Wakefield calculation in the frequency domain

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Motivation

- Long range wakefields
  - Low frequency, long bunches, bunch trains and/or high repetition rate, wall heating
- Approximation of geometry
  - Geometrical details smaller than bunch length, smooth tapering etc.
- Dispersive problems
  - Surface impedance, dielectrics
  - Free-space and waveguide boundary conditions
- Radiation effects
  - Curved beam trajectories and CSR
  - Beams with $\beta < 1$
- Coupler and waveguide signals

Frequency Domain Formulation

- The frequency domain problem

$$\nabla \times \mu^{-1} \nabla \times E - k_0^2 \varepsilon E = -jk_0 Z_0 J_s \qquad J_s(x,y,z,\omega) = \rho(x,y) e^{-i\frac{\omega}{v}z}$$

- Weak FE formulation: find $E \in H(curl)$ such that:

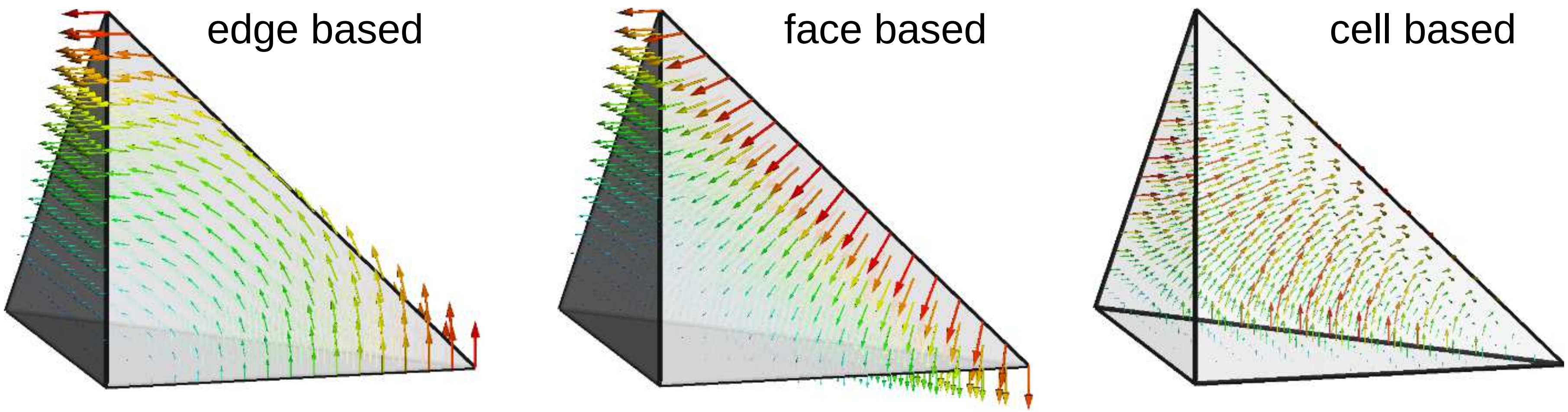
$$\int dV \mu^{-1} \nabla \times E \cdot \nabla \times v_h - k_0^2 \int dV \varepsilon \, E \cdot v_h =$$

$$-jk_0 Z_0 \int dV \, J_s \cdot v_h + \oint_S dS \; n \cdot [v_h \times \mu^{-1} \nabla \times E] \qquad \forall \, v_h \in H(curl)$$

$$\begin{array}{ccccccc} H_1 & \xrightarrow{grad} & \boldsymbol{H(curl)} & \xrightarrow{curl} & H(div) & \xrightarrow{div} & L_2 \\ \cup & & \cup & & \cup & & \cup \\ w_h^{p+1} & \xrightarrow{grad} & \boldsymbol{v_h^p} & \xrightarrow{curl} & q_h^{p-1} & \xrightarrow{div} & s_h^{p-2} \end{array}$$

Frequency Domain Formulation

- High-order hierarchic basis functions*



  - Allows for simple hp-adaption
  - **Supports mesh elements of different type + hybrid meshes**

**M. Ainsworth, J. Coyle: Int. J. of Numerical Methods in Eng., 2003.*

**J. Schöberl, S. Zaglmayr: Int. J. Comp. and Math. in Electrical and Electronic Eng., 2005.*

Frequency Domain Formulation

- Treatment of boundary surfaces

$$\int dV \mu^{-1}\, \nabla \times E \cdot \nabla \times v_h - k_0^2 \int dV \varepsilon\, E \cdot v_h =$$

$$-jk_0 Z_0 \int dV\, J_s \cdot v_h + \underbrace{\int_{S_{SIBC}} \boldsymbol{dS\, n \cdot [v_h \times \mu^{-1}\, \nabla \times E]}}_{\text{resistive wall}} + \underbrace{\int_{S_{WG}} dS n \cdot [v_h \times \mu^{-1}\, \nabla \times E]}_{\text{in \& outgoing pipes}}$$

- SIBC boundaries

$$\oint_{S_{SIBC}} dS\;\, n \cdot [v_h \times \mu^{-1}\, \nabla \times E] = \cdots = j\omega \boldsymbol{Y_S(\omega)} \oint_{S_{SIBC}} dS\;\, v_h \cdot E$$

Simple modification of the system matrix on SIBC surfaces

No fitting of the surface impedance function or ADE/convolution is needed

Frequency Domain Formulation

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- Treatment of boundary surfaces

$$\int dV \mu^{-1}\, \nabla \times E \cdot \nabla \times v_h - k_0^2 \int dV \varepsilon\, E \cdot v_h =$$

$$-jk_0 Z_0 \int dV\, J_s \cdot v_h + \underbrace{\int_{S_{SIBC}} dS\, n \cdot [v_h \times \mu^{-1}\, \nabla \times E]}_{\text{resistive wall}} + \underbrace{\int_{S_{WG}} \boldsymbol{dSn} \cdot [\boldsymbol{v_h} \times \boldsymbol{\mu^{-1}}\, \boldsymbol{\nabla} \times \boldsymbol{E}]}_{\text{in \& outgoing pipes}}$$

