Dipole Mode Simulations for a TESLA 1.3 GHz Cavity

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Outline

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- Motivation
- Dipole-Mode Eigenvalue Calculation
  - Electromagnetic field distribution
  - Single point-charge excitation using shifted trajectories
- Dipole-Mode Excitation
  - Excited complex-valued amplitude wrt. shifted and tilted trajectories
  - Magnitude fitting as used in the experimental data evaluations
  - Sensitivity wrt. shifted and tilted trajectories
- Summary / Outlook

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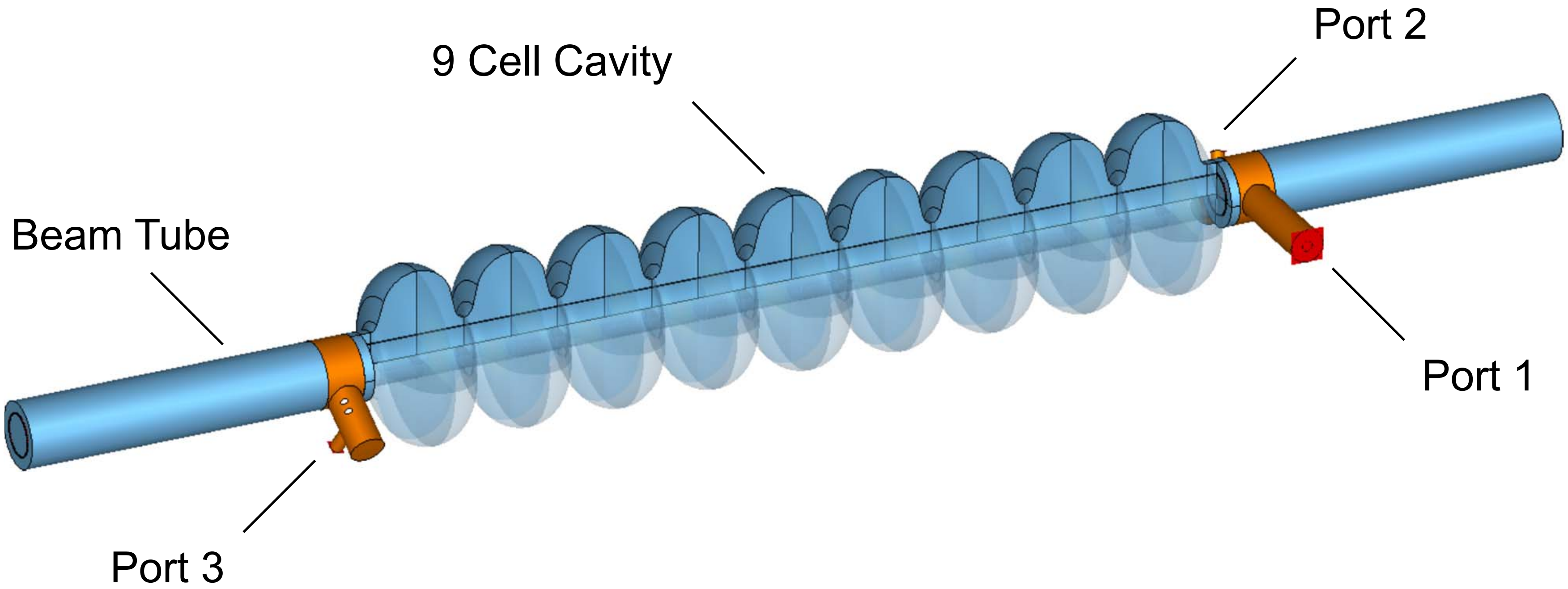
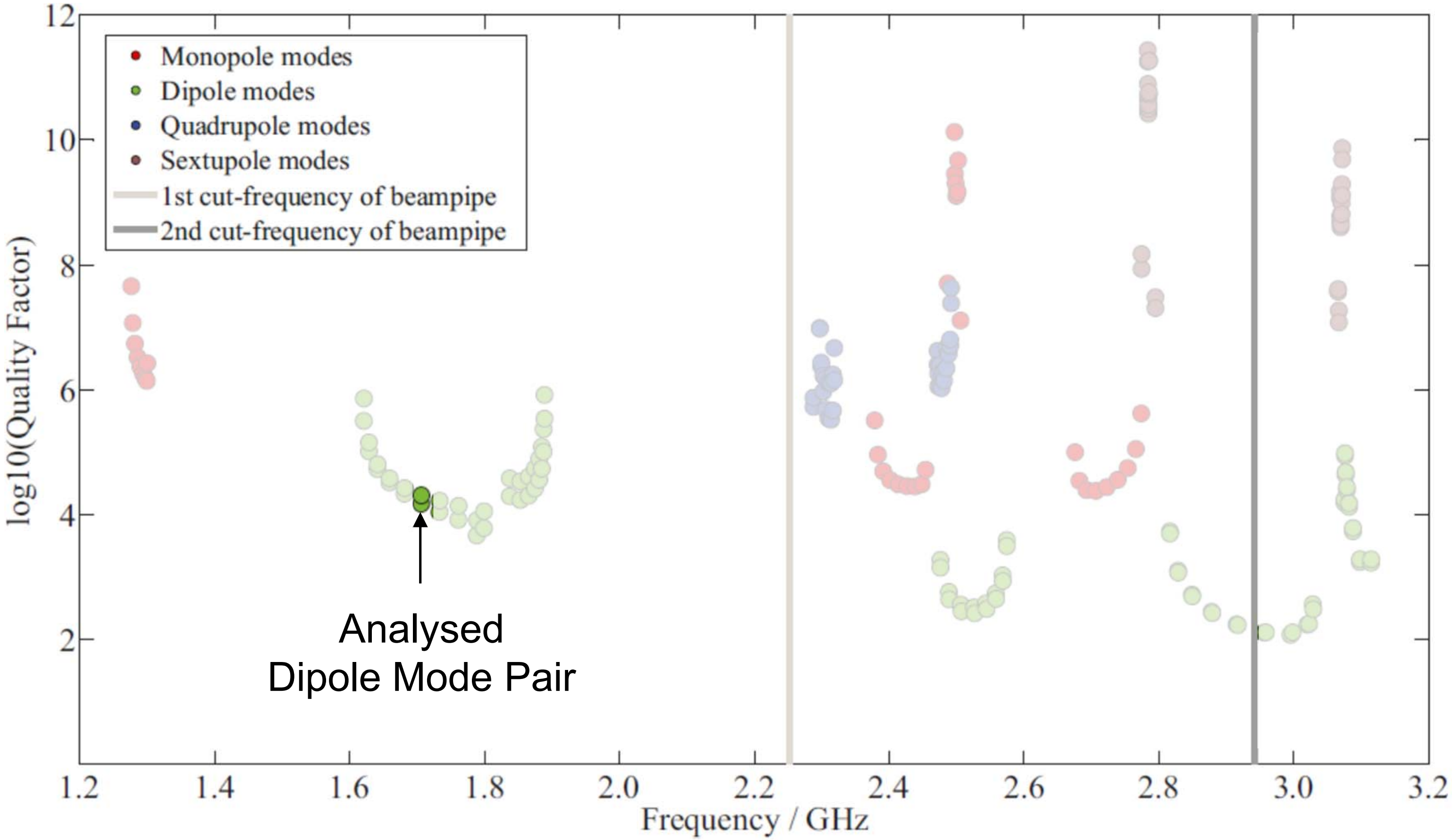
Motivation

- Computational Model
  - 9 Cell Cavity (symmetric mesh in high-field area)
  - Input and HOM Couplers included (local grid refinement)



Motivation

- Mode Compendium



Motivation

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- Mode Compendium (1st + 2nd dipole passband)

Number	Frequency in GHz	Rz / Q in Ohm	Rz in Ohm
1	1.6207	5.08E-07	1.60E-01
2	1.6207	2.79E-08	2.01E-02
3	1.6282	2.59E-06	2.66E-01
4	1.6282	1.63E-07	2.33E-02
5	1.6409	2.86E-07	1.52E-02
6	1.6410	2.63E-06	1.67E-01
7	1.6586	7.00E-06	2.29E-01
8	1.6588	1.03E-06	3.91E-02
9	1.6807	2.57E-06	5.54E-02
10	1.6809	7.19E-06	1.90E-01
11	**1.7061**	**1.77E-05**	**2.65E-01**
12	**1.7064**	**1.36E-06**	**2.76E-02**
13	1.7335	1.53E-05	1.68E-01
14	1.7338	4.00E-06	6.63E-02
15	1.7614	7.19E-06	5.92E-02
16	1.7617	1.13E-05	1.55E-01
17	1.7886	6.18E-05	2.85E-01
18	1.7890	2.02E-05	1.64E-01

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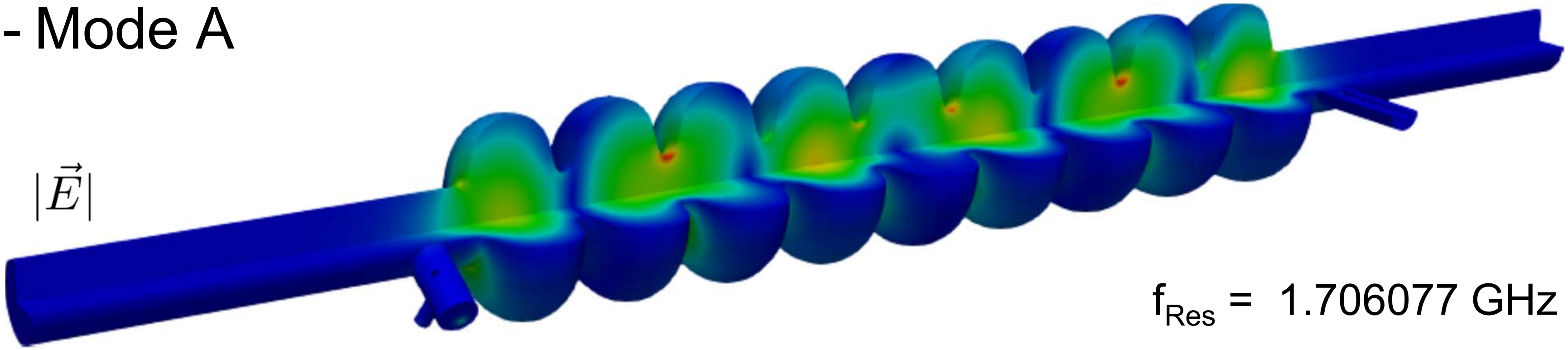
Dipole-Mode Eigenvalue Calculation

- Field Distributions

3.478.532 curved tetrahedrons
21.770.358 complex-valued DOF

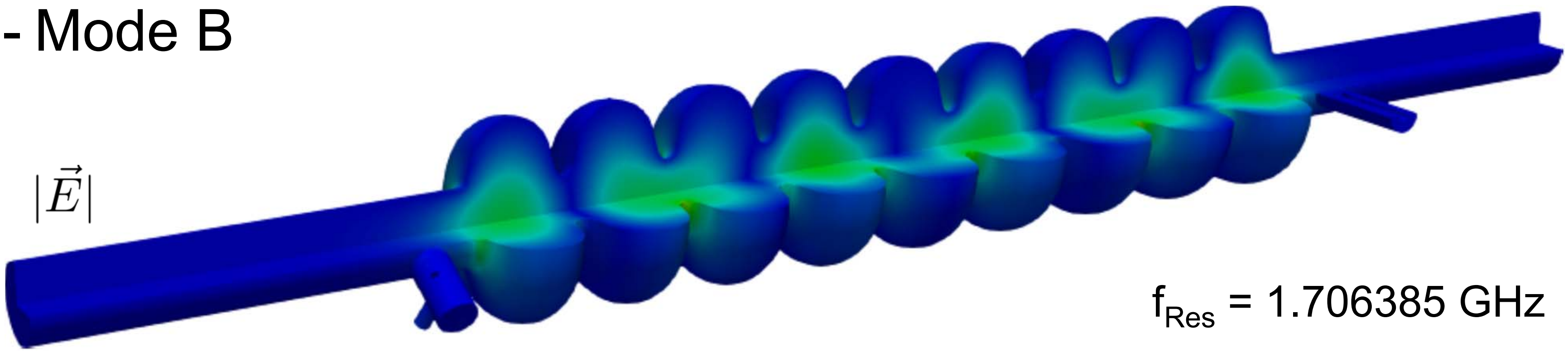
  - Mode A



$|\vec{E}|$

f_{Res} = 1.706077 GHz

  - Mode B



$|\vec{E}|$

f_{Res} = 1.706385 GHz

Dipole-Mode Eigenvalue Calculation

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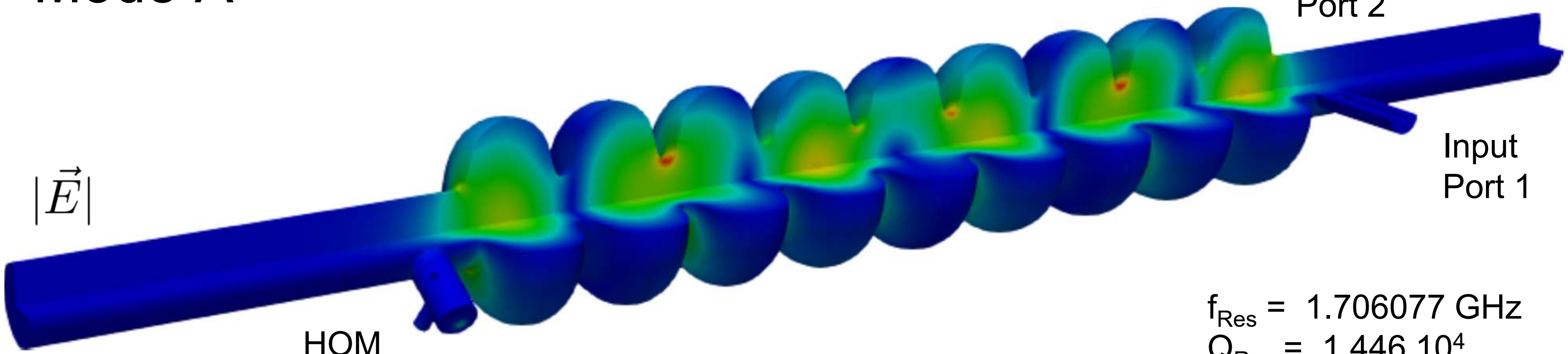
- Extracted Power at Ports
  - Mode A



