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TECHNISCHE  
UNIVERSITÄT  
DARMSTADT

# **DESY/TEMF Meeting - Status 2011**

DESY, Hamburg, 16.12.2011

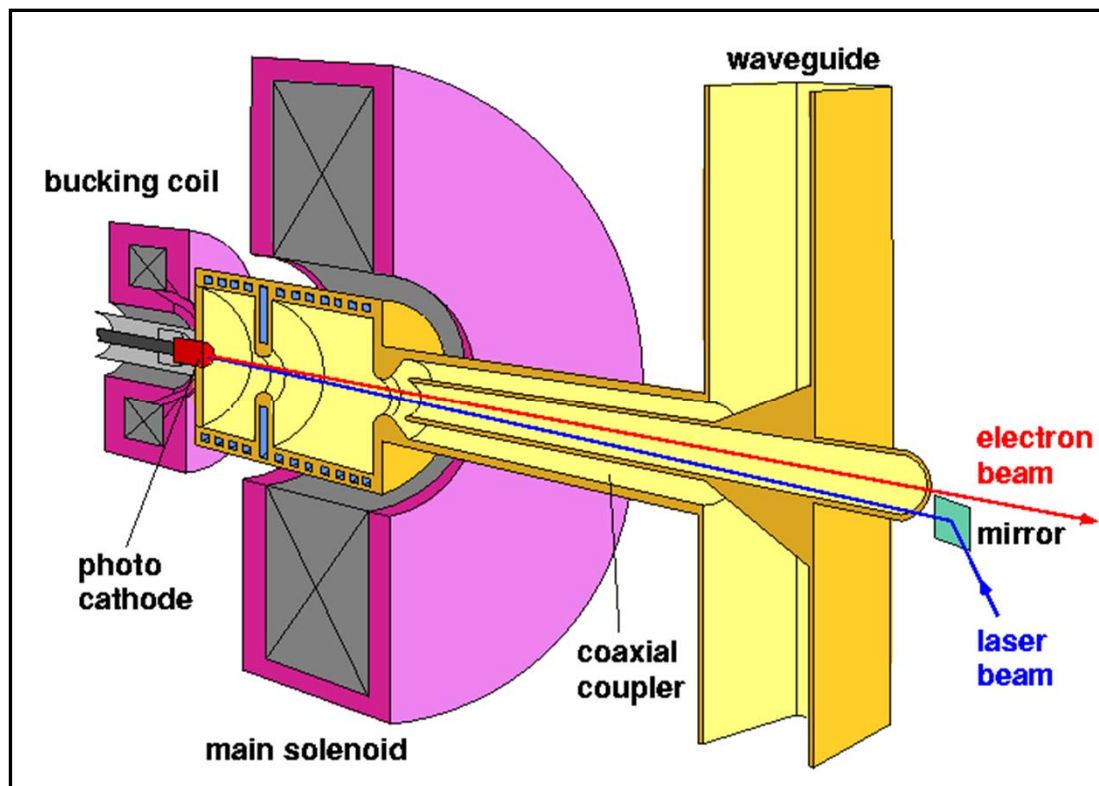
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- Field map calculations
  - Mixed mesh – high order FEM code
  - Field quality study for the 3.9 GHz 3<sup>rd</sup> harmonic cavity
- Summary