- Beam pipe boundaries

$$n \times \nabla \times E = n \times \nabla \times E^{inc} + \sum_m a_m^{TE} \gamma_m^{TE} e_m^{TE} + \sum_m a_m^{TM} \frac{-k_0^2}{\gamma_m^{TM}} e_m^{TM}$$

$$a_m^{TE} = \int_{S_{WG}} dS e_m^{TE} \cdot [E - E^{inc}]$$

Reflection coefficients for each mode

$$a_m^{TM} = \int_{S_{WG}} dS e_m^{TM} \cdot [E - E^{inc}]$$

Frequency Domain Formulation

- Beam pipe boundary conditions

$$\int dV \mu^{-1} \nabla \times E \cdot \nabla \times v_h - k_0^2 \int dV \varepsilon\, E \cdot v_h + \sum_m \boldsymbol{P_m^{TE}(E)} + \sum_m \boldsymbol{P_m^{TM}(E)} =$$

$$-jk_0 Z_0 \int dV\, J_s \cdot v_h + \oint_{S_{WG}} \boldsymbol{dS\ n \cdot [v_h \times \mu^{-1}\, \nabla \times E^{inc}]} + \sum_m \boldsymbol{U_m^{TE}} + \sum_m \boldsymbol{U_m^{TM}}$$

with $P_m^{TE}(E) = -\gamma_m^{TE} \left(\int_{S_{WG}} dS\, v_h \cdot e_m^{TE} \right) \left(\int_{S_{WG}} dS\, e_m^{TE} \cdot E \right), \qquad P_m^{TM}(E) = \cdots$

and matrix representation (TE):

$$P_m^{TE}(E) \;\rightarrow\; \mathbf{P}_m^{TE} \cdot \mathbf{e} = -\gamma_m^{TE} \mathbf{R}^T \cdot \mathbf{M}_m^{TE} \cdot \mathbf{R} \cdot \mathbf{e}$$

$$[\mathbf{R}\,]_{ij} = \int_{S_{WG}} dS\, \varphi_i^{2D} \cdot \varphi_j^{3D}$$

3D-to-2D projection matrix

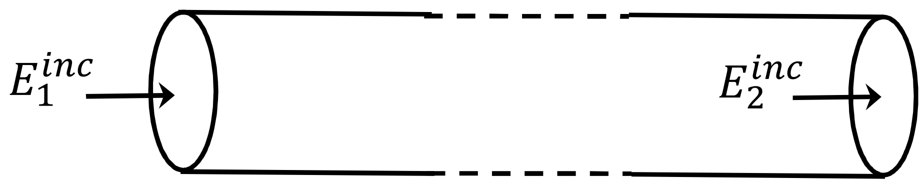
$$\mathbf{M}_m^{TE} = \mathbf{e}\,_m^{TE} \otimes \mathbf{e}\,_m^{TE}$$ dense modal dyadic

Frequency Domain Formulation

- Beam pipe boundary excitation
  - For an ultra-relativistic bunch (same idea for β < 1):

$$\nabla_t \cdot E^{inc} = \frac{1}{\varepsilon_0}\rho(x,y)e^{-ik_0 z_0}$$

$$\nabla \times E^{inc} = 0$$



2D-electrostatic problem at both ends of the pipe

  - Modal contribution to the RHS

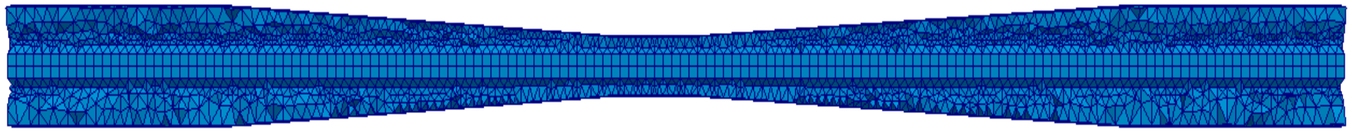
$$U_m^{TE}\left(E^{inc}\right) = -\gamma_m^{TE}\left(\int_{S_{WG}} dS\, v_h \cdot e_m^{TE}\right)\left(\int_{S_{WG}} dS\, e_m^{TE} \cdot E^{inc}\right)$$

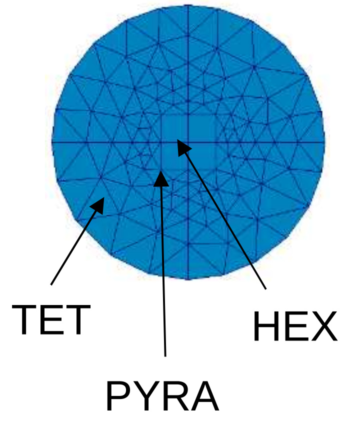
$$U_m^{TE}\left(E^{inc}\right) \rightarrow \cdot\, \mathbf{U}_m^{TE} \cdot \mathbf{e}^{inc} = -\gamma_0^{TE}\mathbf{R}^T \cdot \mathbf{M}_m^{TE} \cdot \mathbf{R}^{2D} \cdot \mathbf{e}^{inc}$$

