

Optics studies at FLASH in 2016-2017.

FEL-Seminar

J. Zemella, M. Vogt

DESY
Hamburg

27.06.2017

Matching of the actual optics to design optics.

Task: Change quadrupoles upstream of the reference point in a way that Twiss parameters of measurement are equal to design Twiss parameters.

- Need:
- ① Optics engine \Rightarrow MAD8 + linac extension = LMAD
 - ② Actual optics of FLASH \Rightarrow read out magnet currents and feed into LMAD \Rightarrow script do this.
 - ③ Matching engine \Rightarrow LMAD (SIMPLEX, MIGRAD, LMDIF)
- Quality factors for comparison of actual and design optics:

$$\text{mismatch parameter: } m_p = \frac{1}{2}(\beta\hat{\gamma} - 2\alpha\hat{\alpha} + \hat{\beta}\gamma)$$

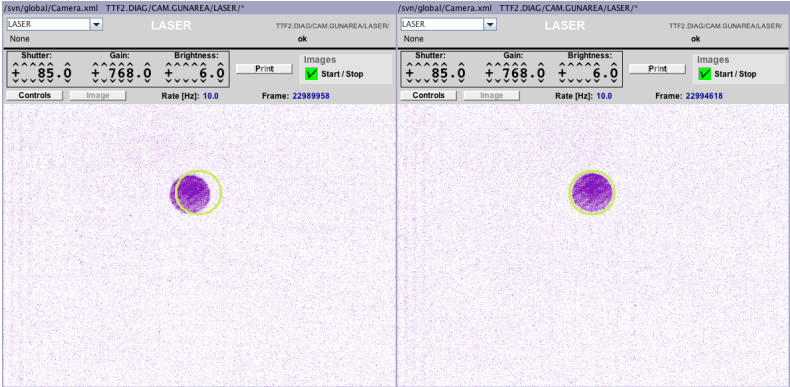
$$\text{mismatch amplitude: } \lambda_p = m_p + \sqrt{m_p^2 - 1}$$

- \Rightarrow The matching is done in a general way.
User can decide which point he wants to match in the beam line (FLASH1/2)

some measurements

some pre-setups before measuring beam sizes

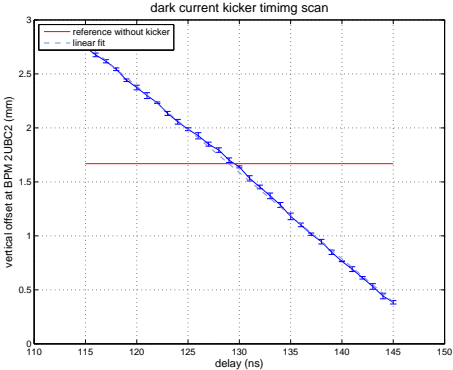
- 1 check laser position on virtual cathode



some measurements

some pre-setups before measuring beam sizes

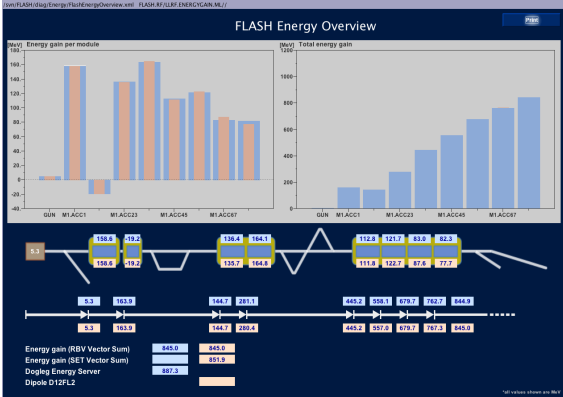
- 1 check laser position on virtual cathode
- 2 dark current kicker timing scan(?)



some measurements

some pre-setups before measuring beam sizes

- ① check laser position on virtual cathode
- ② dark current kicker timing scan(?)
- ③ choose right energy profile



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some pre-setups before measuring beam sizes

- ① check laser position on virtual cathode
- ② dark current kicker timing scan(?)
- ③ choose right energy profile
- ④ check machine optics fit to energy profile

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some pre-setups before measuring beam sizes

- ① check laser position on virtual cathode
- ② dark current kicker timing scan(?)
- ③ choose right energy profile
- ④ check machine optics fit to energy profile
- ⑤ measuring on-crest phases

some measurements

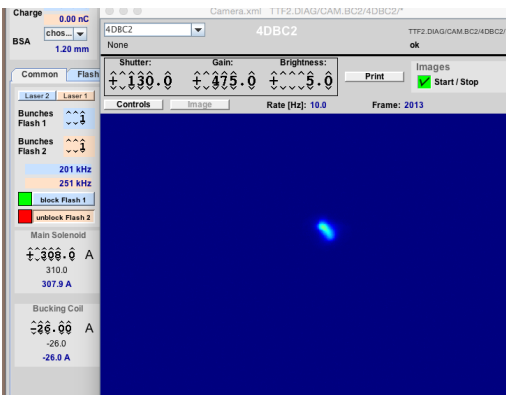
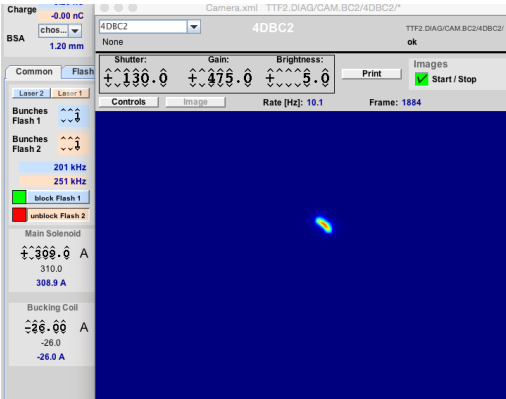
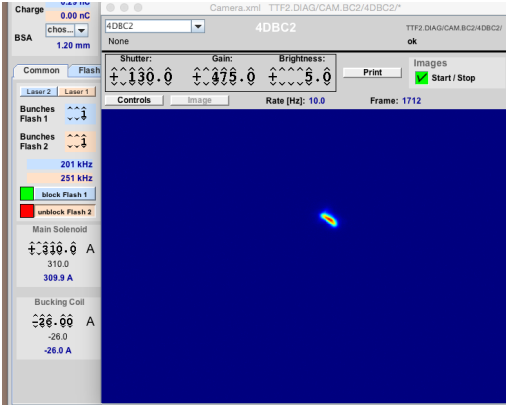
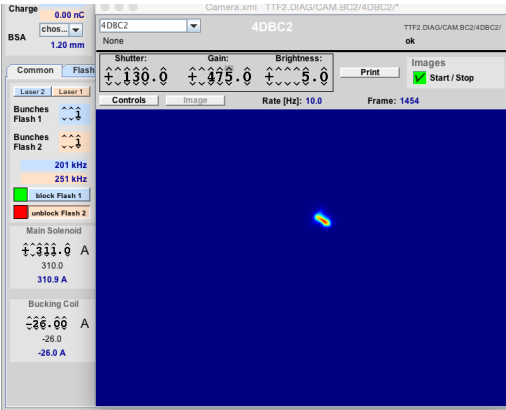
some pre-setups before measuring beam sizes

- ① check laser position on virtual cathode
- ② dark current kicker timing scan(?)
- ③ choose right energy profile
- ④ check machine optics fit to energy profile
- ⑤ measuring on-crest phases
- ⑥ setup charge right, BSA setting (a hint for me, because this is my main fault when setup this)

some measurements

some pre-setups before measuring beam sizes

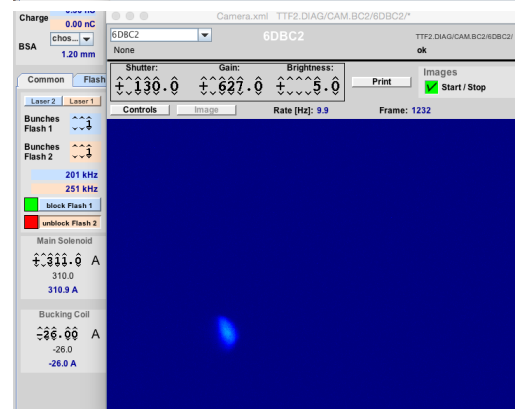
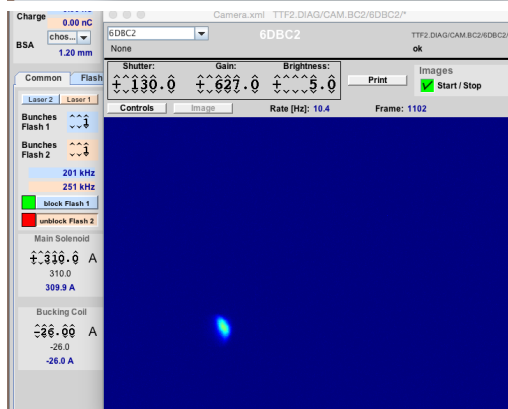
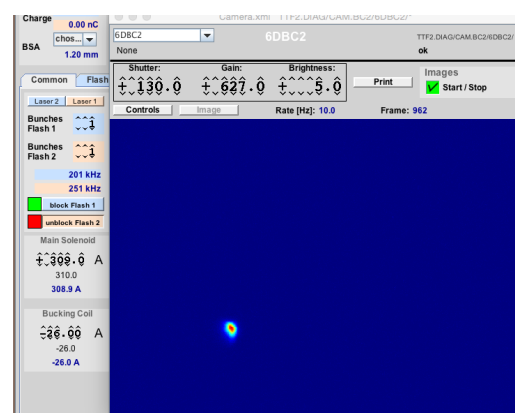
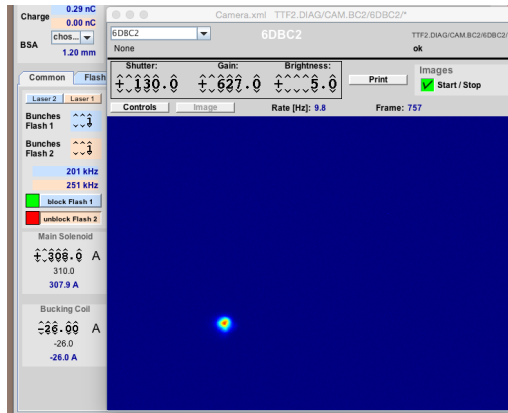
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- 5 measuring on-crest phases
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- 7 steering of the orbit in ACC1



