect match

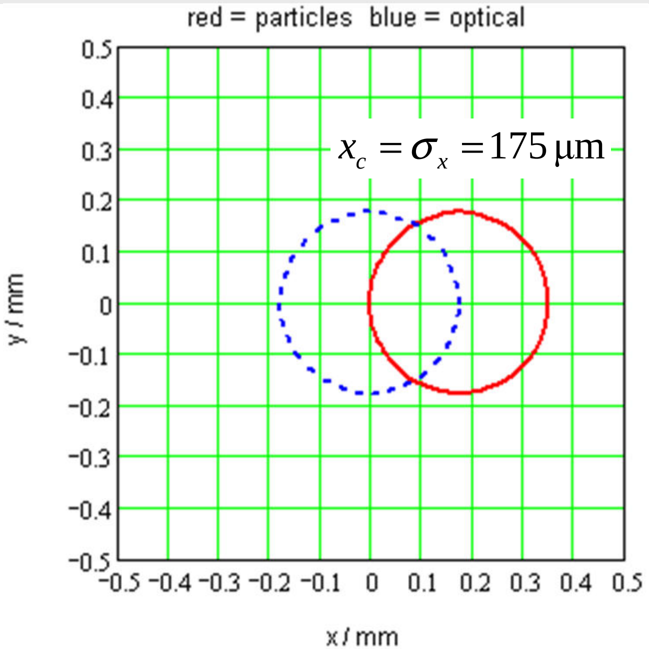


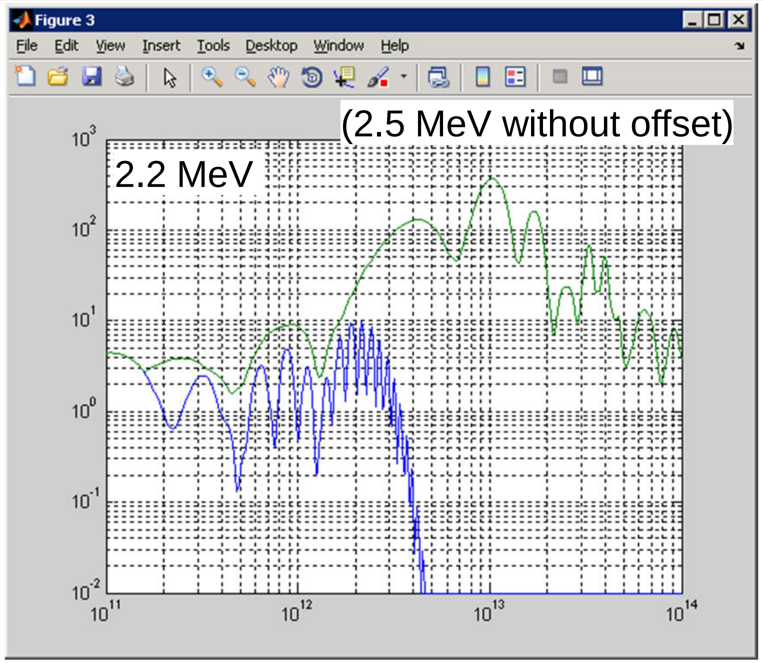


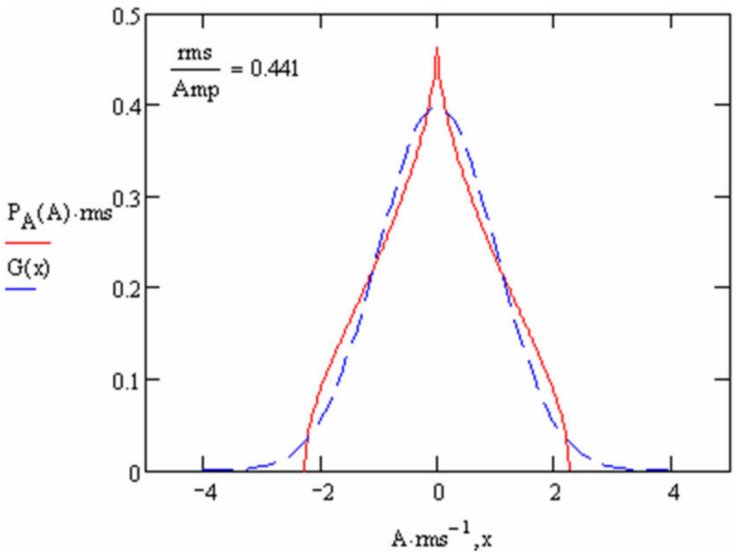


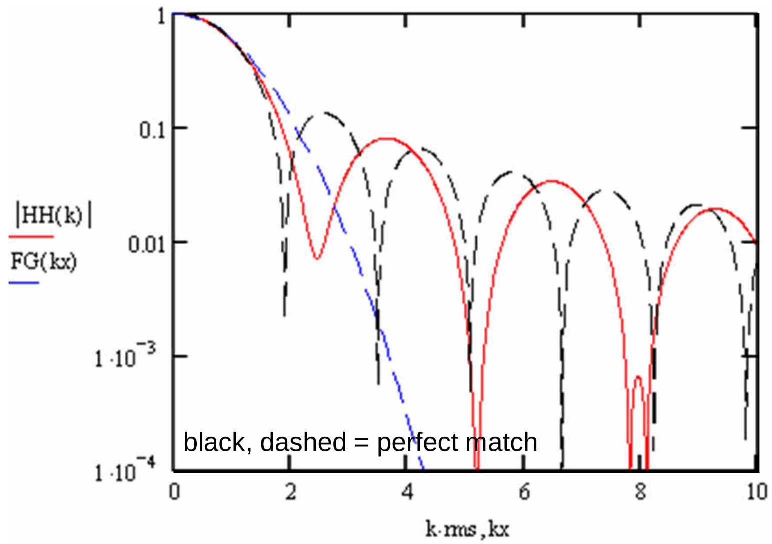


offset of particle slice (correlated energy spread & r61)