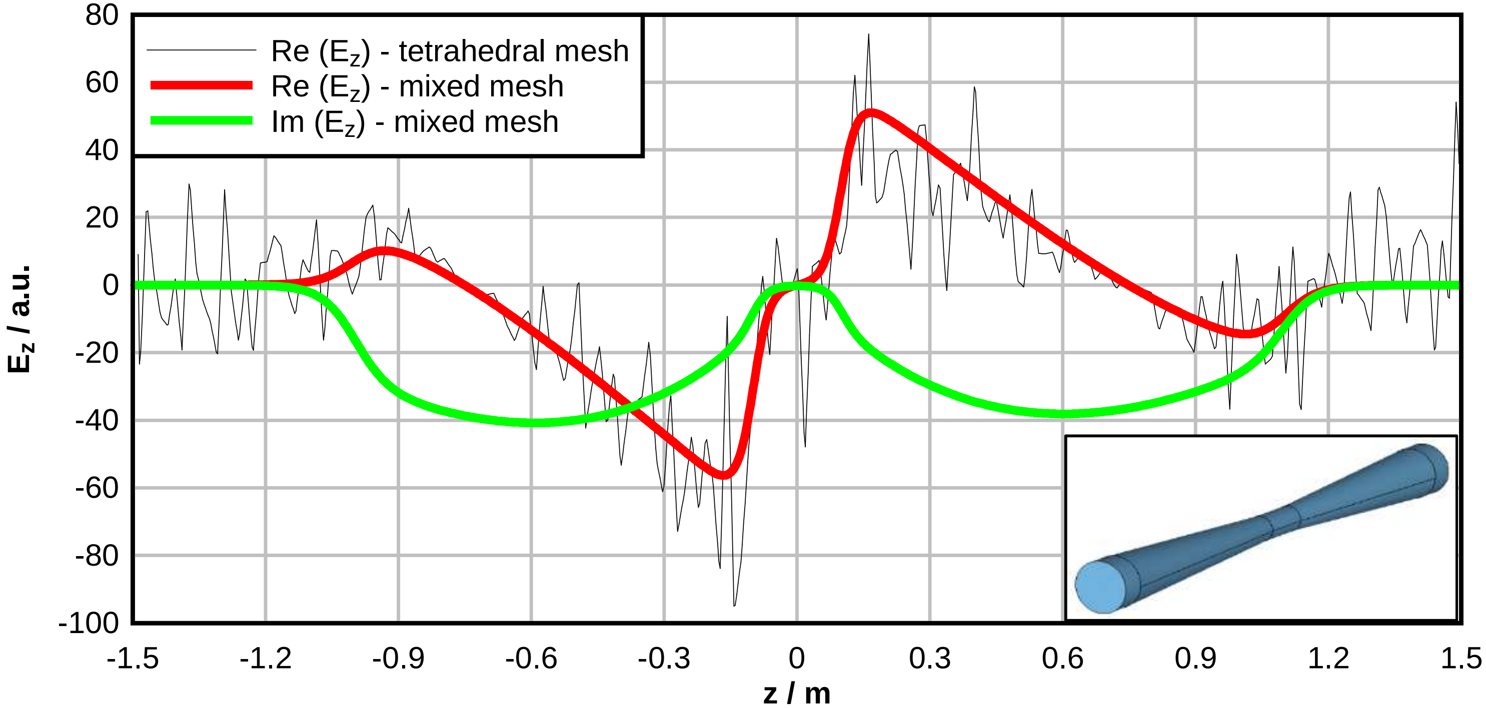
…do this for all waveguide modes supported in the pipe

Results

- Collimator – hybrid meshes



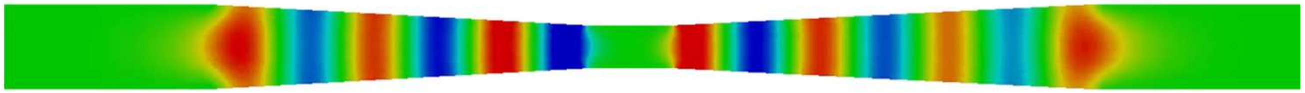




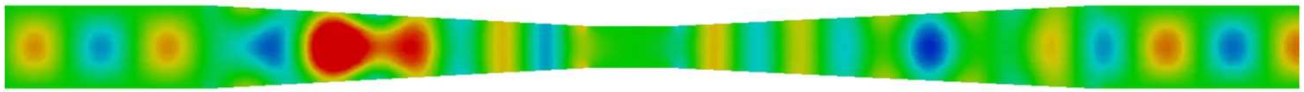
Longitudinal wakefield on axis at 100MHz