some measurements

some pre-setups before measuring beam sizes

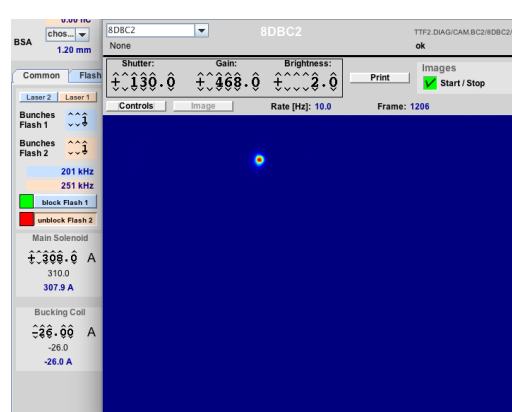
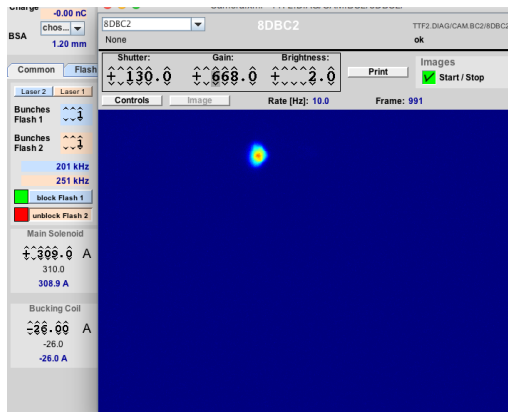
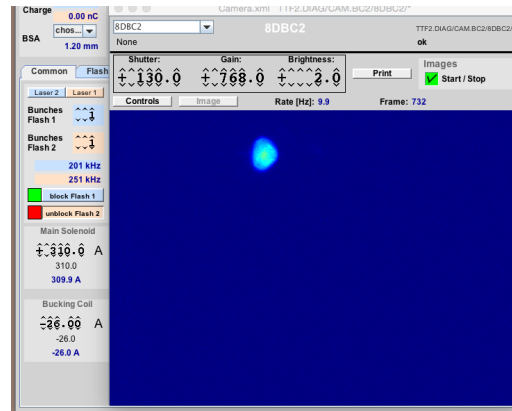
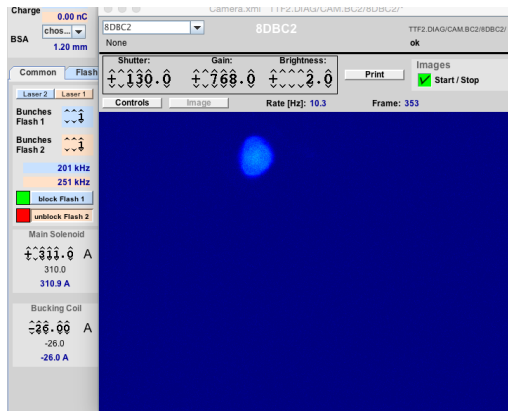
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some pre-setups before measuring bunch sizes

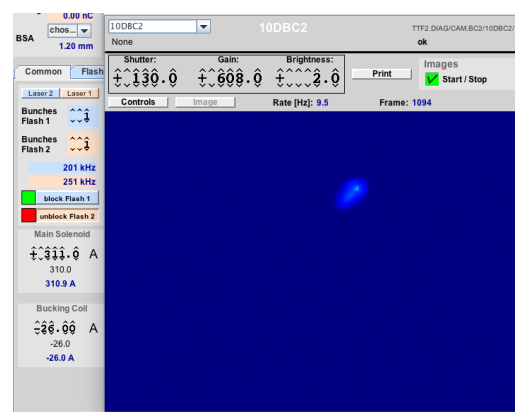
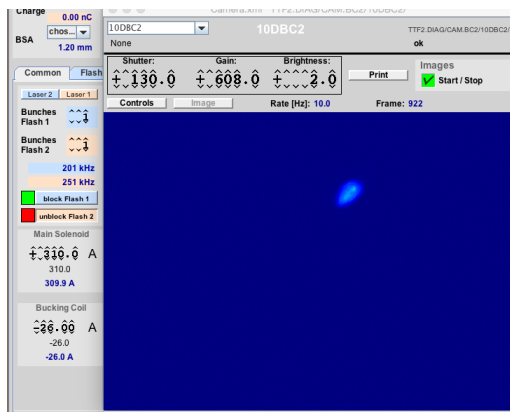
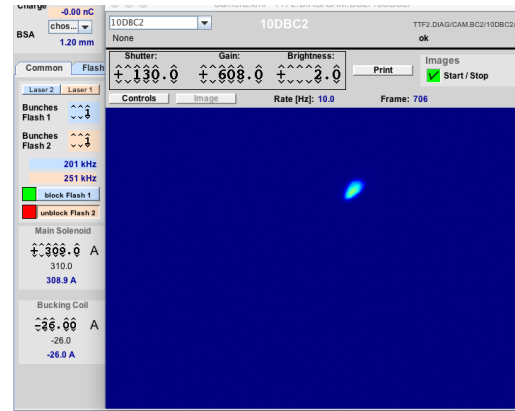
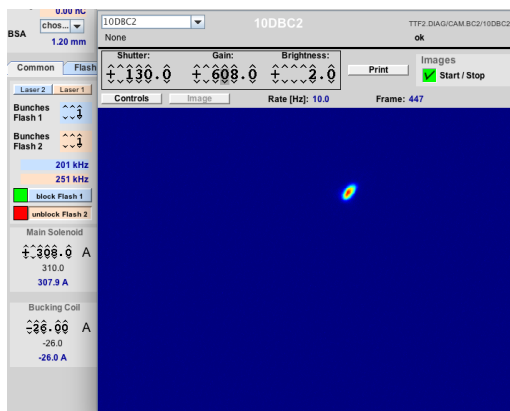
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some measurements

some pre-setups before measuring beam sizes

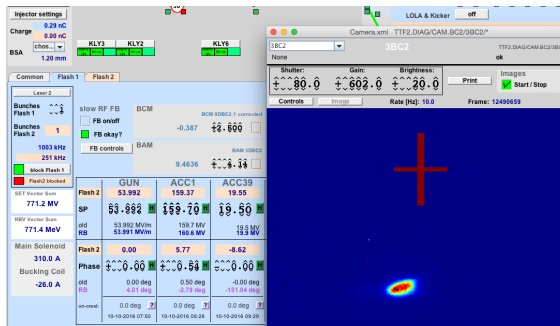
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- 8 setup minimum E-spread



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some pre-setups before measuring beam sizes

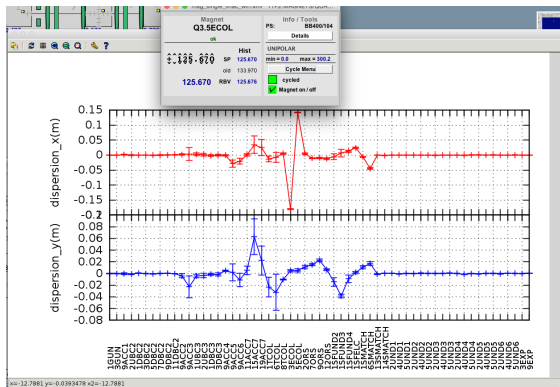
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- 7 steering of the orbit in ACC1
- 8 setup minimum E-spread
- 9 optimizing beam spots on DBC2 screens using solenoid current



some measurements

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- 3 choose right energy profile
- 4 check machine optics fit to energy profile
- 5 measuring on-crest phases
- 6 setup charge right, BSA setting (a hint for me, because this is my main fault when setup this)
- 7 steering of the orbit in ACC1
- 8 setup minimum E-spread
- 9 optimizing beam spots on DBC2 screens using solenoid current
- 10 close dispersion in dogleg and FL2EXTR