Example 2:   0.1nC, typical mismatch

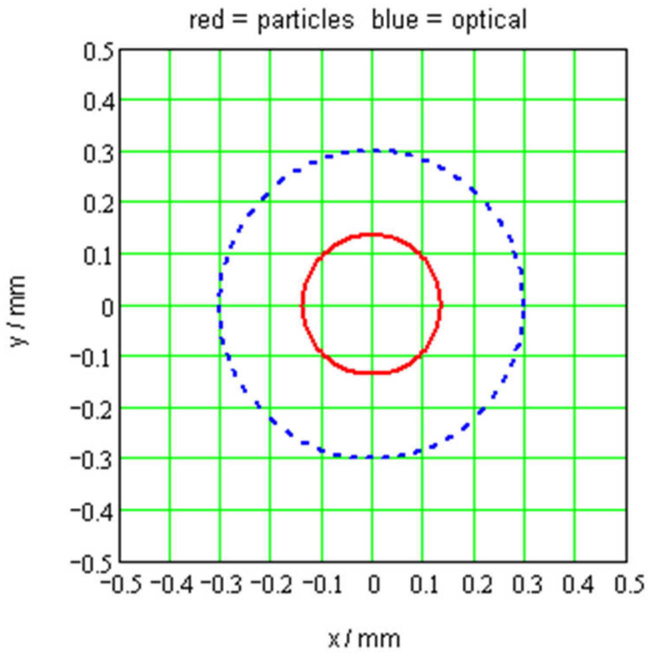
working point for $q = 0.1\,\mathrm{nC}$