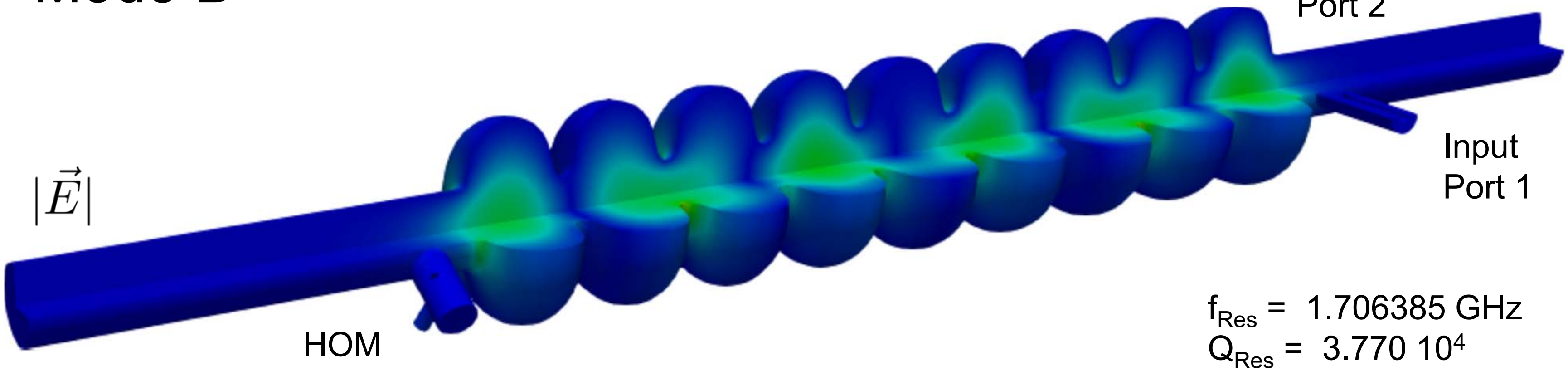
$|\vec{E}|$

f_{Res} = 1.706077 GHz
Q_{Res} = 1.446 10^4
W_{Res} = 1 Ws

Port = 1	Port = 2	Port = 3
P_1 = (**2.224e+01**, 9.474e-14) W	P_1 = (**5.571e+05**, 2.878e-10) W	P_1 = (**1.841e+05**, -1.946e-10) W
P_2 = (-2.270e-07, 5.196e-03) W	P_2 = (-1.424e-07, 4.001e-03) W	P_2 = (-2.203e-09, 6.189e-05) W
P_3 = (-1.192e-06, 2.729e-02) W	P_3 = (-5.345e-07, 1.501e-02) W	P_3 = (-1.453e-07, 4.082e-03) W
P_4 = (-2.578e-16, 7.020e-12) W	P_4 = (-3.865e-09, 1.108e-04) W	P_4 = (-1.555e-10, 4.457e-06) W
P_5 = (-1.646e-15, 4.481e-11) W	P_5 = (-7.305e-10, 2.094e-05) W	P_5 = (-8.945e-11, 2.564e-06) W

Dipole-Mode Eigenvalue Calculation

- Extracted Power at Ports
  - Mode B

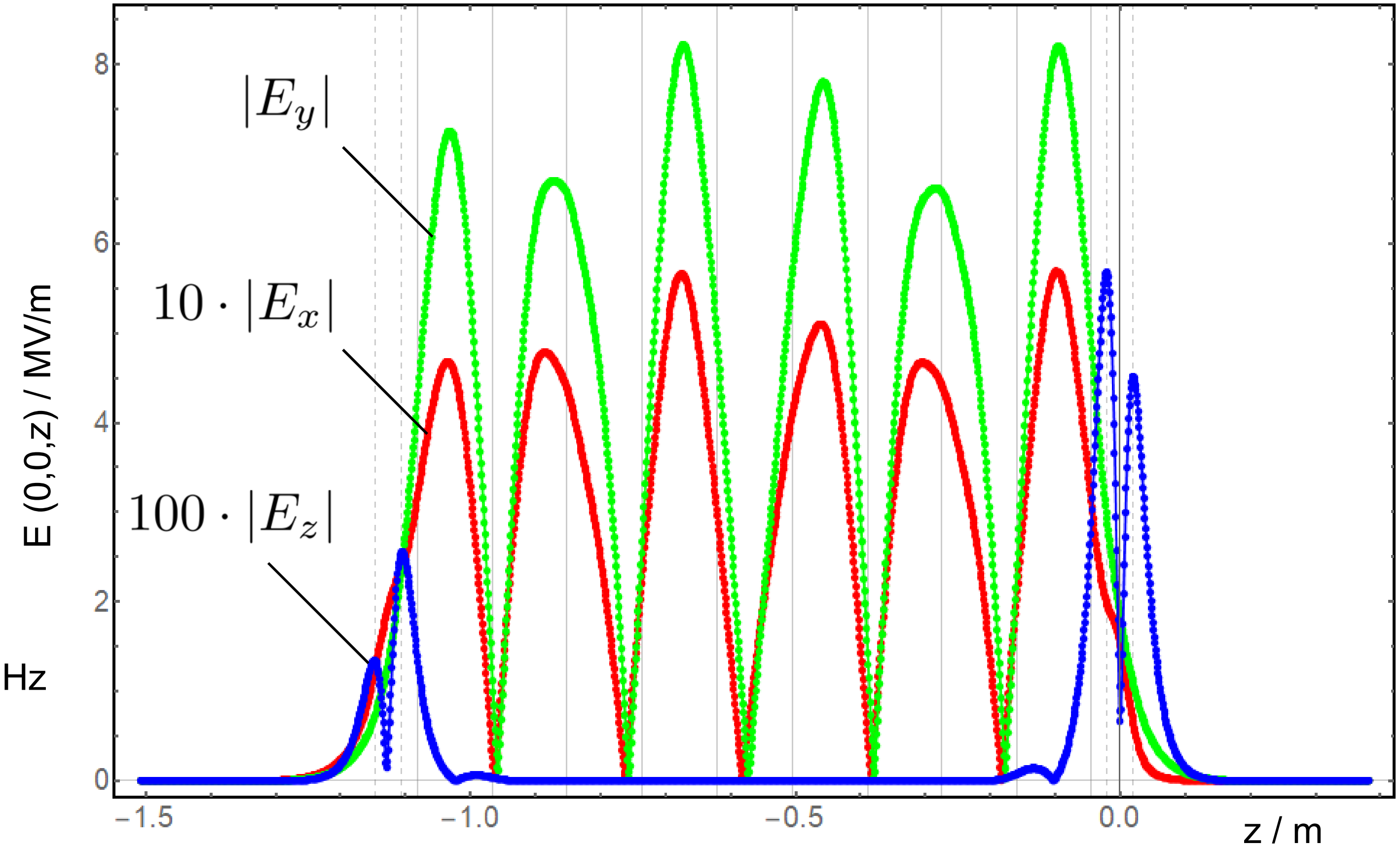


f_{Res} = 1.706385 GHz
Q_{Res} = 3.770 10^4
W_{Res} = 1 Ws

Port = 1	Port = 2	Port = 3
P_1 = (**3.681e+03**, 9.701e-12) W	P_1 = (**4.377e+04**, -1.388e-10) W	P_1 = (**2.369e+05**, 1.511e-09) W
P_2 = (-3.064e-08, 1.829e-03) W	P_2 = (-2.912e-09, 2.132e-04) W	P_2 = (-8.103e-10, 5.934e-05) W
P_3 = (-1.934e-08, 1.154e-03) W	P_3 = (-1.890e-08, 1.384e-03) W	P_3 = (-6.904e-08, 5.056e-03) W
P_4 = (-4.544e-16, 3.225e-11) W	P_4 = (-1.165e-10, 8.706e-06) W	P_4 = (-7.705e-11, 5.757e-06) W
P_5 = (-9.724e-14, 6.902e-09) W	P_5 = (-2.216e-11, 1.656e-06) W	P_5 = (-4.430e-11, 3.310e-06) W

Dipole-Mode Eigenvalue Calculation

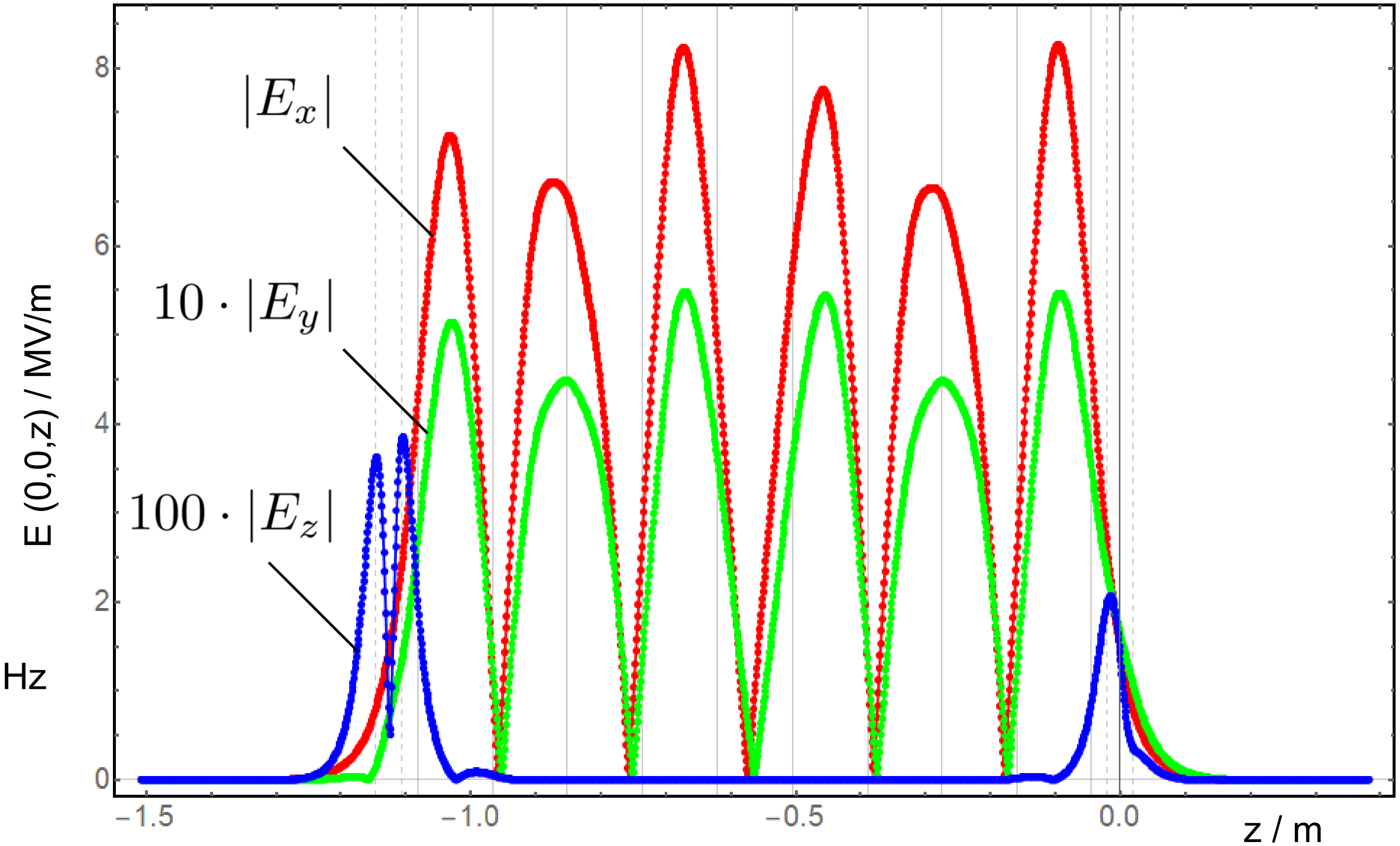
- Field Distribution along the Cavity Axis
  - Mode A



f_{Res} = 1.706077 GHz
Q_{Res} = $1.446\ 10^4$
W_{Res} = 1 Ws

Dipole-Mode Eigenvalue Calculation

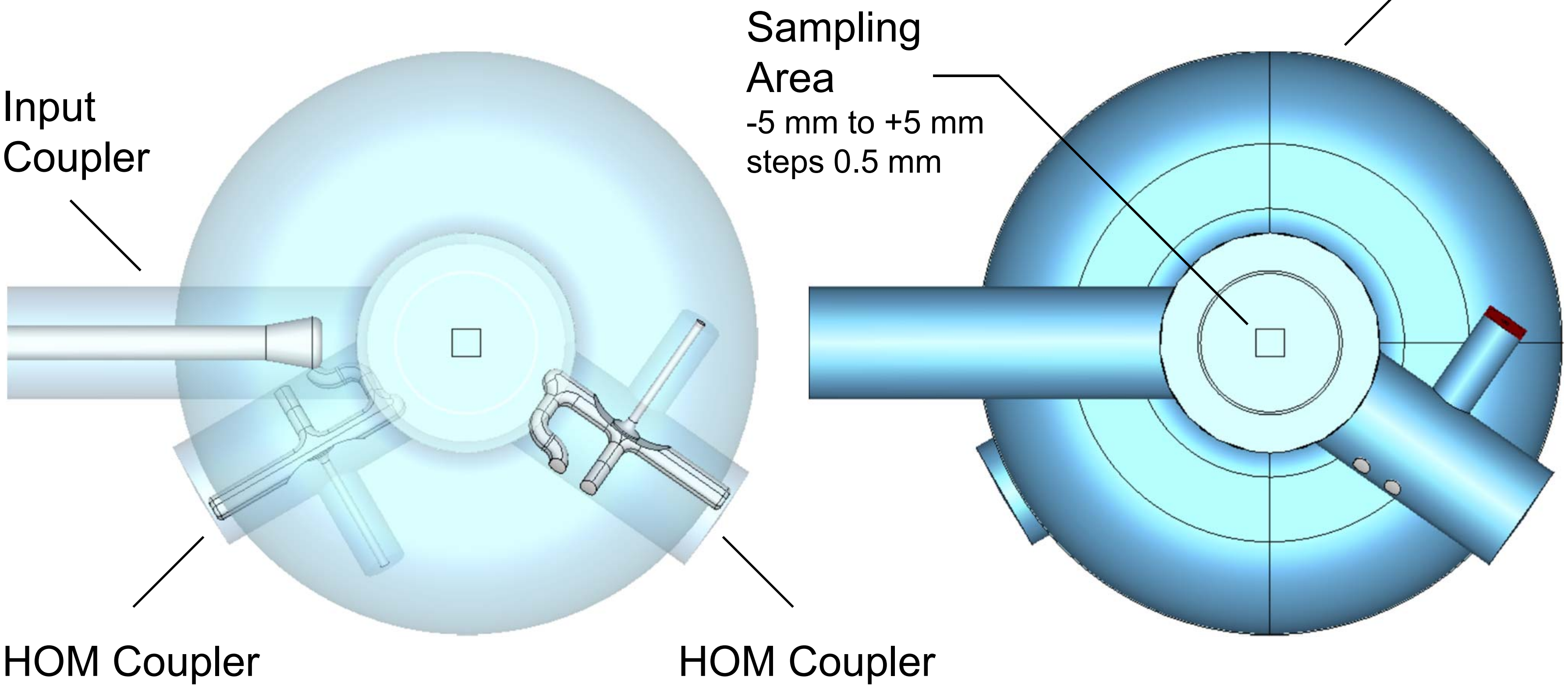
- Field Distribution along the Cavity Axis
  - Mode B



