

# Beam Dynamics Studies for the PITZ Undulator



TECHNISCHE  
UNIVERSITÄT  
DARMSTADT

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- I. Introduction
- II. Numerical Models
  - i. 2D Undulator Model
  - ii. Space Charge Models and Codes
- III. Simulation Results
- IV. Summary and Outlook

# Introduction

## THz SASE FEL at DESY PITZ

### Motivation:

Development of a high power, tunable THz source for European XFEL

⇒ THz radiation at  $\lambda \sim 100 \mu\text{m}$ ,  $\sim 6 \text{ ns}$  pulse length, and  $\sim 38 \text{ MW}$  peak power

### THz-FEL Parameters:

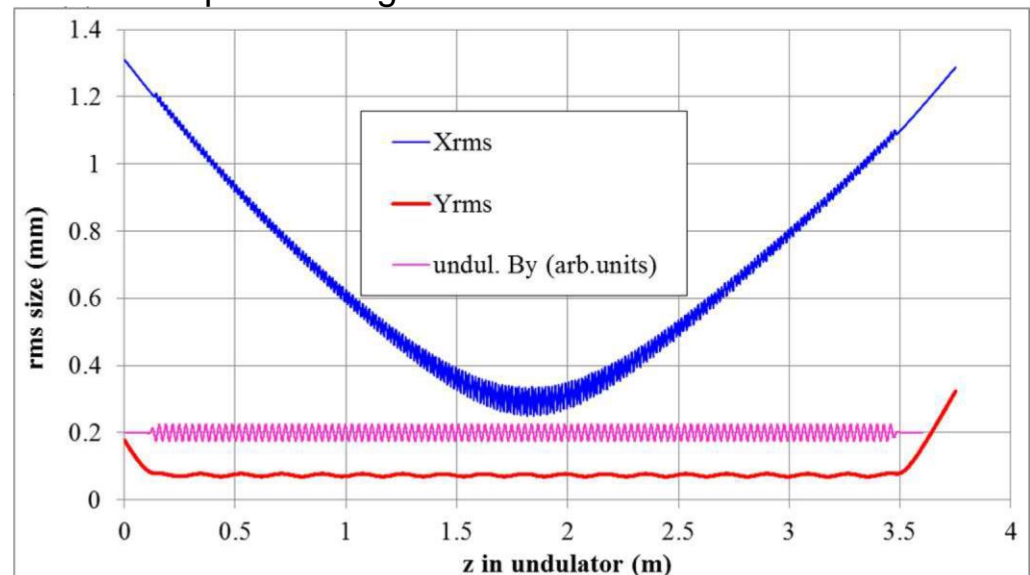
- Bunch charge  $Q_b \sim 4 \text{ nC}$
- Beam energy  $E_b \sim 16.7 \text{ MeV}$
- Und. length  $L_u \sim 3.4 \text{ m}$
- Und. period  $\lambda_U \sim 3 \text{ cm}$

### Challenge:

Transport of SC dominated beam

⇒ SC beam dynamics simulation

Astra Space Charge Simulation of Beam in Undulator



Source: M. Krasilnikov at ICAP 2018

Further Information:

M. Krasilnikov et al., „Start-to-End Simulations of THz SASE FEL Proof-of-Principle Experiment at PITZ”, ICAP'18, Key West

# Numerical Models

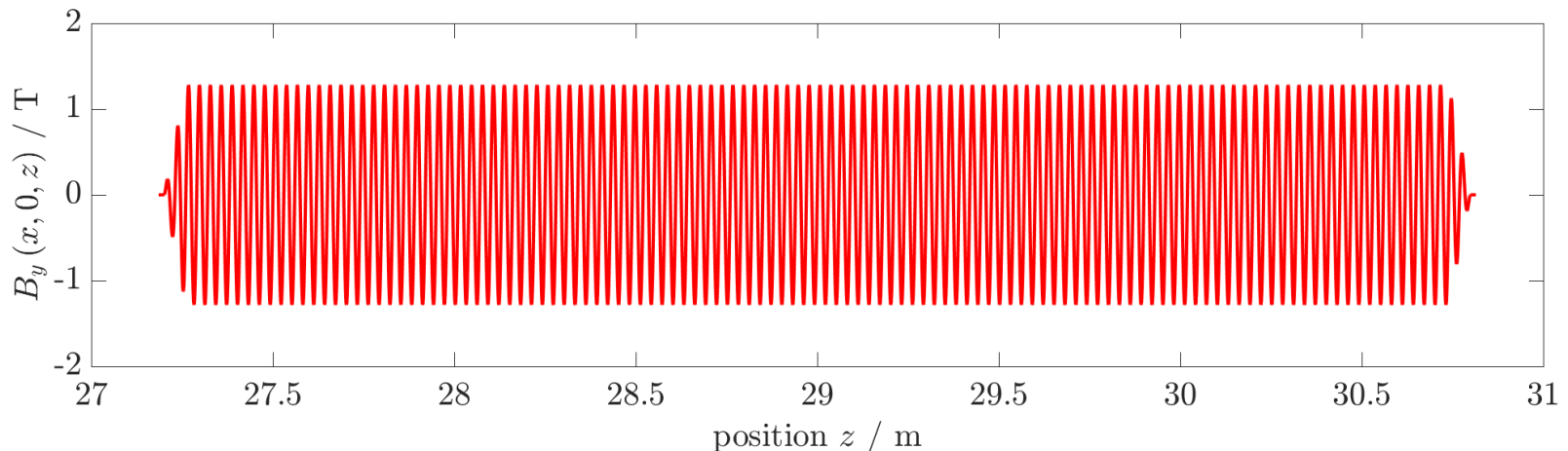
## 2D Undulator Model

### Analytic 2D Undulator Field Model:

$$\vec{B}(\vec{r}) = B_0(z) \cosh\left(\frac{2\pi}{\lambda_u} y\right) \sin\left(\frac{2\pi}{\lambda_u} z\right) \vec{e}_y + B_0(z) \sinh\left(\frac{2\pi}{\lambda_u} y\right) \cos\left(\frac{2\pi}{\lambda_u} z\right) \vec{e}_z$$

with period  $\lambda_u = 3 \text{ cm}$ , total length  $L_u = 120 \lambda_u$ , and tapered  $0 \text{ T} \leq B_0(z) \leq 1.28 \text{ T}$