$\varepsilon\gamma = 0.34\,\mu\mathrm{m}$

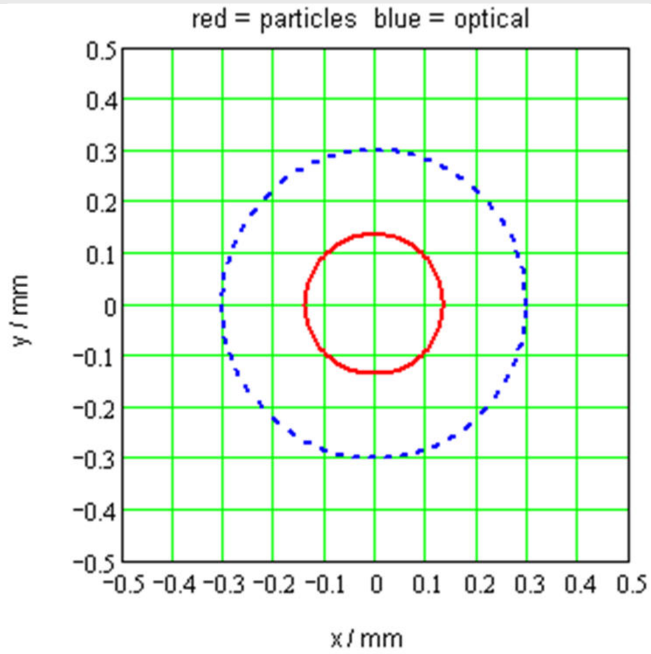
$\mathrm{E} = 130\,\mathrm{MeV}$

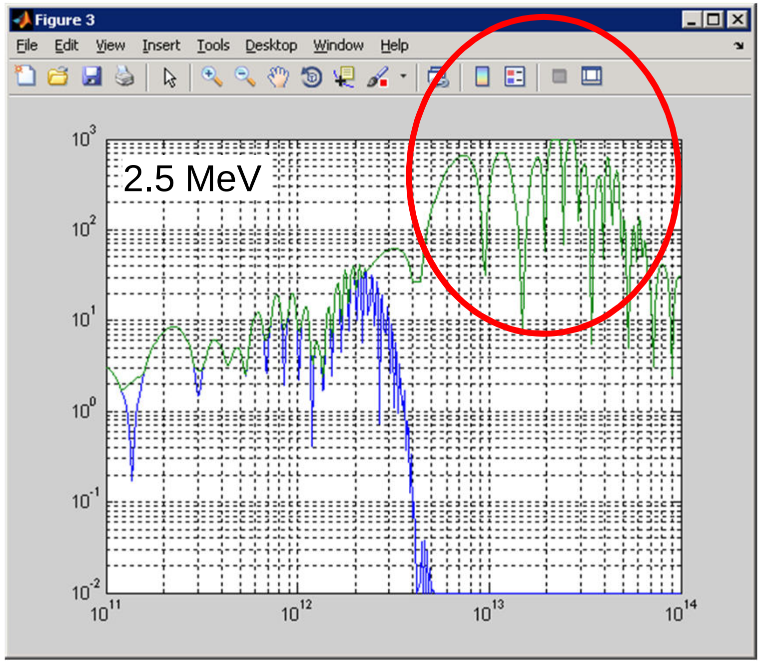
$\beta_x = \beta_y = 15\,\mathrm{m}$

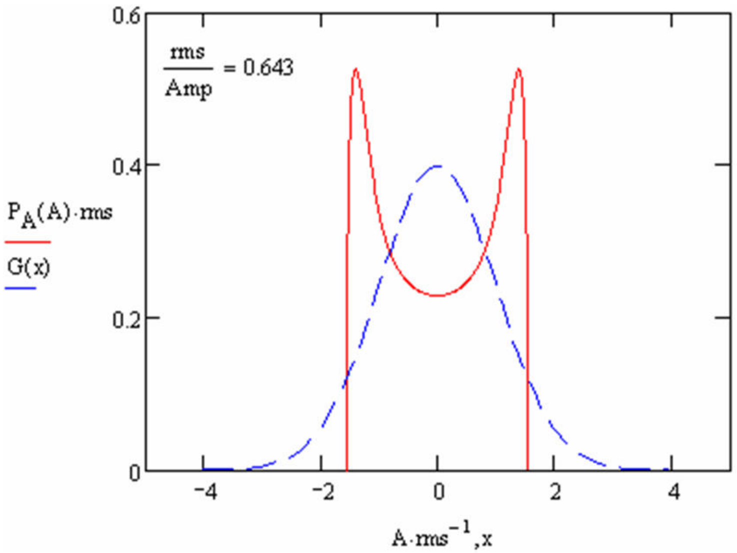
$\sigma_I = 300\,\mu\mathrm{m}$

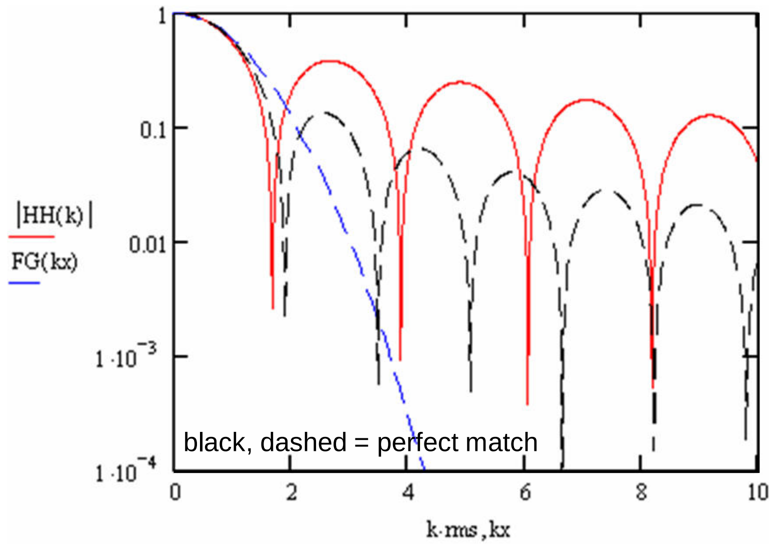


Example 2:   0.1nC, typical mismatch

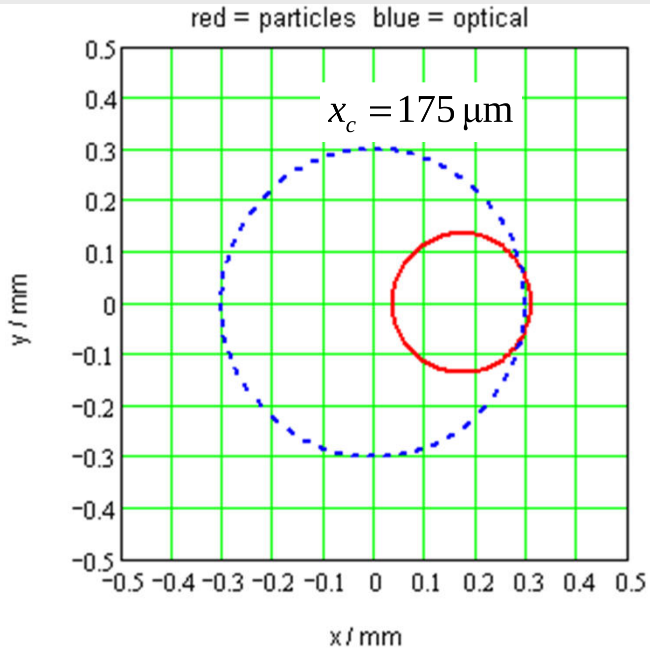