Results

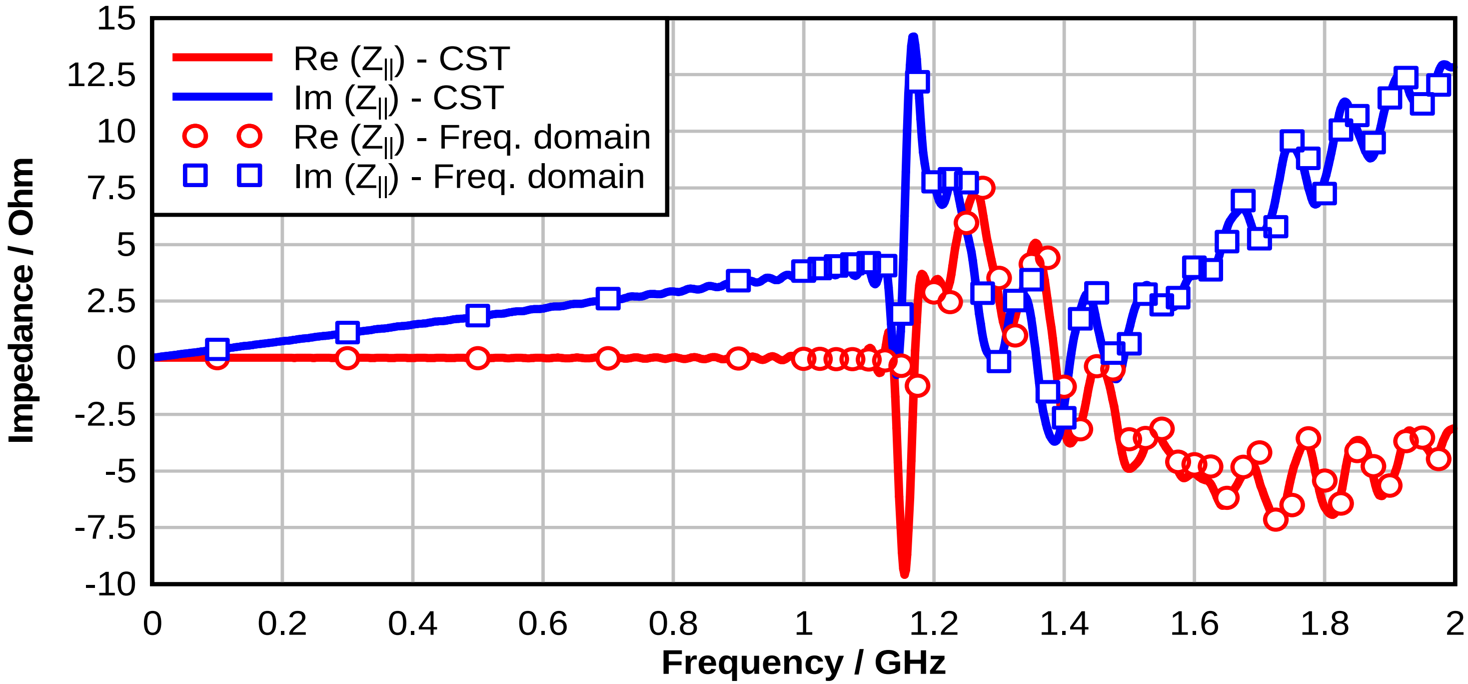
- Collimator – impedance



E_z – 1GHz



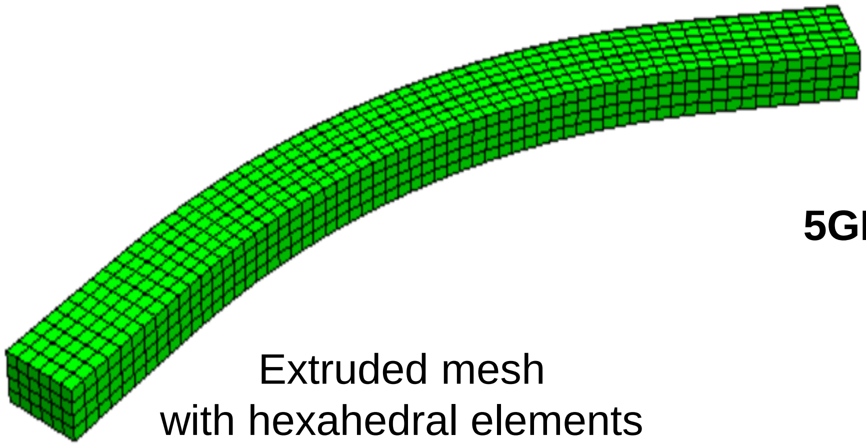
E_z – 1.5GHz



Comparison with CST PS (time domain)