### 2D Undulator Field on Axis:

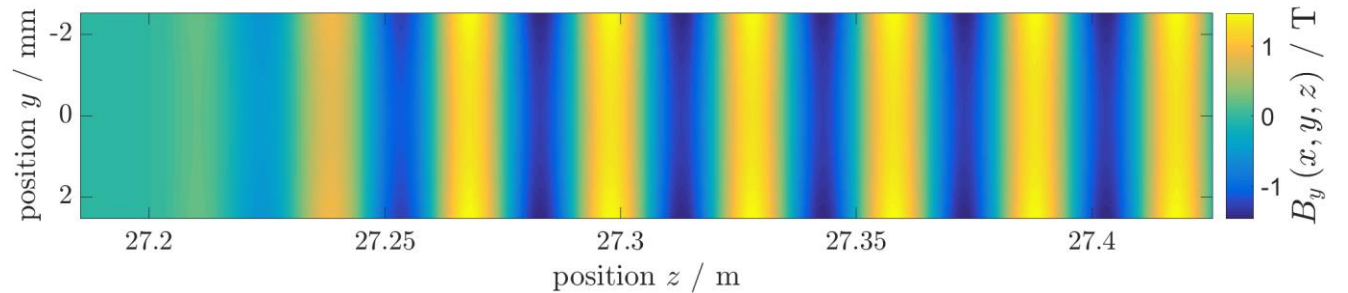


# Numerical Models

## Idealized 2D Undulator

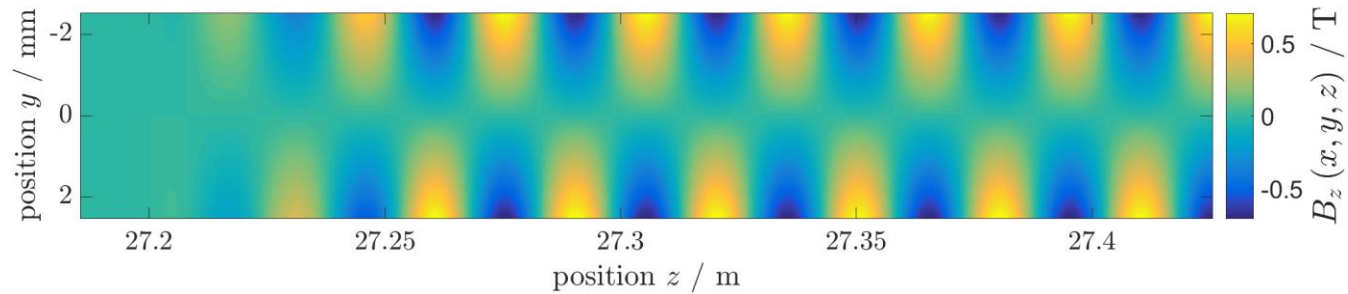
**Periodic field  
map w. tapering:**

$$\Rightarrow B_y(x, y, z)$$



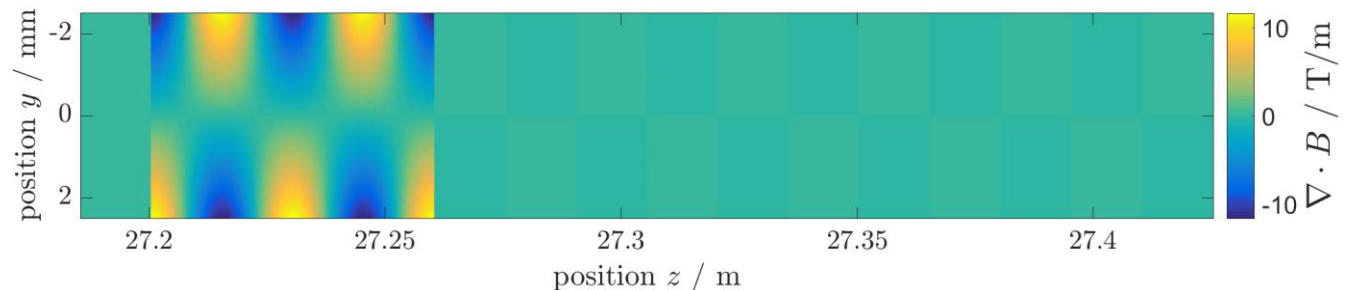
**Focusing in  $y$ :**

$$\Rightarrow B_z(x, y, z)$$



**Linear tapering:**

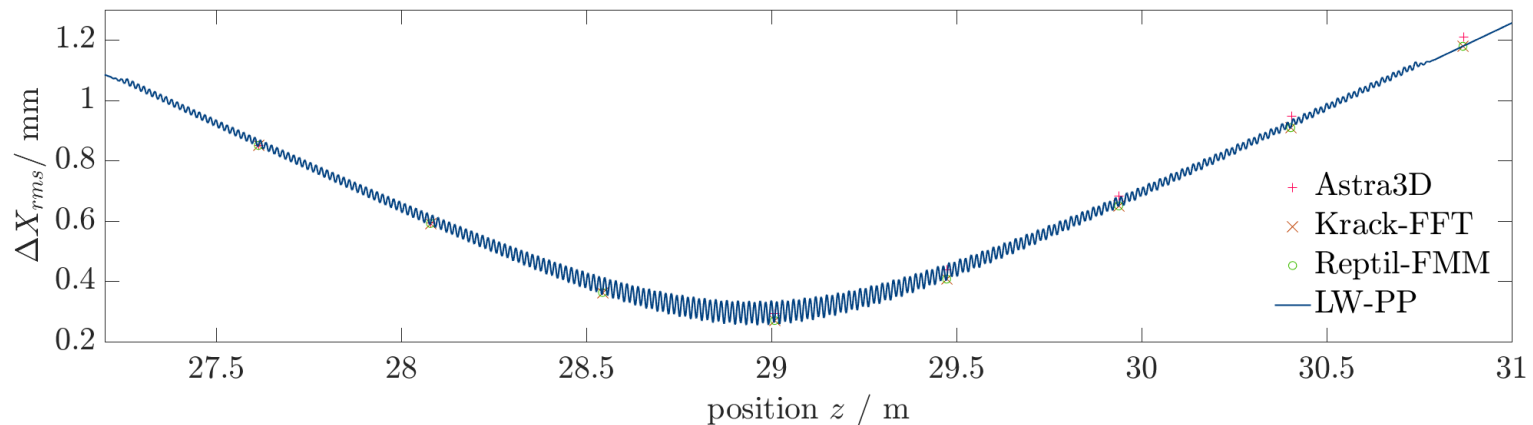
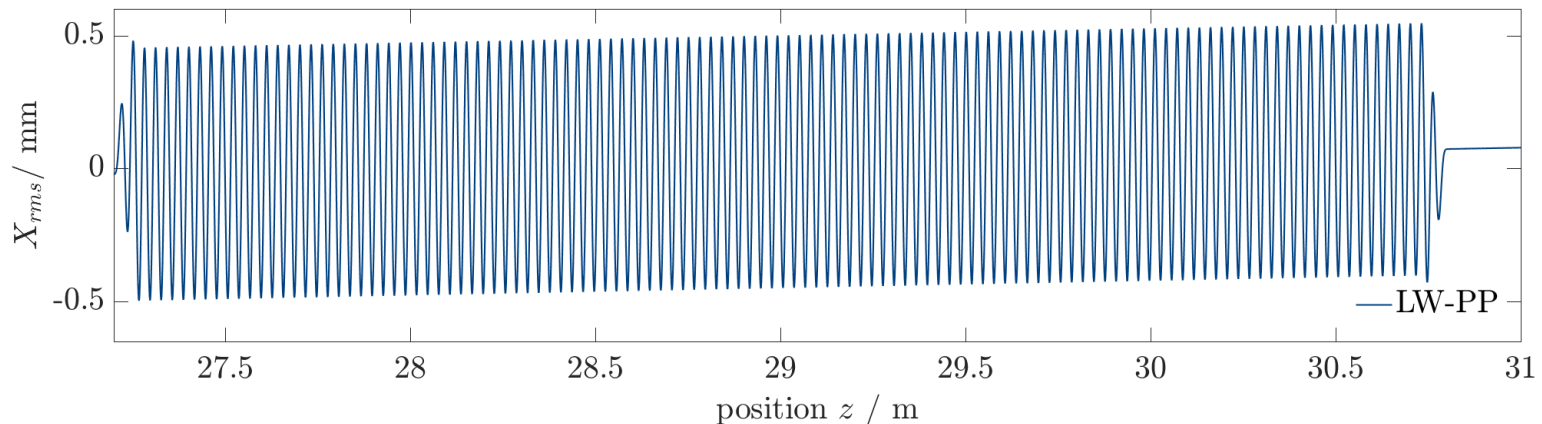
$$\Rightarrow \vec{\nabla} \cdot \vec{B} \neq 0$$



