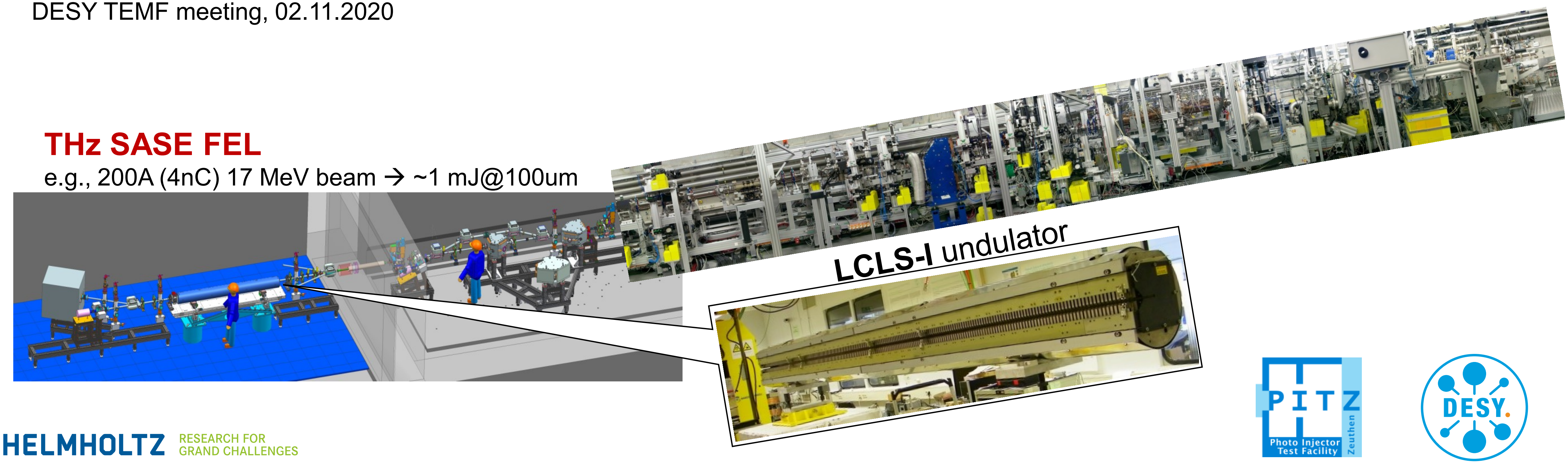
3D field modeling of LCLC-I undulator including horizontal gradient

3D field map reconstruction based on axial field measurements for THz@PITZ

M. Krasilnikov
DESY TEMF meeting, 02.11.2020

THz SASE FEL

e.g., 200A (4nC) 17 MeV beam → ~1 mJ@100um



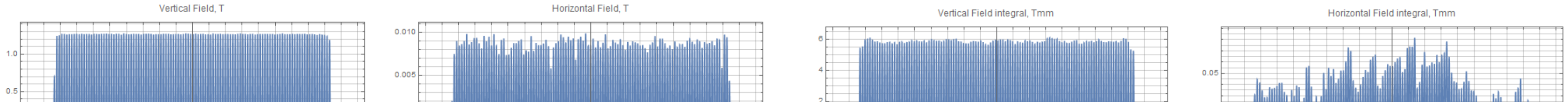
Undulator for THz@PITZ

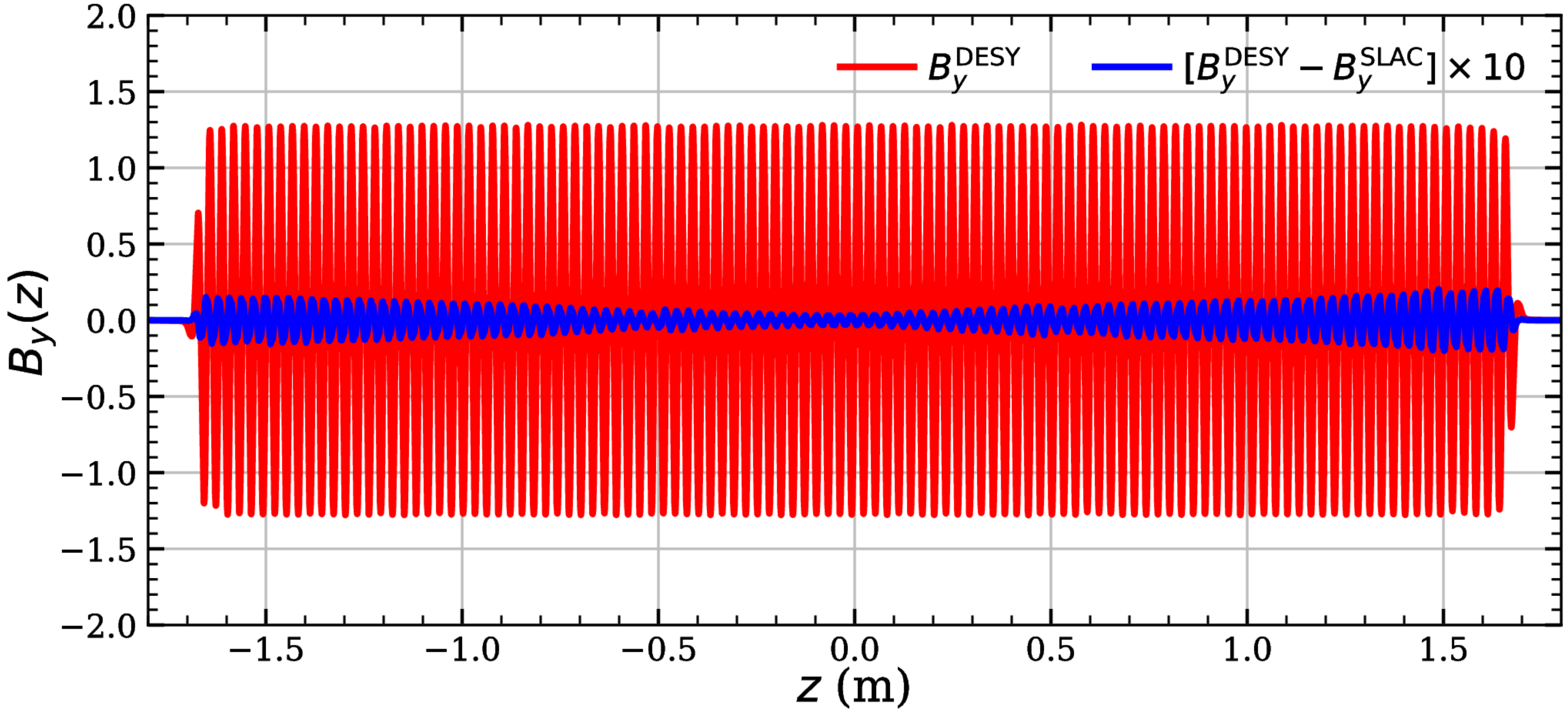
LCLS-I undulators (on loan from SLAC)

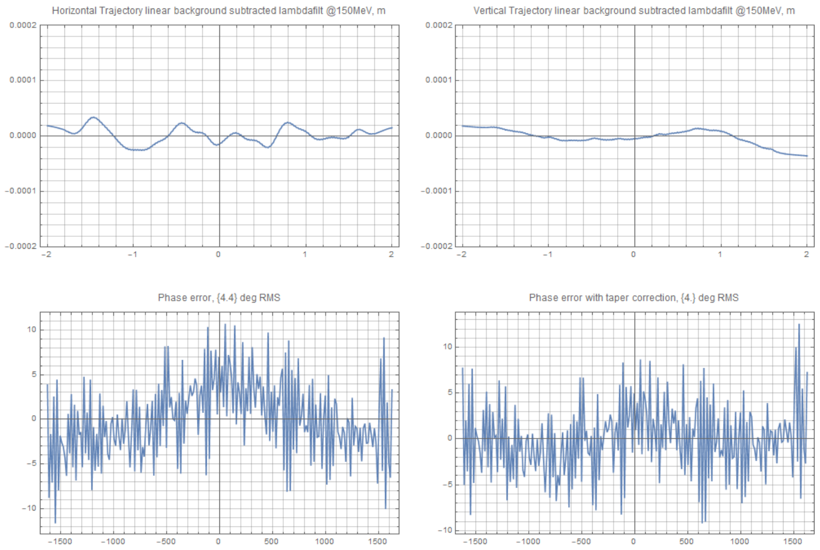
Properties	Details
Type	**fixed gap planar hybrid** (NeFeB)
Nominal gap	6.8 mm
K-value	3.49
Support diameter / length	30 cm / 3.4 m
Vacuum chamber	11 mm x **5 mm**
Period length	30 mm
Poles / a module	226 poles (= 113 periods)
Total weight w/o vac. chamber	1000 kg