f_{Res} = 1.706385 GHz
Q_{Res} = 3.770 10^4
W_{Res} = 1 Ws

Dipole-Mode Eigenvalue Calculation

- Integration Path used for Field Excitation
  - Sampling of the field data



Dipole-Mode Eigenvalue Calculation

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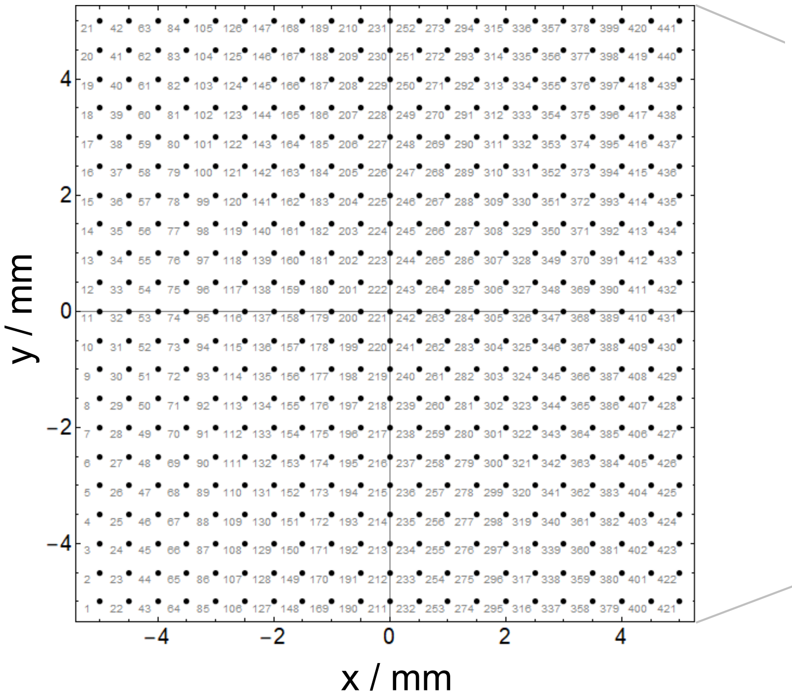

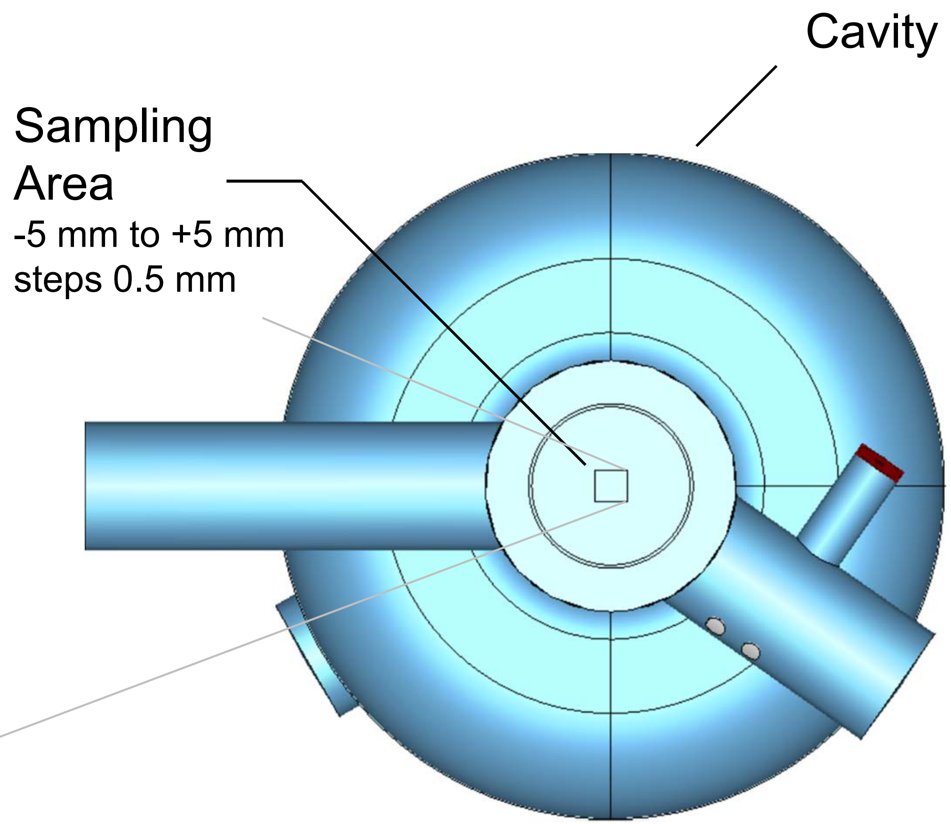

Dipole-Mode Eigenvalue Calculation

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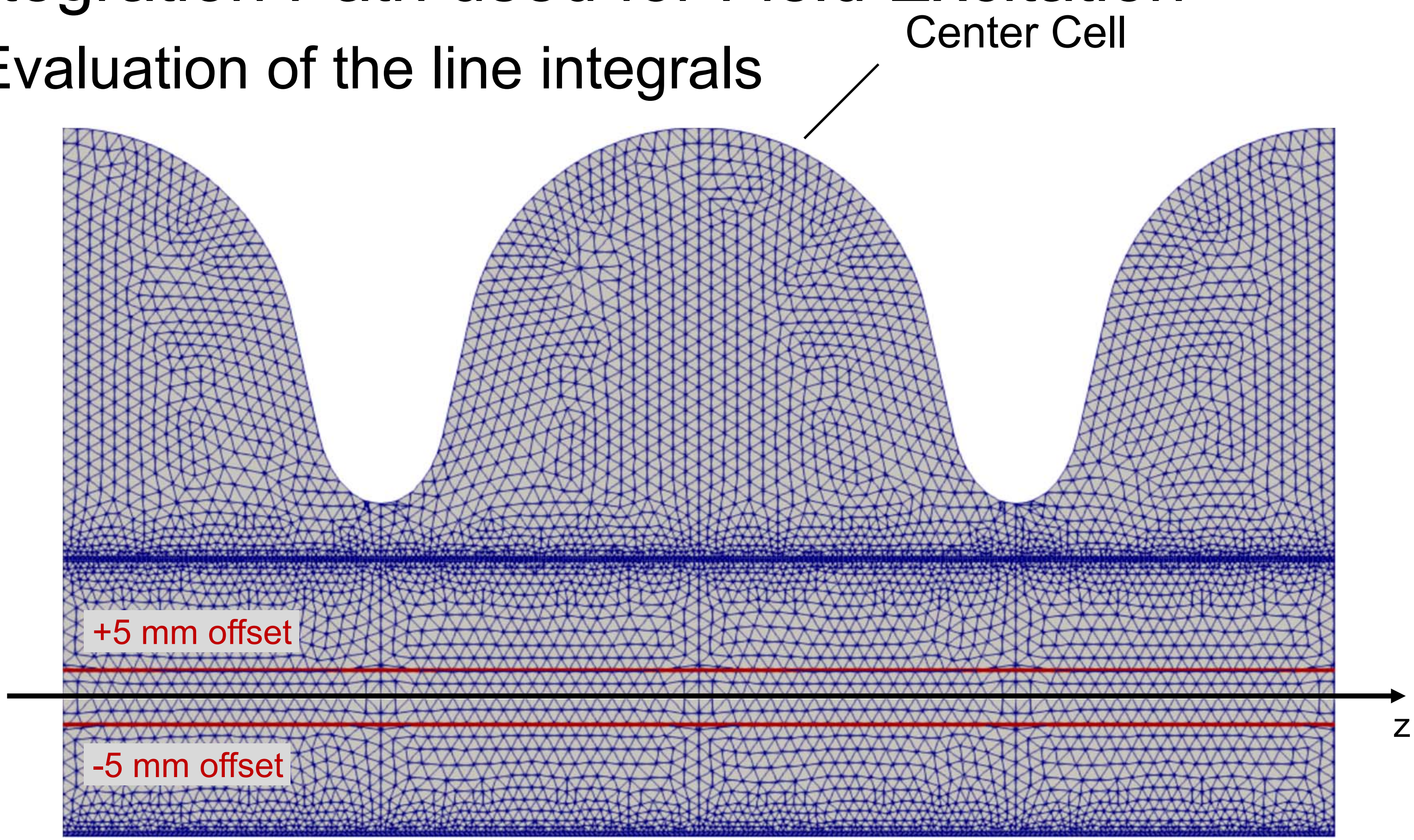
- Integration Path used for Field Excitation
  - Evaluation of the line integrals



Computational Modeling

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- Maxwell's Equations

$$\mathrm{curl}\,\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\mathrm{curl}\,\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\mathrm{div}\,\vec{D} = \varrho$$

$$\mathrm{div}\,\vec{B} = 0$$

$$\vec{D} = \varepsilon \vec{E}$$

$$\vec{B} = \mu \vec{H}$$

$$\frac{\partial \varepsilon}{\partial t} = 0$$

$$\frac{\partial \mu}{\partial t} = 0$$

https://de.wikipedia.org

Combine both curl equations:

$$\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} + \mathrm{curl}(\frac{1}{\mu}\,\mathrm{curl}\vec{E}) = -\frac{\partial \vec{J}}{\partial t}$$

Ansatz:

$$\vec{E}(\vec{r},t) = \vec{E}_0(\vec{r},t) + \sum_{\nu=1}^{\infty} \alpha_\nu(t)\,\vec{E}_\nu(\vec{r})$$

Computational Modeling

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- Time Evolution of the Electromagnetic Field
  - Excited mode magnitude and phase

$$\underline{c}_\nu = \frac{-i}{\omega_\nu} \int_{-\infty}^{\infty} f_\nu(\tau)\, e^{-i\omega_\nu \tau}\, \mathrm{d}\tau \qquad f_\nu(t) = -\frac{1}{2W} \iiint_V \frac{\partial \vec{J}}{\partial t} \cdot \vec{E}_\nu \,\mathrm{d}V$$

  - Simplification using a point-charge excitation

$$\vec{J} = \delta(x-x_0)\,\delta(y-y_0)\,\delta(z-vt)\; j_0\, \vec{e}_z = \varrho\, v\, \vec{e}_z \qquad [j_0] = \mathrm{Am}$$

$$\varrho = \delta(x-x_0)\,\delta(y-y_0)\,\delta(z-vt)\; q_0 \qquad [q_0] = \mathrm{As}$$

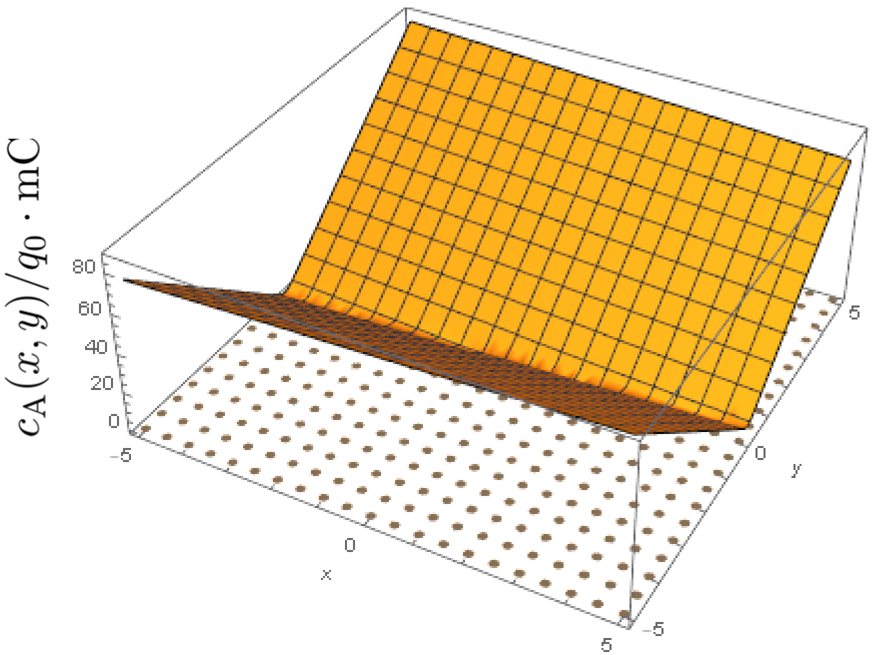
$$[\underline{c}_\nu] = 1$$

$$\underline{c}_\nu = \frac{-q_0}{2W} \int_{-\infty}^{\infty} (\vec{e}_z \cdot \vec{E}_\nu)\, e^{-i\omega_\nu \frac{z}{v}}\, \mathrm{d}z = q_0\, \frac{\underline{U}_0}{2W} \qquad \alpha_\nu(t) = \Re(\underline{c}_\nu\, e^{i\omega_\nu t})$$