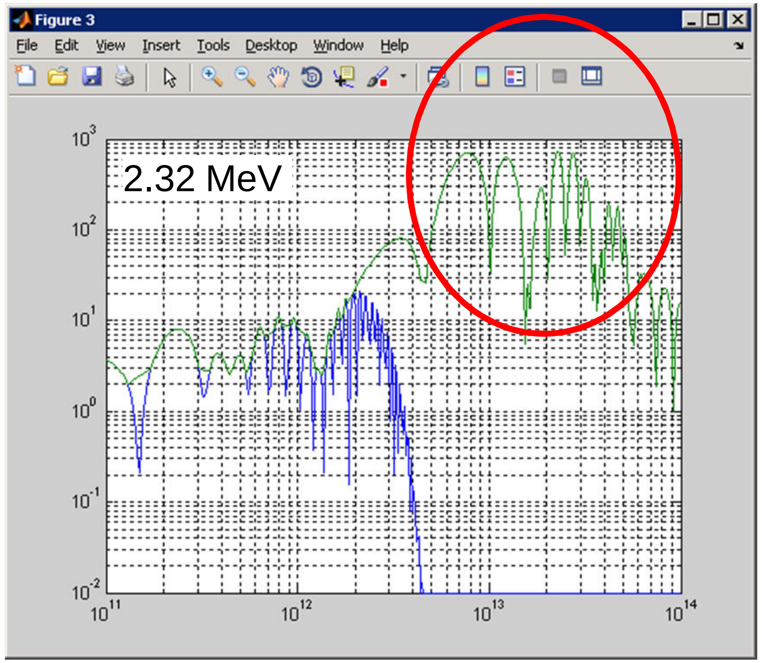


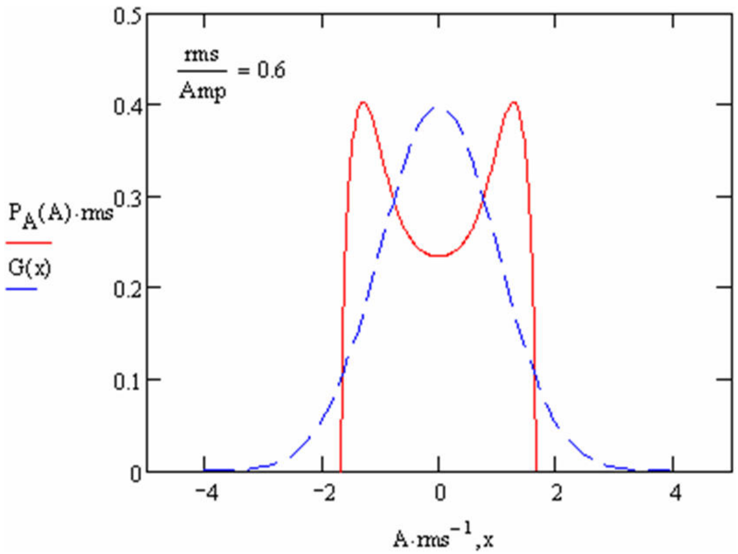


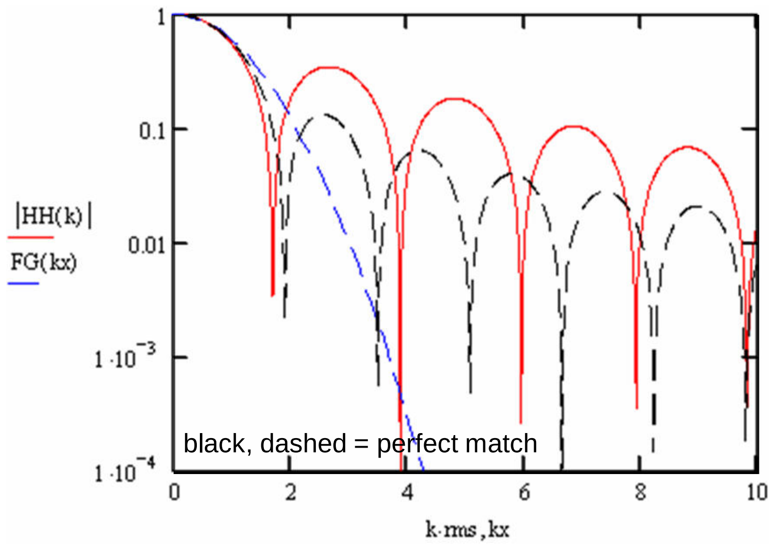


Example 2:   0.1nC, typical mismatch + offset









Example 3: 0.1nC, much more vertical offset

working point for $q = 0.1\,\mathrm{nC}$

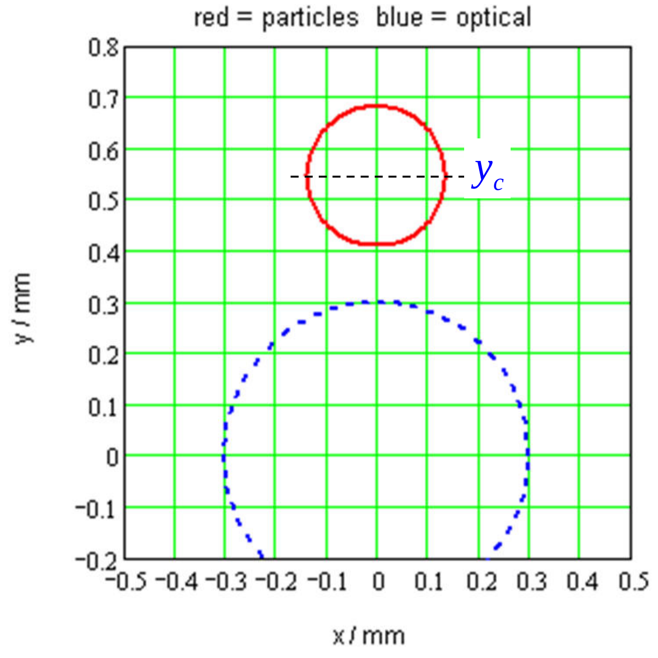
$\varepsilon\gamma = 0.34\,\mu\mathrm{m}$