LCLS-I Undulator Field

Recent measurements of LCLS-I module 26 at DESY in Hamburg (M. Tischer, T. Vielitz, P. Vagin)







Two data sets measured at:

- DESY (*fld26.txt*)
- LCLS (*SN26 x+00000_y+000_bscanz.dat*)

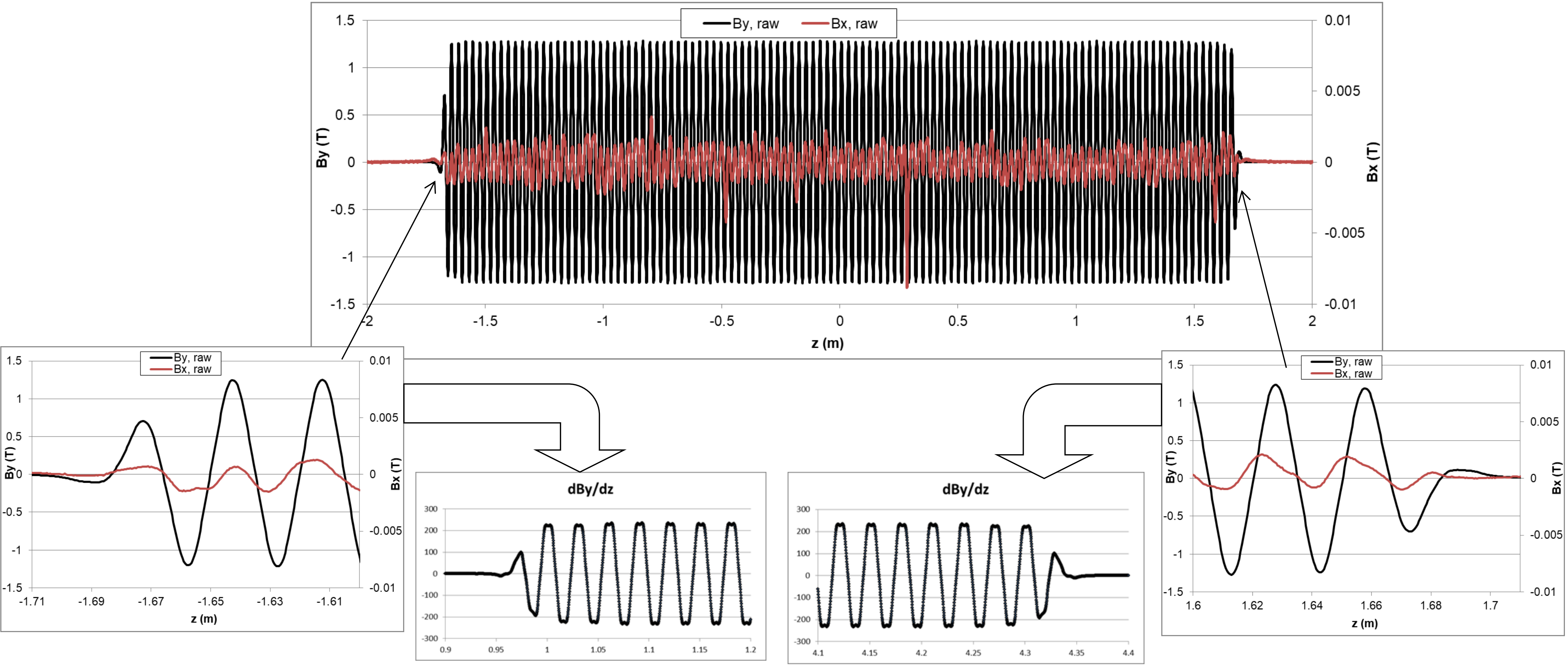
And field integrals (*fldInt26.txt*) measured with the stretched wire.

Measured **transverse taper** is
K(x) = 3.47716 + 0.00264755*x[mm],
or K/K0(x) = 1 + 0.000761412*x[mm].

- Two **LCLS-I** undulators have arrived at Hamburg in 08/ 2019
- The fields of the undulator L143-112000-**26** have been re-measured at DESY Hamburg and are **consistent** with SLAC measurement (discrepancy < 0.02 T)

LCLS-I Undulator Field

Recent measurements of LCLS-I module 26 at DESY in Hamburg



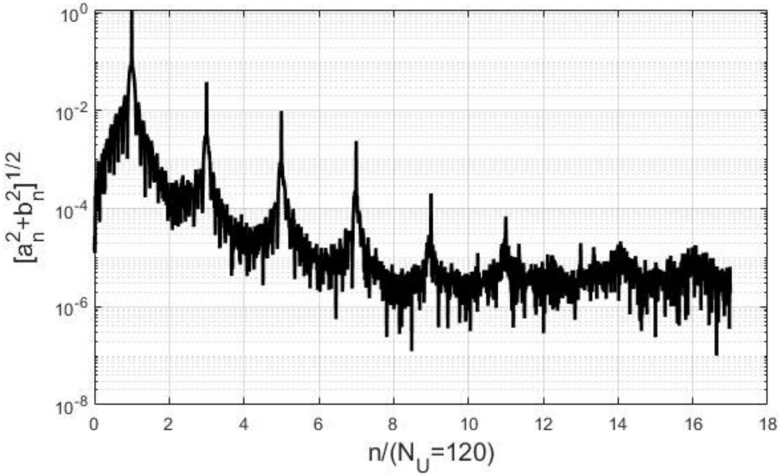
LCLS-I Undulator Field → Analysis

Recent measurements of LCLS-I module 26 at DESY in Hamburg

$$a_n = \frac{2}{L}\int_{-\frac{L}{2}}^{\frac{L}{2}} B_{y,m}(x=0,y=0,z)\cos\left(\frac{2\pi nz}{N_U\lambda_U}\right)dz,$$

$$a_0 = \frac{1}{L}\int_{-\frac{L}{2}}^{\frac{L}{2}} B_{y,m}(x=0,y=0,z)\,dz \to 0,$$

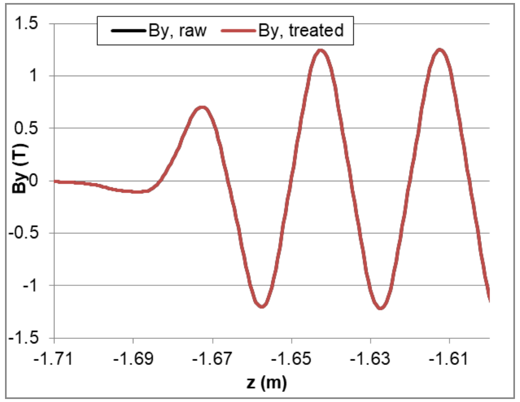
$$b_n = \frac{2}{L}\int_{-\frac{L}{2}}^{\frac{L}{2}} B_{y,m}(x=0,y=0,z)\sin\left(\frac{2\pi nz}{N_U\lambda_U}\right)dz.$$