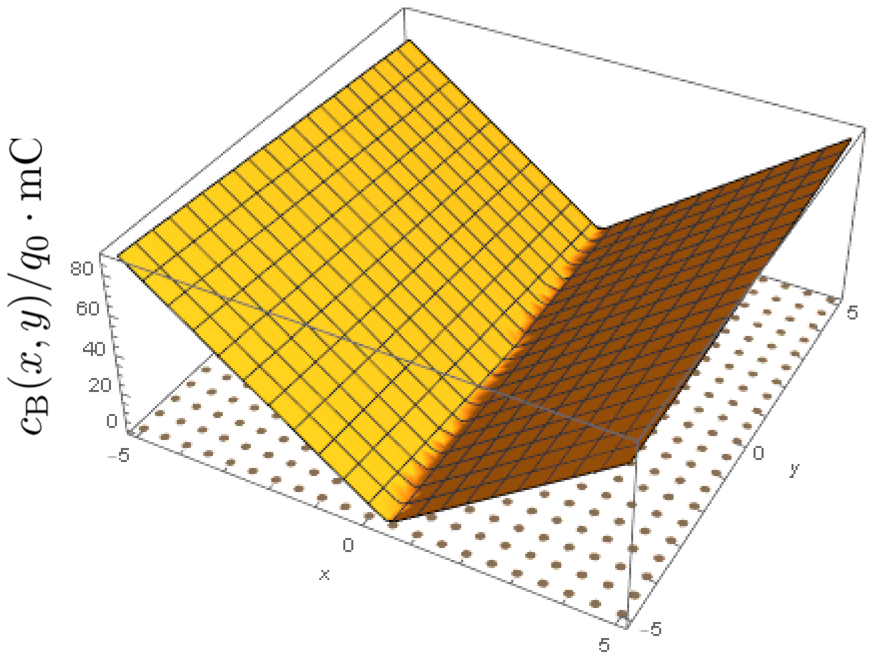
Dipole-Mode Excitation

- Excited Mode Amplitude
  - Single point-charge excitation at various offsets

Mode A



Mode B



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- Excited Mode Amplitude
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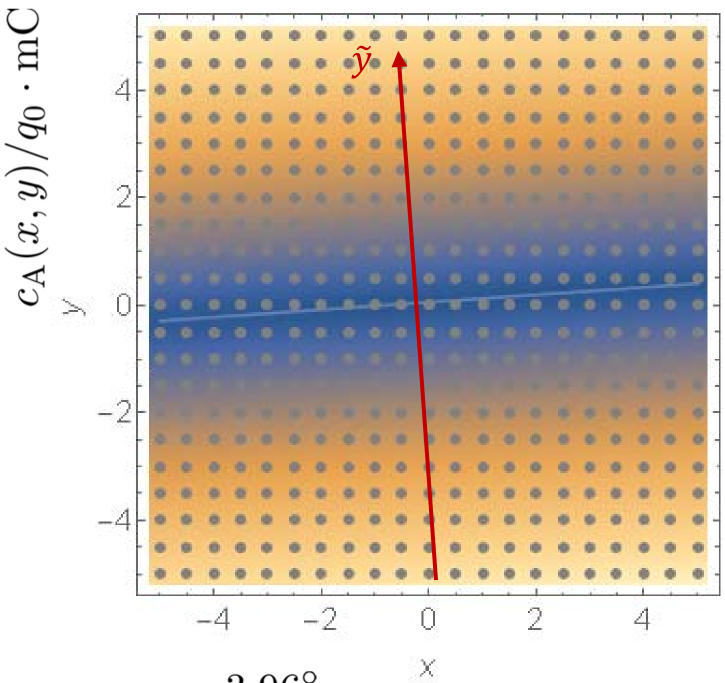
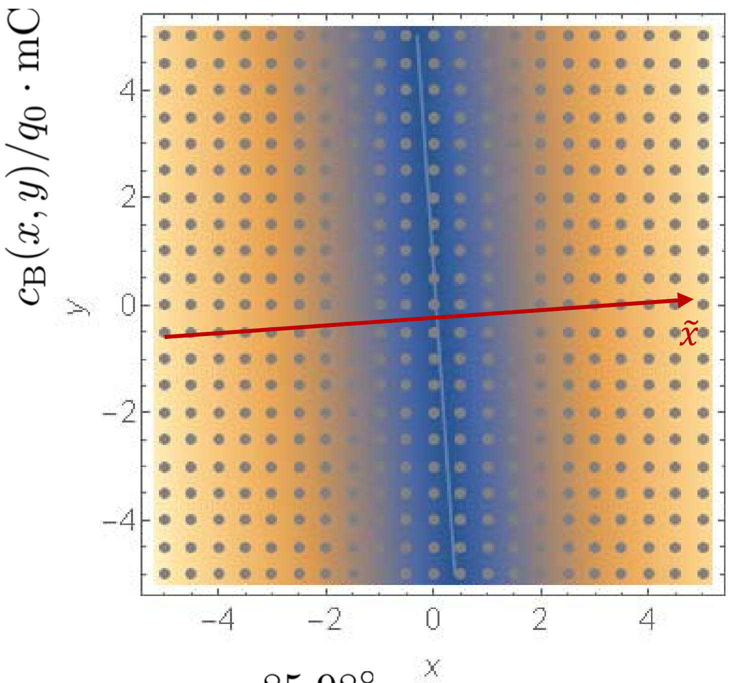
Mode A



Mode B



$\varphi_{\mathrm{A}} = 3.96°$ $\quad \varphi_{\mathrm{A}} - \varphi_{\mathrm{B}} = 89.94°$ $\quad \varphi_{\mathrm{B}} = -85.98°$

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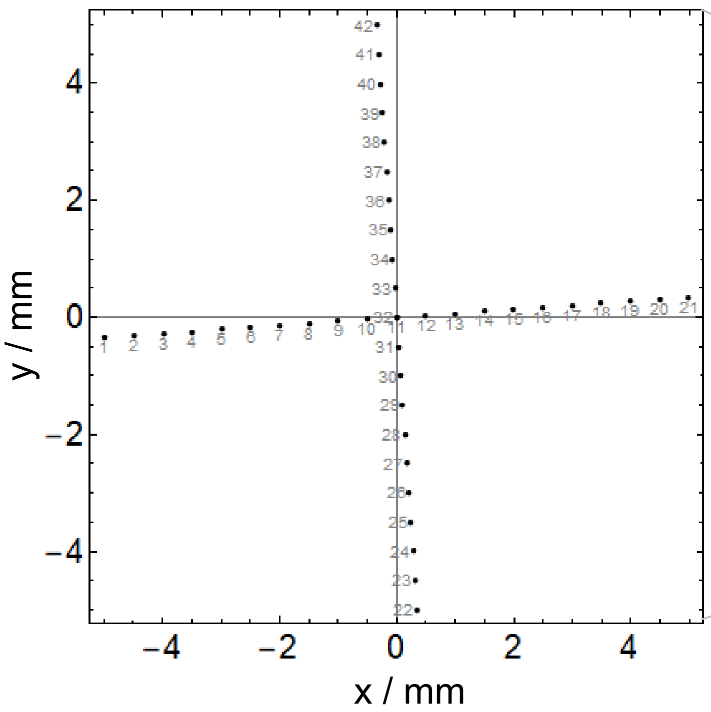
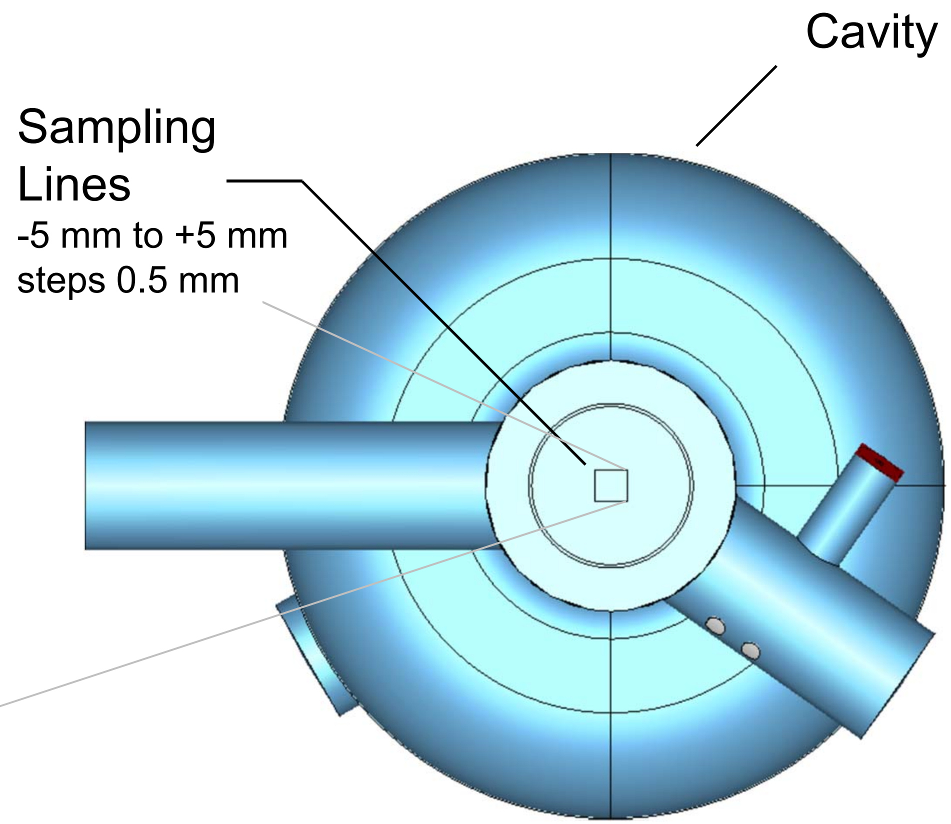
Dipole-Mode Excitation

- Integration Path used for Field Excitation
  - Sampling of the field data

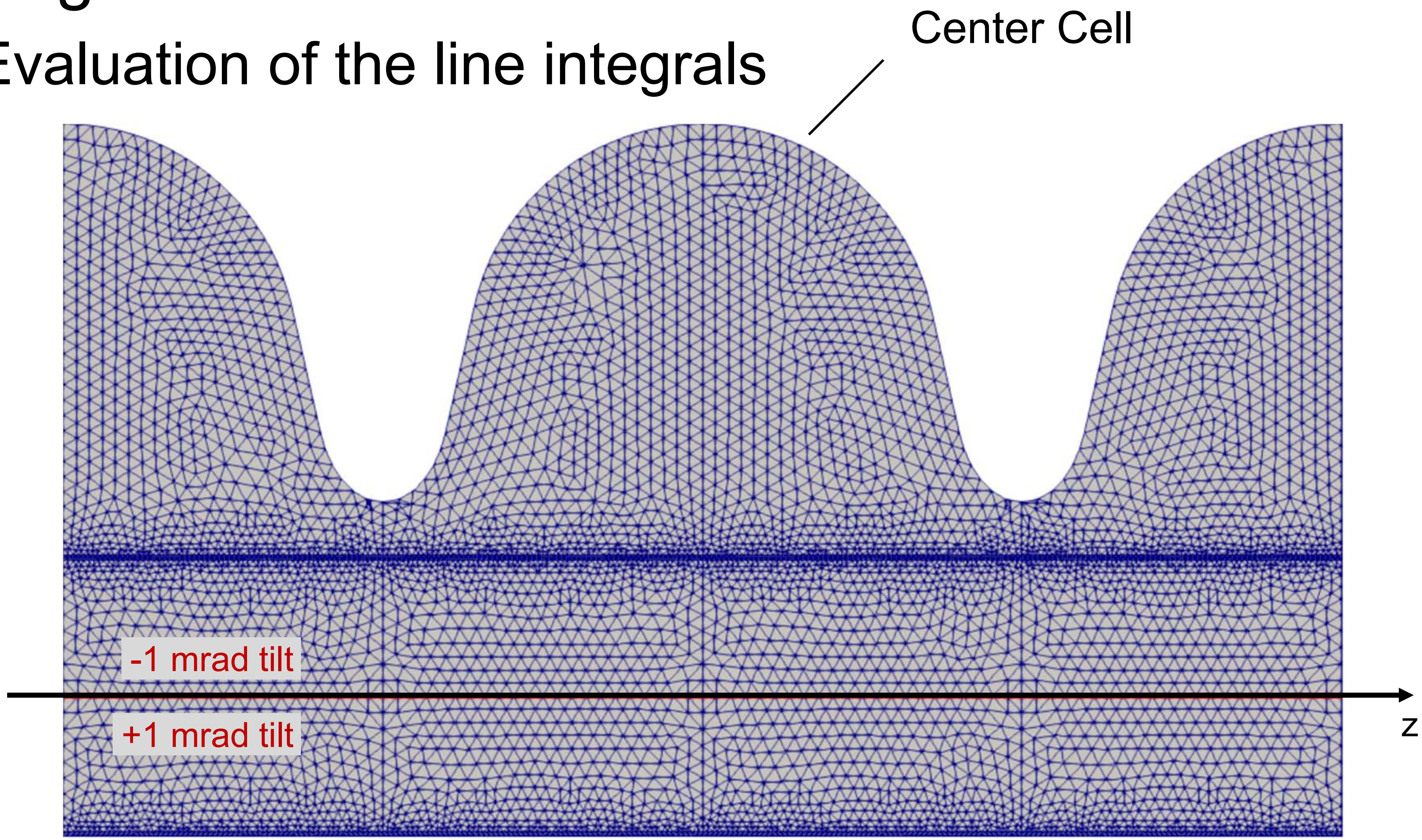




Each line is tilted within the rotated coordinates from -1 mrad to +1 mrad in steps of 0.1 mrad

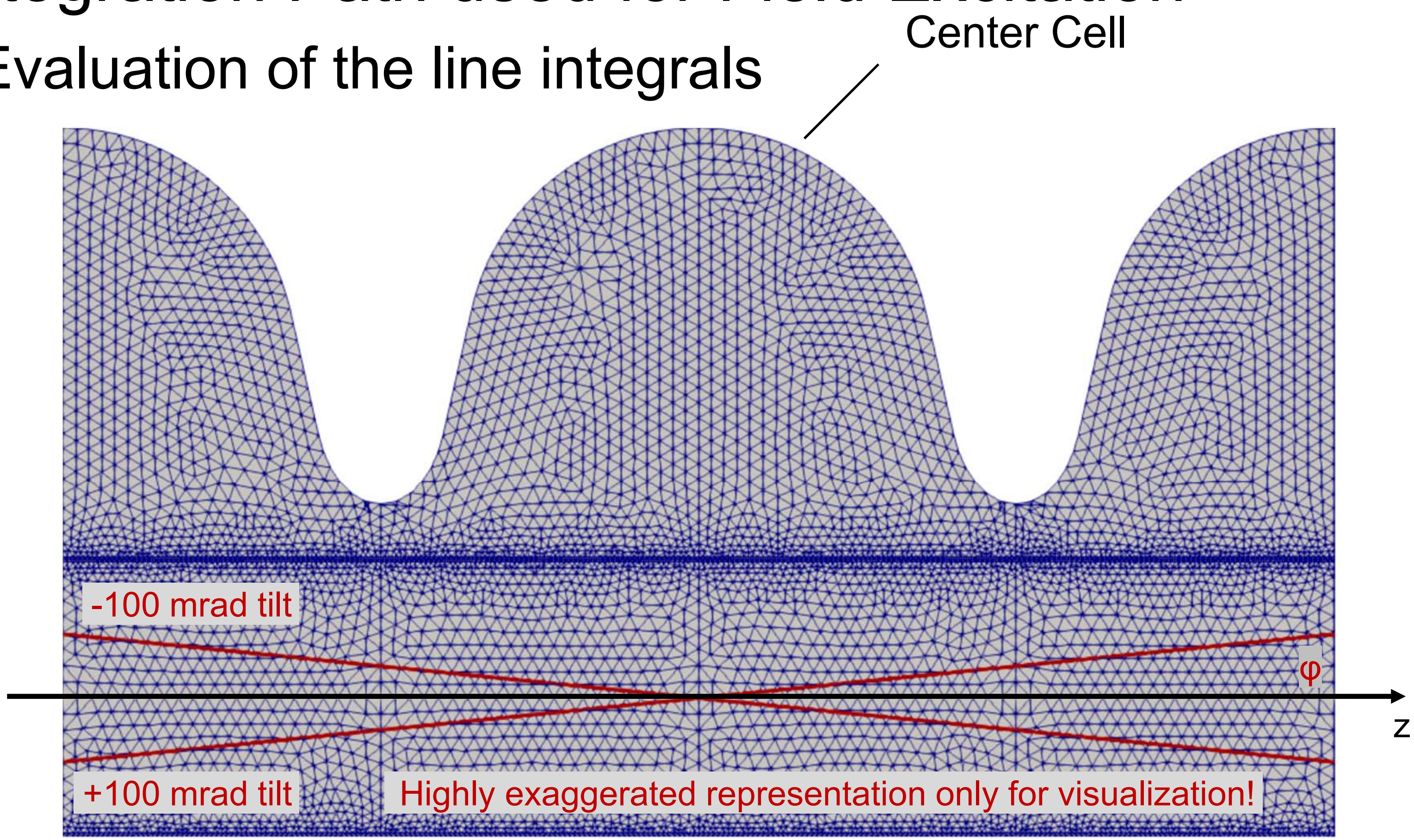
Dipole-Mode Excitation

- Integration Path used for Field Excitation
  - Evaluation of the line integrals