some measurements

injector matching for different charges

charge: 0.29 nC, BSA: 1.2 mm

4DBC2 6DBC2 8DBC2 10DBC2

$\beta_T = 2.45$ m $\beta_M = 2.68 \pm 0.07$ m

$\alpha_T = -1.18$ $\alpha_M = -1.30 \pm 0.04$

$\varepsilon_T = 0.49$ μm $\varepsilon_M = 0.49 \pm 0.01$ μm

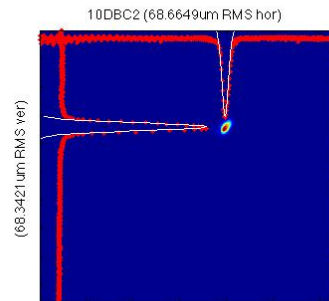
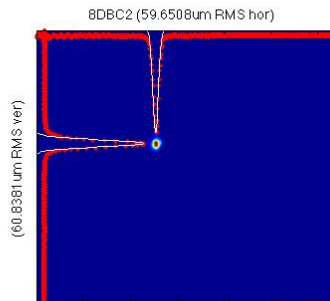
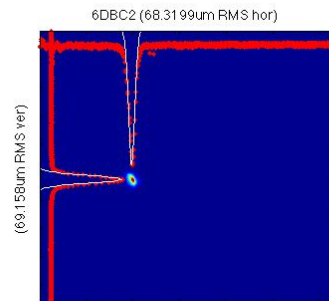
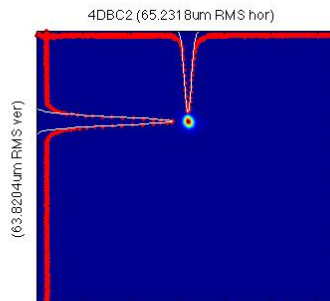
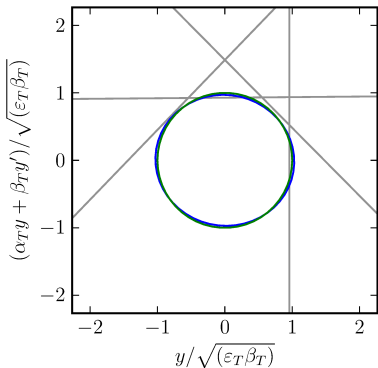
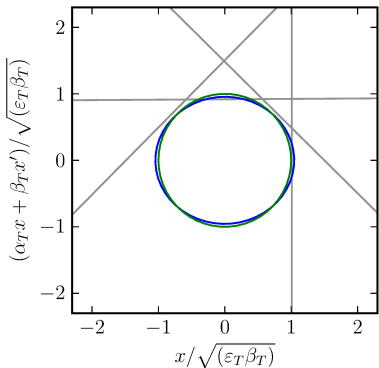
$m_P = 1.00$ $\lambda_P = 1.09$

$\beta_T = 2.58$ m $\beta_M = 2.73 \pm 0.06$ m

$\alpha_T = 1.24$ $\alpha_M = 1.34 \pm 0.05$

$\varepsilon_T = 0.49$ μm $\varepsilon_M = 0.49 \pm 0.01$ μm

$m_P = 1.00$ $\lambda_P = 1.06$



some measurements

injector matching for different charges

charge: 0.41 nC, BSA: 1.2 mm

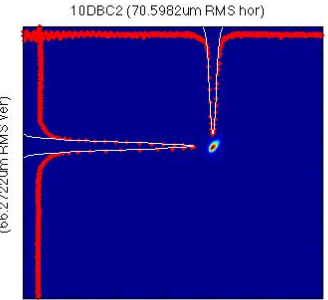
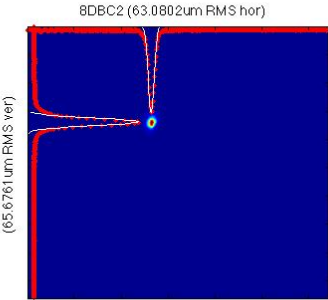
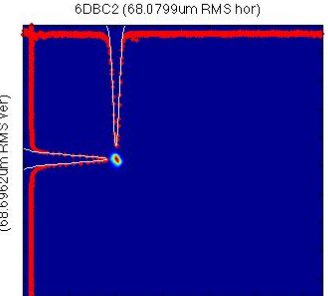
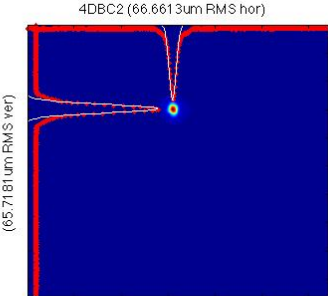
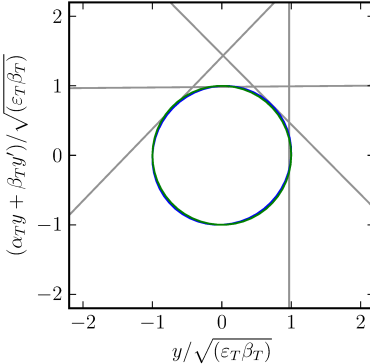
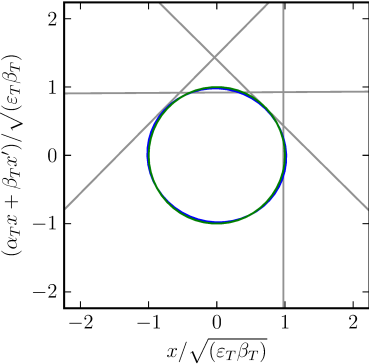
4DBC2 6DBC2 8DBC2 10DBC2

$\beta_T = 2.45 \text{ m}$ $\beta_M = 2.55 \pm 0.06 \text{ m}$
 $\alpha_T = -1.18$ $\alpha_M = -1.20 \pm 0.03$
 $\epsilon_T = 0.54 \text{ } \mu\text{m}$ $\epsilon_M = 0.54 \pm 0.01 \text{ } \mu\text{m}$

$m_P = 1.00$ $\lambda_P = 1.05$

$\beta_T = 2.58 \text{ m}$ $\beta_M = 2.58 \pm 0.07 \text{ m}$
 $\alpha_T = 1.24$ $\alpha_M = 1.20 \pm 0.05$
 $\epsilon_T = 0.51 \text{ } \mu\text{m}$ $\epsilon_M = 0.51 \pm 0.02 \text{ } \mu\text{m}$

$m_P = 1.00$ $\lambda_P = 1.04$



some measurements

injector matching for different charges

charge: 0.47 nC, BSA: 1.3 mm

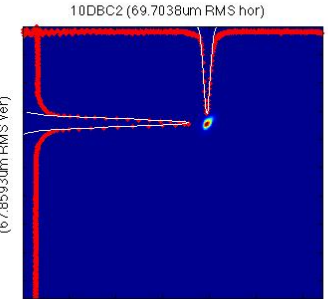
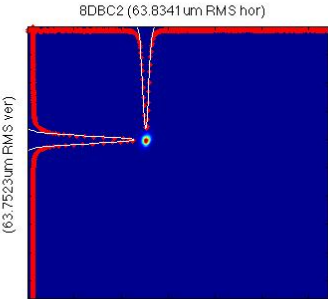
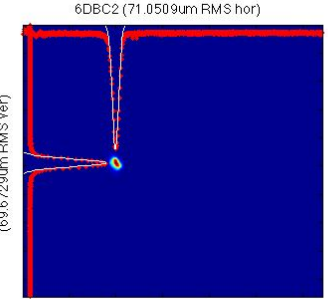
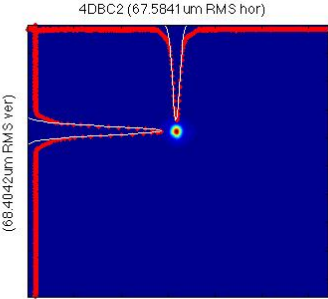
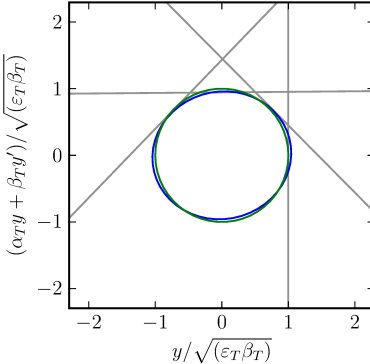
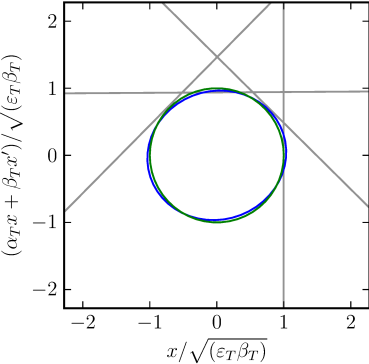
4DBC2 6DBC2 8DBC2 10DBC2

$\beta_T = 2.45 \text{ m}$ $\beta_M = 2.63 \pm 0.08 \text{ m}$
 $\alpha_T = -1.18$ $\alpha_M = -1.32 \pm 0.04$
 $\varepsilon_T = 0.53 \text{ } \mu\text{m}$ $\varepsilon_M = 0.53 \pm 0.01 \text{ } \mu\text{m}$

$m_P = 1.00$ $\lambda_P = 1.09$

$\beta_T = 2.58 \text{ m}$ $\beta_M = 2.80 \pm 0.08 \text{ m}$
 $\alpha_T = 1.24$ $\alpha_M = 1.31 \pm 0.06$
 $\varepsilon_T = 0.52 \text{ } \mu\text{m}$ $\varepsilon_M = 0.52 \pm 0.02 \text{ } \mu\text{m}$