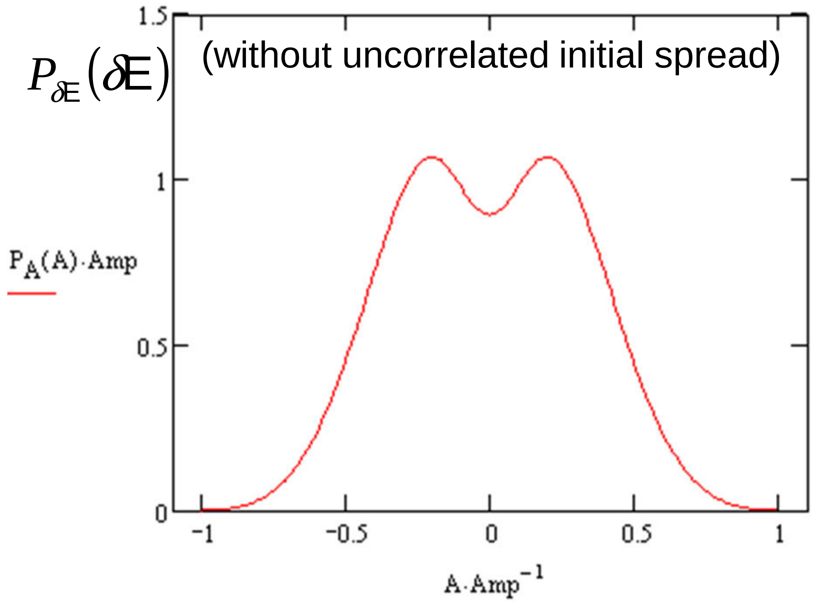
$\mathrm{E} = 130\,\mathrm{MeV}$

$\beta_x = \beta_y = 15\,\mathrm{m}$

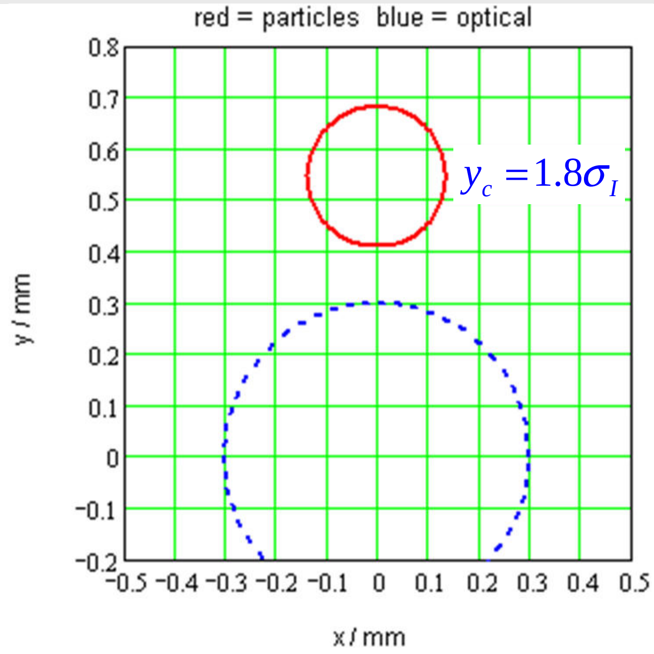
$\sigma_I = 300\,\mu\mathrm{m}$

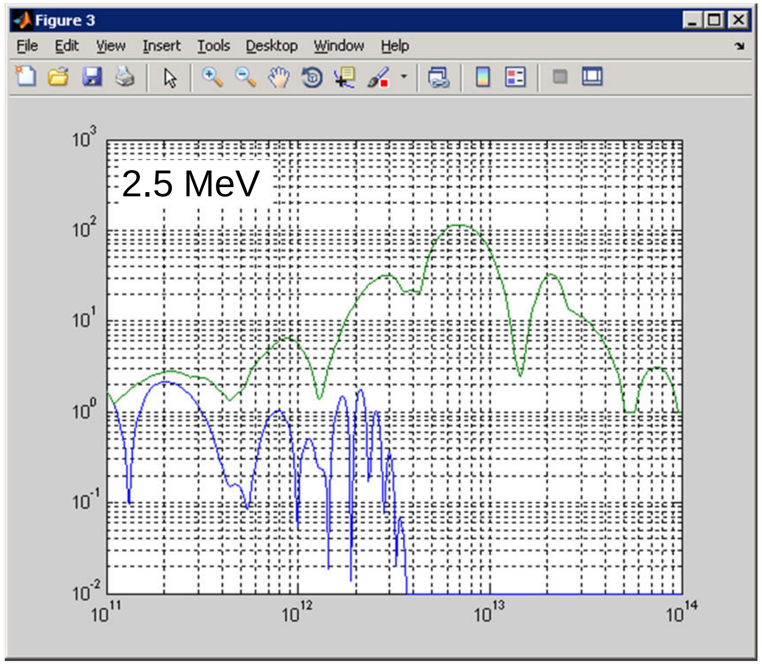
$y_c = 1.8\sigma_I$

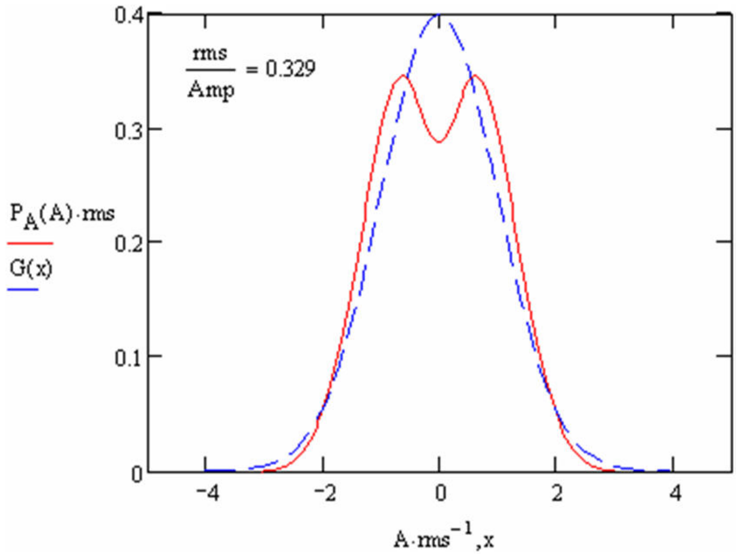


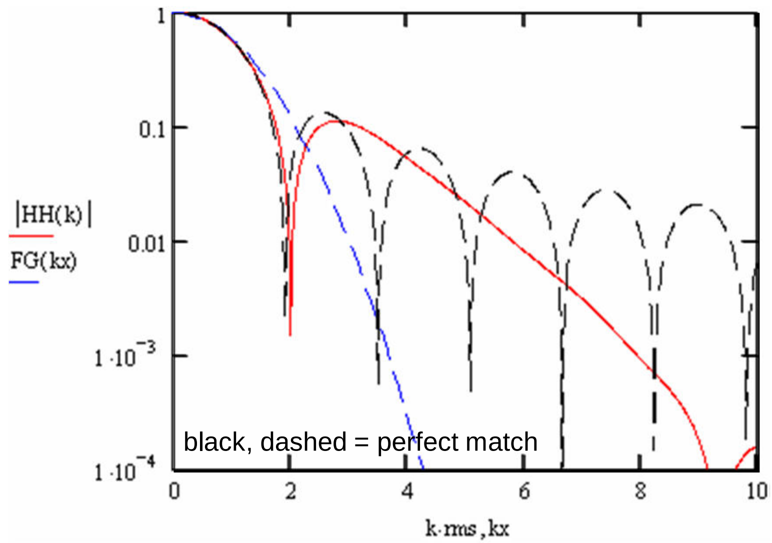


Example 3: 0.1nC, much more vertical offset

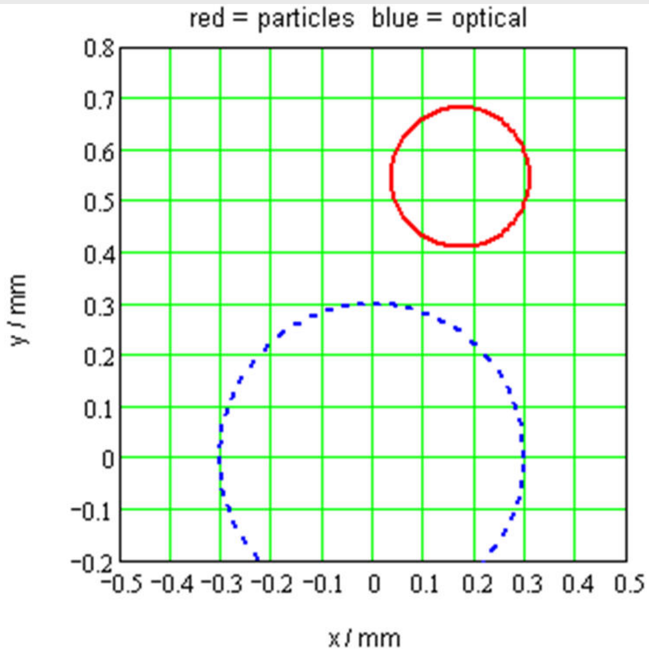


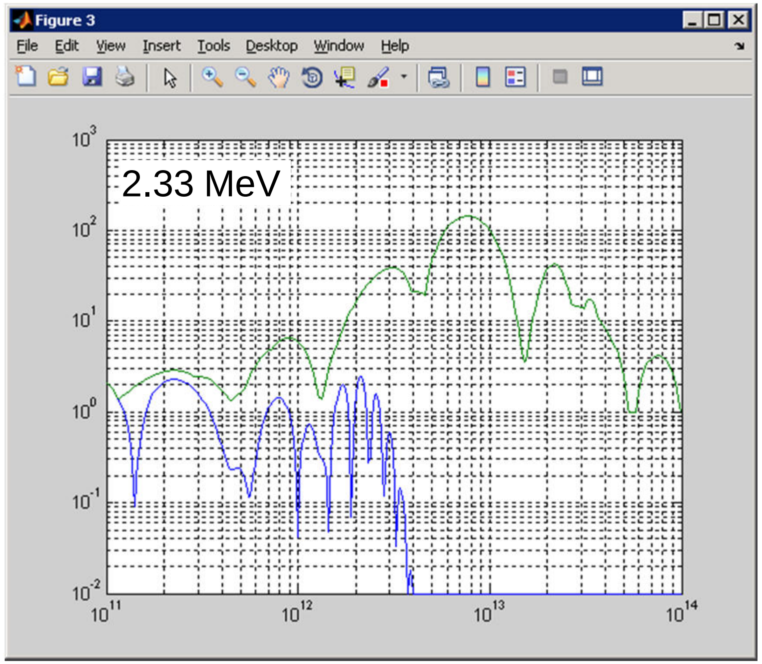


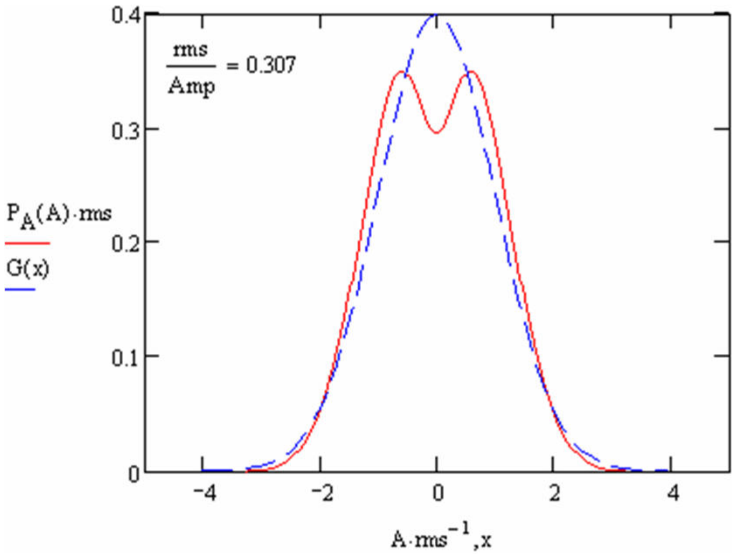


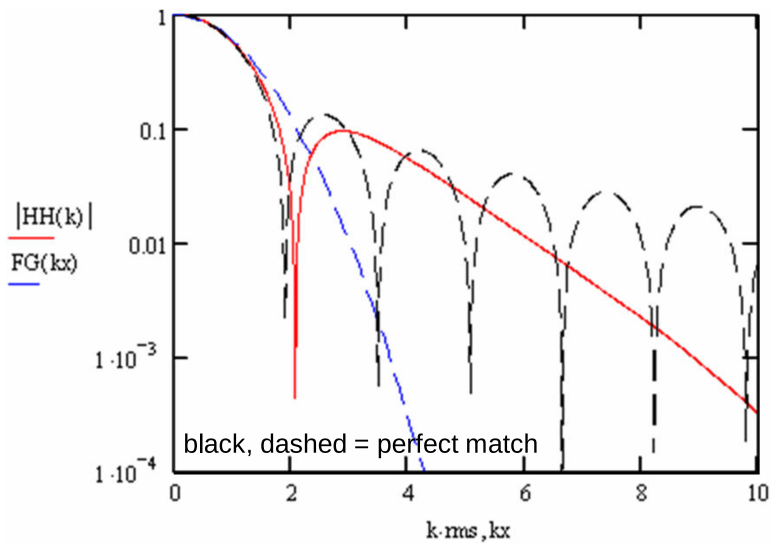


Example 3: 0.1nC, much more vertical offset









Conclusion

electron/photon beam matching in laser heater for low charge operation needs care:

spectrum and damping is critical