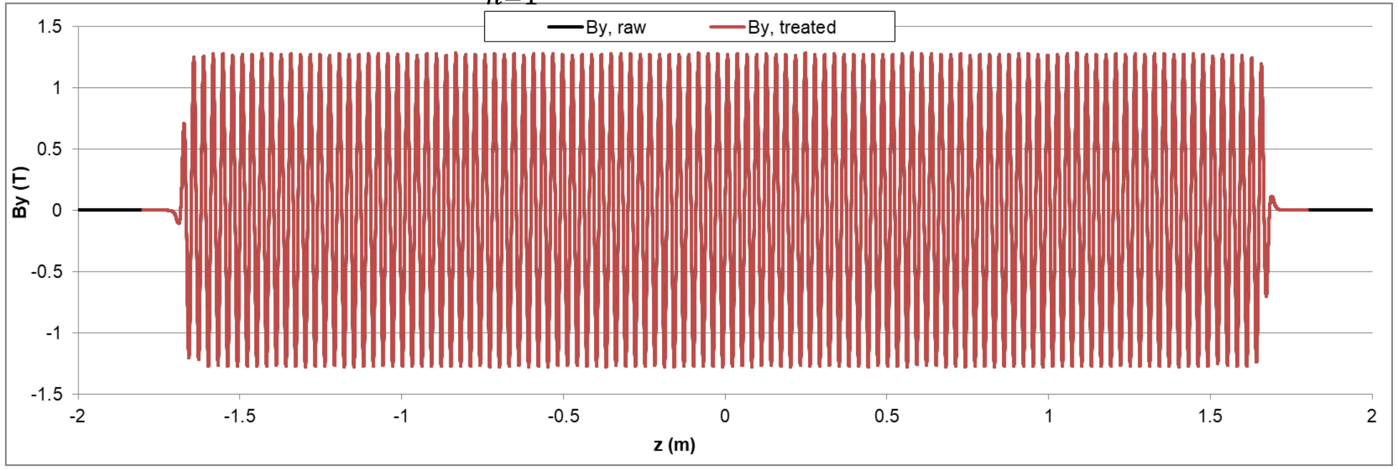


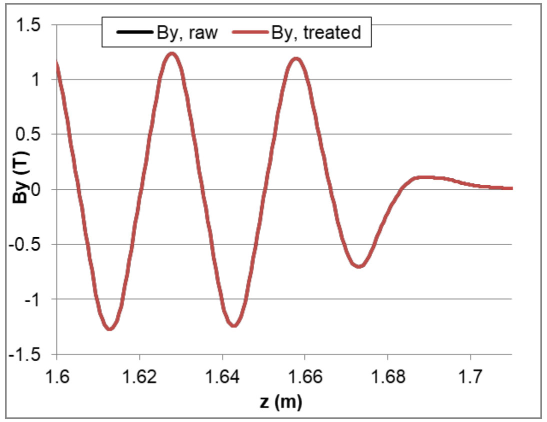
$$L = N_U\lambda_U$$

$$k_{zn} = \frac{2\pi n}{N_U\lambda_U}$$

$$B_y(x=0,y=0,z) = \sum_{n=1}^{N_h\cdot N_U}\left\{a_n\cos\left(\frac{2\pi nz}{N_U\lambda_U}\right) + b_n\sin\left(\frac{2\pi nz}{N_U\lambda_U}\right)\right\}$$







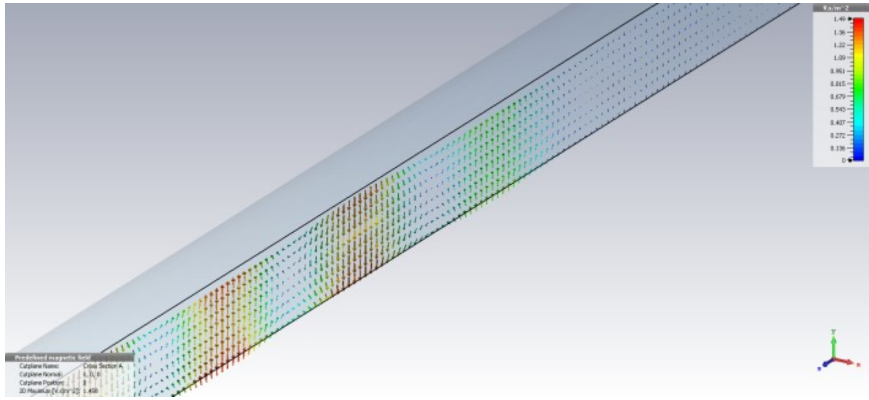
$$N_h = 17;\ N_U = 120$$

LCLS-I Undulator field w/o horizontal gradient

3D field map generation

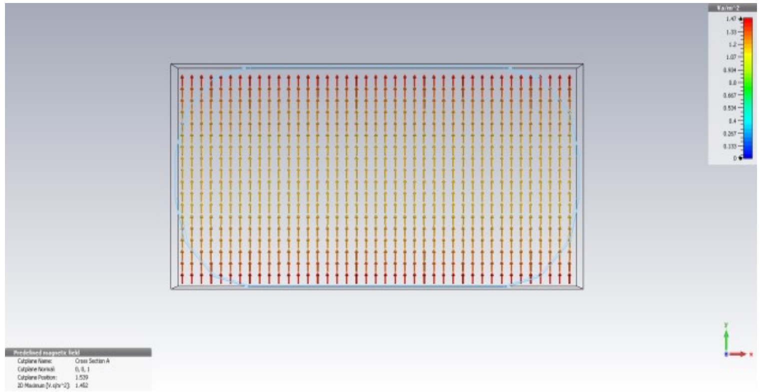
Vertical and longitudinal components of undulator magnetic field:

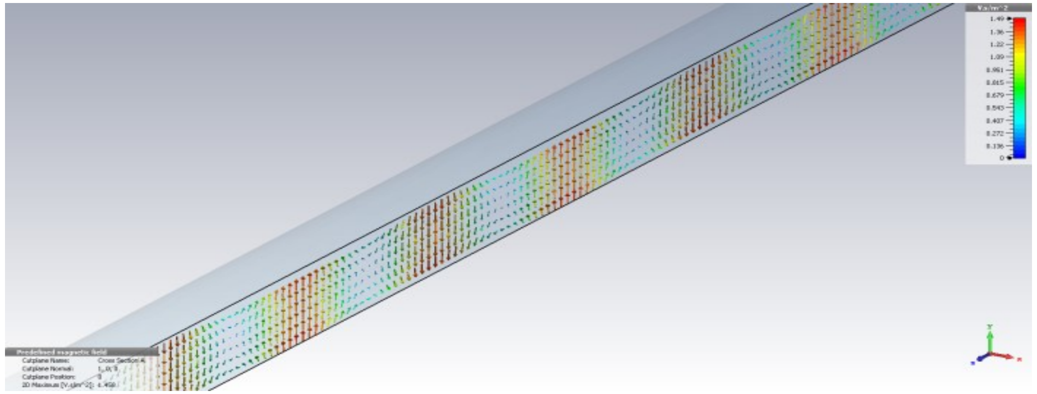
$$B_y(x,y,z) = \sum_{n=1}^{N_h\cdot N_U}[\{a_n\cos(k_n z)+b_n\sin(k_n z)\}\cdot\cosh(k_n y)],$$
$$B_z(x,y,z) = \sum_{n=1}^{N_h\cdot N_U}[\{-a_n\sin(k_n z)+b_n\cos(k_n z)\}\cdot\sinh(k_n y)],$$



where $k_n = \frac{2\pi n}{N_U\lambda_U}$ is the wavenumber of the *n*-th Fourier harmonic.

$$\begin{bmatrix}a\\b\end{bmatrix}_n = \frac{2}{N_U\lambda_U}\int_{-\frac{N_U\lambda_U}{2}}^{\frac{N_U\lambda_U}{2}} B_{y,m}(x=0,y=0,z)\begin{bmatrix}\cos\\\sin\end{bmatrix}\left(\frac{2\pi nz}{N_U\lambda_U}\right)dz,$$



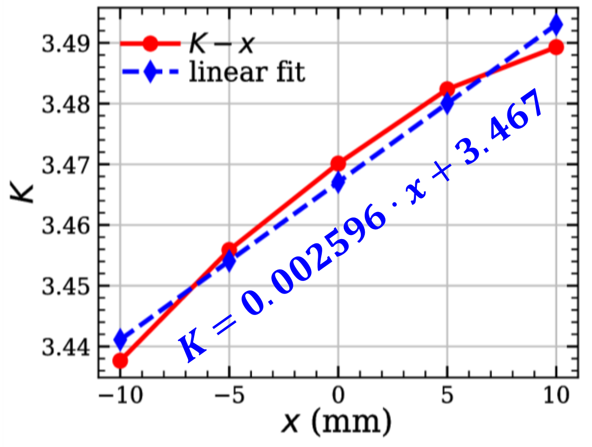


$$N_h = 17;\ N_U = 120$$

How to implement a horizontal gradient?