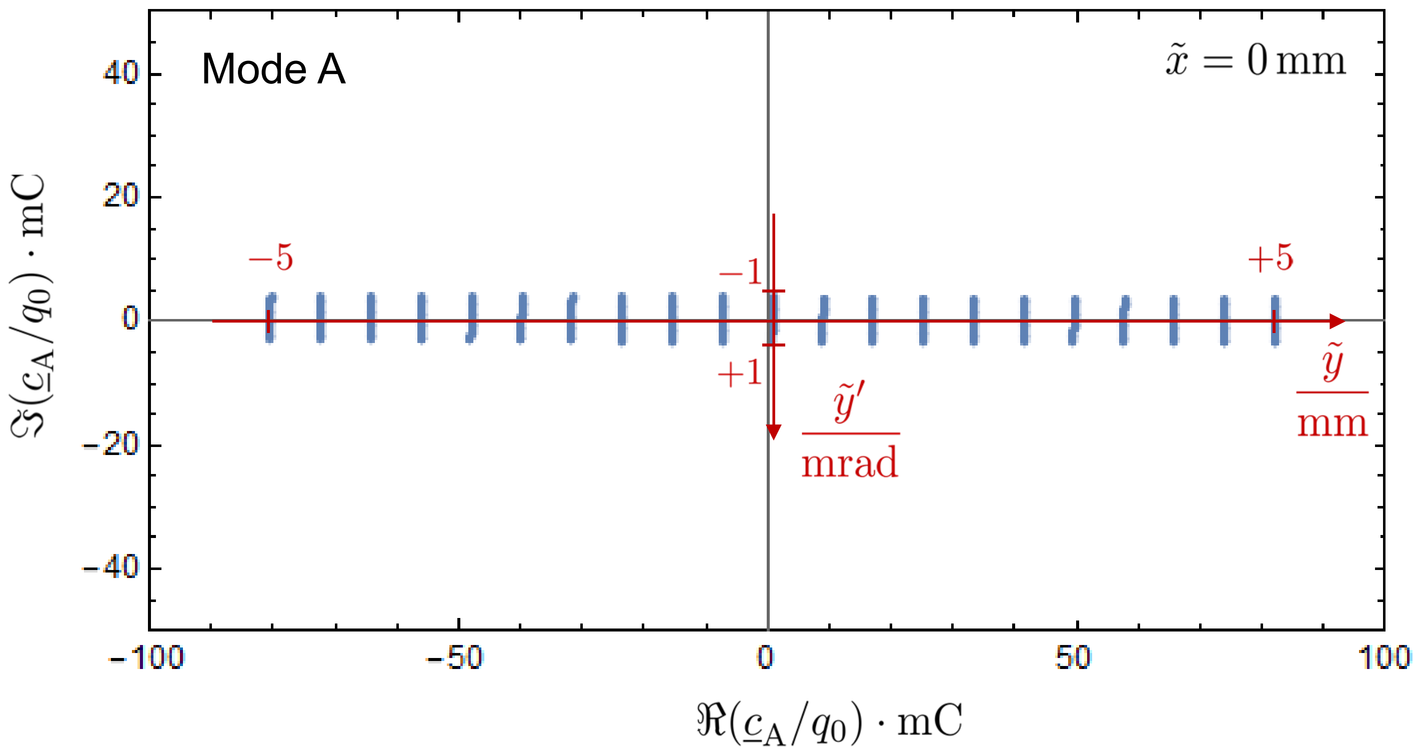
Dipole-Mode Excitation

- Integration Path used for Field Excitation
  - Evaluation of the line integrals



Dipole-Mode Excitation

- Excited Mode Amplitude (real and imaginary part)
  - Single point-charge excitation



Dipole-Mode Excitation

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- Excited Mode Amplitude (magnitude)
  - Single point-charge excitation of mode A

$$\underline{c}_\mathrm{A} = \underline{c}_\mathrm{A0} + \underline{c}_\mathrm{A1}\,\tilde{y} + \underline{c}_\mathrm{A2}\,\tilde{y}'$$

(shifted, non-orthogonal coordinate system)

$$\begin{aligned} c_\mathrm{A} &= |\underline{c}_\mathrm{A}| \\ &= \sqrt{\left(c^\mathrm{re}_\mathrm{A0} + c^\mathrm{re}_\mathrm{A1}\,\tilde{y} + c^\mathrm{re}_\mathrm{A2}\,\tilde{y}'\right)^2 + \left(c^\mathrm{im}_\mathrm{A0} + c^\mathrm{im}_\mathrm{A1}\,\tilde{y} + c^\mathrm{im}_\mathrm{A2}\,\tilde{y}'\right)^2} \\ &= \sqrt{\tilde{c}_\mathrm{A1}\,(\tilde{y}-\tilde{y}_0)^2 + \tilde{c}_\mathrm{A2}\,(\tilde{y}'-\tilde{y}'_0)^2 + \tilde{c}_\mathrm{A12}\,(\tilde{y}-\tilde{y}_0)\,(\tilde{y}'-\tilde{y}'_0)} \end{aligned}$$

Magnitude

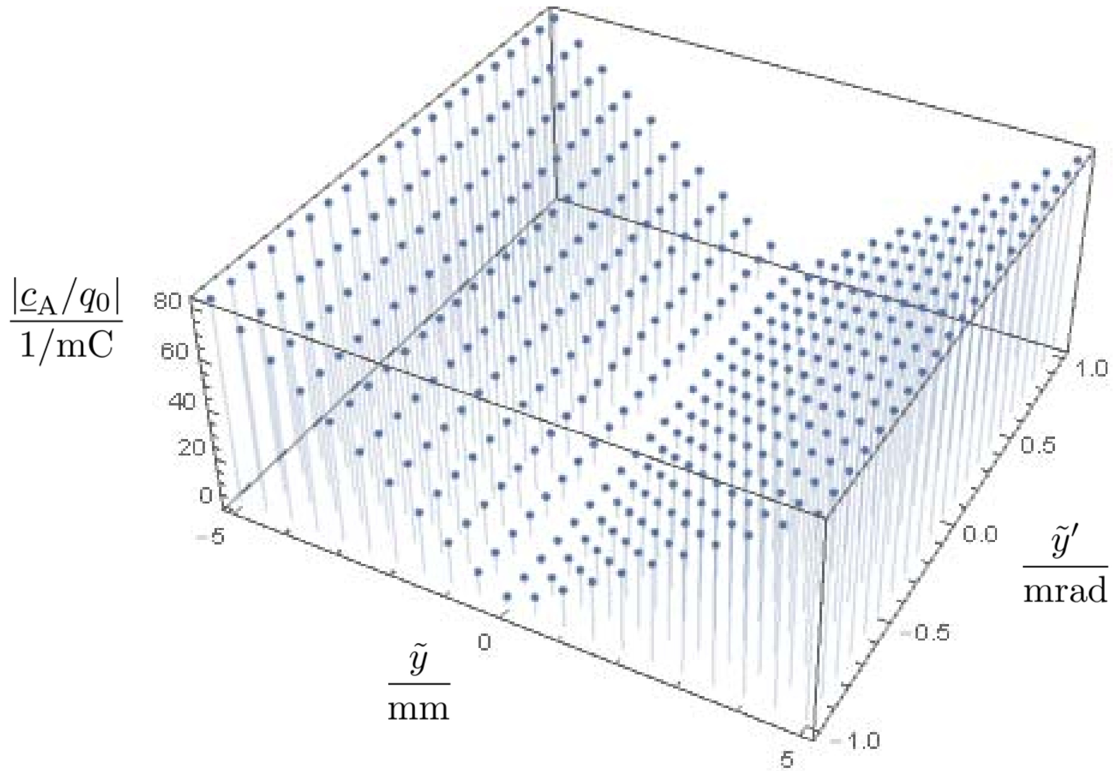
$$\text{with } \tilde{c}_\mathrm{A1} = |\underline{c}_\mathrm{A1}|^2\,, \quad \tilde{c}_\mathrm{A2} = |\underline{c}_\mathrm{A2}|^2\,, \quad \tilde{c}_\mathrm{A12} = 2\left(c^\mathrm{re}_\mathrm{A1}\,c^\mathrm{re}_\mathrm{A2} + c^\mathrm{im}_\mathrm{A1}\,c^\mathrm{im}_\mathrm{A2}\right)$$

Scalar Product

$$\text{and } \tilde{y}_0 = \frac{c^\mathrm{im}_\mathrm{A0}\,c^\mathrm{re}_\mathrm{A2} - c^\mathrm{re}_\mathrm{A0}\,c^\mathrm{im}_\mathrm{A2}}{c^\mathrm{re}_\mathrm{A1}\,c^\mathrm{im}_\mathrm{A2} - c^\mathrm{im}_\mathrm{A1}\,c^\mathrm{re}_\mathrm{A2}}\,, \quad \tilde{y}'_0 = \frac{c^\mathrm{re}_\mathrm{A0}\,c^\mathrm{im}_\mathrm{A1} - c^\mathrm{im}_\mathrm{A0}\,c^\mathrm{re}_\mathrm{A1}}{c^\mathrm{re}_\mathrm{A1}\,c^\mathrm{im}_\mathrm{A2} - c^\mathrm{im}_\mathrm{A1}\,c^\mathrm{re}_\mathrm{A2}}$$

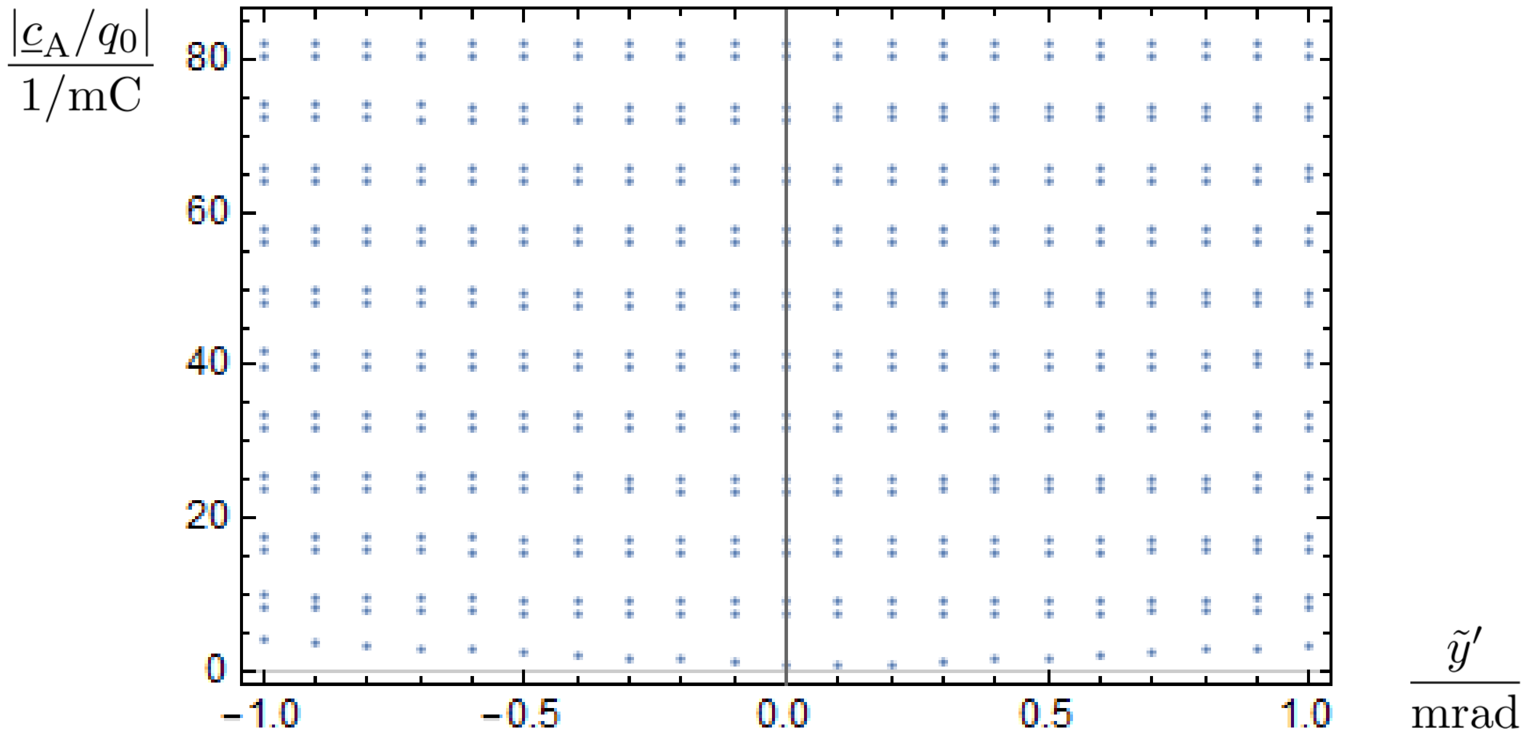
Dipole-Mode Excitation

- Excited Mode Amplitude (magnitude)
  - Single point-charge excitation of mode A



Dipole-Mode Excitation

- Excited Mode Amplitude (magnitude, projected)
  - Single point-charge excitation of mode A