$m_P = 1.00$ $\lambda_P = 1.10$



some measurements

injector matching for different charges

charge: 0.57 nC, BSA: 1.5 mm

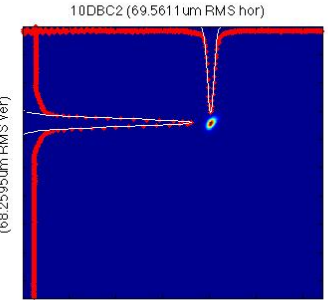
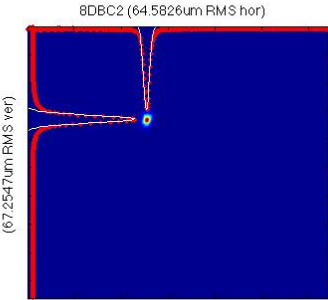
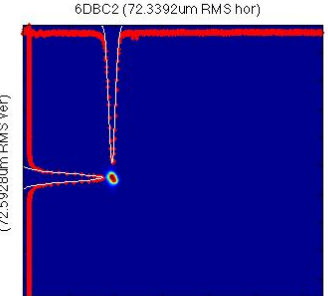
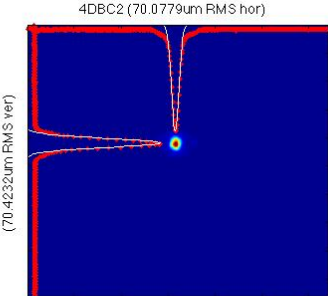
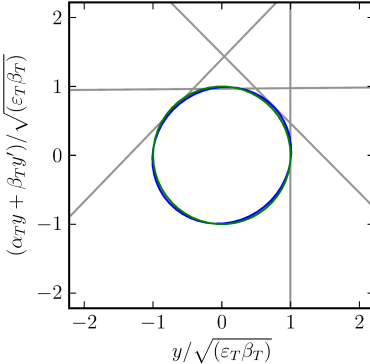
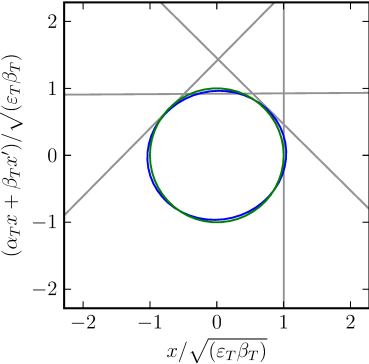
4DBC2 6DBC2 8DBC2 10DBC2

$\beta_T = 2.45 \text{ m}$ $\beta_M = 2.64 \pm 0.09 \text{ m}$
 $\alpha_T = -1.18$ $\alpha_M = -1.31 \pm 0.05$
 $\epsilon_T = 0.57 \text{ } \mu\text{m}$ $\epsilon_M = 0.57 \pm 0.01 \text{ } \mu\text{m}$

$m_P = 1.00$ $\lambda_P = 1.09$

$\beta_T = 2.58 \text{ m}$ $\beta_M = 2.63 \pm 0.07 \text{ m}$
 $\alpha_T = 1.24$ $\alpha_M = 1.20 \pm 0.05$
 $\epsilon_T = 0.55 \text{ } \mu\text{m}$ $\epsilon_M = 0.55 \pm 0.01 \text{ } \mu\text{m}$

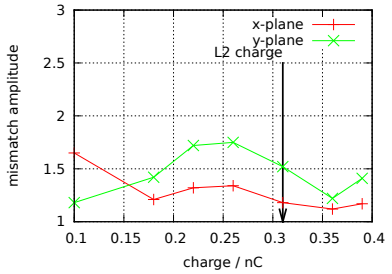
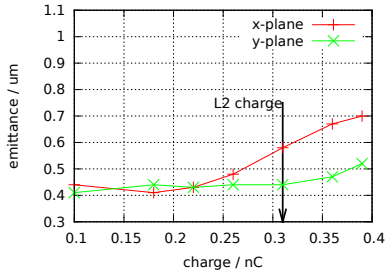
$m_P = 1.00$ $\lambda_P = 1.07$



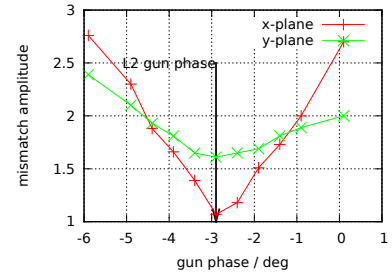
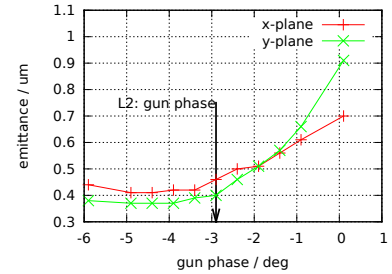
some measurements

Injector match of LASER 1 while LASER 2 is matched using free parameters

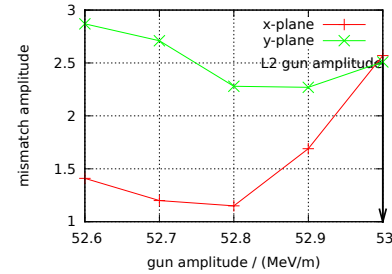
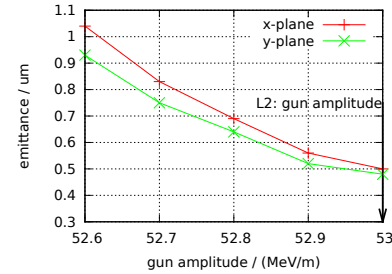
LASER 1 pulse energy
 ⇒ bunch charge



GUN phase



GUN gradient



some measurements

comparing different beam size estimations methods

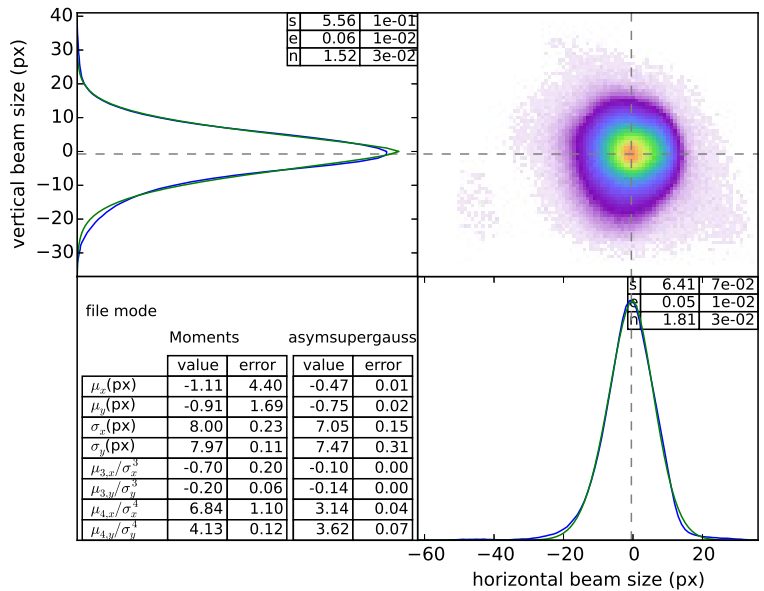
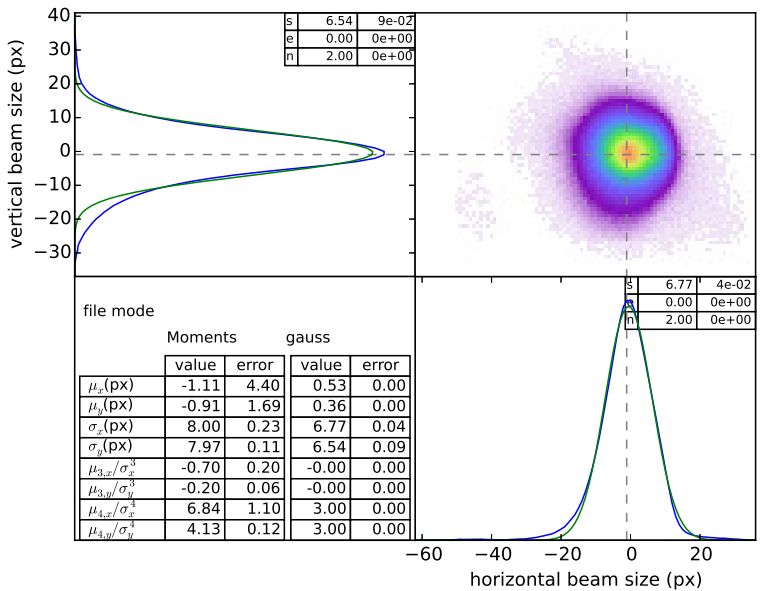
OTR8DBC2

gauss: $f(x) = A \exp\left[-\frac{(x-x_0)^2}{2s^2}\right]$

asymptogauss: $g(x) = A \exp\left[-\frac{|x-x_0|^n}{2|s(1+\text{sign}(x-x_0)e)|^n}\right]$

ver/beamprofile_server/MeasurementData/2016-10-11T140035-OTR.DBC2/Image-OTR.8DBC2-2(

ver/beamprofile_server/MeasurementData/2016-10-11T140035-OTR.DBC2/Image-OTR.8DBC2-