# Numerical Models

## 2D Undulator Model

### Beam Dynamics in the Undulator w/o SC:



# Numerical Models

## Space Charge Models and Codes

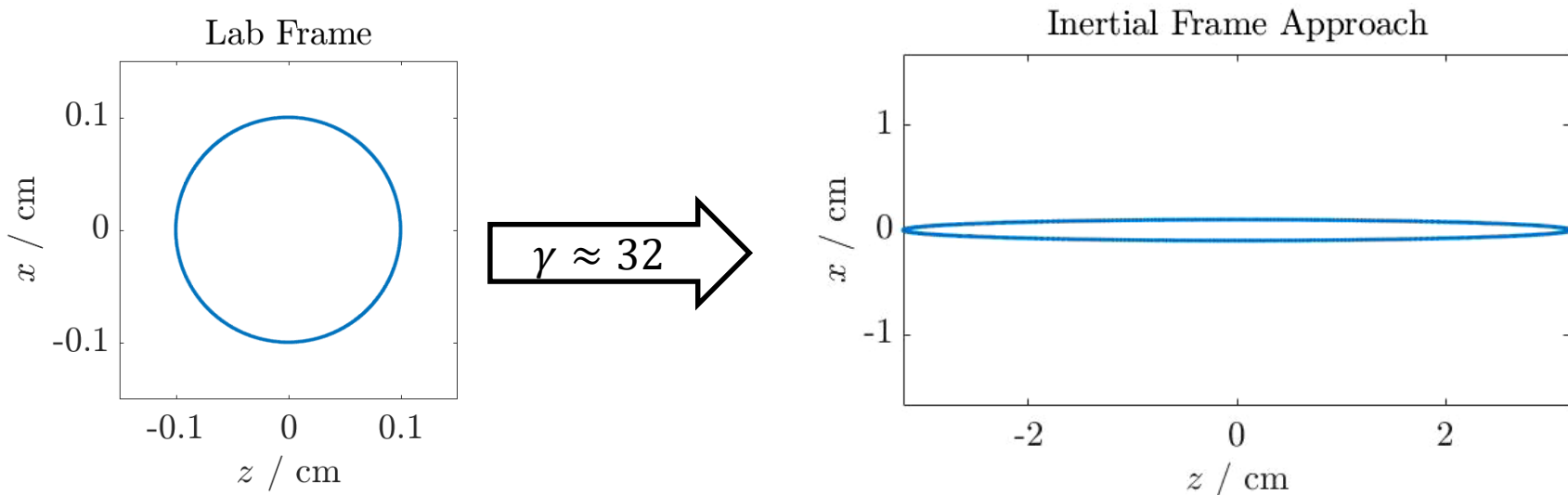
### Inertial Frame Approach:

Electrostatic solver in bunch „rest“ frame

⇒ Approximation exact if  $\beta = \text{const.}$  and  $\Delta\beta = 0$

⇒ Missing effects: Non-inertial frame, velocity dispersion, radiation

⇒ Codes: Astra3D (PIC-FFT), Krack (PIC-FFT), TEMF-Code (PP), REPTIL (FMM)



# Numerical Models

## Space Charge Models and Codes

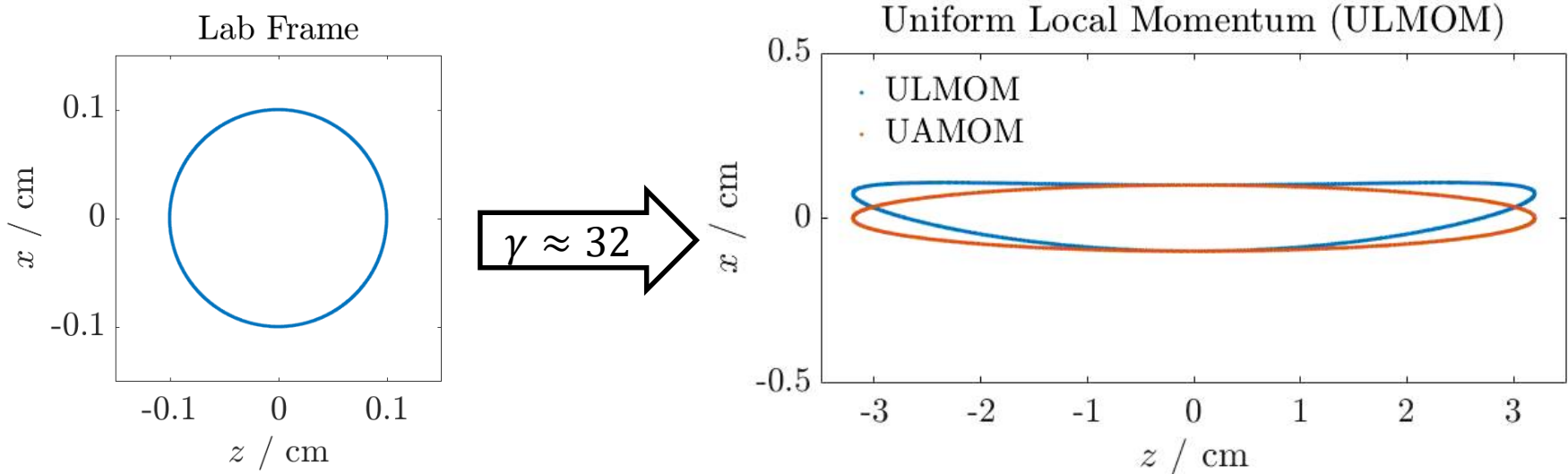
### Local Inertial Frame Approach:

Electrostatic solver for local particle frame

⇒ Approximation exact if  $\beta_{particle} = const.$

⇒ Missing effects: Nonlinear trajectory, radiation

⇒ Codes: TEMF-Code (PP), “REPTIL (w. energy binning)”