small emittance $\rightarrow$ compromise between electron optics (beta function)
and photon optics (focus & Rayleigh length)

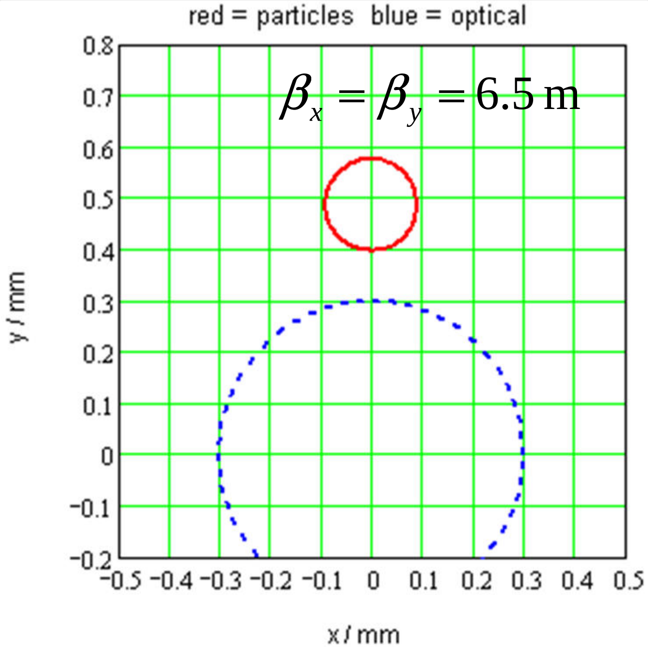
good results for perfect matching even with some offset

mismatch $\sigma_l/\sigma_x \approx 1.5$ causes high uB-amplification between BC1 and BC2

electron/photon beam without matching + vertical offset improves situation:

spectrum & damping for optimized (!) parameter set has been shown
geometrical tolerances less critical, week effect of horizontal offset
LH-amplitude requirement can be fulfilled

More


red = particles blue = optical
$\beta_x = \beta_y = 6.5\,\mathrm{m}$
y / mm
x / mm

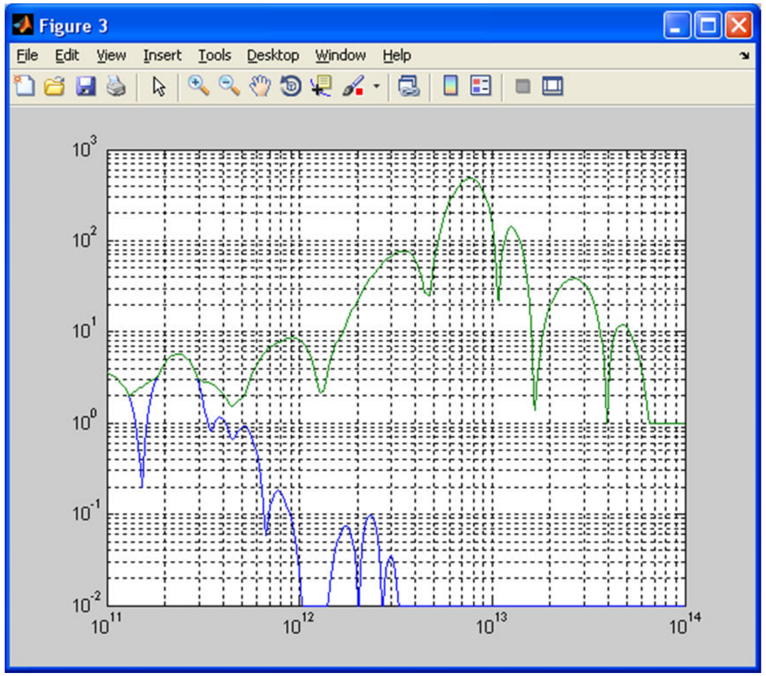
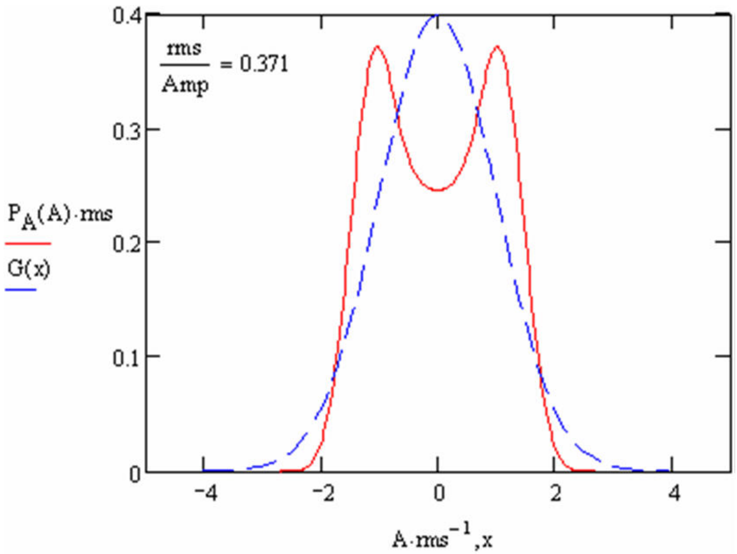
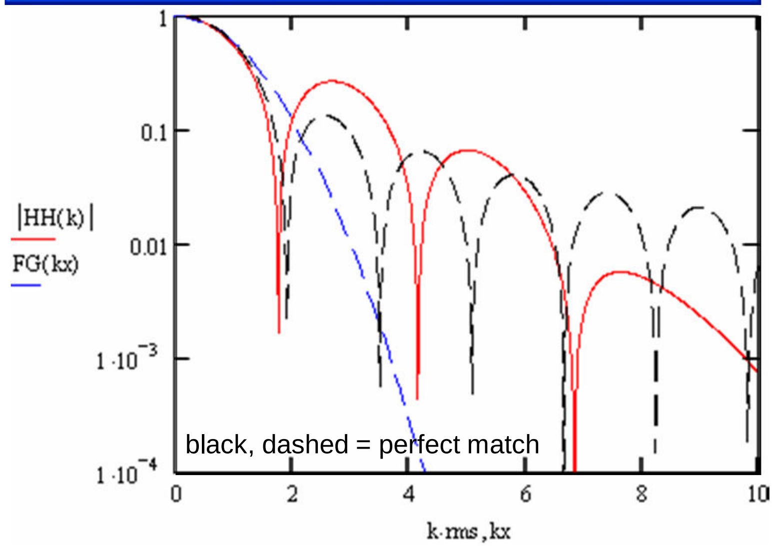