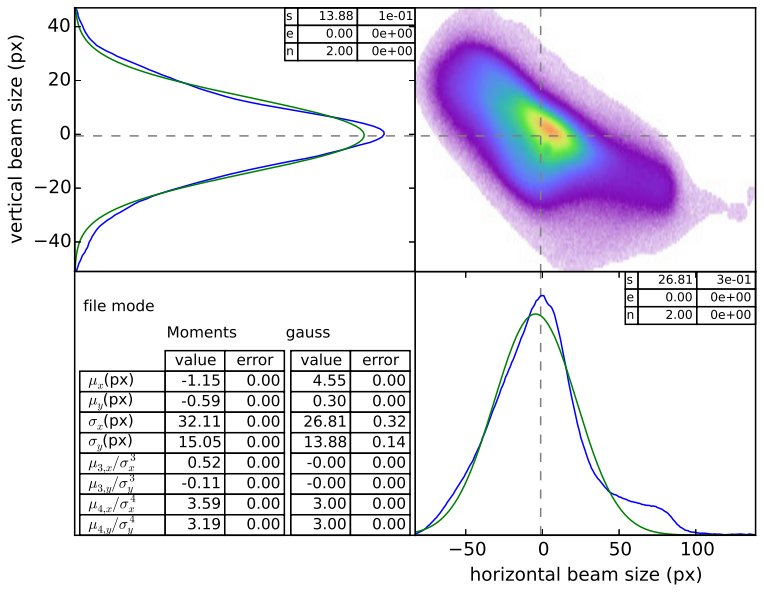
some measurements

comparing different beam size estimations methods

OTR5DBC3

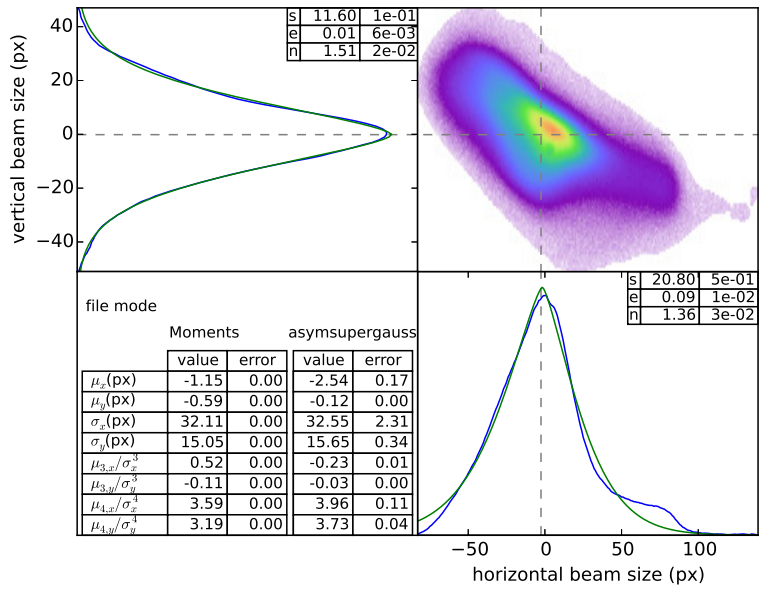
gauss: $f(x) = A \exp\left[-\frac{(x-x_0)^2}{2s^2}\right]$

BeamSize_ROI_0010.dat



asymsupergauss: $g(x) = A \exp\left[-\frac{|x-x_0|^n}{2|s(1+\text{sign}(x-x_0)e)|^n}\right]$

BeamSize_ROI_0010.dat



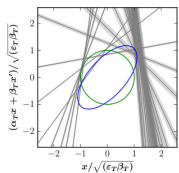
some measurements

BC3 matching

theory optics Q1DBC3 → OTR5DBC3

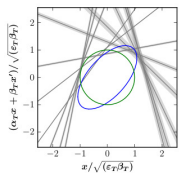
$\beta_T = 10.33 \text{ m}$ $\beta_M = 12.54 + 1.69 \text{ m}$
 $\alpha_T = -0.91$ $\alpha_M = -1.93 + 0.21$
 $e_T = 4.27 \text{ }\mu\text{m}$ $e_M = 4.27 + 0.40 \text{ }\mu\text{m}$

$m_P = 1.30$ $\lambda_P = 2.13$



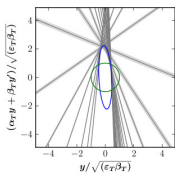
$\beta_T = 10.33 \text{ m}$ $\beta_M = 12.09 + 2.49 \text{ m}$
 $\alpha_T = -0.91$ $\alpha_M = -1.82 + 0.33$
 $e_T = 4.14 \text{ }\mu\text{m}$ $e_M = 4.14 + 0.62 \text{ }\mu\text{m}$

$m_P = 1.26$ $\lambda_P = 2.02$



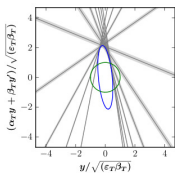
$\beta_T = 19.60 \text{ m}$ $\beta_M = 4.15 + 0.10 \text{ m}$
 $\alpha_T = -1.26$ $\alpha_M = -0.04 + 0.02$
 $e_T = 3.72 \text{ }\mu\text{m}$ $e_M = 3.72 + 0.09 \text{ }\mu\text{m}$

$m_P = 2.59$ $\lambda_P = 4.97$



$\beta_T = 19.60 \text{ m}$ $\beta_M = 5.01 + 0.23 \text{ m}$
 $\alpha_T = -1.26$ $\alpha_M = 0.09 + 0.03$
 $e_T = 3.74 \text{ }\mu\text{m}$ $e_M = 3.74 + 0.18 \text{ }\mu\text{m}$

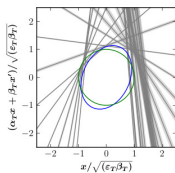
$m_P = 2.41$ $\lambda_P = 4.60$



theory optics Q2DBC3 → OTR5DBC3

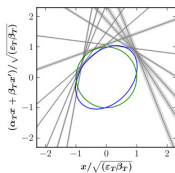
$\beta_T = 9.34 \text{ m}$ $\beta_M = 7.74 + 0.30 \text{ m}$
 $\alpha_T = 1.43$ $\alpha_M = 0.91 + 0.04$
 $e_T = 5.11 \text{ }\mu\text{m}$ $e_M = 5.11 + 0.14 \text{ }\mu\text{m}$

$m_P = 1.06$ $\lambda_P = 1.42$



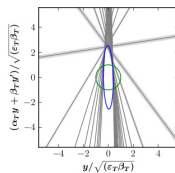
$\beta_T = 9.34 \text{ m}$ $\beta_M = 9.62 + 0.53 \text{ m}$
 $\alpha_T = 1.43$ $\alpha_M = 1.12 + 0.08$
 $e_T = 5.63 \text{ }\mu\text{m}$ $e_M = 5.63 + 0.26 \text{ }\mu\text{m}$

$m_P = 1.06$ $\lambda_P = 1.41$



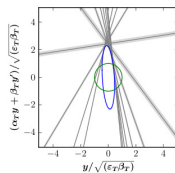
$\beta_T = 25.30 \text{ m}$ $\beta_M = 3.98 + 0.12 \text{ m}$
 $\alpha_T = -6.86$ $\alpha_M = -0.97 + 0.03$
 $e_T = 4.74 \text{ }\mu\text{m}$ $e_M = 4.74 + 0.12 \text{ }\mu\text{m}$

$m_P = 3.29$ $\lambda_P = 6.43$



$\beta_T = 25.30 \text{ m}$ $\beta_M = 5.14 + 0.47 \text{ m}$
 $\alpha_T = -6.86$ $\alpha_M = -1.12 + 0.15$
 $e_T = 4.77 \text{ }\mu\text{m}$ $e_M = 4.77 + 0.42 \text{ }\mu\text{m}$

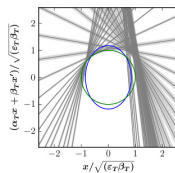
$m_P = 2.74$ $\lambda_P = 5.30$



theory optics Q3DBC3 → OTR5DBC3

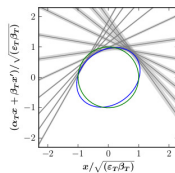
$\beta_T = 6.33 \text{ m}$ $\beta_M = 4.59 + 0.61 \text{ m}$
 $\alpha_T = 2.86$ $\alpha_M = 2.08 + 0.22$
 $e_T = 5.38 \text{ }\mu\text{m}$ $e_M = 5.38 + 0.49 \text{ }\mu\text{m}$

$m_P = 1.05$ $\lambda_P = 1.38$



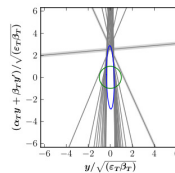
$\beta_T = 6.33 \text{ m}$ $\beta_M = 6.92 + 4.10 \text{ m}$
 $\alpha_T = 2.86$ $\alpha_M = 2.90 + 1.50$
 $e_T = 6.64 \text{ }\mu\text{m}$ $e_M = 6.64 + 3.18 \text{ }\mu\text{m}$

$m_P = 1.03$ $\lambda_P = 1.27$



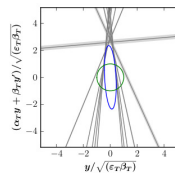
$\beta_T = 39.59 \text{ m}$ $\beta_M = 4.90 + 0.25 \text{ m}$
 $\alpha_T = -16.40$ $\alpha_M = -1.88 + 0.09$
 $e_T = 6.03 \text{ }\mu\text{m}$ $e_M = 6.03 + 0.24 \text{ }\mu\text{m}$

$m_P = 4.20$ $\lambda_P = 8.27$



$\beta_T = 39.59 \text{ m}$ $\beta_M = 7.67 + 2.82 \text{ m}$
 $\alpha_T = -16.40$ $\alpha_M = -2.91 + 0.91$
 $e_T = 5.72 \text{ }\mu\text{m}$ $e_M = 5.72 + 1.65 \text{ }\mu\text{m}$

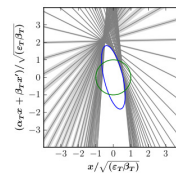
$m_P = 2.87$ $\lambda_P = 5.55$



symmetrized QS Q3DBC3 → OTR5DBC3

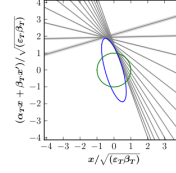
$\beta_T = 5.38 \text{ m}$ $\beta_M = 2.46 + 0.20 \text{ m}$
 $\alpha_T = 2.60$ $\alpha_M = 1.89 + 0.13$
 $e_T = 3.39 \text{ }\mu\text{m}$ $e_M = 3.39 + 0.20 \text{ }\mu\text{m}$

$m_P = 1.86$ $\lambda_P = 3.44$



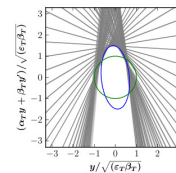
$\beta_T = 5.38 \text{ m}$ $\beta_M = 2.87 + 1.26 \text{ m}$
 $\alpha_T = 2.60$ $\alpha_M = 2.33 + 0.92$
 $e_T = 3.87 \text{ }\mu\text{m}$ $e_M = 3.87 + 1.36 \text{ }\mu\text{m}$