# Numerical Models

## Space Charge Models and Codes

### Liénard-Wiechert Approach:

Full electromagnetic solver

⇒ Evaluation of time-retardation  $|\vec{r}_i - \vec{r}_j| = c (t_i - t_j)$

⇒ Liénard-Wiechert fields include radiation

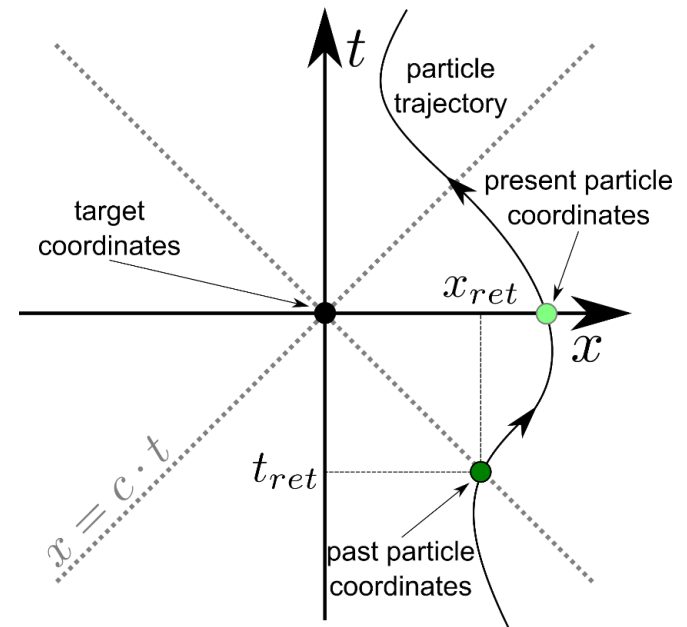
⇒ Codes: TEMF-Code (PP)

For  $t_j < t_0$  rigid bunch initialization:

$$\Rightarrow z_j(t_j) = z_j(t_0) - c \beta_z(t_0) (t_0 - t_j)$$

### Radiation Off:

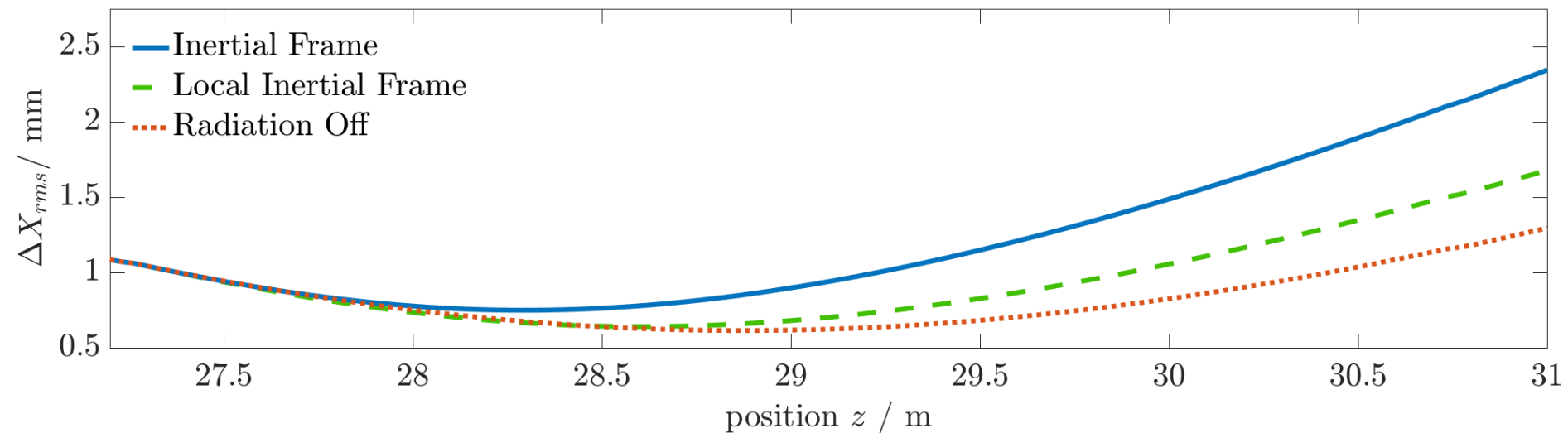
⇒ Setting  $\frac{d\vec{\beta}}{dt} = 0$  neglects radiation field



# Simulation Results

## Retardation and Radiation Effects

### Transversal Bunch Size Growth:

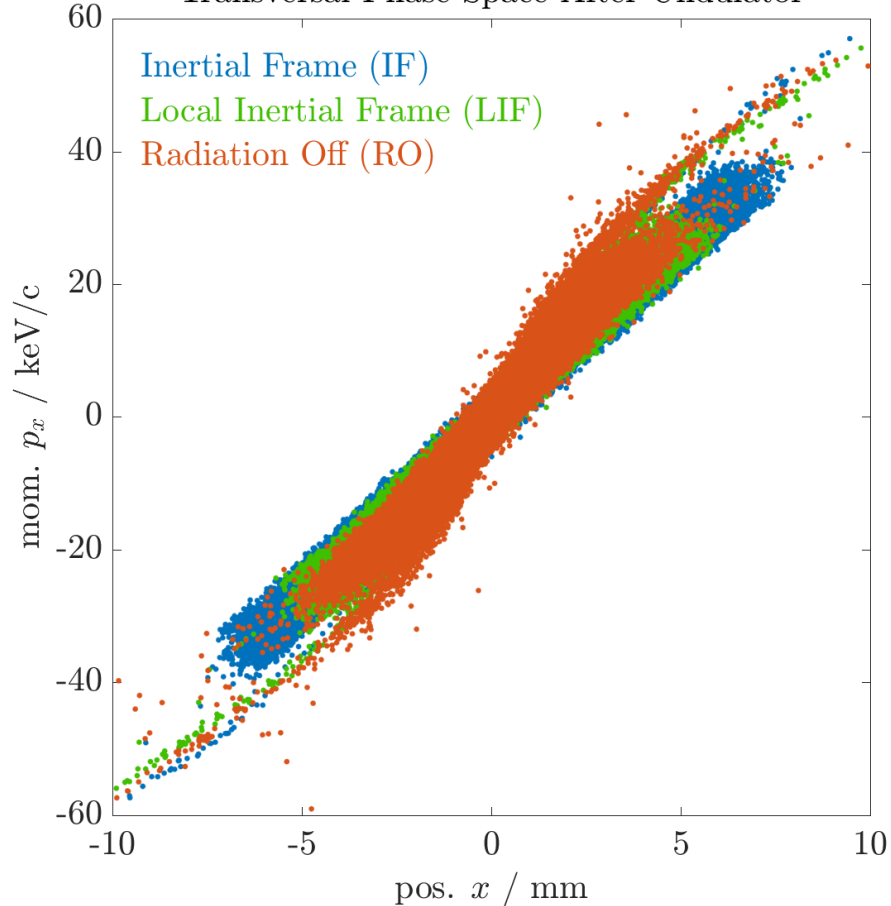


- Bunch size  $\Delta X_{rms}$  for IF model larger than for LIF model  
⇒ Artificial increase of static space charge effects
- Bunch size  $\Delta X_{rms}$  for RO smallest  
⇒ Static space charge effects not dominating