# Emittance studies for the PITZ injector

## Problem description

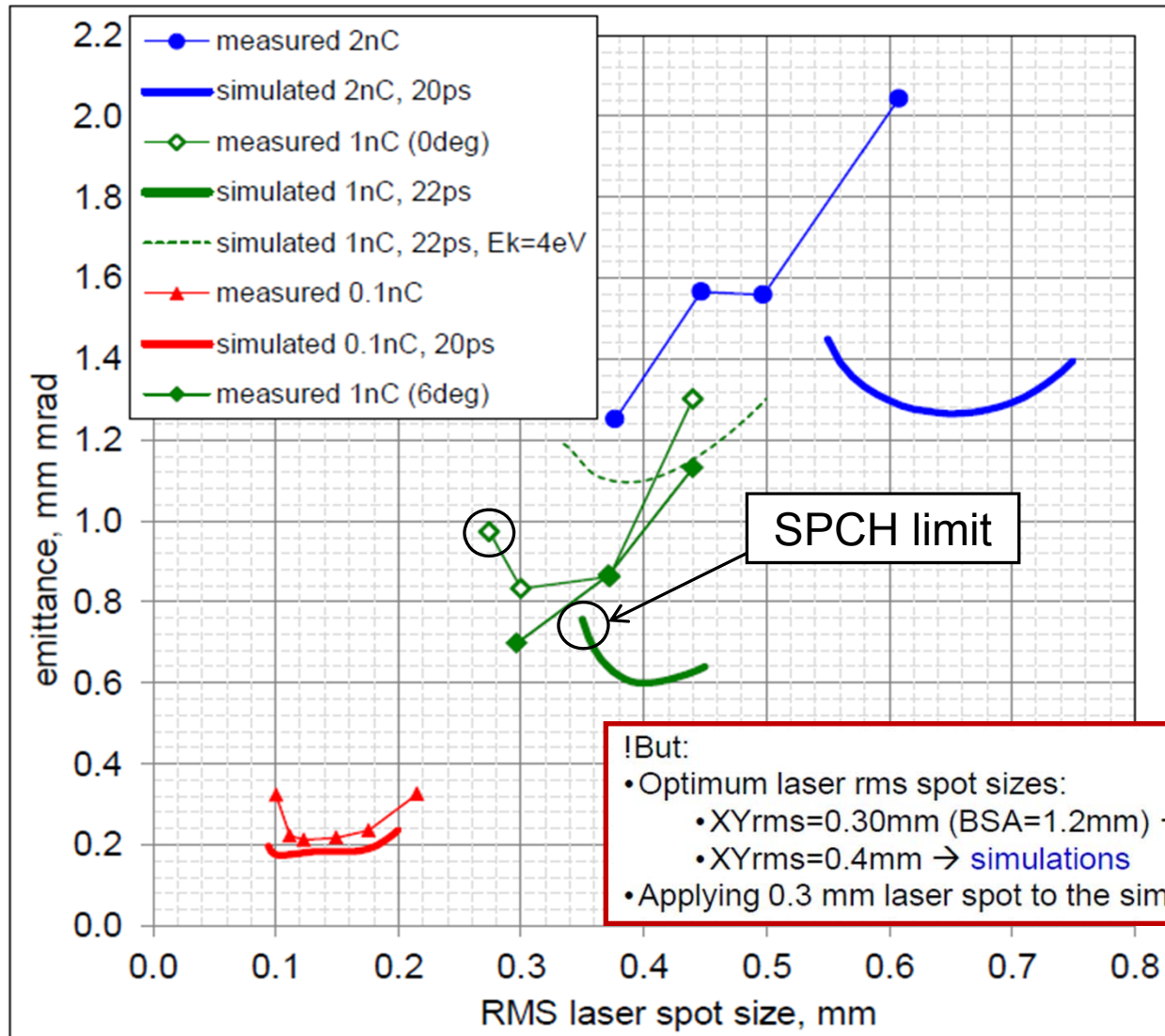


*talk from M. Krasilnikov,  
Zeuthen, 2011*

## Optimized machine parameters for $Q = 1\text{nC}$ (simulations)

	parameter	unit	value
cathode laser	temporal	profile	flat-top
	transverse	distribution	rad.homogen
	rt/FWHM\ft	ps	2/22\2
	XYrms	mm	0,401
	Ek	eV	0,55
	th.emit.	mm mrad	0,34
RF-gun	Ecath	MV/m	60,58
	phase	deg	-1,116
	maxBz	T	-0,22808
CDS boost	maxE	MV/m	20,6
	phase	deg	0
e-beam @EMSY1	charge	nC	1
	momentum	MeV/c	24,64
	proj.emit.	mm mrad	0,60
	th./proj.em.	%	57%
	<sl.emit.>	mm mrad	0,53

# Emittance studies for the PITZ injector



• Optimum machine parameters (laser spot size, gun phase):  
experiment  $\neq$  simulations