Dipole-Mode Excitation

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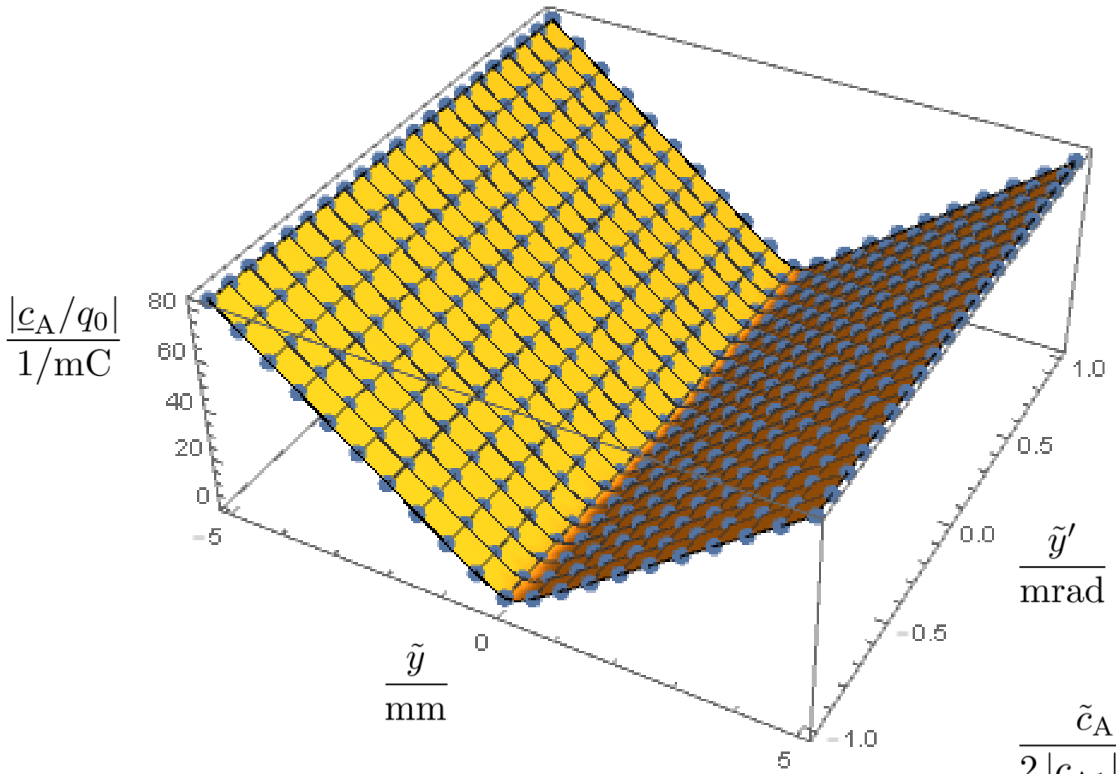
- Excited Mode Amplitude (magnitude fitting)
  - Single point-charge excitation of mode A



$$|\underline{c}_{\mathrm{A1}}/q_0| = \frac{16.248}{\mathrm{mC\ mm}}$$

$$|\underline{c}_{\mathrm{A2}}/q_0| = \frac{3.517}{\mathrm{mC\ mrad}}$$

$$\tilde{c}_{\mathrm{A12}}/q_0^2 = \frac{2.326}{\mathrm{mC^2\ mm\ mrad}}$$

$$\tilde{y}_0 = 0.049\,\mathrm{mm}$$

$$\tilde{y}_0' = 0.097\,\mathrm{mrad}$$

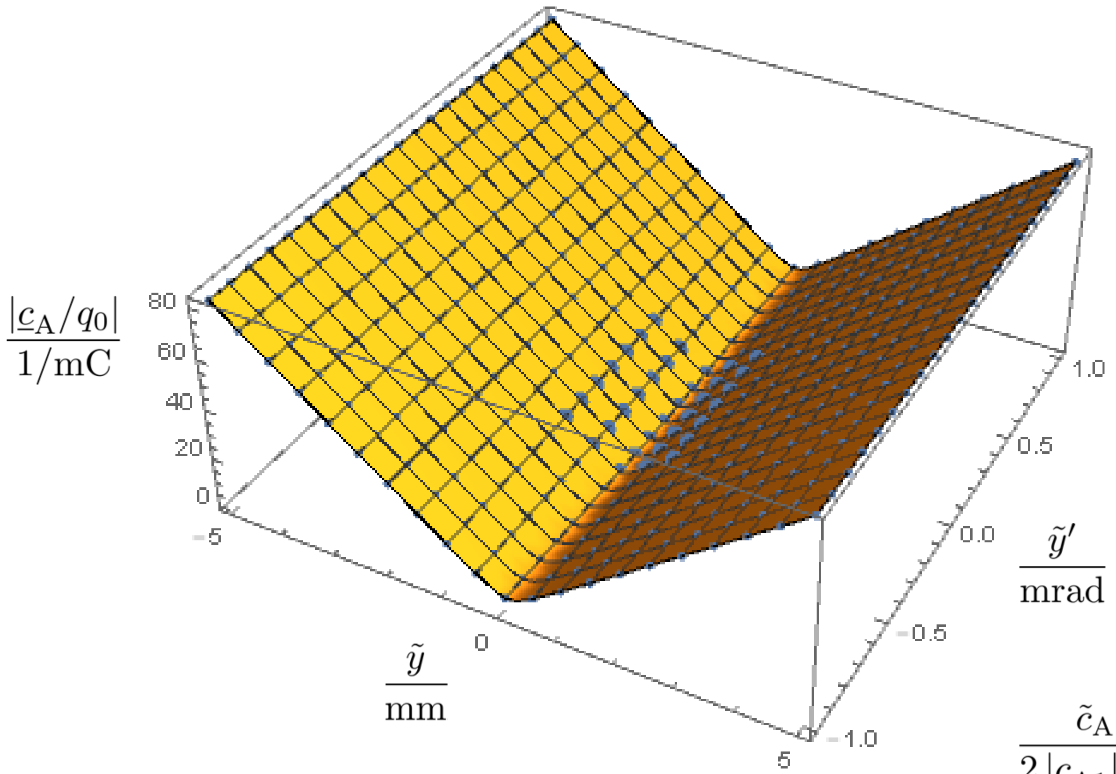
$$\frac{|\underline{c}_{\mathrm{A2}}|}{|\underline{c}_{\mathrm{A1}}|} = 0.216\,\frac{\mathrm{mm}}{\mathrm{mrad}}$$

Sensitivity

$$\frac{\tilde{c}_{\mathrm{A12}}}{2\,|\underline{c}_{\mathrm{A1}}|\,|\underline{c}_{\mathrm{A2}}|} = \cos(88.8°)$$

Dipole-Mode Excitation

- Excited Mode Amplitude (magnitude fitting, reduced)
  - Single point-charge excitation of mode A



$$|\underline{c}_{\mathrm{A1}}/q_0| = \frac{16.248}{\mathrm{mC\ mm}}$$

$$|\underline{c}_{\mathrm{A2}}/q_0| = \frac{3.516}{\mathrm{mC\ mrad}}$$

$$\tilde{c}_{\mathrm{A12}}/q_0^2 = \frac{2.354}{\mathrm{mC}^2\ \mathrm{mm\ mrad}}$$

$$\tilde{y}_0 = 0.049\,\mathrm{mm}$$

$$\tilde{y}'_0 = 0.097\,\mathrm{mrad}$$

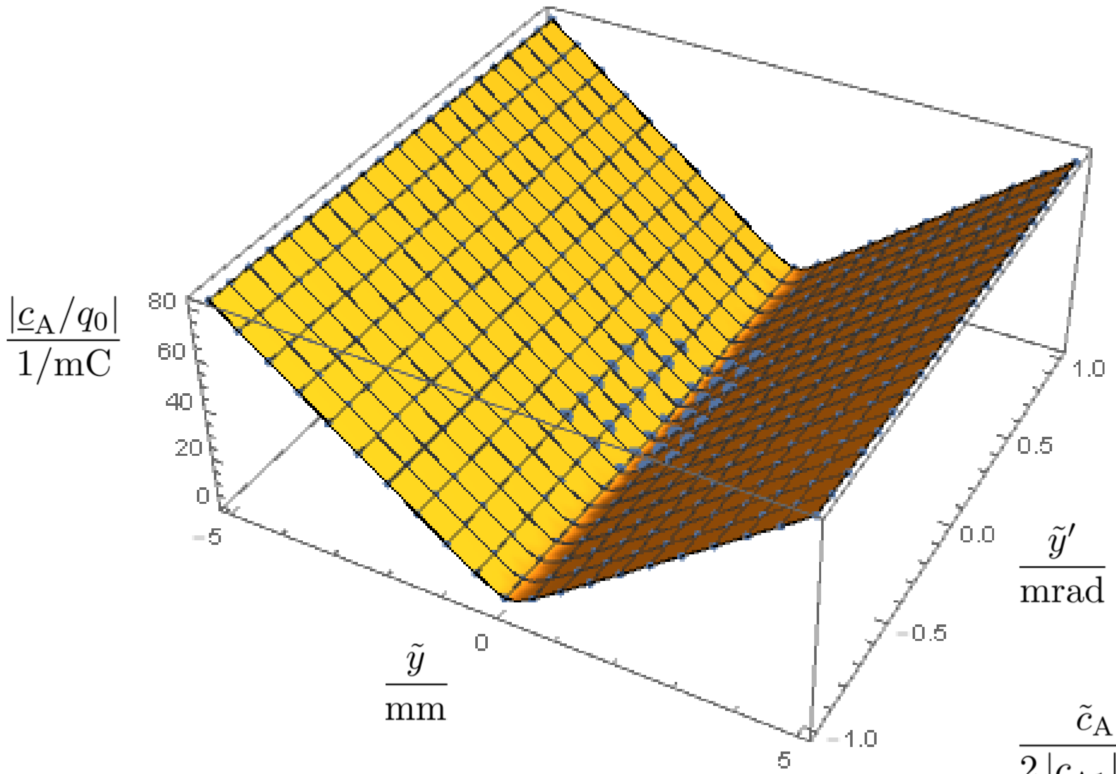
$$\frac{|\underline{c}_{\mathrm{A2}}|}{|\underline{c}_{\mathrm{A1}}|} = 0.216\,\frac{\mathrm{mm}}{\mathrm{mrad}}$$

Sensitivity

$$\frac{\tilde{c}_{\mathrm{A12}}}{2\,|\underline{c}_{\mathrm{A1}}|\,|\underline{c}_{\mathrm{A2}}|} = \cos(88.8°)$$

Dipole-Mode Excitation

- Excited Mode Amplitude (magnitude fitting, reduced)
  - Single point-charge excitation of mode A



$$|\underline{c}_\mathrm{A1}/q_0| = \frac{16.248}{\mathrm{mC\ mm}}$$

$$|\underline{c}_\mathrm{A2}/q_0| = \frac{3.494}{\mathrm{mC\ mrad}}$$

$$\tilde{c}_\mathrm{A12}/q_0^2 \stackrel{!}{=} \frac{0}{\mathrm{mC^2\ mm\ mrad}}$$

$$\tilde{y}_0 = 0.050\,\mathrm{mm}$$

$$\tilde{y}'_0 = 0.102\,\mathrm{mrad}$$

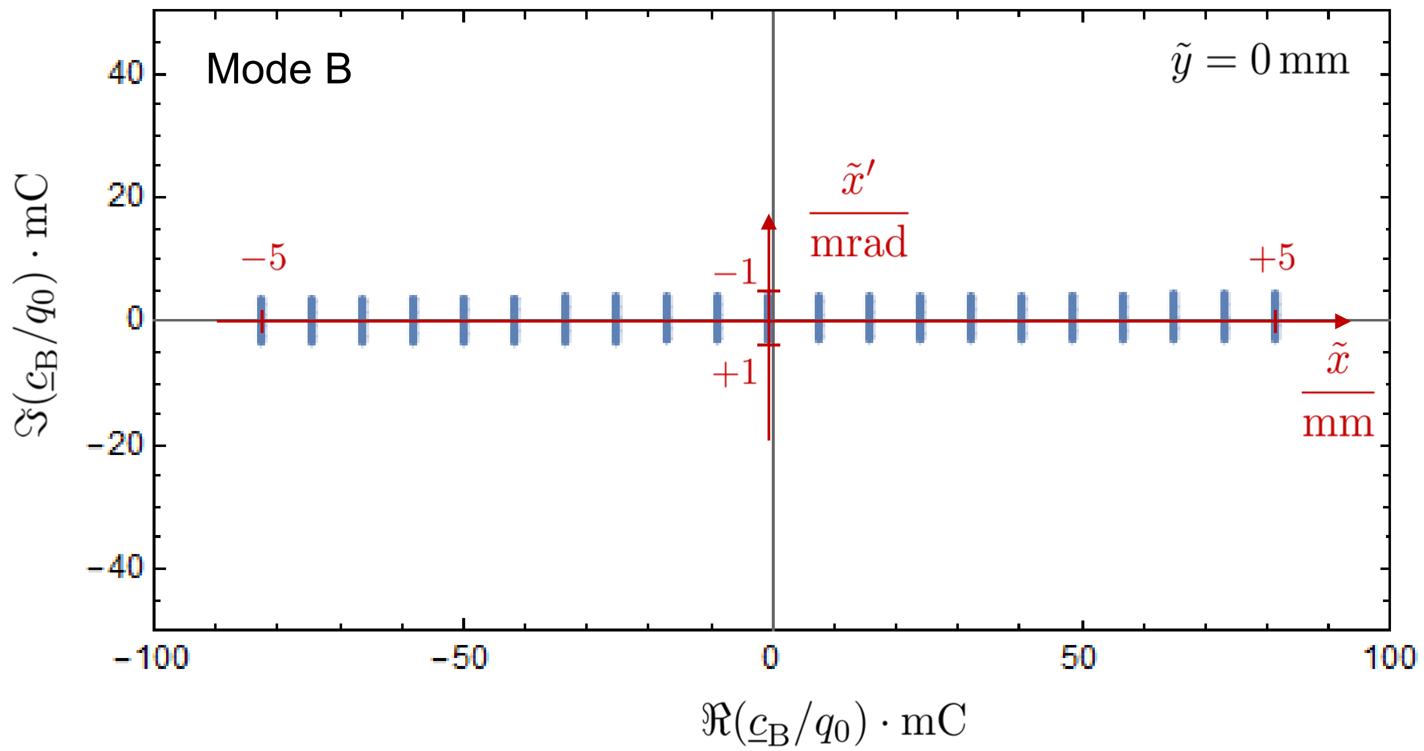
$$\frac{|\underline{c}_\mathrm{A2}|}{|\underline{c}_\mathrm{A1}|} = 0.215\,\frac{\mathrm{mm}}{\mathrm{mrad}}$$

Sensitivity

$$\frac{\tilde{c}_\mathrm{A12}}{2\,|\underline{c}_\mathrm{A1}|\,|\underline{c}_\mathrm{A2}|} = \cos(90.0^\circ)$$

Dipole-Mode Excitation

- Excited Mode Amplitude (real and imaginary part)
  - Single point-charge excitation