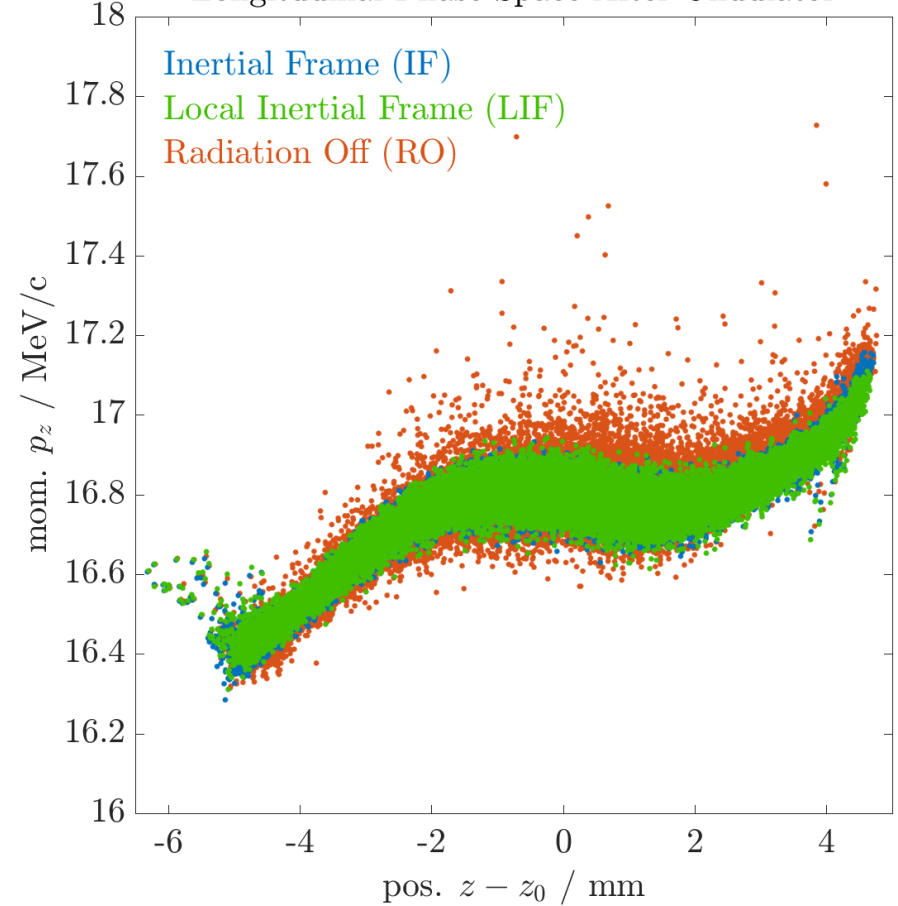
# Simulation Results

## Retardation and Radiation Effects

Transversal Phase Space After Undulator



Longitudinal Phase Space After Undulator

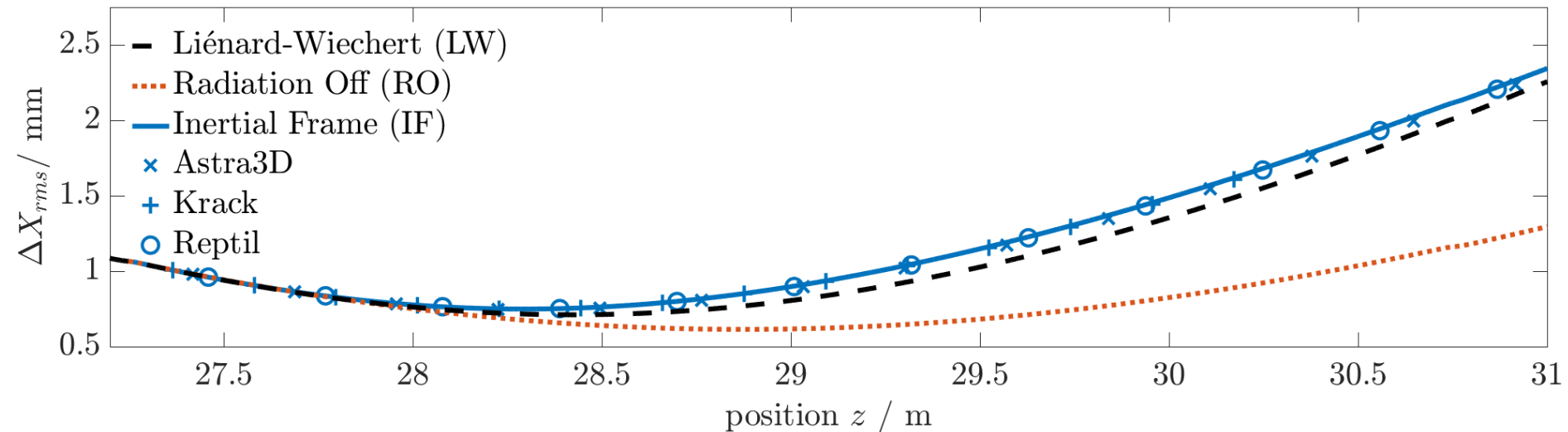


⇒ Inertial frame approaches overestimate static space charge field

# Simulation Results

## Retardation and Radiation Effects

### Transversal Bunch Size Growth:

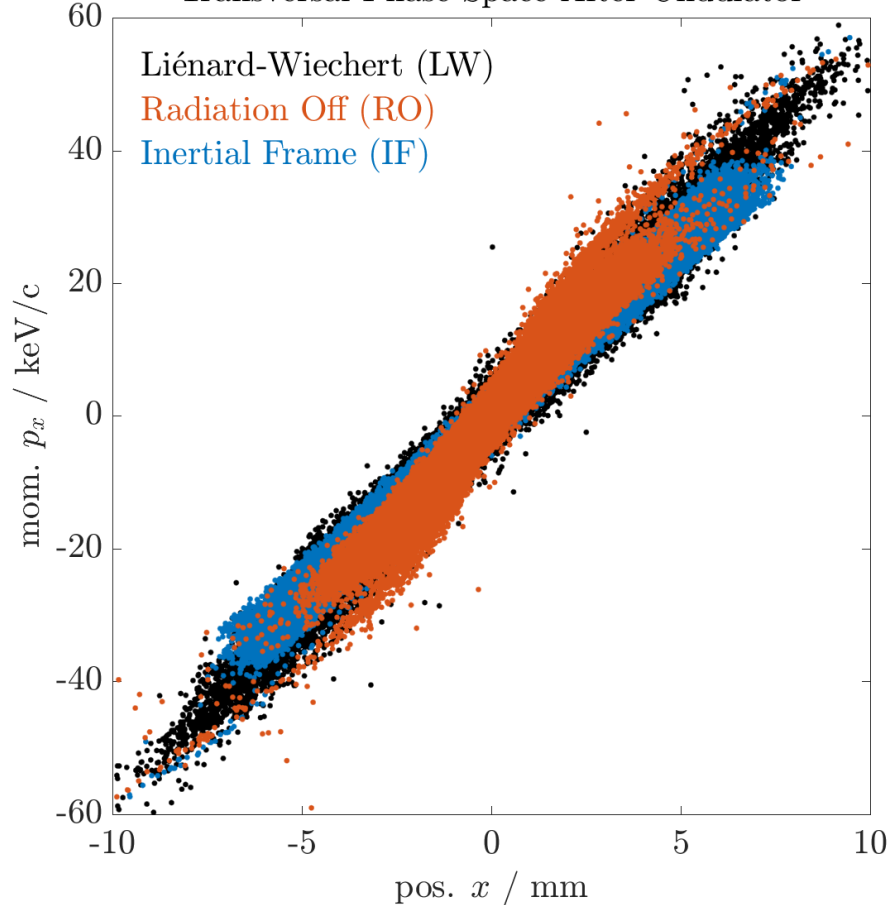


- Good agreement of IF, Astra3D, Krack and Reptil simulations
- Small difference in  $\Delta X_{rms}$  between IF and LW in bunch size
- Bunch size for RO significantly smaller than for LW  
⇒ Radiation leads to higher emittance