Figure 3
File Edit View Insert Tools Desktop Window Help


$\frac{\text{rms}}{\text{Amp}} = 0.371$
$P_A(A)\cdot \text{rms}$
$G(x)$
$A \cdot \text{rms}^{-1}, x$


$|HH(k)|$
$FG(kx)$
black, dashed = perfect match
$k \cdot \text{rms}, kx$

large vertical offset → 2 parameter problem

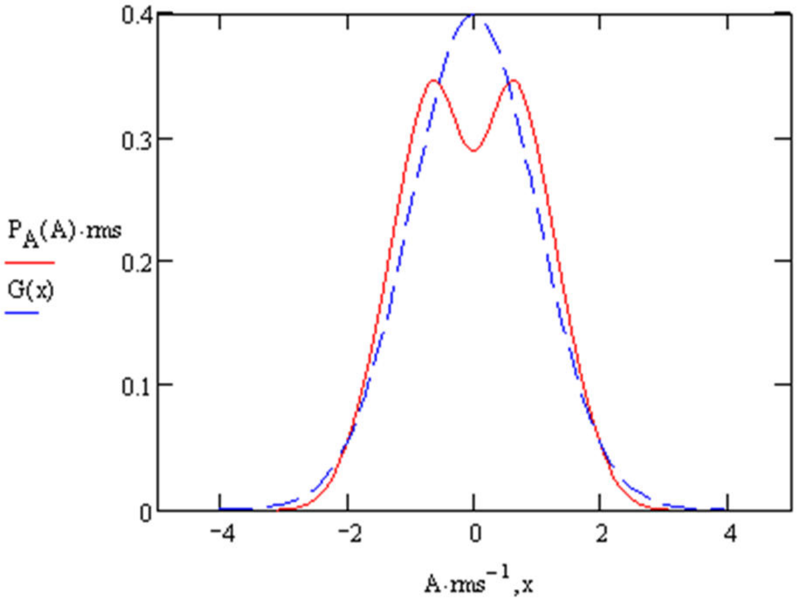
$$P_r(r) \rightarrow \frac{1}{\sqrt{2\pi}\sigma_y} \exp\left(-\frac{1}{2} \left(\frac{r - y_c}{\sigma_y} \right)^2 \right)$$

with $\boxed{\hat{y}_c = y_c / \sigma_I} >> 1$ $\boxed{\hat{\sigma}_y = \sigma_y / \sigma_I}$

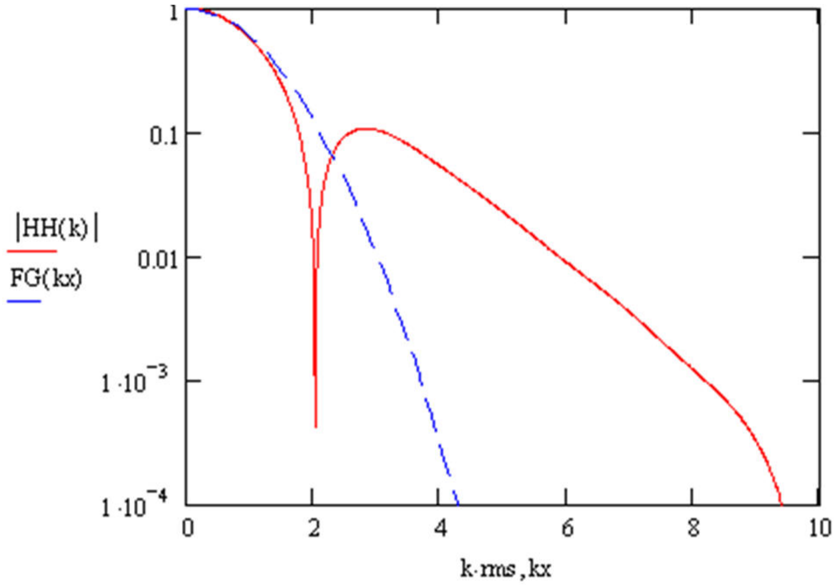
$$\hat{P}(\hat{r}) \rightarrow \frac{1}{\sqrt{2\pi}\hat{\sigma}_y} \exp\left(-\frac{1}{2} \left(\frac{\hat{r} - \hat{y}_c}{\hat{\sigma}_y} \right)^2 \right)$$