• Difference in the optimum laser spot size is bigger for higher charges  
(good agreement for 100pC)

• Artificial increase of the thermal kinetic energy at the cathode (from 0.55eV to 4eV) did not improve the situation

**talk from M. Krasilnikov,  
Zeuthen, 2011**

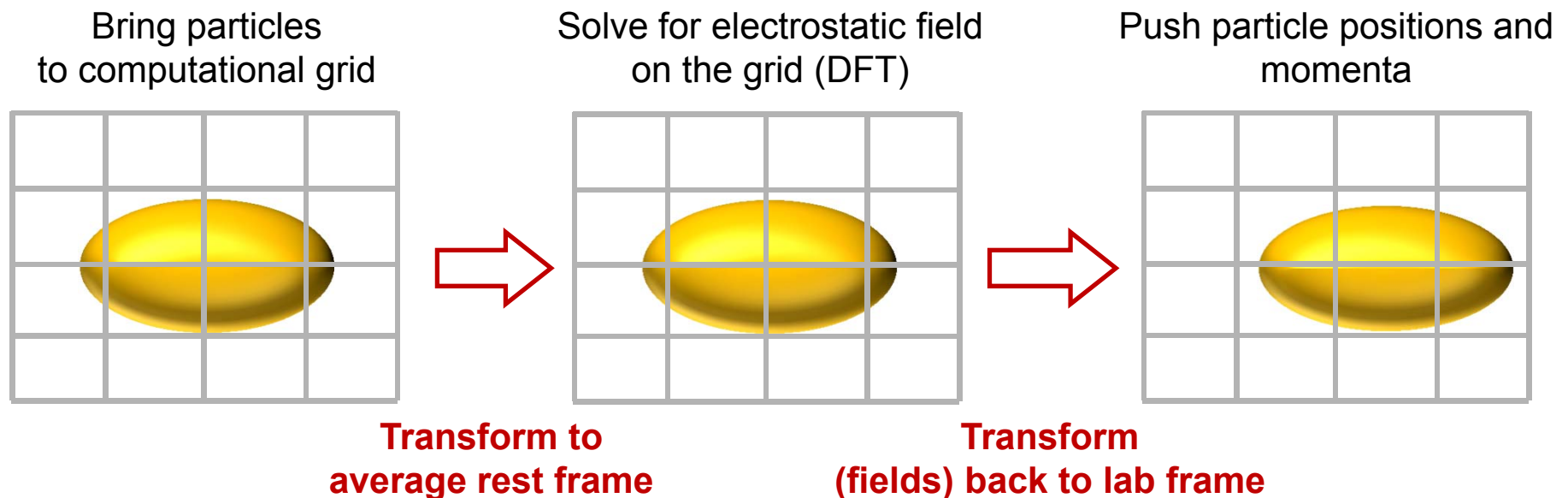
# Emittance studies for the PITZ injector

## Problem description

- Sources of discrepancy:
  - Thermal energy spread at cathode
  - Oxide layer effects and cathode impurities
  - Limited knowledge of machine conditions during measurement
  - Emittance measurement – slit scan technique
  - Wakefields
  - ...
- Improper modeling of radiation fields?
- Inaccurate simulation of the emission process?

# Emittance studies for the PITZ injector

## Problem description



- Boosted frame approach (ASTRA):
  - Retardation effects due to relative motion within bunch?
  - Acceleration radiation?
  - Relativistic correction for mirror charge fields?