Consistent implementation of the $B_y(x)$ taper

Besides longitudinal profile By(z) **transverse gradient** By(x) has been measured



$$\left.\begin{array}{l} div\,\vec{B} = 0 \\ curl\,\vec{B} = 0 \end{array}\right. \Rightarrow \vec{B} = -\frac{\partial \chi}{\partial \vec{r}} \Rightarrow \Delta\chi = 0$$

$$\Downarrow$$

$$k_x^2 + k_y^2 = k_z^2$$

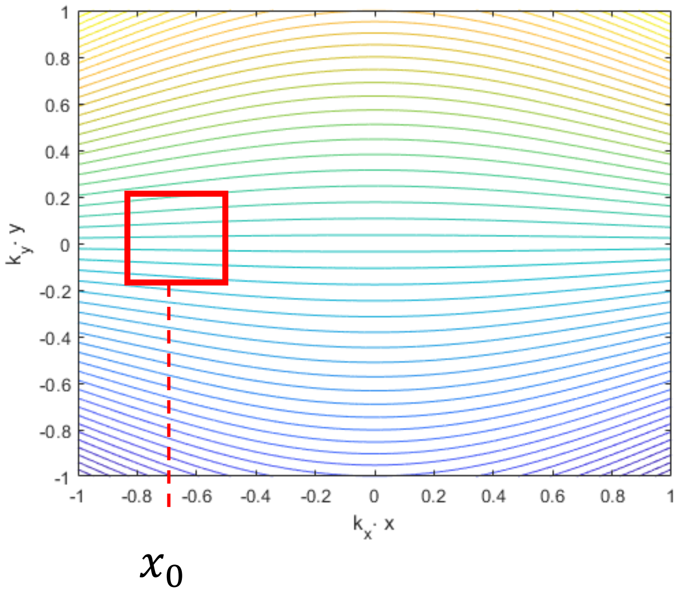
$$\Uparrow$$

$$\chi(x,y,z) \propto -\frac{B_0}{k_y} \cdot \cosh(k_x x) \cdot \sinh(k_y y) \cdot \cos(k_z z)$$

$$\frac{B_x(x,y,z)}{B_0} \propto \frac{k_x}{k_y} \cdot \sinh(k_x x) \cdot \sinh(k_y y) \cdot \cos(k_z z)$$

$$\frac{B_y(x,y,z)}{B_0} \propto \cosh(k_x x) \cdot \cosh(k_y y) \cdot \cos(k_z z)$$

$$-\frac{k_y}{B_0}\chi(x,y,z=0)$$



$$\boxed{} \Rightarrow \chi(x,y,z) \propto \cosh[k_x(x_0 + x)] \cdot \sinh(k_y y) \cdot \cos(k_z z)$$

How to implement a horizontal gradient?

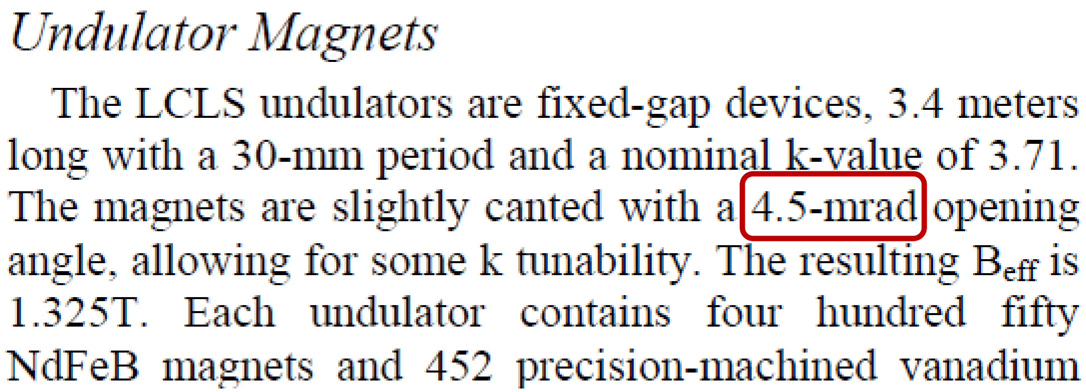
Consistent implementation of the $B_y(x)$ taper

K/K0(x) = 1 + α * x;   α =0. 761412 m^{-1}

$$\frac{B_y(x,y,z)}{B_0} \propto \cosh[k_x(x_0+x)] \approx \cosh[k_x x_0] + \sinh[k_x x_0]\cdot k_x x \sim \boxed{1+\tanh[k_x x_0]\cdot k_x} x$$

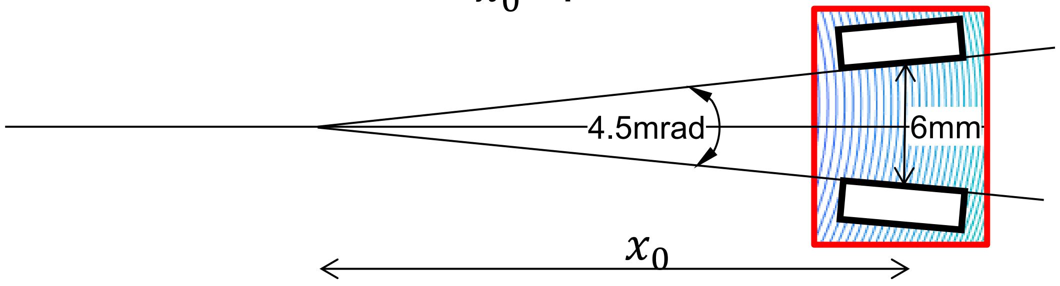
$$\alpha = \tanh[k_x x_0]\cdot k_x$$

$$x_0 - ?$$

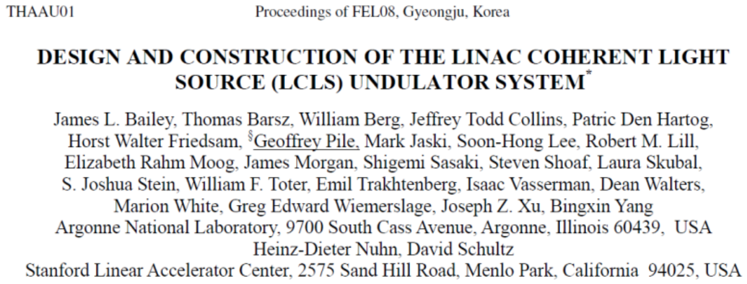


Undulator Magnets

The LCLS undulators are fixed-gap devices, 3.4 meters long with a 30-mm period and a nominal k-value of 3.71. The magnets are slightly canted with a 4.5-mrad opening angle, allowing for some k tunability. The resulting B_{eff} is 1.325T. Each undulator contains four hundred fifty NdFeB magnets and 452 precision-machined vanadium



$$x_0 = \frac{6/2mm}{\tan\left[\frac{4.5}{2}mrad\right]} \approx 1.33m$$



THAAU01   Proceedings of FEL08, Gyeongju, Korea

DESIGN AND CONSTRUCTION OF THE LINAC COHERENT LIGHT SOURCE (LCLS) UNDULATOR SYSTEM[*]

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Horst Walter Friedsam, [§]Geoffrey Pile, Mark Jaski, Soon-Hong Lee, Robert M. Lill,
Elizabeth Rahm Moog, James Morgan, Shigemi Sasaki, Steven Shoaf, Laura Skubal,
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$$\alpha = \tanh[k_x x_0]\cdot k_x \Rightarrow 0.761412 = \tanh[1.33\cdot k_x]\cdot k_x \Rightarrow k_x \approx 0.916\ m^{-1}$$