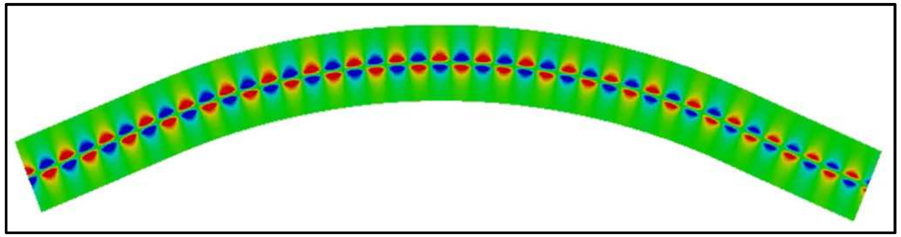
Results

- Waveguide bend

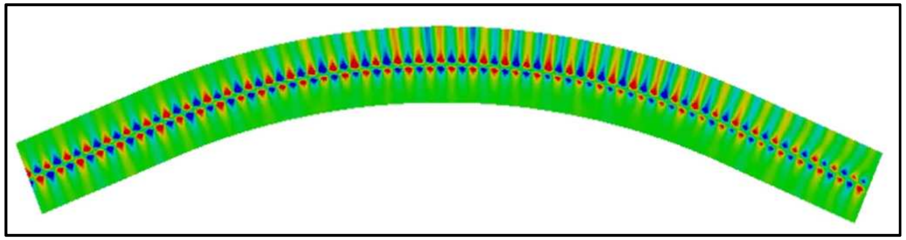


Extruded mesh
with hexahedral elements

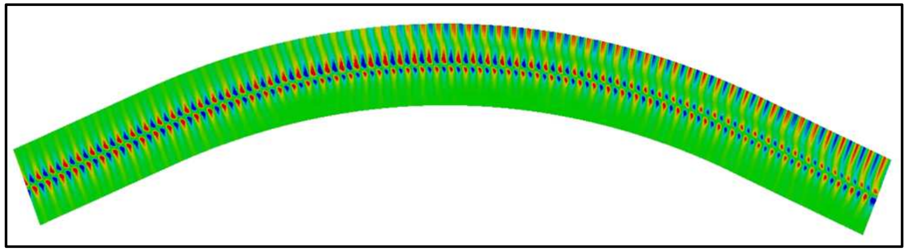
5GHz



10GHz



15GHz



Bend radius: 1m

Bend length: 0.825m

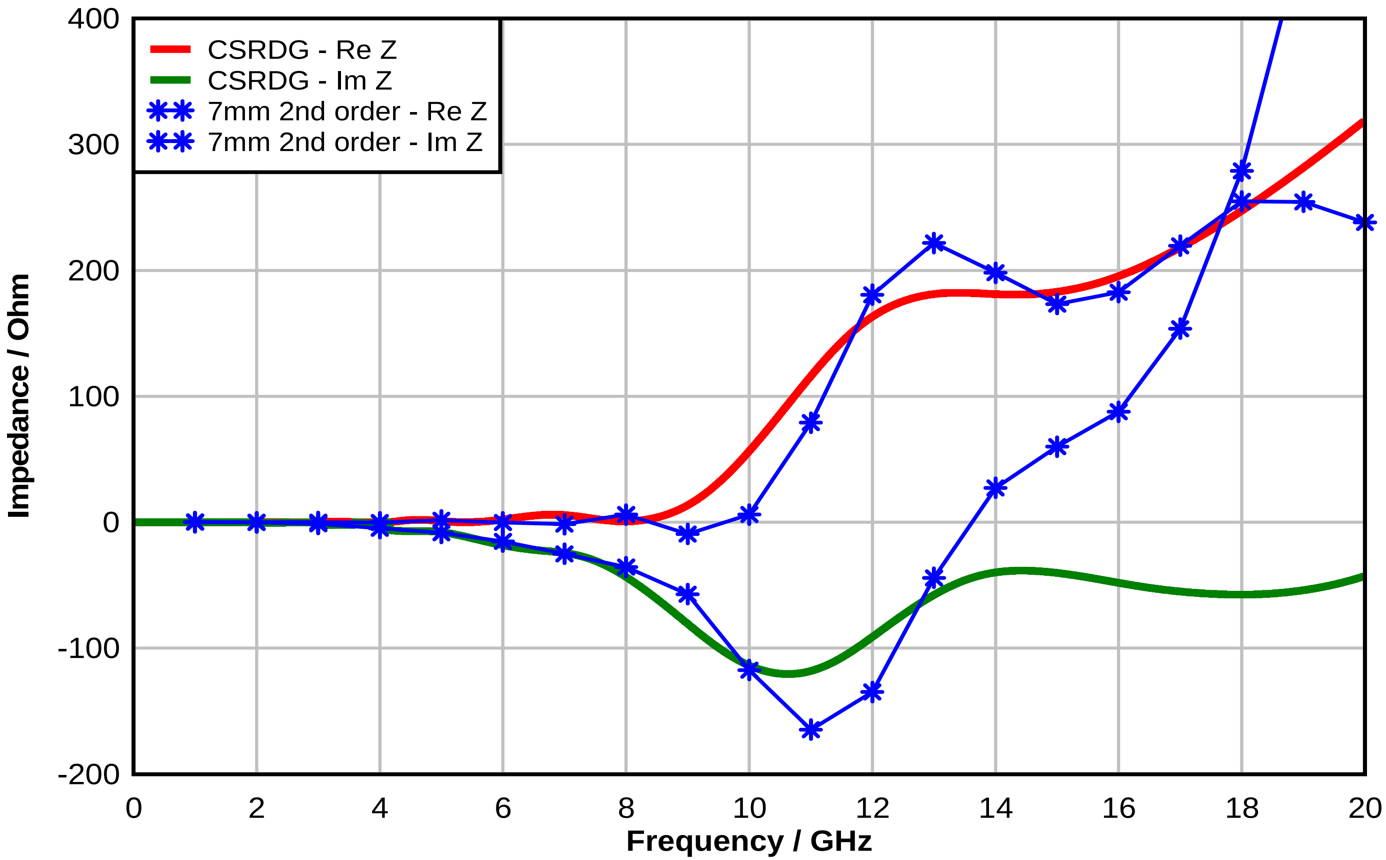
Straight sections: 0.2m

Waveguide: 100x50mm

(see also talk of D. Bizzozero)