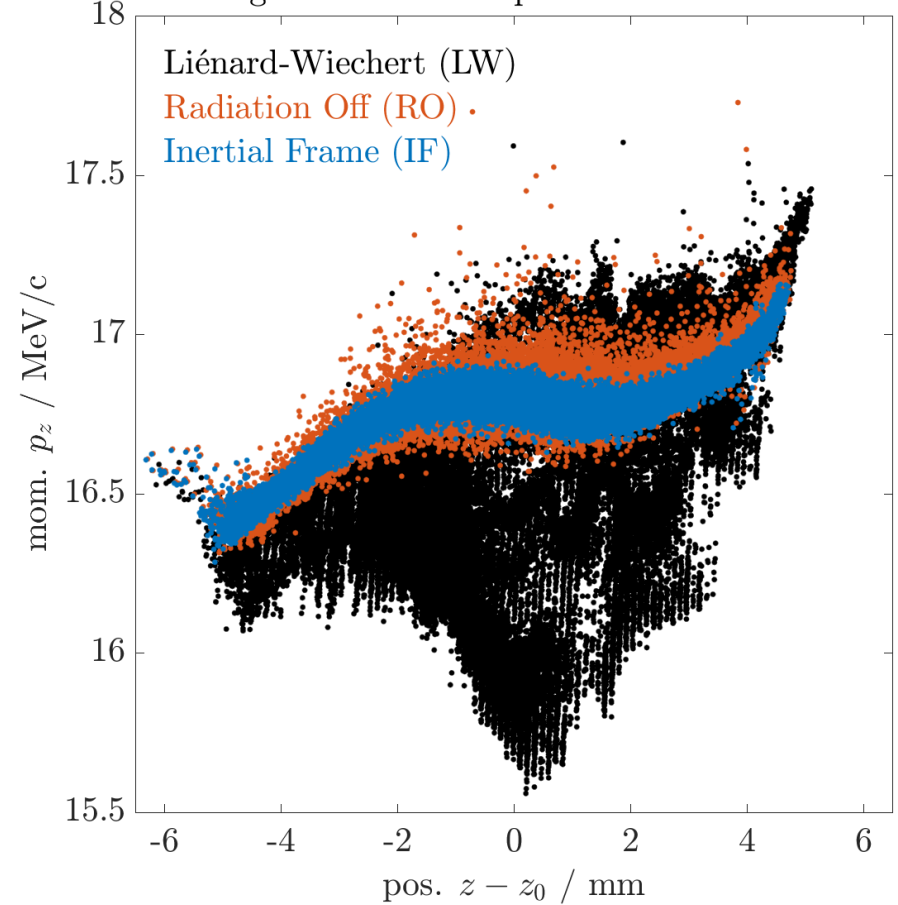
# Simulation Results

## Retardation and Radiation Effects

Transversal Phase Space After Undulator



Longitudinal Phase Space After Undulator



⇒ Radiation fields dominate space charge beam dynamics

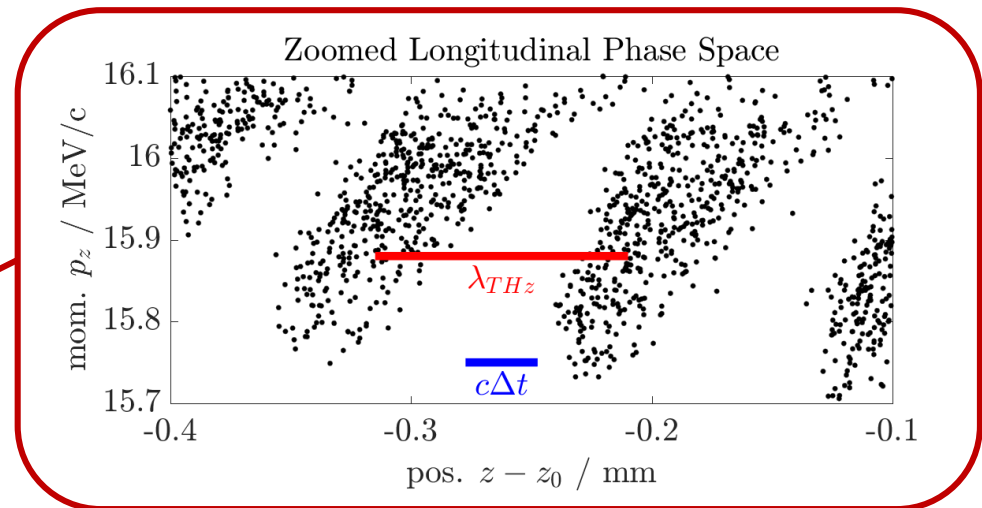
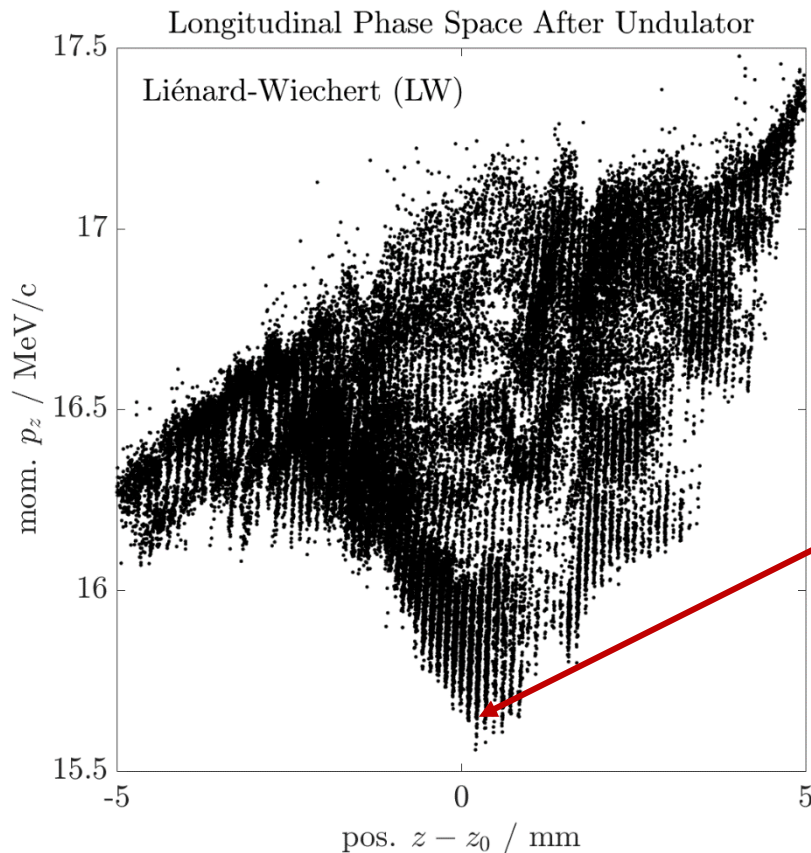
# Simulation Results