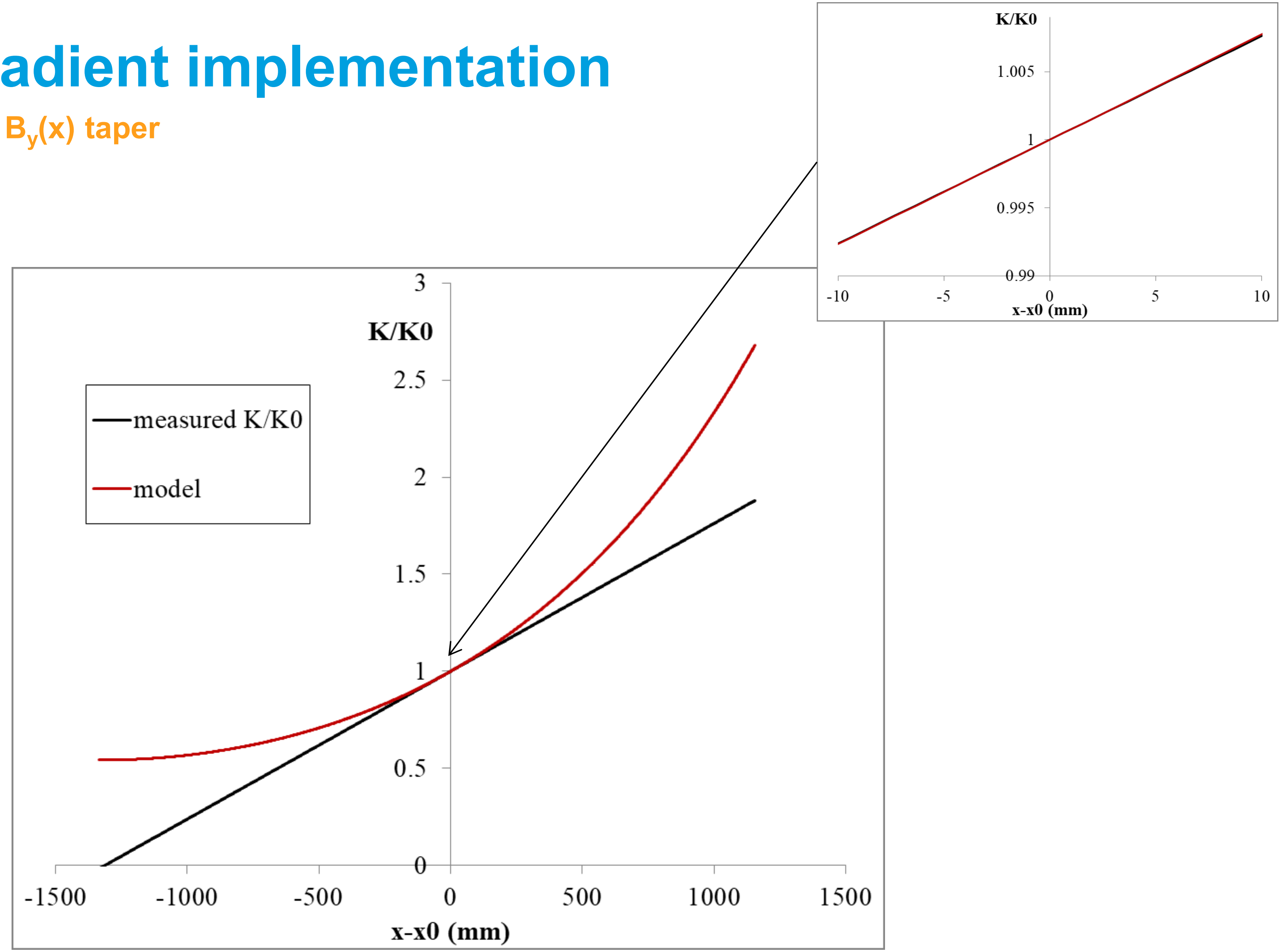
Horizontal field gradient implementation

Consistent implementation of the $B_y(x)$ taper

$$\frac{B_y(x,y,z)}{B_0} \propto \frac{\cosh[k_x(x_0+x)]}{\cosh[k_x x_0]}$$

$x_0 \approx 1.33m$
$k_x \approx 0.916\,m^{-1}$

Measured:
K/K0(x) = 1+0.000761412*x[mm]



3D Field with horizontal gradient

Based on axial field measurements and transverse taper modeling

$$\chi(x,y,z) = -\frac{\cosh[k_x(x_0+x)]}{\cosh[k_x x_0]} \cdot \sum_{n=1}^{N_h \cdot N_U} \{a_n \cos(k_{zn}z) + b_n \sin(k_{zn}z)\} \cdot \frac{\sinh(k_{yn}y)}{k_{yn}}$$

$$k_x^2 + k_{yn}^2 = k_{zn}^2$$

NB: The same x-dependence for all modes!

$$B_x(x,y,z) = \frac{\sinh[k_x(x_0+x)]}{\cosh[k_x x_0]} \cdot \sum_{n=1}^{N_h \cdot N_U} \{a_n \cos(k_{zn}z) + b_n \sin(k_{zn}z)\} \cdot \frac{k_x}{k_{yn}} \cdot \sinh(k_{yn}y)$$

$$B_y(x,y,z) = \frac{\cosh[k_x(x_0+x)]}{\cosh[k_x x_0]} \cdot \sum_{n=1}^{N_h \cdot N_U} \{a_n \cos(k_{zn}z) + b_n \sin(k_{zn}z)\} \cdot \cosh(k_{yn}y)$$

$$B_z(x,y,z) = \frac{\cosh[k_x(x_0+x)]}{\cosh[k_x x_0]} \cdot \sum_{n=1}^{N_h \cdot N_U} \{-a_n \sin(k_{zn}z) + b_n \cos(k_{zn}z)\} \cdot \frac{k_{zn}}{k_{yn}} \cdot \sinh(k_{yn}y)$$

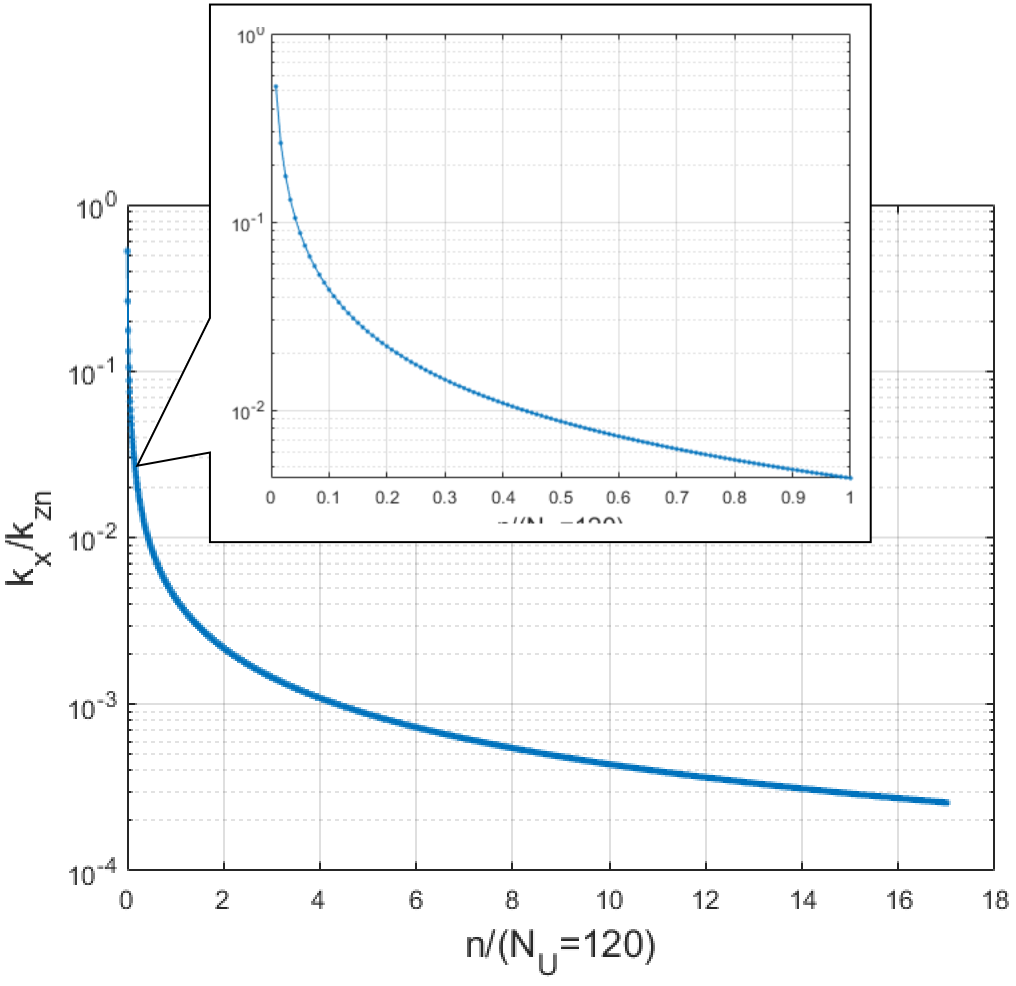
$$x_0 \approx 1.33m$$
$$k_x \approx 0.916\, m^{-1}$$

$$k_{zn} = \frac{2\pi n}{N_U \lambda_U} \qquad \lambda_U = 30mm \qquad k_{zN_U} \cong 209\, m^{-1}$$