Dipole-Mode Excitation

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- Excited Mode Amplitude (magnitude)
  - Single point-charge excitation of mode B

$$\underline{c}_{\mathrm{B}} = \underline{c}_{\mathrm{B0}} + \underline{c}_{\mathrm{B1}}\,\tilde{y} + \underline{c}_{\mathrm{B2}}\,\tilde{y}'$$

(shifted, non-orthogonal coordinate system)

Magnitude

$$\begin{aligned} c_{\mathrm{B}} &= |\underline{c}_{\mathrm{B}}| \\ &= \sqrt{\left(c^{\mathrm{re}}_{\mathrm{B0}} + c^{\mathrm{re}}_{\mathrm{B1}}\,\tilde{y} + c^{\mathrm{re}}_{\mathrm{B2}}\,\tilde{y}'\right)^2 + \left(c^{\mathrm{im}}_{\mathrm{B0}} + c^{\mathrm{im}}_{\mathrm{B1}}\,\tilde{y} + c^{\mathrm{im}}_{\mathrm{B2}}\,\tilde{y}'\right)^2} \\ &= \sqrt{\tilde{c}_{\mathrm{B1}}\,\left(\tilde{y}-\tilde{y}_0\right)^2 + \tilde{c}_{\mathrm{B2}}\,\left(\tilde{y}'-\tilde{y}'_0\right)^2 + \tilde{c}_{\mathrm{B12}}\,\left(\tilde{y}-\tilde{y}_0\right)\left(\tilde{y}'-\tilde{y}'_0\right)} \end{aligned}$$

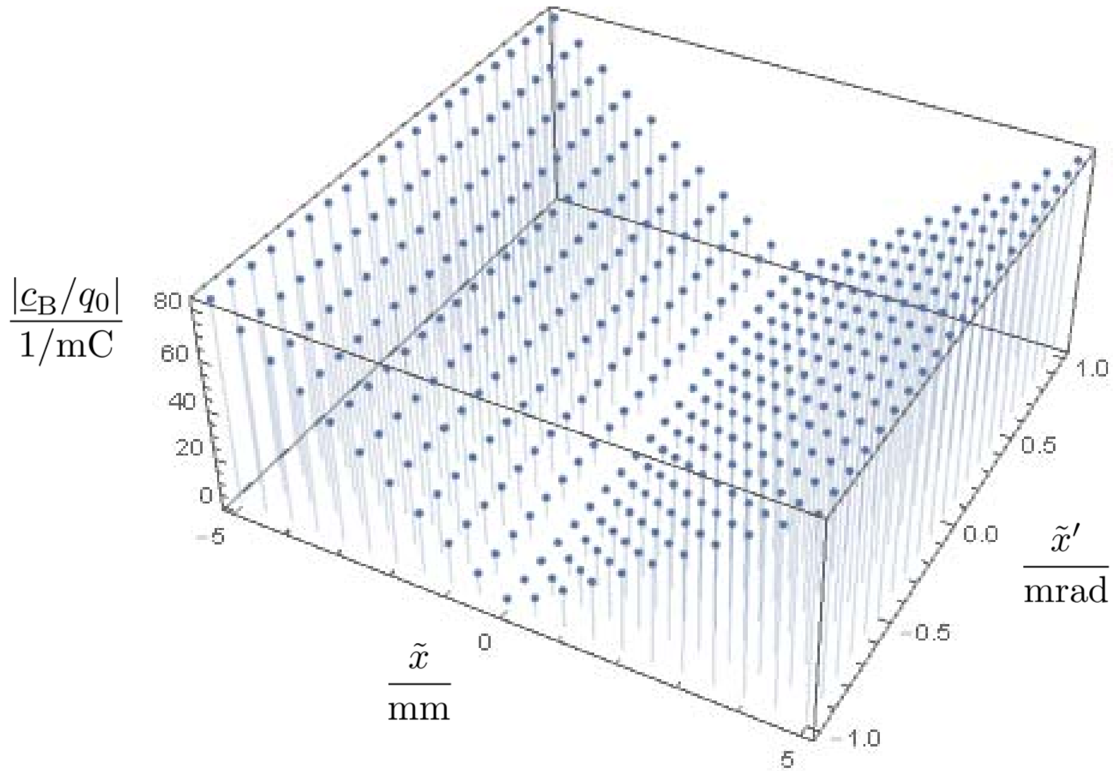
Scalar Product

$$\text{with } \tilde{c}_{\mathrm{B1}} = |\underline{c}_{\mathrm{B1}}|^2\,, \quad \tilde{c}_{\mathrm{B2}} = |\underline{c}_{\mathrm{B2}}|^2\,, \quad \tilde{c}_{\mathrm{B12}} = 2\left(c^{\mathrm{re}}_{\mathrm{B1}}\,c^{\mathrm{re}}_{\mathrm{B2}} + c^{\mathrm{im}}_{\mathrm{B1}}\,c^{\mathrm{im}}_{\mathrm{B2}}\right)$$

$$\text{and } \tilde{y}_0 = \frac{c^{\mathrm{im}}_{\mathrm{B0}}\,c^{\mathrm{re}}_{\mathrm{B2}} - c^{\mathrm{re}}_{\mathrm{B0}}\,c^{\mathrm{im}}_{\mathrm{B2}}}{c^{\mathrm{re}}_{\mathrm{B1}}\,c^{\mathrm{im}}_{\mathrm{B2}} - c^{\mathrm{im}}_{\mathrm{B1}}\,c^{\mathrm{re}}_{\mathrm{B2}}}\,, \quad \tilde{y}'_0 = \frac{c^{\mathrm{re}}_{\mathrm{B0}}\,c^{\mathrm{im}}_{\mathrm{B1}} - c^{\mathrm{im}}_{\mathrm{B0}}\,c^{\mathrm{re}}_{\mathrm{B1}}}{c^{\mathrm{re}}_{\mathrm{B1}}\,c^{\mathrm{im}}_{\mathrm{B2}} - c^{\mathrm{im}}_{\mathrm{B1}}\,c^{\mathrm{re}}_{\mathrm{B2}}}$$

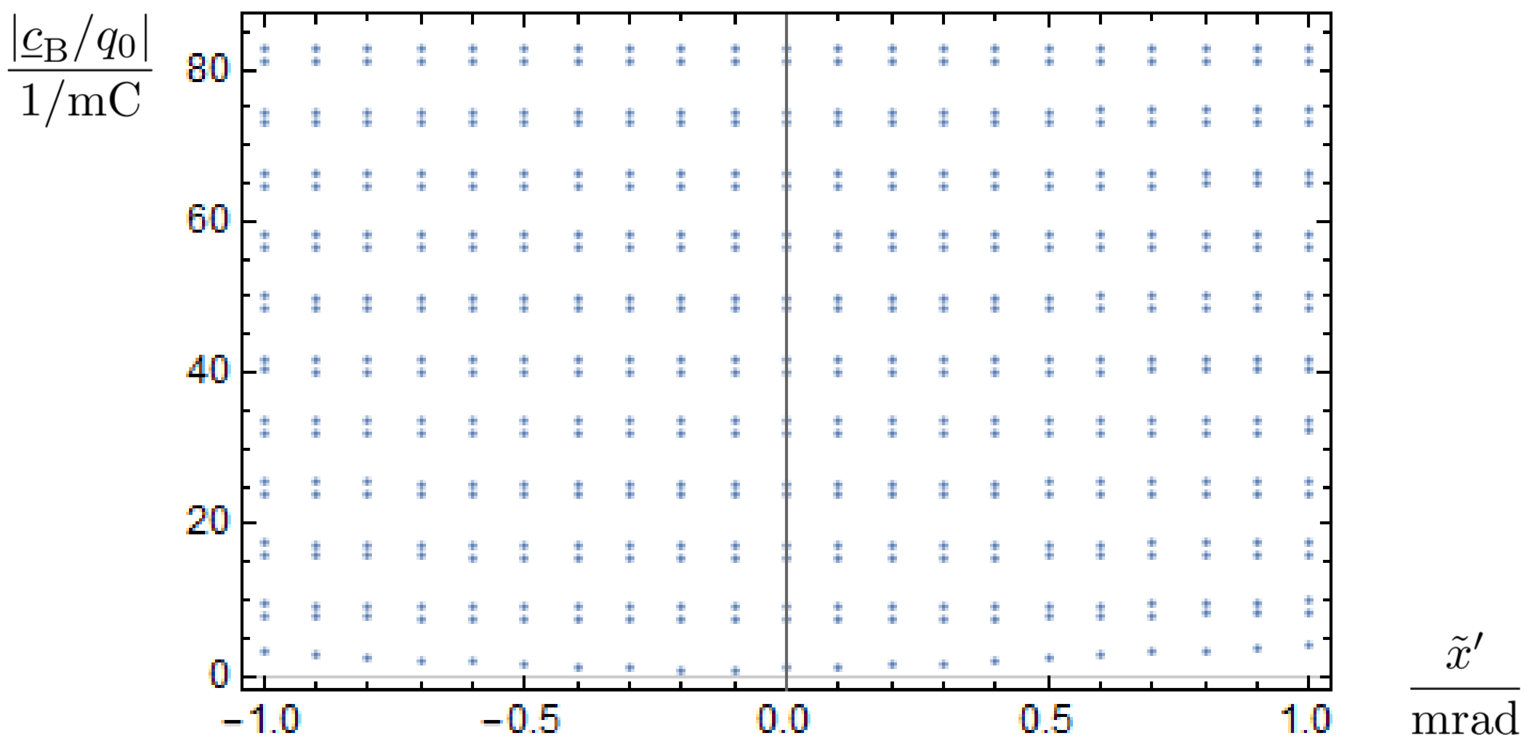
Dipole-Mode Excitation

- Excited Mode Amplitude (magnitude)
  - Single point-charge excitation of mode B



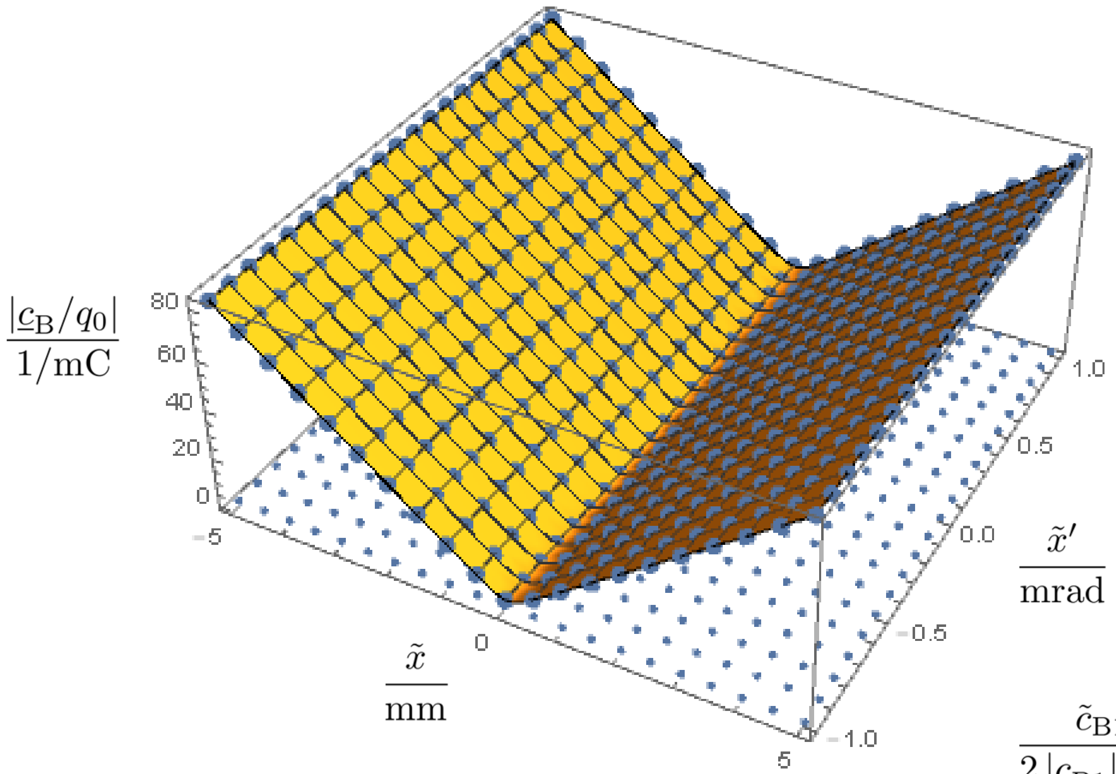
Dipole-Mode Excitation

- Excited Mode Amplitude (magnitude, projected)
  - Single point-charge excitation of mode B



Dipole-Mode Excitation

- Excited Mode Amplitude (magnitude fitting)
  - Single point-charge excitation of mode B



$$|\underline{c}_\mathrm{B1}/q_0| = \frac{16.376}{\mathrm{mC\ mm}}$$

$$|\underline{c}_\mathrm{B2}/q_0| = \frac{3.492}{\mathrm{mC\ mrad}}$$

$$\tilde{c}_\mathrm{B12}/q_0^2 = \frac{-0.241}{\mathrm{mC}^2\ \mathrm{mm\ mrad}}$$

$$\tilde{x}_0 = 0.048\,\mathrm{mm}$$

$$\tilde{x}'_0 = -0.143\,\mathrm{mrad}$$

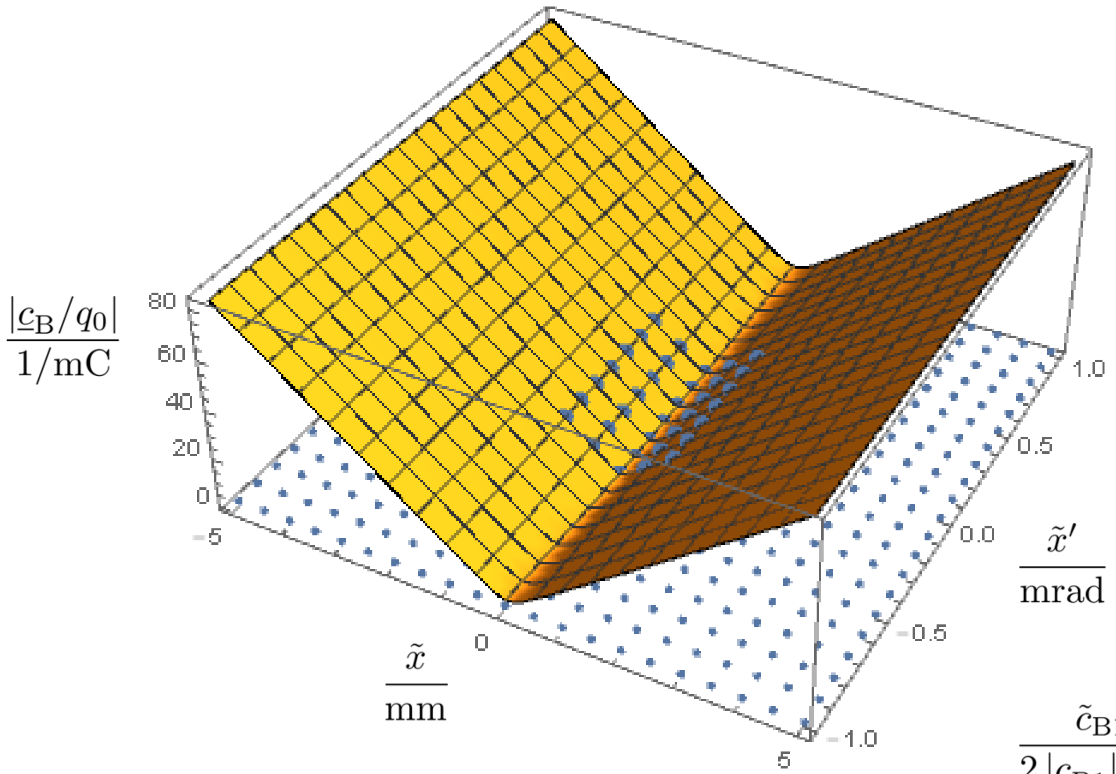
$$\frac{|\underline{c}_\mathrm{B2}|}{|\underline{c}_\mathrm{B1}|} = 0.213\,\frac{\mathrm{mm}}{\mathrm{mrad}}$$