# Beam dynamics in the boosted frame

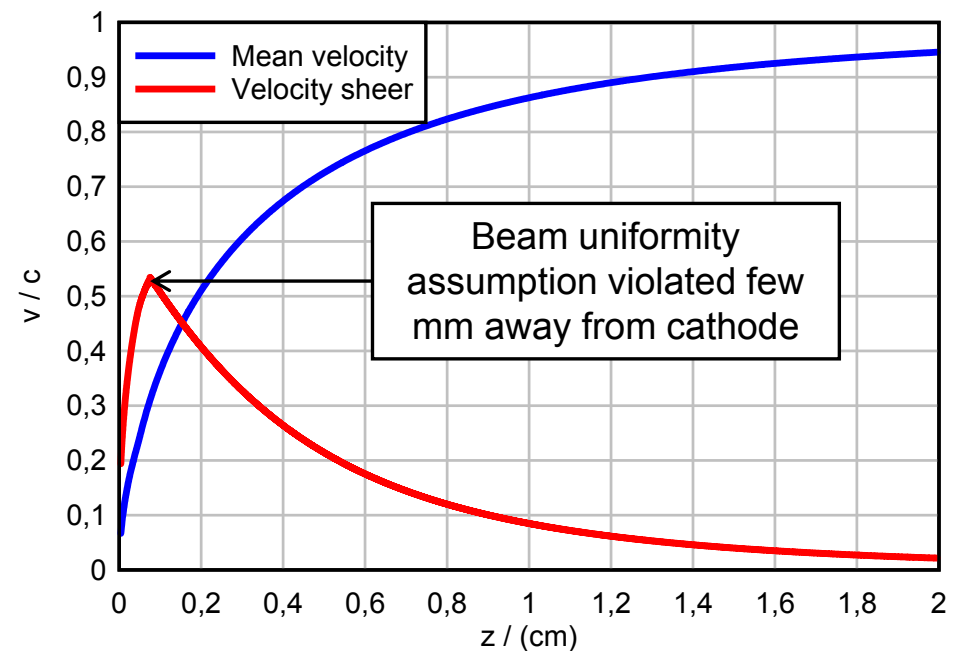
## Problem description

- Field of accelerated point charge:

$$\mathbf{E} = \frac{q}{4\pi\epsilon_0} \left[ \underbrace{\frac{(\mathbf{n} - \boldsymbol{\beta})(1 - |\boldsymbol{\beta}|^2)}{(1 - \boldsymbol{\beta} \cdot \mathbf{n})^3 R^2}}_{\text{retardation}} + \underbrace{\frac{\mathbf{n} \times (\mathbf{n} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}}{(1 - \boldsymbol{\beta} \cdot \mathbf{n})^3 R}}_{\text{acceleration}} \right]_{t=t_r}$$

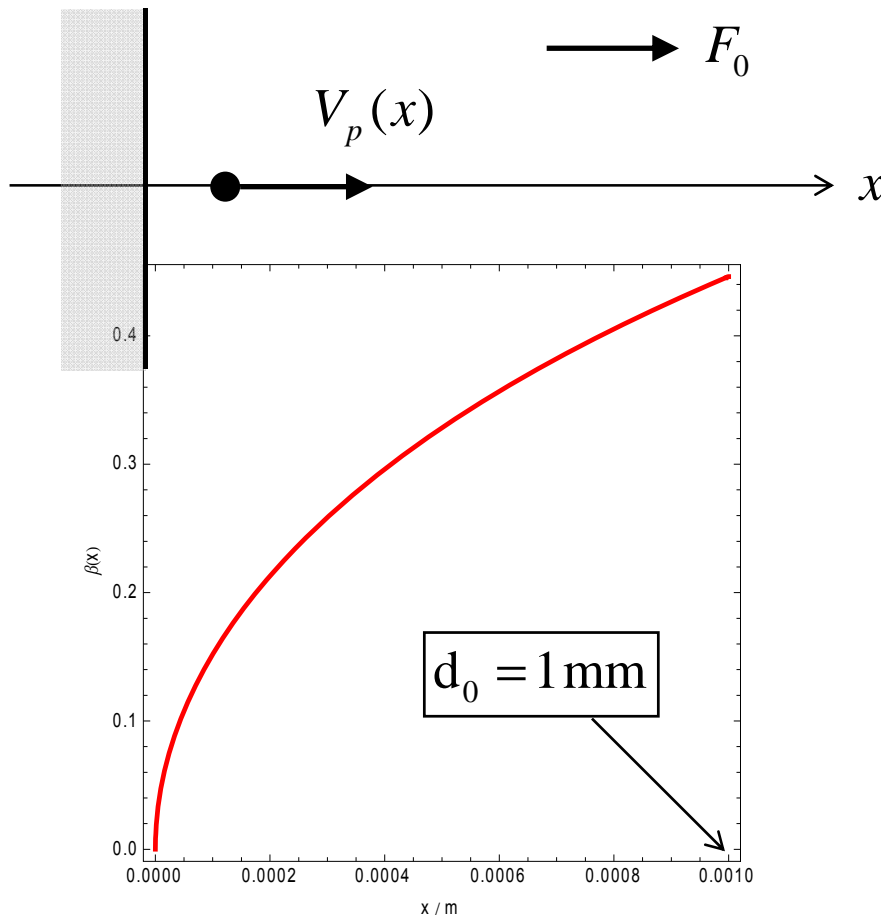
- Retardation only wrt. mean bunch motion
- Effect is stronger when space charge is included
- Effect is stronger when mirror charges are considered