$$k_{yn} = \sqrt{k_{zn}^2 - k_x^2}$$

3D Field with horizontal gradient: transverse wave vectors

Based on axial field measurements and transverse taper modeling

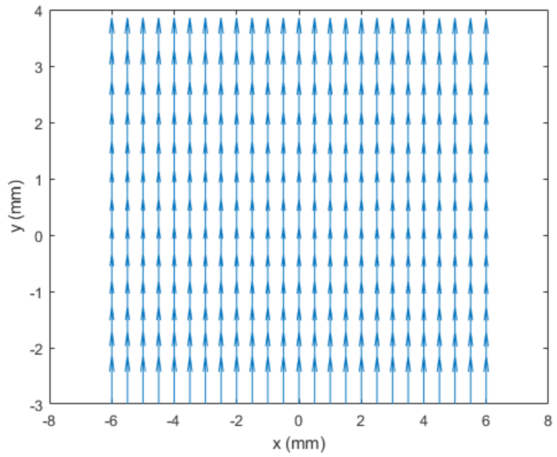


$$k_x \approx 0.916\, m^{-1}$$

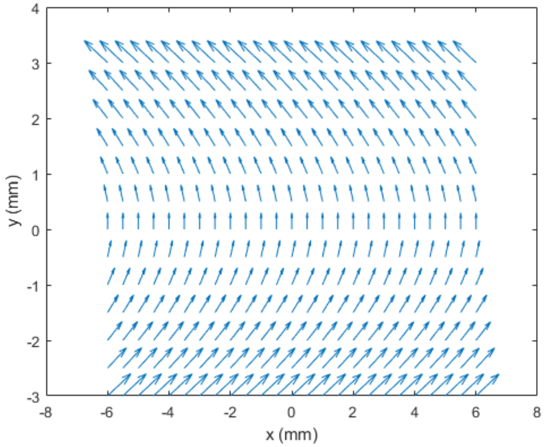
$$k_{zn} = \frac{2\pi n}{N_U \lambda_U}$$

$$k_{yn} = \sqrt{k_{zn}^2 - k_x^2} \sim k_{zn}$$

$B_x(x, y, z = 0.64m)$; $B_y(x, y, z = 0.64m)$



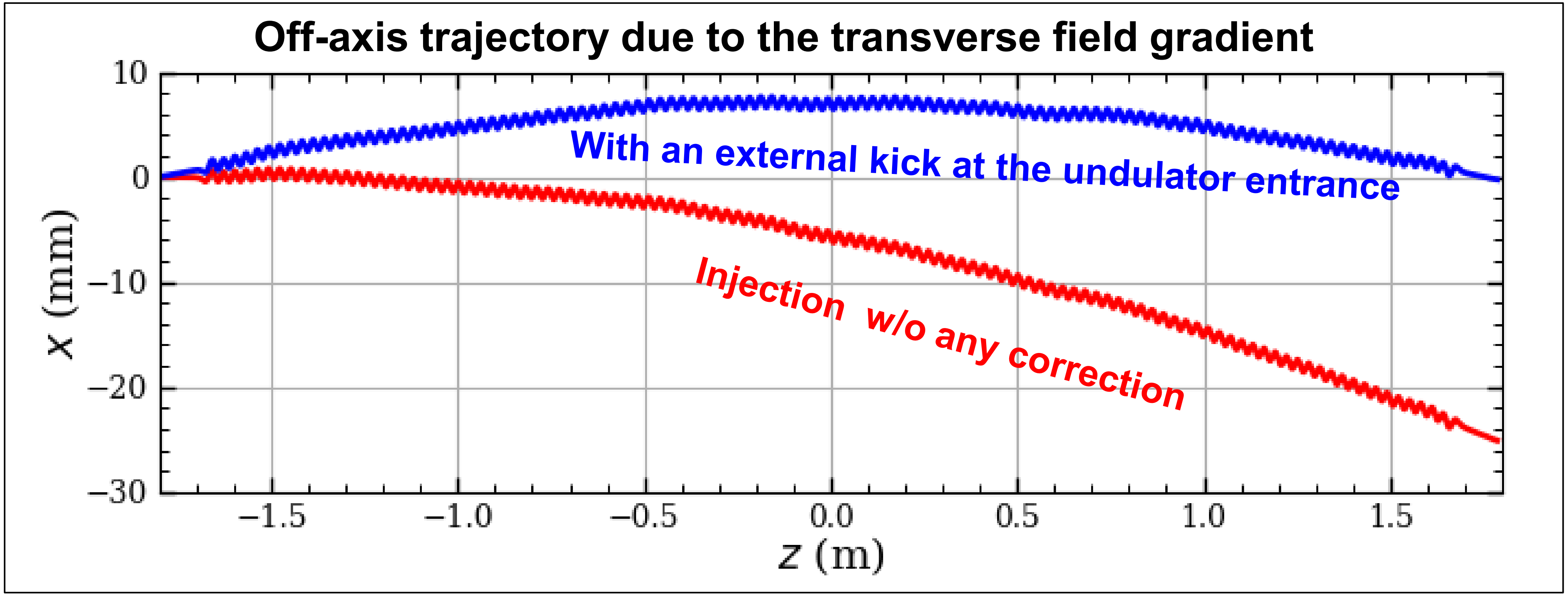
$B_x(x, y, z = 0.64m) \times \mathbf{1000}$; $B_y(x, y, z = 0.64m)$

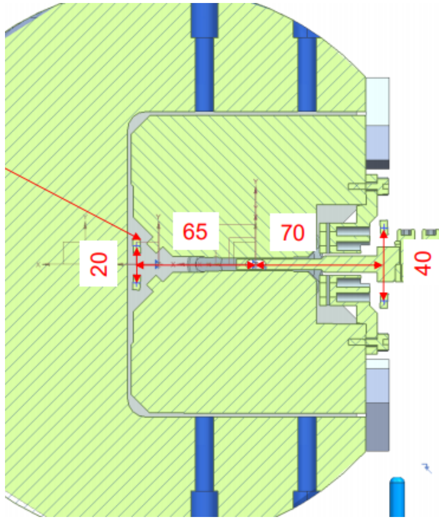


Design and modeling of correction coils

Horizontal undulator gradient impact onto beam transport

- **Transverse gradien**t will lead to an off-axis (~25 mm) trajectory (~17MeV) in the horizontal plane





?**steering coils are** considered to correct it

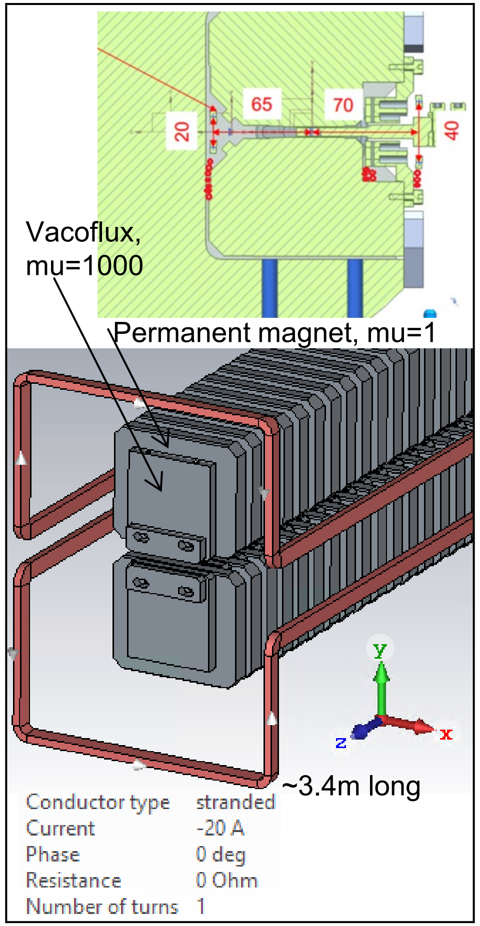
Courtesy Xiangkun Li

Courtesy Anusorn Lueangaramwong

Air Coil Field Simulated by CST2020

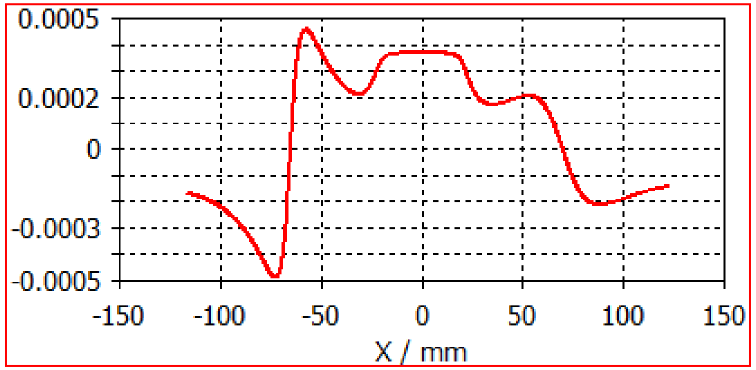
Use of Hexahedral mesh model and field results