$m_P = 2.03$ $\lambda_P = 3.79$



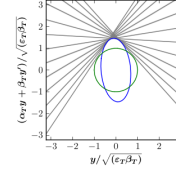
$\beta_T = 4.35 \text{ m}$ $\beta_M = 1.99 + 0.07 \text{ m}$
 $\alpha_T = 2.11$ $\alpha_M = 1.12 + 0.05$
 $e_T = 3.01 \text{ }\mu\text{m}$ $e_M = 3.01 + 0.10 \text{ }\mu\text{m}$

$m_P = 1.35$ $\lambda_P = 2.26$



$\beta_T = 4.35 \text{ m}$ $\beta_M = 2.07 + 0.24 \text{ m}$
 $\alpha_T = 2.11$ $\alpha_M = 1.14 + 0.18$
 $e_T = 3.15 \text{ }\mu\text{m}$ $e_M = 3.15 + 0.36 \text{ }\mu\text{m}$

$m_P = 1.31$ $\lambda_P = 2.15$



some measurements

BC3 matching, back-tracking measurements to STARTACC2 to compare reconstruction results

full data set:		Q1DBC2→OTR5DBC3		Q2DBC2→OTR5DBC3		Q3DBC2→OTR5DBC3		symm. QS: Q3DBC2→OTR5DBC3	
Location	Design	Reconstructed	Measured	Reconstructed	Measured	Reconstructed	Measured	Reconstructed	Measured
	STARTACC2	STARTACC2	MQ1DBC3.U	STARTACC2	MQ2DBC3.U	STARTACC2	MQ3DBC3.U	STARTACC2	MQ3DBC3.U
Beta x (m)	+7.237	+8.394	+12.537	+6.731	+7.742	+7.280	+4.593	+16.233	+2.456
Alpha x	-0.178	-0.925	-1.927	-0.281	+0.915	+0.025	+2.076	-1.965	+1.891
Beta y (m)	+6.046	+7.907	+4.149	+9.379	+3.983	+12.441	+4.902	+3.042	+1.985
Alpha y	-0.093	+1.576	-0.041	+2.082	-0.967	+2.798	-1.875	-0.690	+1.124

limited data set: 11 points around the waist

Location	Design	Reconstructed	Measured	Reconstructed	Measured	Reconstructed	Measured	Reconstructed	Measured
	STARTACC2	STARTACC2	MQ1DBC3.U	STARTACC2	MQ2DBC3.U	STARTACC2	MQ3DBC3.U	STARTACC2	MQ3DBC3.U
Beta x (m)	+7.237	+8.222	+12.088	+8.019	+9.625	+8.700	+6.917	+16.450	+2.872
Alpha x	-0.178	-0.860	-1.822	-0.477	+1.123	-0.426	+2.897	-2.182	+2.326
Beta y (m)	+6.046	+8.786	+5.013	+9.223	+5.138	+10.068	+7.669	+3.164	+2.072
Alpha y	-0.093	+1.540	+0.087	+1.772	-1.124	+1.905	-2.906	-0.681	+1.141

some measurements

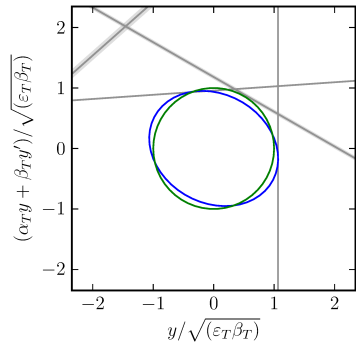
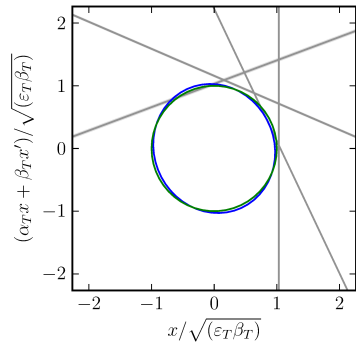
SFUND matching

charge: 0.41 nC, BSA: 1.2 mm

1SFUND1 1SFUND2 1SFUND3 1SFUND4

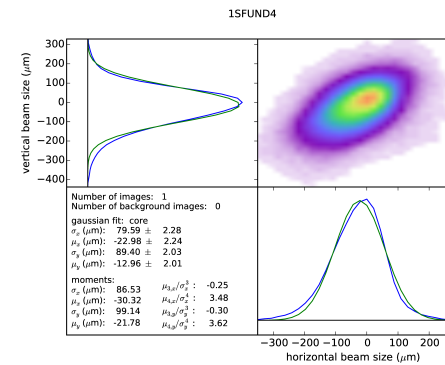
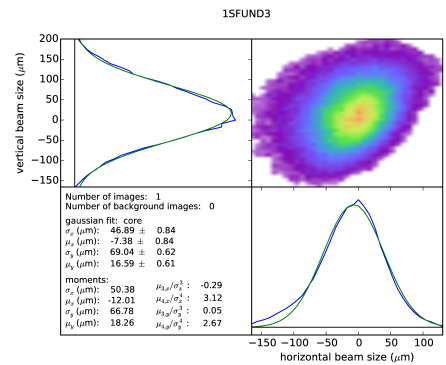
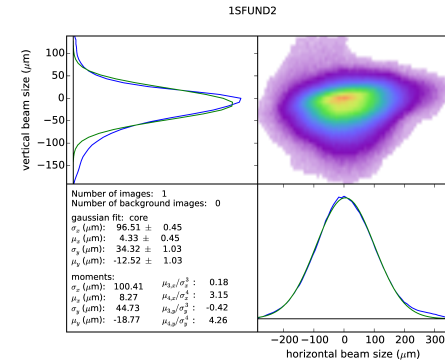
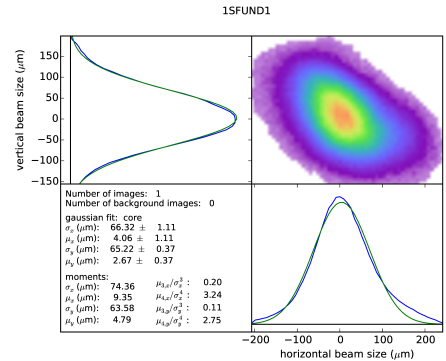
$\beta_T = 4.20$ m $\beta_M = 3.98 \pm 0.05$ m
 $\alpha_T = -0.36$ $\alpha_M = -0.28 \pm 0.03$
 $\varepsilon_T = 1.49$ μ m $\varepsilon_M = 1.49 \pm 0.04$ μ m

$m_P = 1.00$ $\lambda_P = 1.09$



$\beta_T = 6.74$ m $\beta_M = 7.67 \pm 0.62$ m
 $\alpha_T = 1.04$ $\alpha_M = 1.37 \pm 0.17$
 $\varepsilon_T = 0.85$ μ m $\varepsilon_M = 0.85 \pm 0.07$ μ m

$m_P = 1.02$ $\lambda_P = 1.24$



some measurements

undulator match

charge: 0.41 nC, BSA: 1.2 mm

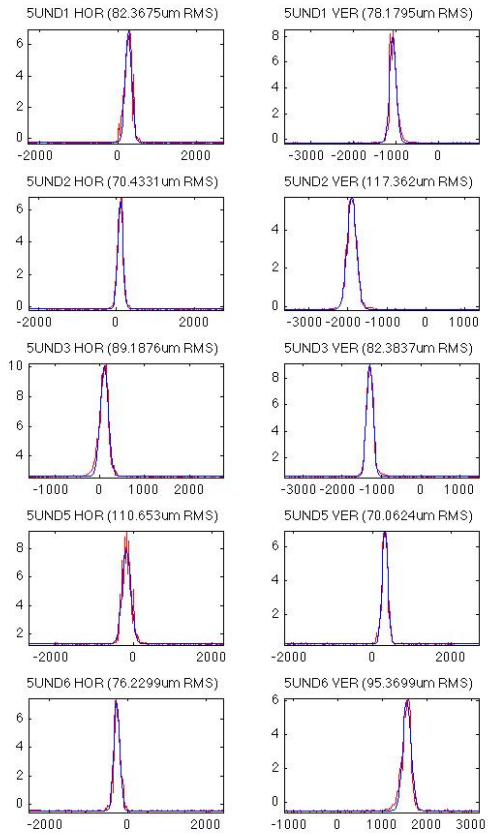
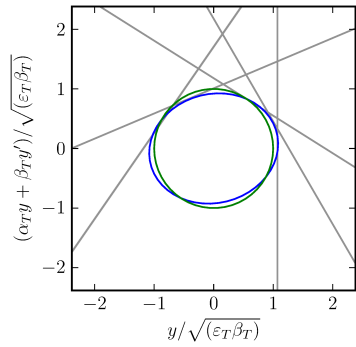
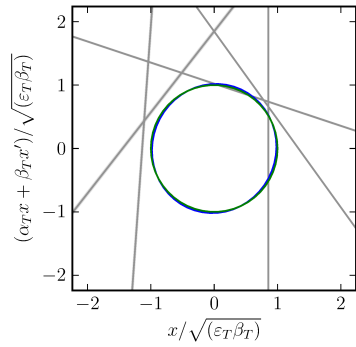
WIRE5UND1 WIRE5UND2 WIRE5UND3 WIRE5UND5 WIRE5UND6