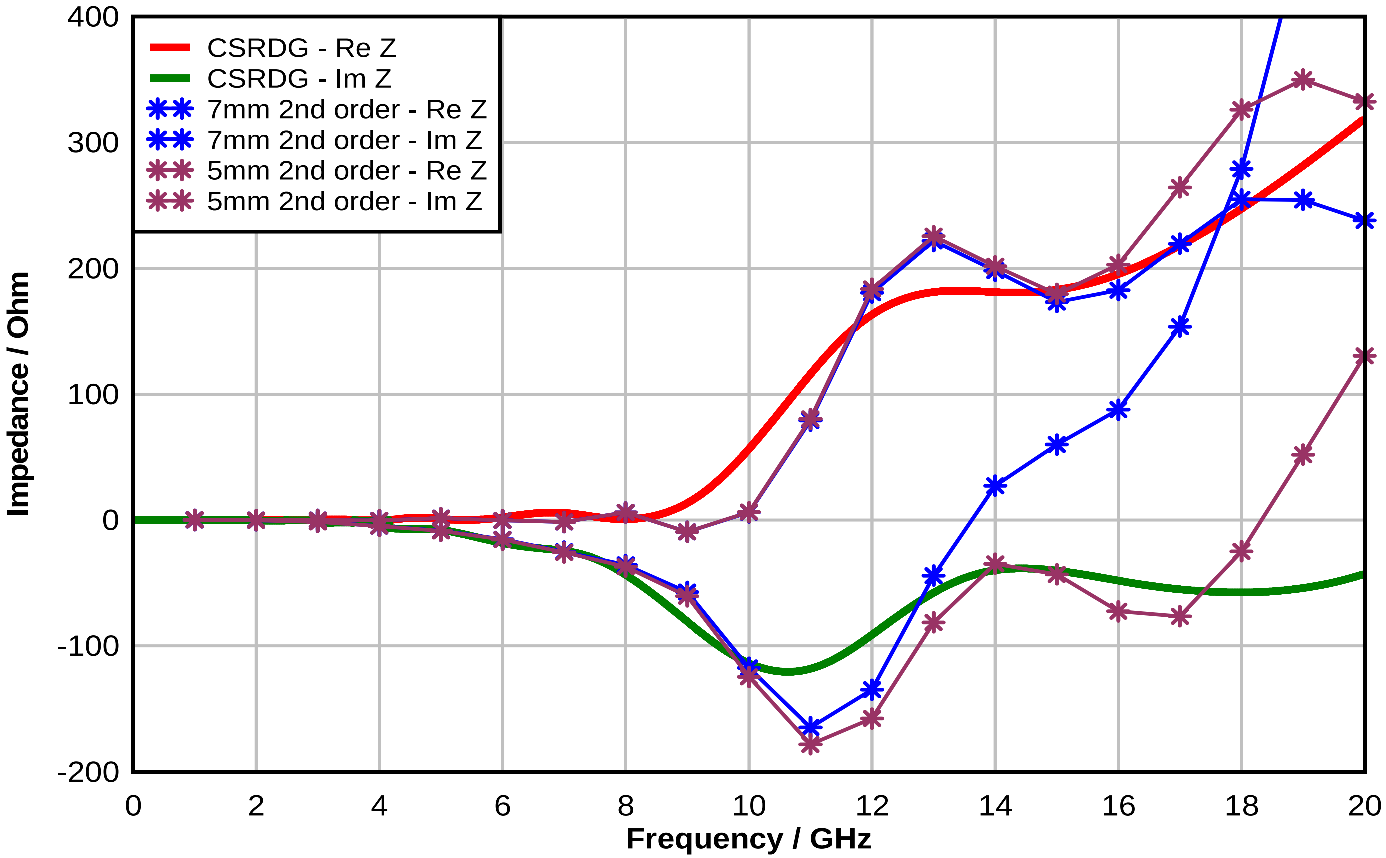
Results

- Waveguide bend



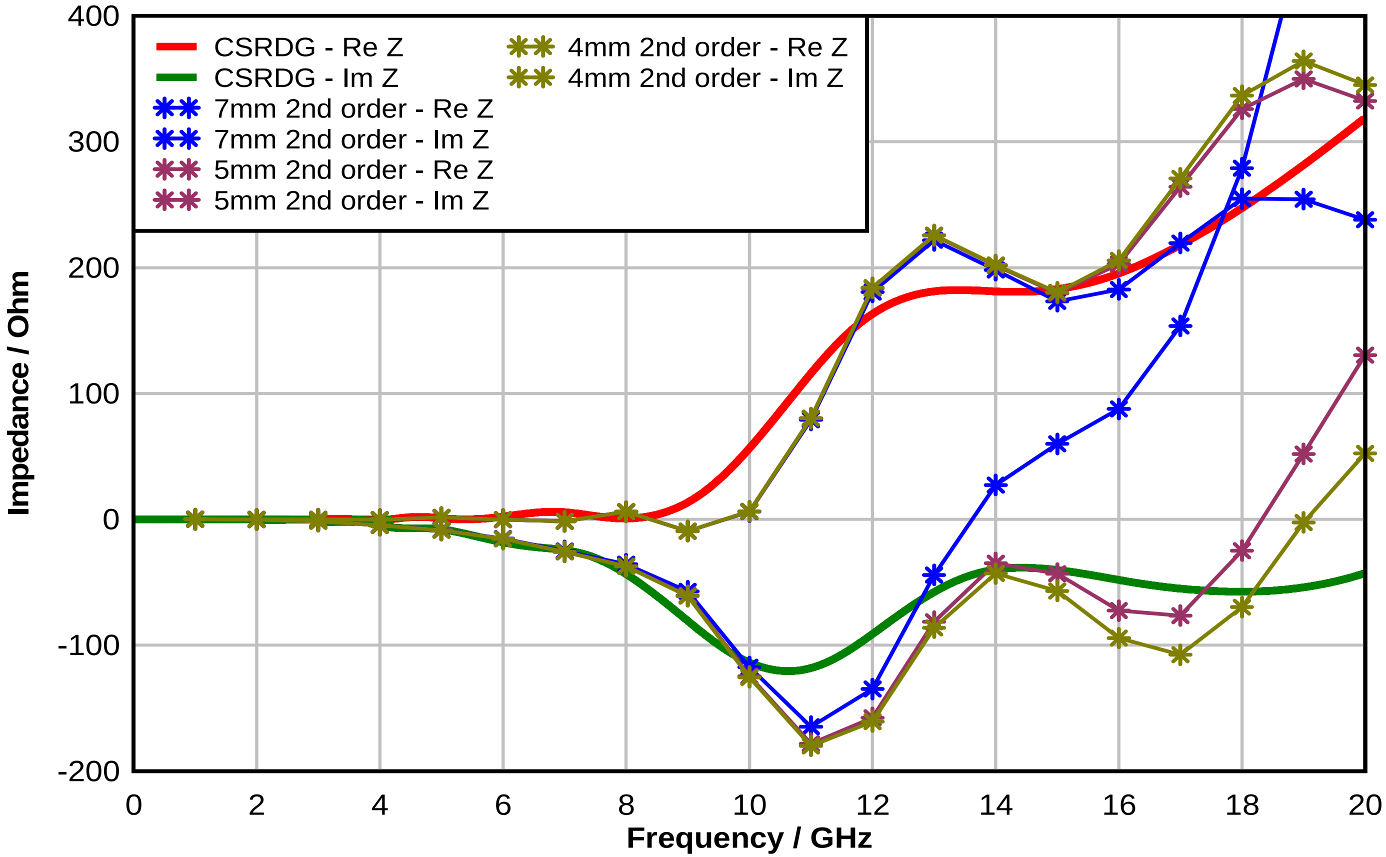
Results

- Waveguide bend



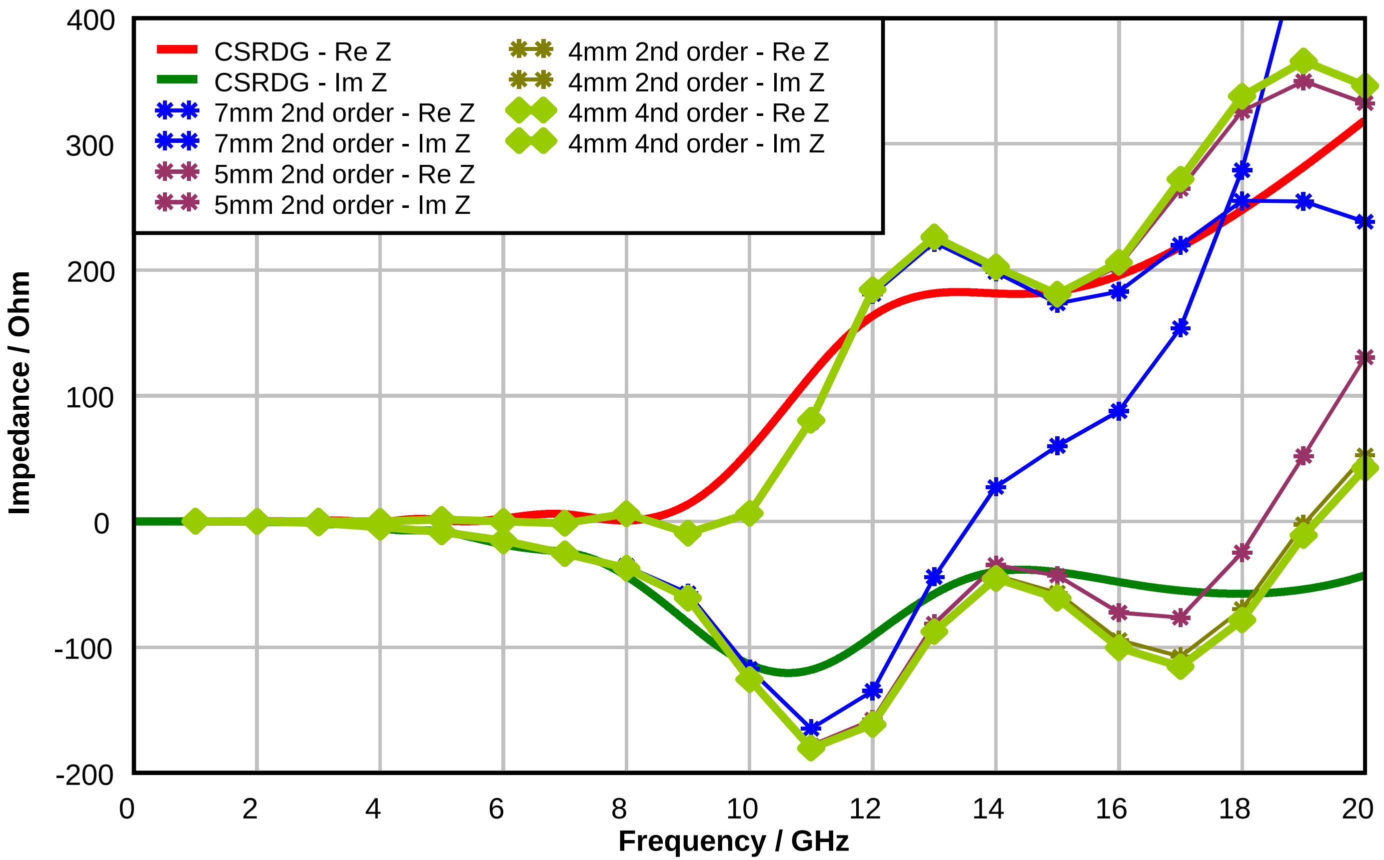
Results

- Waveguide bend



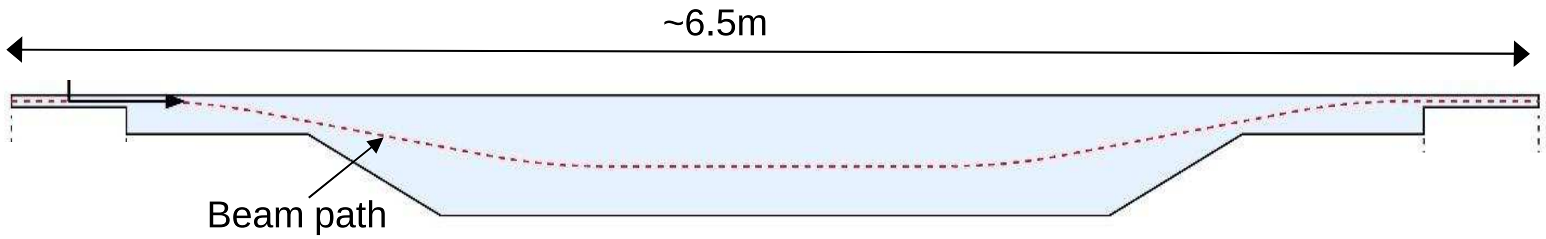
Results

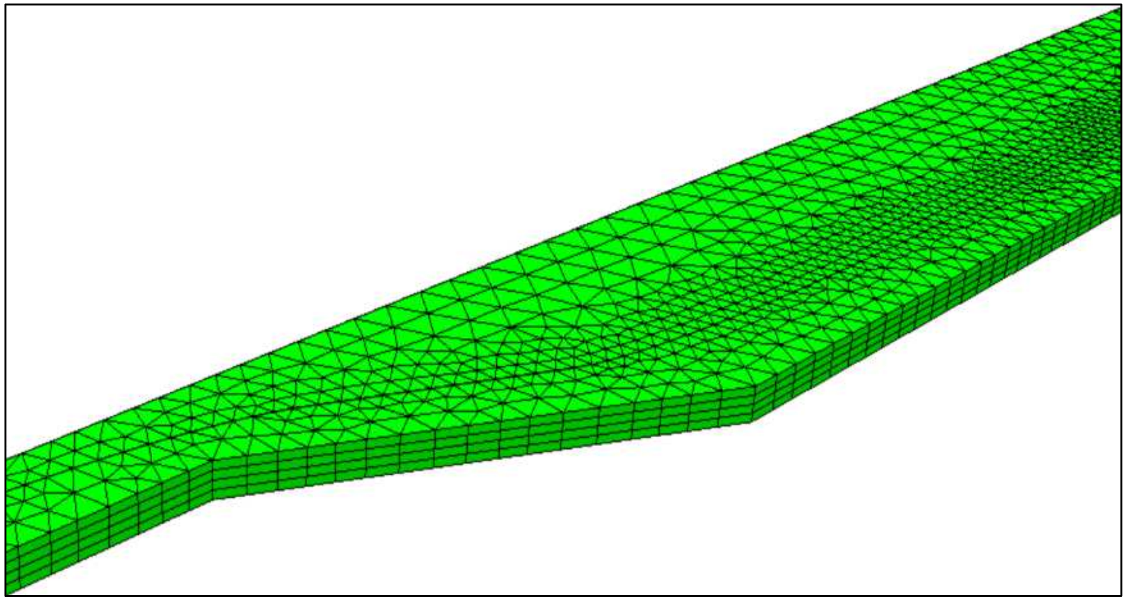
- Waveguide bend



Results

- XFEL bunch compressor





Prismatic element mesh:

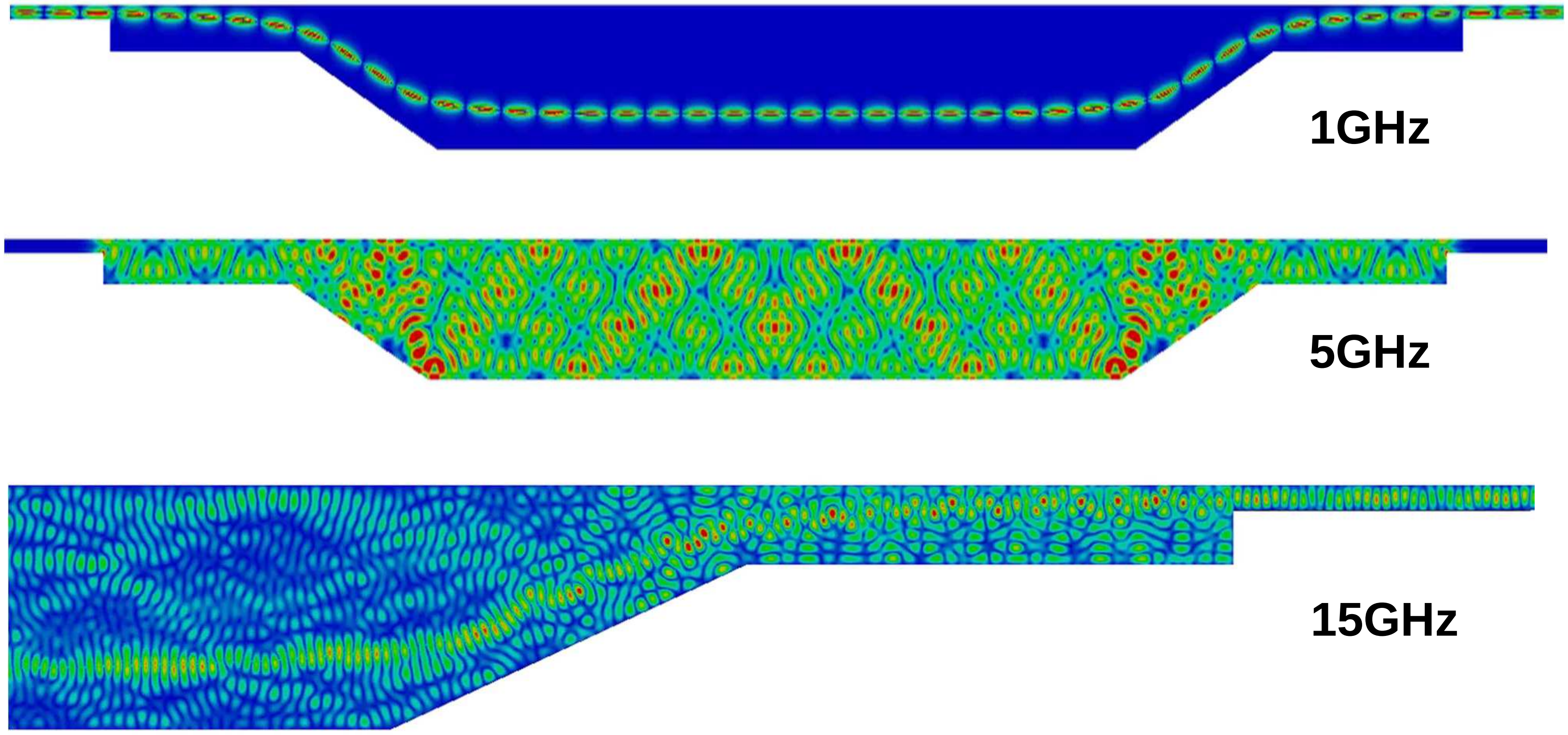
$\Delta \approx 5$mm

600k cells

4th order FEM

Results

- XFEL bunch compressor



Discussion & Outlook

- The frequency domain approach
  - Purpose: Fill the gap for some important wakefield/impedance problems
    - Complicated geometry
    - Long range wakefields – Joule losses
    - Resistive, rough surfaces, dispersive materials, waveguide openings
    - Curved beam trajectories and CSR. Validation of other CSR approaches
  - Status: Implementation of main code finished
    - Mixed, high-order elements, parallelization,…
    - Waveguide operators
  - ToDo: Enable larger problems and faster solutions
    - Domain decomposition based solver, multigrid, …
    - Fast spectral evaluation by model order reduction
  - Limitation: Huge size of discrete problem for ultra-high frequencies
    - Estimated upper limit using proper solvers and computing power ~100 GHz

Thank You for your attention