Sensitivity

$$\frac{\tilde{c}_\mathrm{B12}}{2\,|\underline{c}_\mathrm{B1}|\,|\underline{c}_\mathrm{B2}|} = \cos(90.1^\circ)$$

Dipole-Mode Excitation

- Excited Mode Amplitude (magnitude fitting, reduced)
  - Single point-charge excitation of mode B



$$|\underline{c}_\mathrm{B1}/q_0| = \frac{16.376}{\mathrm{mC\ mm}}$$

$$|\underline{c}_\mathrm{B2}/q_0| = \frac{3.492}{\mathrm{mC\ mrad}}$$

$$\tilde{c}_\mathrm{B12}/q_0^2 = \frac{-0.239}{\mathrm{mC^2\ mm\ mrad}}$$

$$\tilde{x}_0 = 0.048\,\mathrm{mm}$$

$$\tilde{x}'_0 = -0.143\,\mathrm{mrad}$$

$$\frac{|\underline{c}_\mathrm{B2}|}{|\underline{c}_\mathrm{B1}|} = 0.213\,\frac{\mathrm{mm}}{\mathrm{mrad}}$$

Sensitivity

$$\frac{\tilde{c}_\mathrm{B12}}{2\,|\underline{c}_\mathrm{B1}|\,|\underline{c}_\mathrm{B2}|} = \cos(90.1^\circ)$$

Dipole-Mode Excitation

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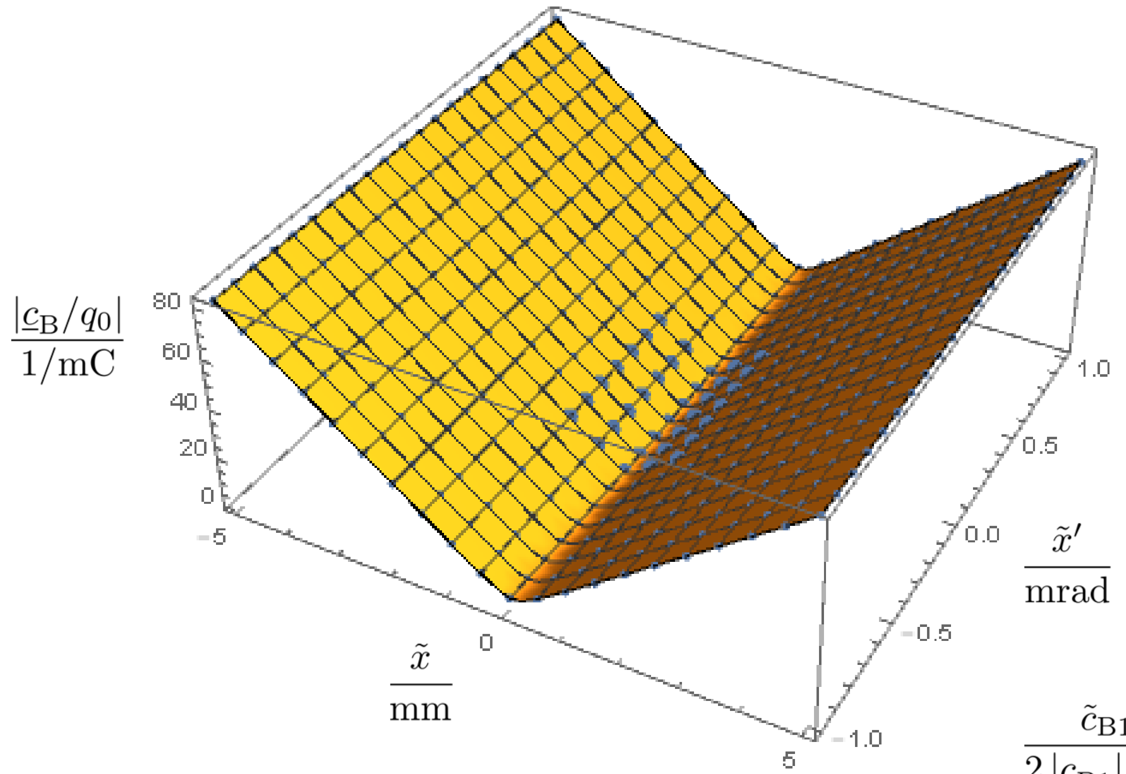
- Excited Mode Amplitude (magnitude fitting, reduced)
  - Single point-charge excitation of mode B



$$|\underline{c}_{B1}/q_0| = \frac{16.376}{\text{mC mm}}$$

$$|\underline{c}_{B2}/q_0| = \frac{3.489}{\text{mC mrad}}$$

$$\tilde{c}_{B12}/q_0^2 \overset{!}{=} \frac{0}{\text{mC}^2\text{ mm mrad}}$$

$$\tilde{x}_0 = 0.048\,\text{mm}$$

$$\tilde{x}'_0 = -0.143\,\text{mrad}$$

$$\frac{|\underline{c}_{B2}|}{|\underline{c}_{B1}|} = 0.213\,\frac{\text{mm}}{\text{mrad}}$$

Sensitivity

$$\frac{\tilde{c}_{B12}}{2\,|\underline{c}_{B1}|\,|\underline{c}_{B2}|} = \cos(90.0^\circ)$$

Dipole Mode Excitation

- Sensitivities of Field Magnitude

Mode A

$$\left|\frac{\partial \underline{c}_{\mathrm{A}}/q_0}{\partial \tilde{y}}\right| = |\underline{c}_{\mathrm{A1}}/q_0| = \frac{16.248}{\mathrm{mC\ mm}}$$

$$\left|\frac{\partial \underline{c}_{\mathrm{A}}/q_0}{\partial \tilde{y}'}\right| = |\underline{c}_{\mathrm{A2}}/q_0| = \frac{3.517}{\mathrm{mC\ mrad}}$$

Mode B

$$\left|\frac{\partial \underline{c}_{\mathrm{B}}/q_0}{\partial \tilde{x}}\right| = |\underline{c}_{\mathrm{B1}}/q_0| = \frac{16.376}{\mathrm{mC\ mm}}$$

$$\left|\frac{\partial \underline{c}_{\mathrm{B}}/q_0}{\partial \tilde{x}'}\right| = |\underline{c}_{\mathrm{B2}}/q_0| = \frac{3.492}{\mathrm{mC\ mrad}}$$

- Ratio of Sensitivities

$$\vec{E}(\vec{r},t) = \sum_{\nu=1}^{\infty} \Re(\underline{c}_{\nu}\, e^{i\omega_{\nu} t})\, \vec{E}_{\nu}(\vec{r})$$

$$\frac{\left|\frac{\partial \underline{c}_{\mathrm{A}}}{\partial \tilde{y}'}\right|}{\left|\frac{\partial \underline{c}_{\mathrm{A}}}{\partial \tilde{y}}\right|} = 0.216\ \frac{\mathrm{mm}}{\mathrm{mrad}}$$

$$\frac{\left|\frac{\partial \underline{c}_{\mathrm{B}}}{\partial \tilde{x}'}\right|}{\left|\frac{\partial \underline{c}_{\mathrm{B}}}{\partial \tilde{x}}\right|} = 0.213\ \frac{\mathrm{mm}}{\mathrm{mrad}}$$

Dipole Mode Excitation

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- Sensitivities of Port Voltages wrt. Shifted Trajectories

Mode A

$$\frac{\partial U_{\text{Port 1}}/q_0}{\partial \tilde{y}} = 0.640\ \frac{\text{mV}}{\text{nC mm}}$$

$$\frac{\partial U_{\text{Port 2}}/q_0}{\partial \tilde{y}} = 116.9\ \frac{\text{mV}}{\text{nC mm}}$$

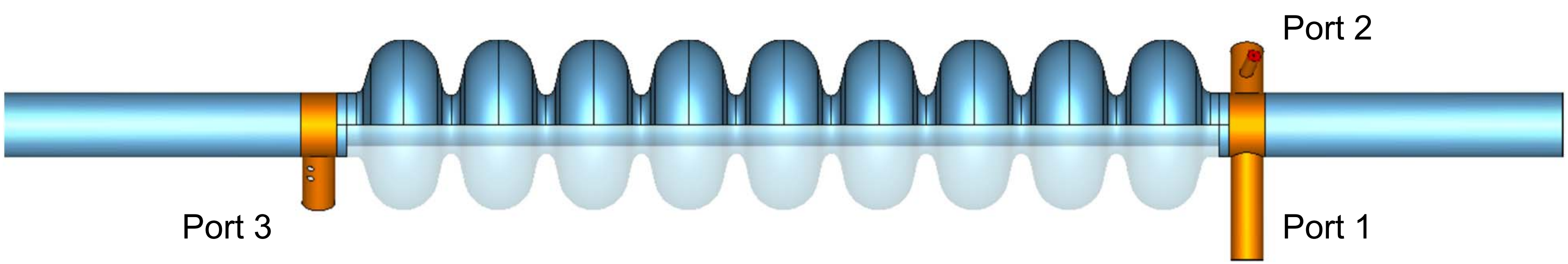
$$\frac{\partial U_{\text{Port 3}}/q_0}{\partial \tilde{y}} = 67.18\ \frac{\text{mV}}{\text{nC mm}}$$

Mode B

$$\frac{\partial U_{\text{Port 1}}/q_0}{\partial \tilde{x}} = 8.297\ \frac{\text{mV}}{\text{nC mm}}$$

$$\frac{\partial U_{\text{Port 2}}/q_0}{\partial \tilde{x}} = 33.01\ \frac{\text{mV}}{\text{nC mm}}$$

$$\frac{\partial U_{\text{Port 3}}/q_0}{\partial \tilde{x}} = 76.81\ \frac{\text{mV}}{\text{nC mm}}$$



Dipole Mode Excitation

- Sensitivities of Port Voltages wrt. Tilted Trajectories

Mode A

$$\frac{\partial U_{\mathrm{Port\ 1}}/q_0}{\partial \tilde{y}'} = 0.139\ \frac{\mathrm{mV}}{\mathrm{nC\ mrad}}$$

$$\frac{\partial U_{\mathrm{Port\ 2}}/q_0}{\partial \tilde{y}'} = 25.30\ \frac{\mathrm{mV}}{\mathrm{nC\ mrad}}$$

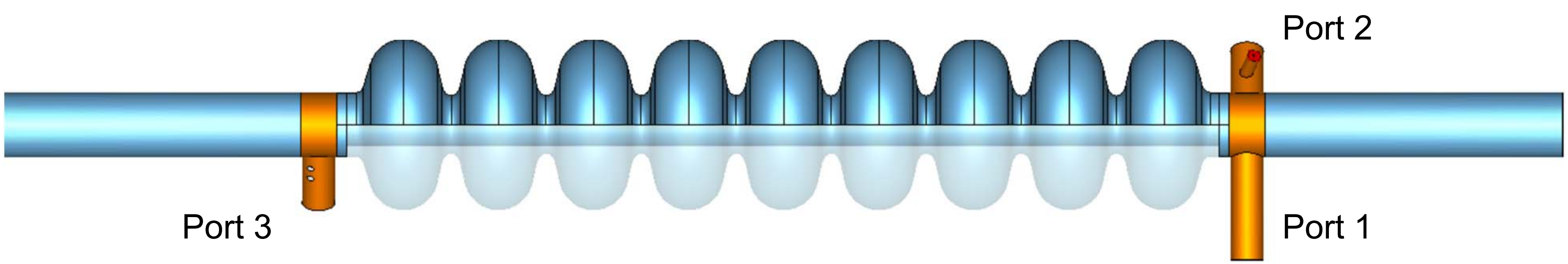
$$\frac{\partial U_{\mathrm{Port\ 3}}/q_0}{\partial \tilde{y}'} = 14.54\ \frac{\mathrm{mV}}{\mathrm{nC\ mrad}}$$

Mode B

$$\frac{\partial U_{\mathrm{Port\ 1}}/q_0}{\partial \tilde{x}'} = 1.769\ \frac{\mathrm{mV}}{\mathrm{nC\ mrad}}$$

$$\frac{\partial U_{\mathrm{Port\ 2}}/q_0}{\partial \tilde{x}'} = 7.040\ \frac{\mathrm{mV}}{\mathrm{nC\ mrad}}$$

$$\frac{\partial U_{\mathrm{Port\ 3}}/q_0}{\partial \tilde{x}'} = 16.38\ \frac{\mathrm{mV}}{\mathrm{nC\ mrad}}$$



Outline

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- Motivation
- Dipole-Mode Eigenvalue Calculation
  - Electromagnetic field distribution
  - Single point-charge excitation using shifted trajectories
- Dipole-Mode Excitation
  - Excited complex-valued amplitude wrt. shifted and tilted trajectories
  - Magnitude fitting as used in the experimental data evaluations
  - Sensitivity wrt. shifted and tilted trajectories
- **Summary / Outlook**

Summary / Outlook

- Summary
  - Precise eigenmode calculation for the two dipole modes resonating at 1.706 GHz including frequency, quality factor and field distributions
  - Single point-charge excitation using shifted and tilted trajectories
  - Fitting of the excited magnitude to an analytically derived model (nonzero shift and tilt obtained even for an ideal cavity setup)
- Outlook
  - Magnitude fitting as used in the experimental data evaluations are sensitive to measurement errors
  - Evaluating additional phase information can avoid such type of errors