$\beta_T = 13.06$ m $\beta_M = 12.60 \pm 0.57$ m
 $\alpha_T = 1.18$ $\alpha_M = 1.11 \pm 0.09$
 $\varepsilon_T = 1.08$ μm $\varepsilon_M = 1.08 \pm 0.06$ μm

$m_P = 1.00$ $\lambda_P = 1.05$

$\beta_T = 6.30$ m $\beta_M = 7.40 \pm 0.12$ m
 $\alpha_T = -0.66$ $\alpha_M = -0.85 \pm 0.03$
 $\varepsilon_T = 1.28$ μm $\varepsilon_M = 1.28 \pm 0.03$ μm

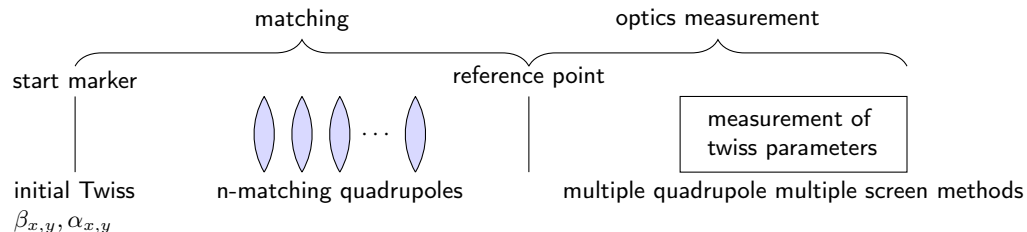
$m_P = 1.02$ $\lambda_P = 1.19$



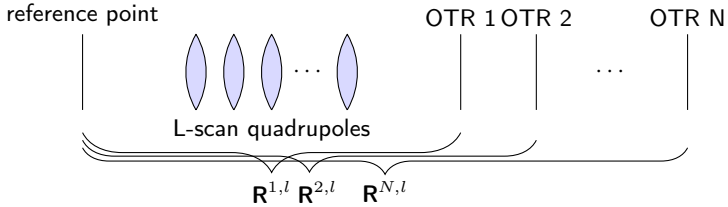
Appendix.

Matching of the actual optics to design optics.

- Procedure:
- 1 Measurement of the current Twiss parameters at a reference point.
 - 2 Reconstruct Twiss parameters at a start marker upstream of the matching quadrupoles.
↳ This is done by 'matching' the initial Twiss parameter using LMAD.
 - 3 Match the actual optics to design optics at reference point by user defined quadrupoles.
↳ This is done by LMAD.
 - 4 Generate a current list of the new quadrupole strengths.



Emittance and optics measurement.



Special cases:

- ① Single quad scan: L=1, N=1
- ② 4-screen method: L=0, N=4

Set of equations has to be solved: (with (n,l)-th measured beam size $\sigma_{n,l}$)

$$\mathbf{M}_{(n,l),1\dots 3} = \left(\left(\mathbf{R}_{l \leftarrow 0}^{(n)} \right)_{q,q}^2, 2 \left(\mathbf{R}_{l \leftarrow 0}^{(n)} \right)_{q,q} \left(\mathbf{R}_{l \leftarrow 0}^{(n)} \right)_{q,p}, \left(\mathbf{R}_{l \leftarrow 0}^{(n)} \right)_{q,p}^2 \right)$$

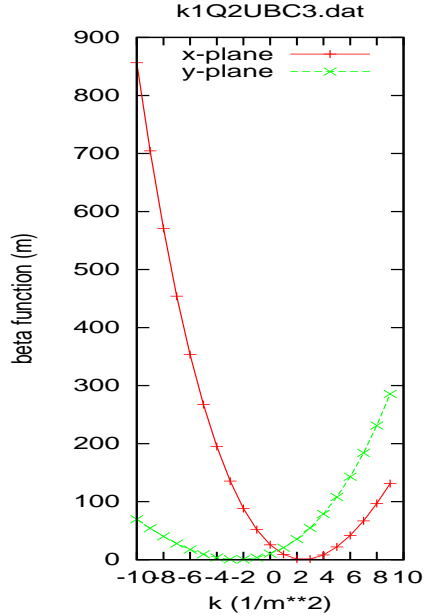
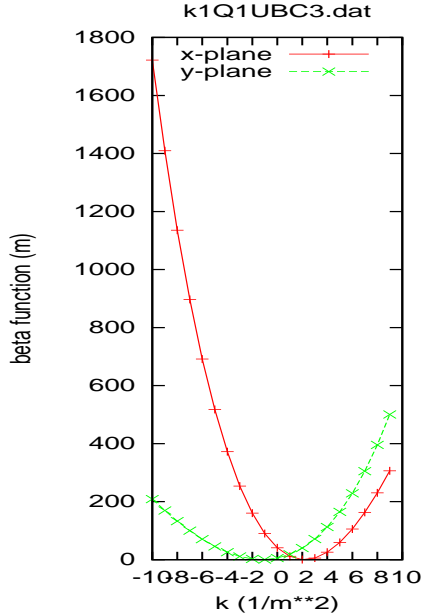
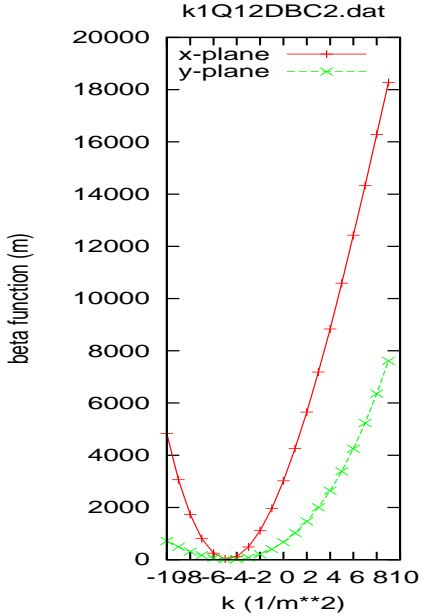
$$\mathbf{M} \begin{pmatrix} \sigma_0^2 \\ \text{Cov}(q,p)_0 \\ \sigma_{p,0}^2 \end{pmatrix} = \begin{pmatrix} \sigma_{n=1,l=1}^2 \\ \dots \\ \sigma_{n,l}^2 \\ \dots \\ \sigma_{n=N,l=L}^2 \end{pmatrix}$$

$$\begin{aligned} \sigma_0^2 &= \beta_0 \varepsilon \\ \text{Cov}(q,p)_0 &= -\alpha \varepsilon \\ \sigma_{p,0}^2 &= \gamma_0 \varepsilon \\ \varepsilon^2 &= \sigma_0^2 \sigma_{p,0}^2 - \text{Cov}(q,p)_0^2 \end{aligned}$$

- Transfer matrix \mathbf{R}
- x-plane: (q,p) = (1,2)
- y-plane: (q,p) = (3,4)

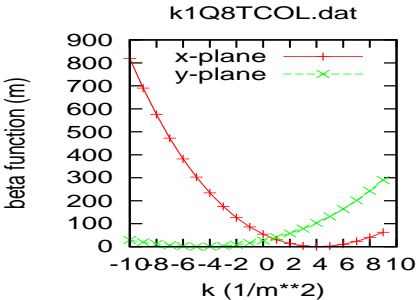
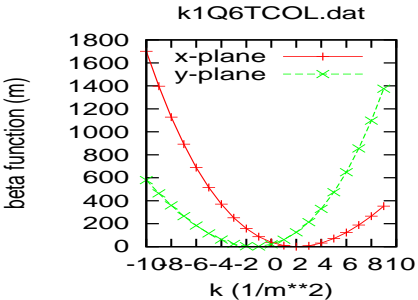
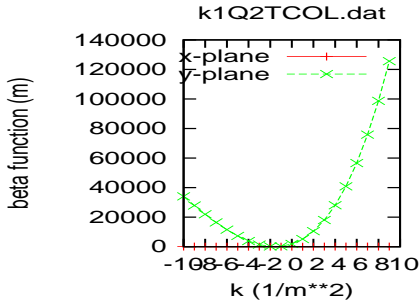
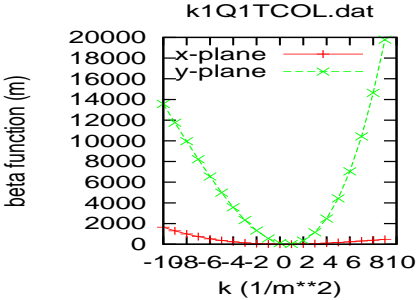
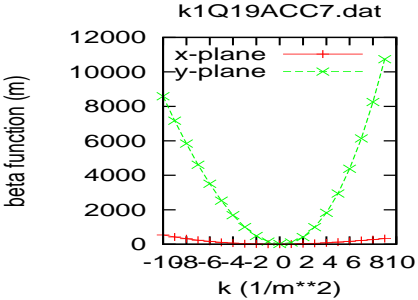
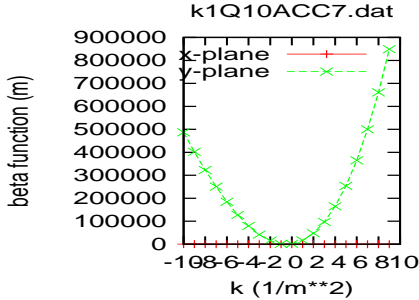
some more quad scans