## Retardation and Radiation Effects

### Longitudinal Bunching:

$$\text{THz-Wavelength } \lambda_{\text{THz}} = \frac{\lambda_U}{2\gamma^2} \left(1 + \frac{K^2}{2}\right) \approx 105 \mu\text{m}$$

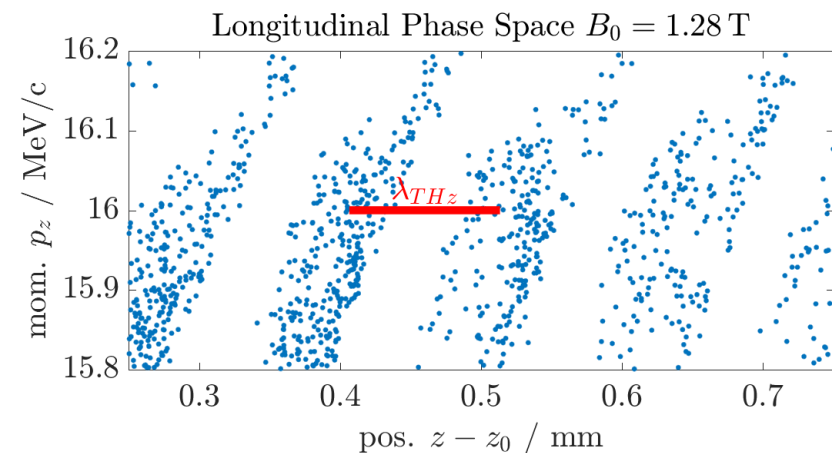
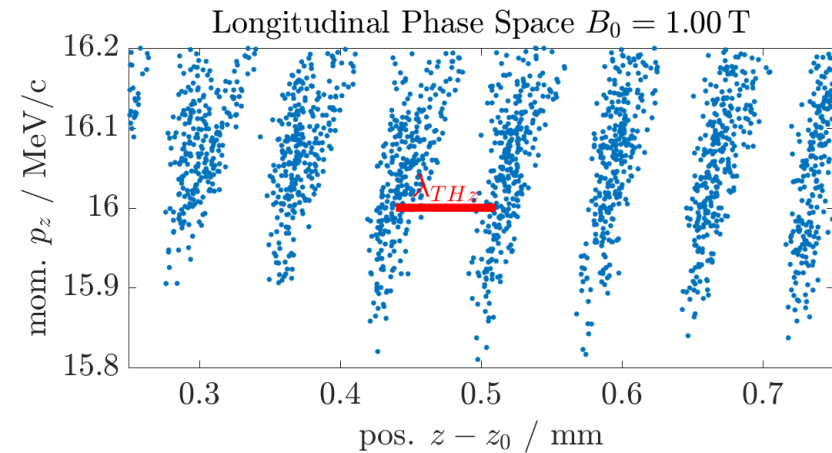
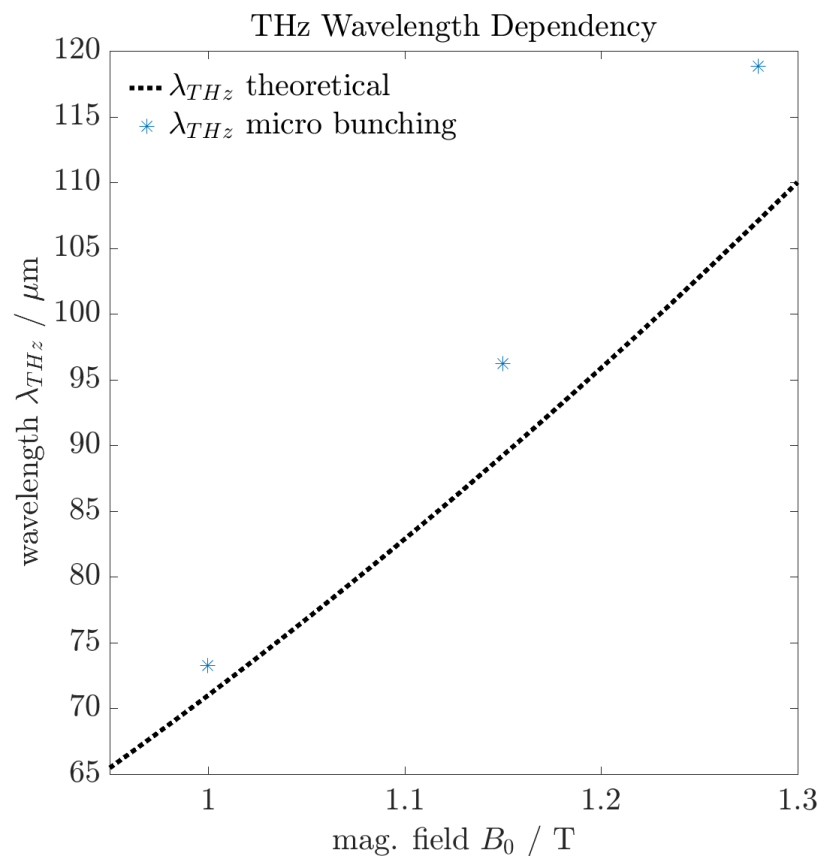
Micro-bunching consistent with  $\lambda_{\text{THz}}$



# Simulation Results

## Retardation and Radiation Effects

### THz-Wavelength Dependency:



# Simulation Results

## Retardation and Radiation Effects

### Particle Bunch at Undulator Exit:

Quantity	LW-model	IF-model	Rel. Dev.
$\Delta x_{rms}$	3.1 mm	3.1 mm	+0.5%
$\Delta y_{rms}$	2.6 mm	2.6 mm	-0.9%
$\Delta z_{rms}$	2.2 mm	2.2 mm	-2.3%
$\delta E_{rms}$	347.4 keV	95.7 keV	-72.5%
$\epsilon_x$	17 $\pi$ mrad mm	13 $\pi$ mrad mm	-24.6%
$\epsilon_y$	10 $\pi$ mrad mm	7 $\pi$ mrad mm	-30.5%
$\epsilon_z$	1366 $\pi$ mrad mm	282 $\pi$ mrad mm	-79.4%

- IF-model provides reasonable estimate for bunch size
- IF-model underestimates transversal emittance
- Strong deviation for energy spread and long. emittance