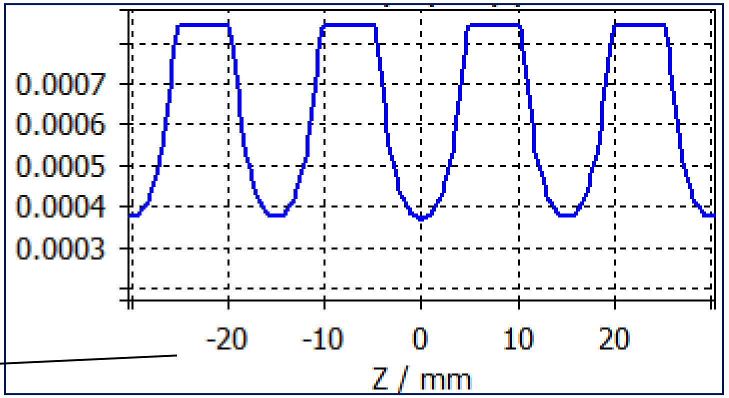
- We simulated magnetic field of the air coil with a model including
  - Permanent magnet holders with relative permeability of 1000
- The air coil ends are lifted to lower edge field
- Permanent magnet holders are periodic and provide field enhancement periodically along z-axis



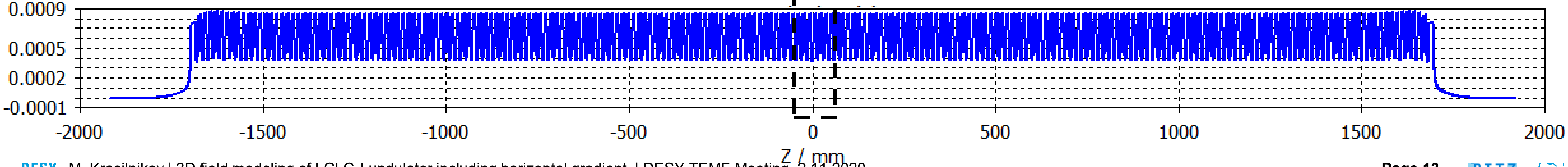
Conductor type	stranded
Current	-20 A
Phase	0 deg
Resistance	0 Ohm
Number of turns	1

- By (T) vs x (mm) @y=0,z=0,I =20A





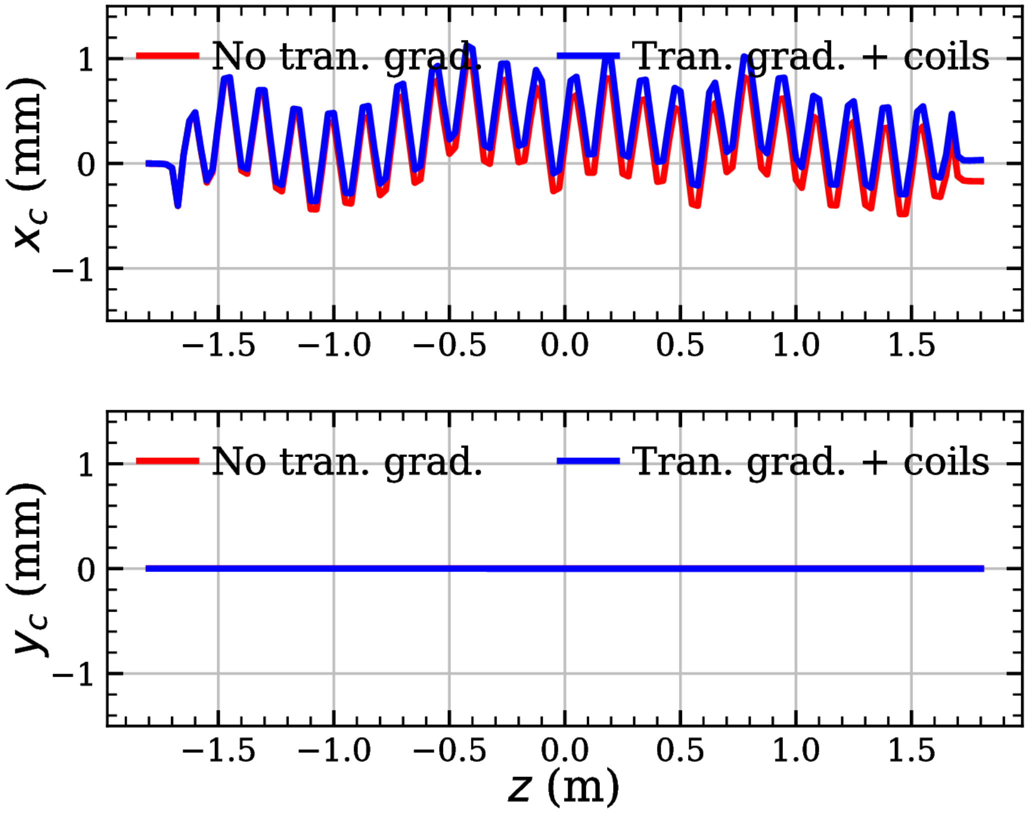
- By (T) vs z (mm) @x=0,y=0, I =20A



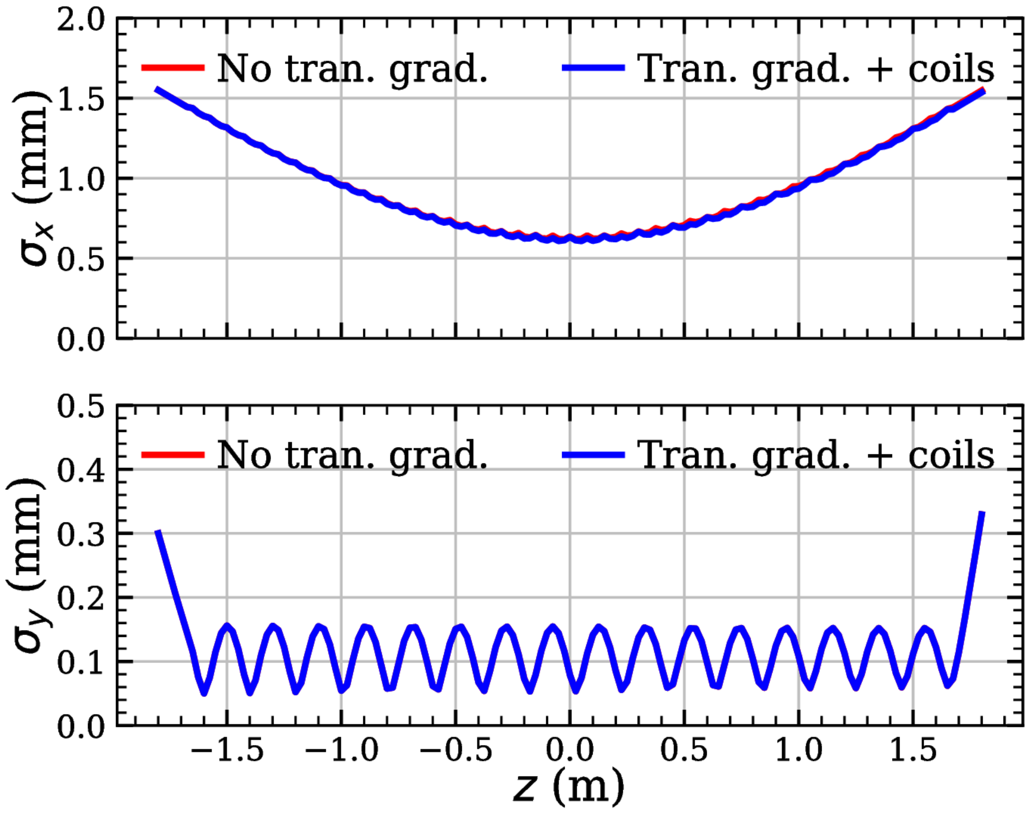
Particle tracking with correction coils by Astra

Courtesy Xiangkun Li

Maximum on-axis field of By is 322.9 uT (7.4 A turns)