in theory



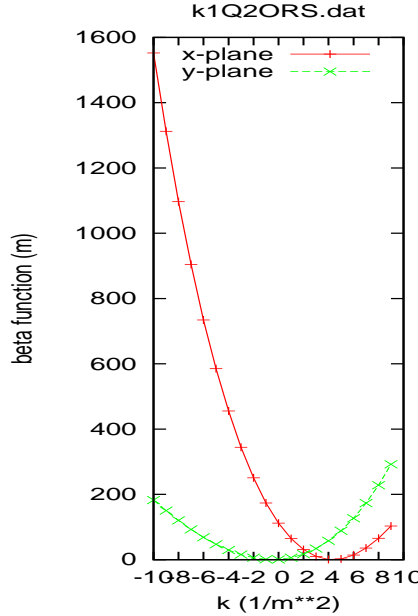
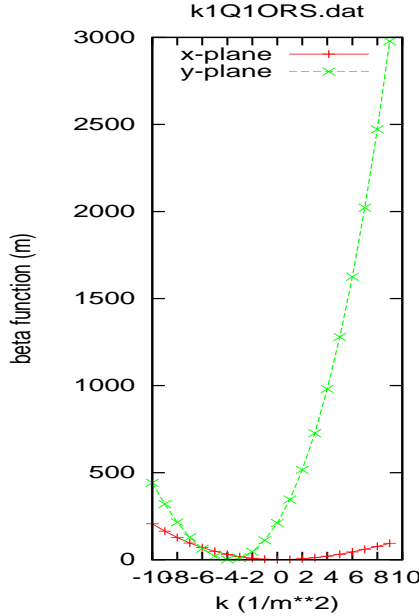
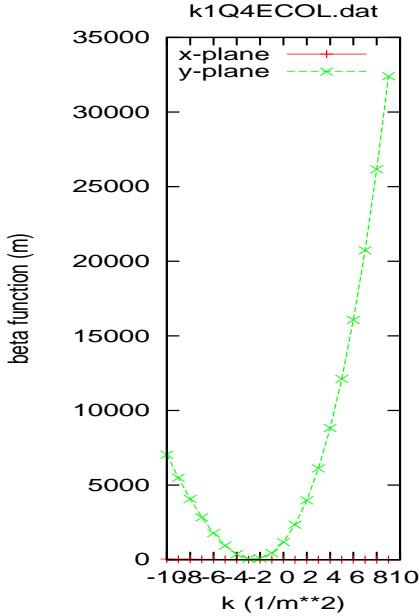
some more quad scans

in theory



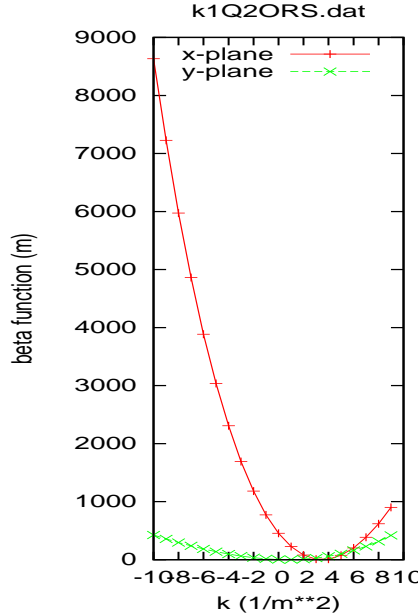
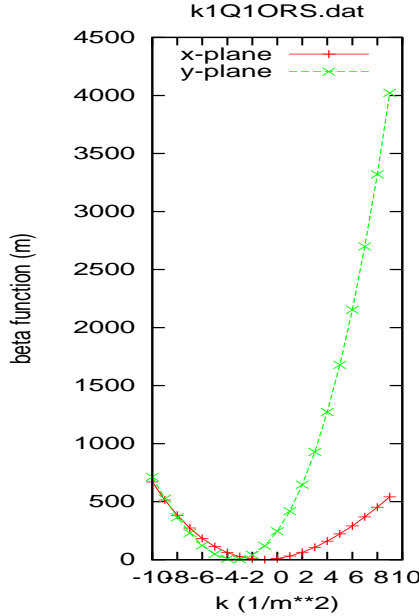
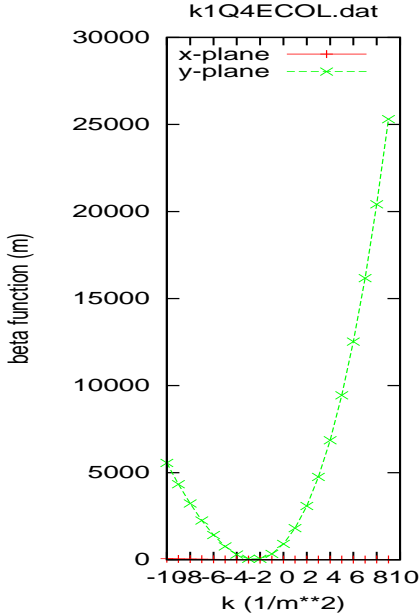
some more quad scans

in theory



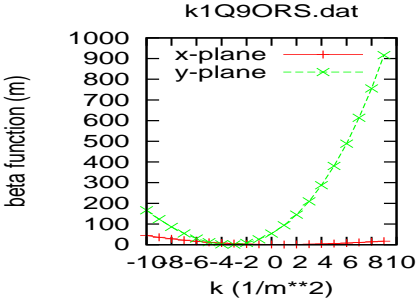
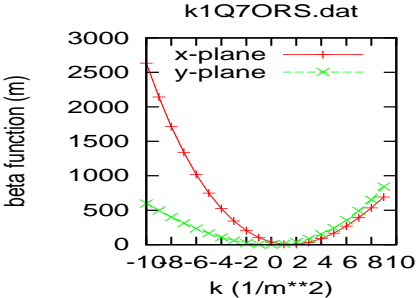
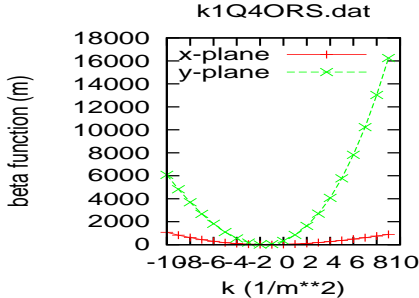
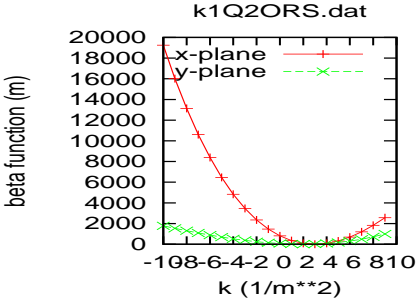
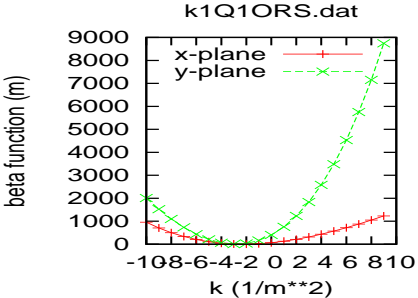
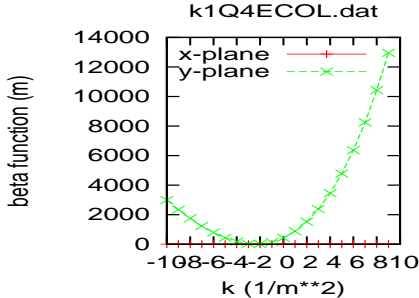
some more quad scans

in theory



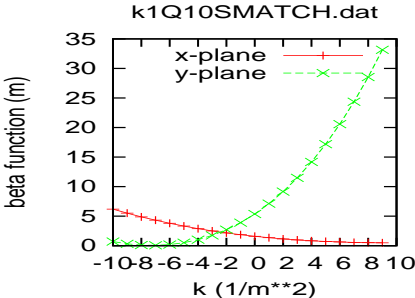
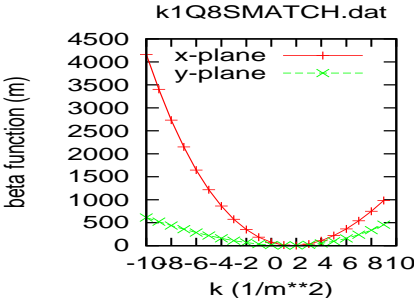
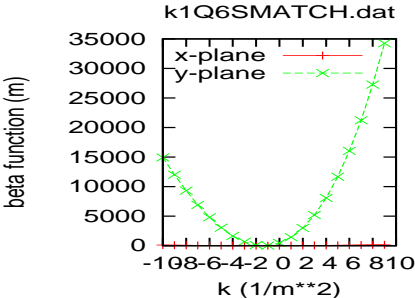
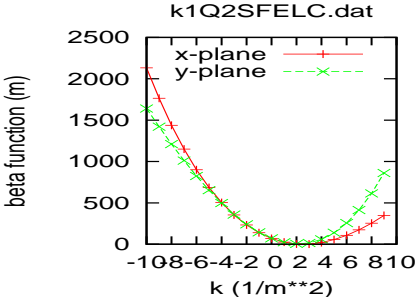
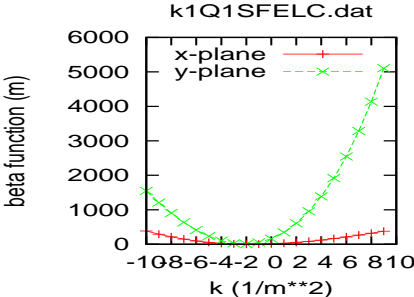
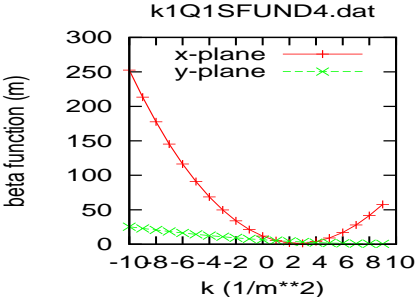
some more quad scans

in theory



some more quad scans

in theory



some more quad scans

in theory