# Simulation Results

## Retardation and Radiation Effects

### Radiation Field:

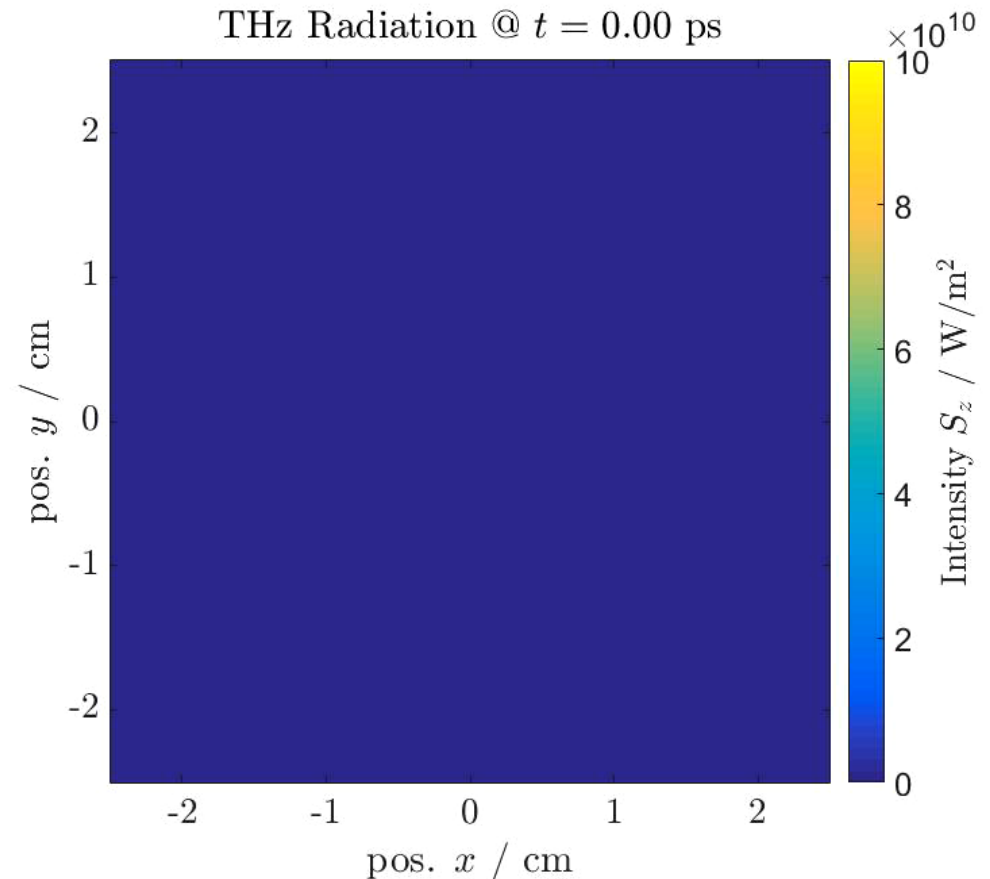
Screen at  $z = 30.8$  m

Spatial res.  $\Delta x = 100$   $\mu\text{m}$   
( $\lambda_{\text{THZ}} \approx 105$   $\mu\text{m}$ )

Temporal res.  $\Delta t = 0.1$  ps  
( $T_{\text{THZ}} \approx 0.35$  ps)

⇒ Memory limitation for  $\Delta t$

... work in progress



# Simulation Results

## Retardation and Radiation Effects

### Radiation Field:

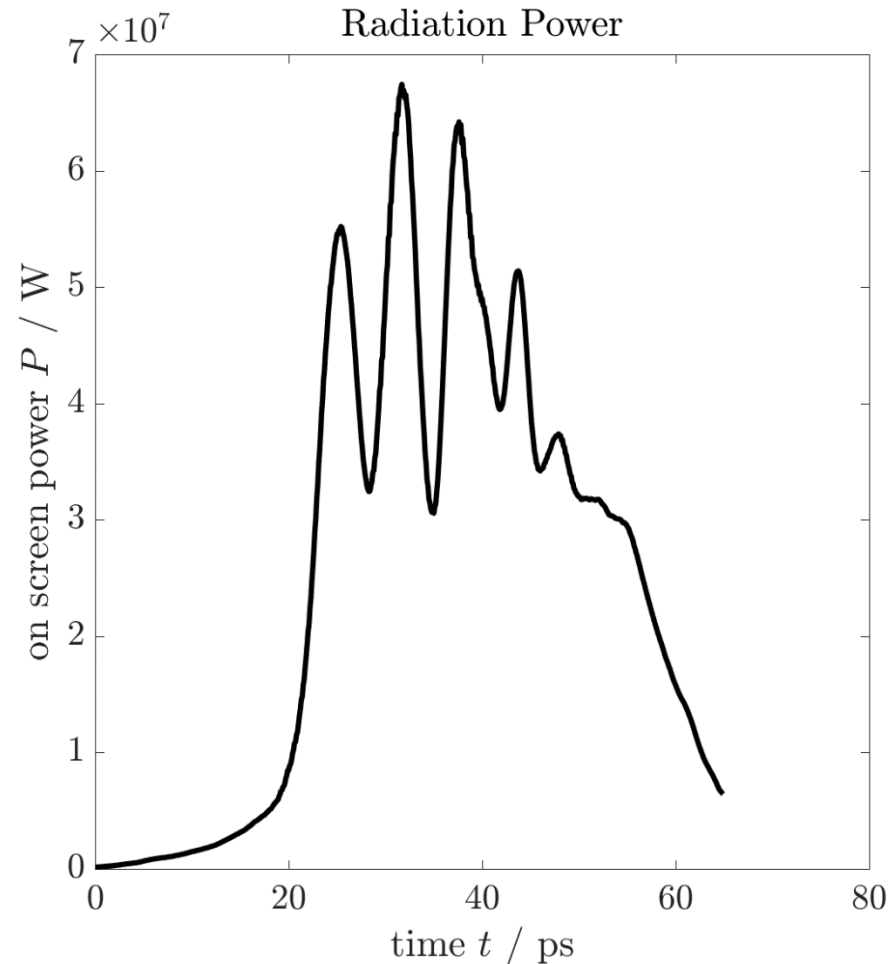
Screen at  $z = 30.8$  m

Spatial res.  $\Delta x = 100 \mu\text{m}$   
( $\lambda_{THZ} \approx 105 \mu\text{m}$ )

Temporal res.  $\Delta t = 0.1$  ps  
( $T_{THZ} \approx 0.35$  ps)

⇒ Memory limitation for  $\Delta t$

... work in progress



# Summary & Outlook

## Summary:

- Liénard-Wiechert simulations of THz-FEL undulator with  $N \leq 100k$  particles
- Inertial frame models overestimate static space charge effects
- Radiation effects dominate space charge beam dynamics
- Astra3D, Krack, and Reptil provide reasonable estimates for beam size, but cannot reproduce momentum space (no general statement)

## Outlook:

- Validation of Liénard-Wiechert simulations with CST EM-PIC
- Implementation of realistic undulator field map
- Study of bunch parameters: charge, size, etc...
- Approximation of particle world lines  $\Rightarrow$  red. LW-code memory requirements
- Approximation of retardation & retardation effects in Reptil  $\Rightarrow$  red. runtime

**...thanks to M. Krasilnikov for the provided data.**