Beam centroid along the undulator

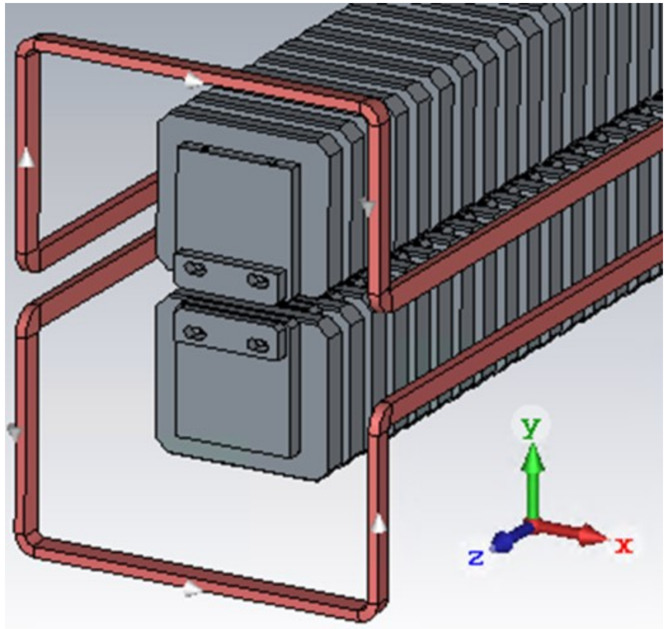


RMS beam size along the undulator

Design, modeling and preliminary test of correction coils

Iron in the undulator boosts the magnetic field of the steering coils significantly

CST Modeling (Anusorn Lueangaramwong, PITZ)



Maximum on-axis of By is 322.9 uT (7.4 A turns) for a good transport of 17MeV beam

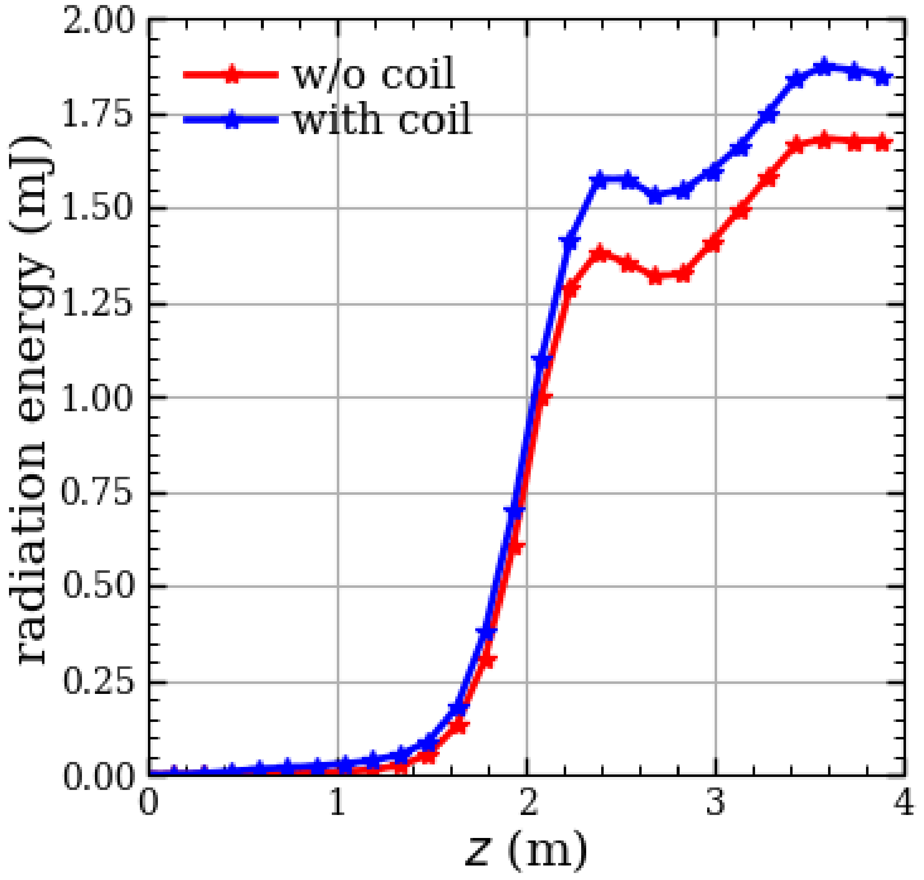
Test Setup at DESY in Hamburg (Pavel Vagin, FS-US)

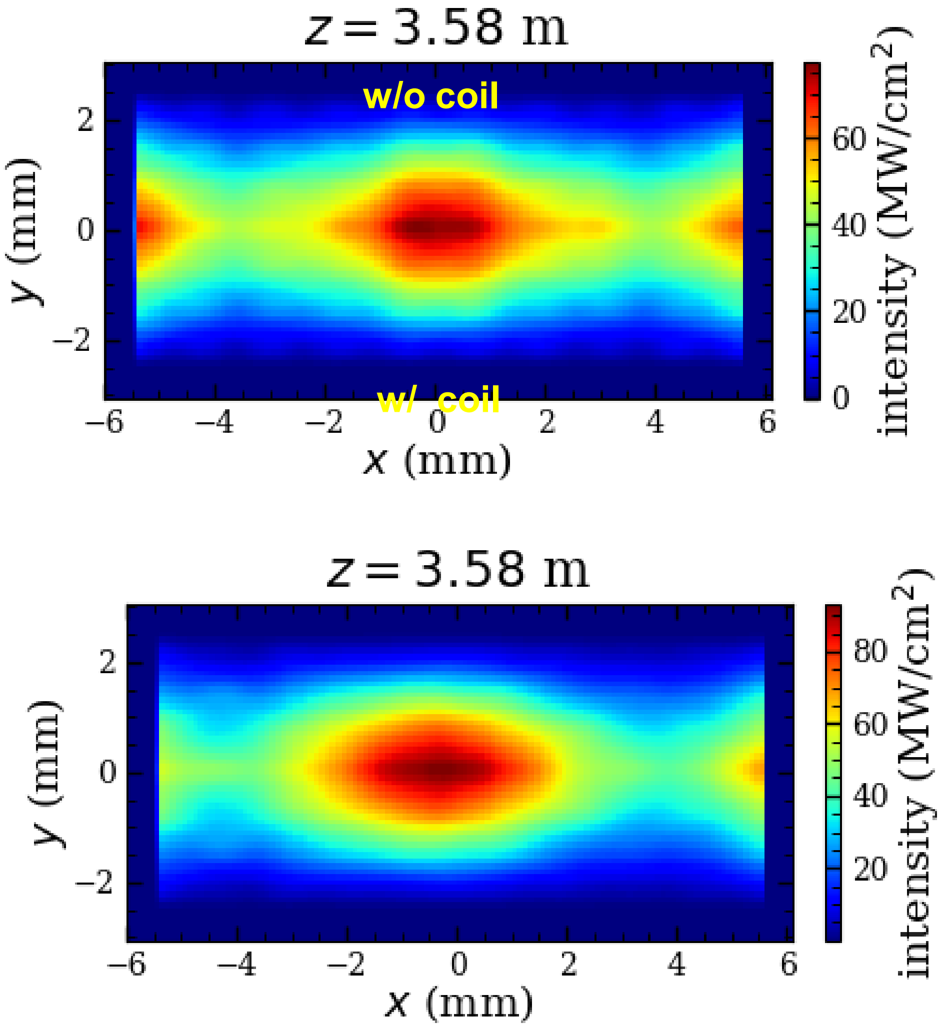
Preliminary measurements with a single wire with 10A → estimated field 140 uT

Horizontal undulator gradient impact onto THz SASE FEL

W/o and with coils

FEL simulations by Warp (Xiangkun Li)





Conclusions

3D field map reconstruction based on axial field measurements

- Method to reconstruct a 3D field map of the LCLS-I undulator including horizontal gradient (taper) has been proposed
- It is based on magnetic potential approach using $cosh(k_x*x)$ dependence assuming a significant offset from the field center ($|x|<<x_0$)
- In order to estimate taper parameters (k_x and x_0) a 4.5 mrad opening angle of the canted magnets was utilized
- This resulted in a 3D field model, where all 3 field components (B_x, B_y, B_z) are treated in a consistent way
  - For x_0 =1.33 m and k_x = 0.9 m^{-1} → $k_x << k_{zn}$ → the field horizontal gradient (taper) is fairly weak
- A matlab script for the 3D field map generation has been created (supporting formats: ASTRA cavity field and CST external field)
- Transverse gradient will lead to an off-axis (~25 mm) trajectory in the horizontal plane
- Steering coils are under design to correct the steering effect, preliminary experimental tests at DESY in Hamburg have been done
- Simulations of beam trajectory (ASTRA) and THz SASE FEL (WARP) with coils → should work!