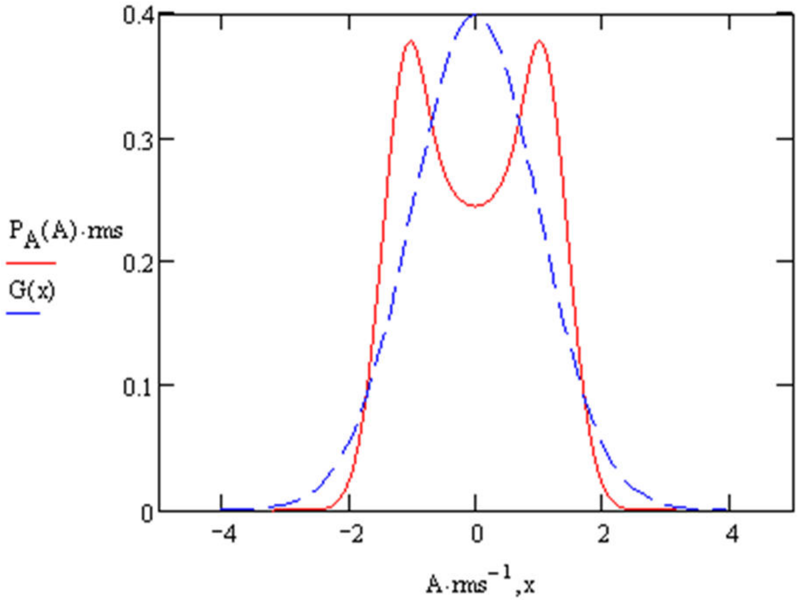
$$P_{\delta E}(\delta E) = \hat{E}^{-1} \int d\hat{r} \times Q\left(\delta E / \hat{E}, \exp\left(-(\hat{r}/2)^2\right)\right) \hat{P}_r(\hat{r})$$

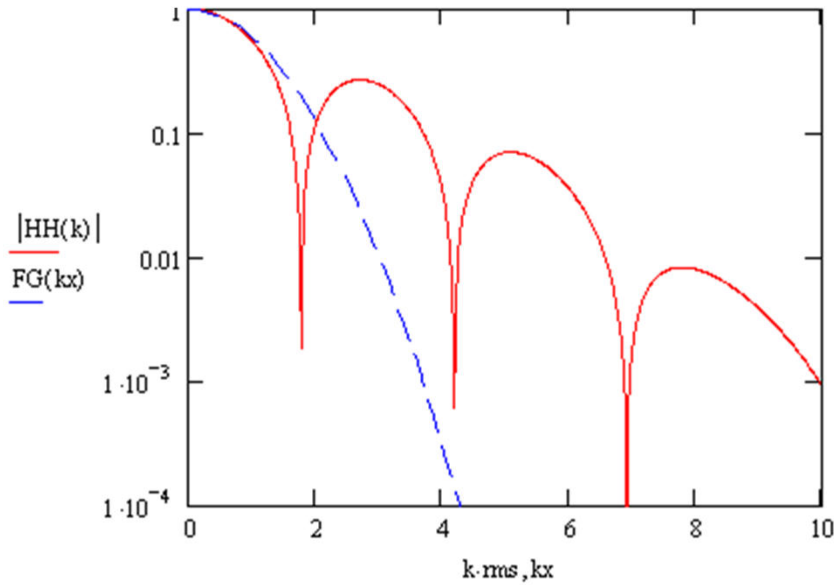
$$rms(\hat{y}_c, \hat{\sigma}_y) = \frac{\exp\left(\frac{-\hat{y}_c^2}{4\left(1 + \hat{\sigma}_y^2\right)} \right)}{\sqrt{2\sqrt{1 + \hat{\sigma}_y^2}}}$$



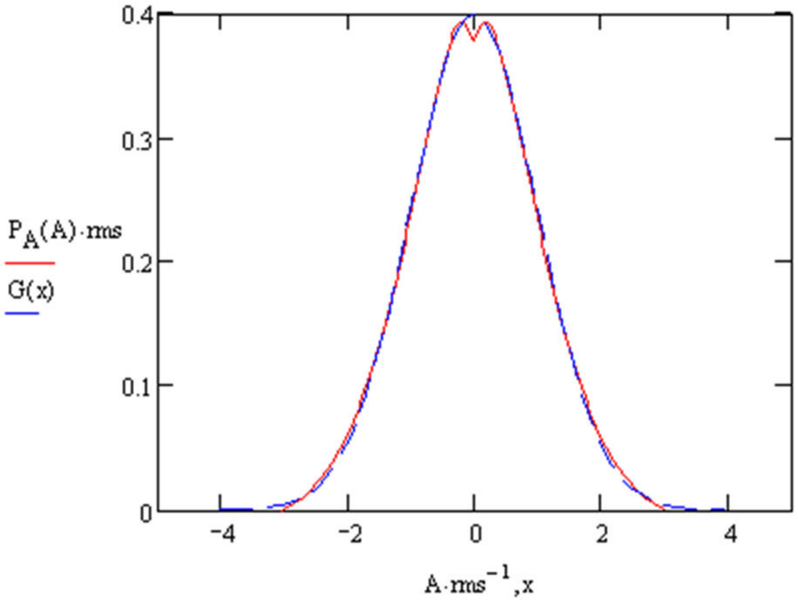
$\frac{ys}{\sigma_I} = 1.821$ $\frac{\sigma_y}{\sigma_I} = 0.455$ $\frac{rms}{Amp} = 0.324$

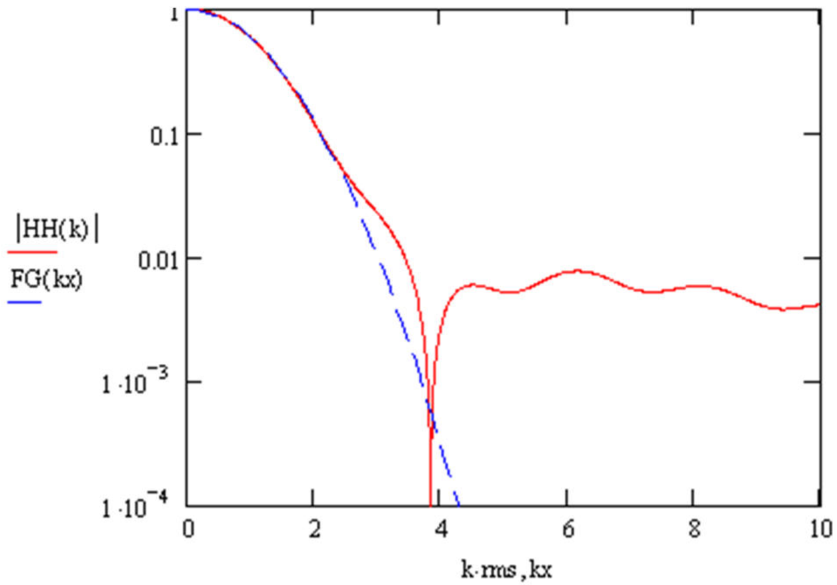






$\frac{ys}{\sigma_I} = 1.821$ $\frac{\sigma_y}{\sigma_I} = 0.263$ $\frac{rms}{Amp} = 0.315$





$\frac{ys}{\sigma_I} = 1.821$ $\frac{\sigma_y}{\sigma_I} = 0.644$ $\frac{rms}{Amp} = 0.331$