Velocity shear  
(space charge free simulation)



# Emittance studies for the PITZ injector

## Effect of retardation for single electron fields



$$\begin{aligned} m &= m_0 \\ q &= -e_0 \\ F_0 &= e_0 E_0 \\ E_0 &= 60 \text{ MV/m} \end{aligned}$$

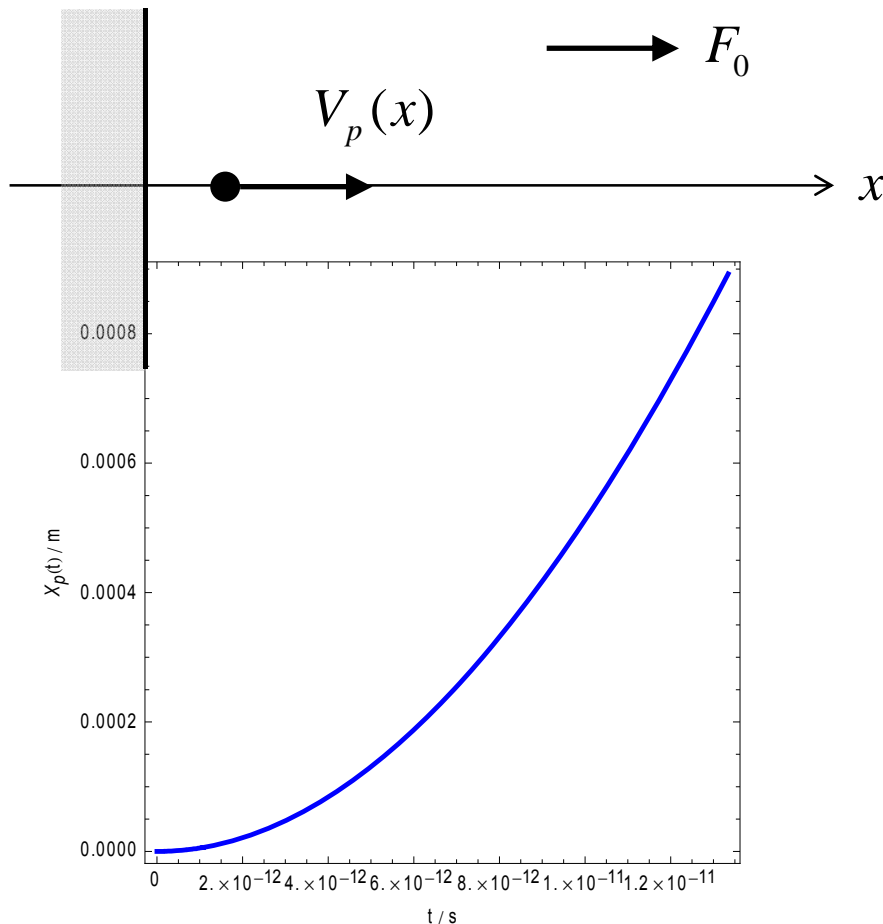
$$\frac{m v(x) v'(x)}{\left(1 - \frac{v(x)^2}{c^2}\right)^{3/2}} = F_0, \quad v(0) = 0$$

$$\Rightarrow V_p(x) = \frac{c \sqrt{F_0 x (2c^2 m + F_0 x)}}{c^2 m + F_0 x}$$



# Emittance studies for the PITZ injector

## Effect of retardation for single electron fields



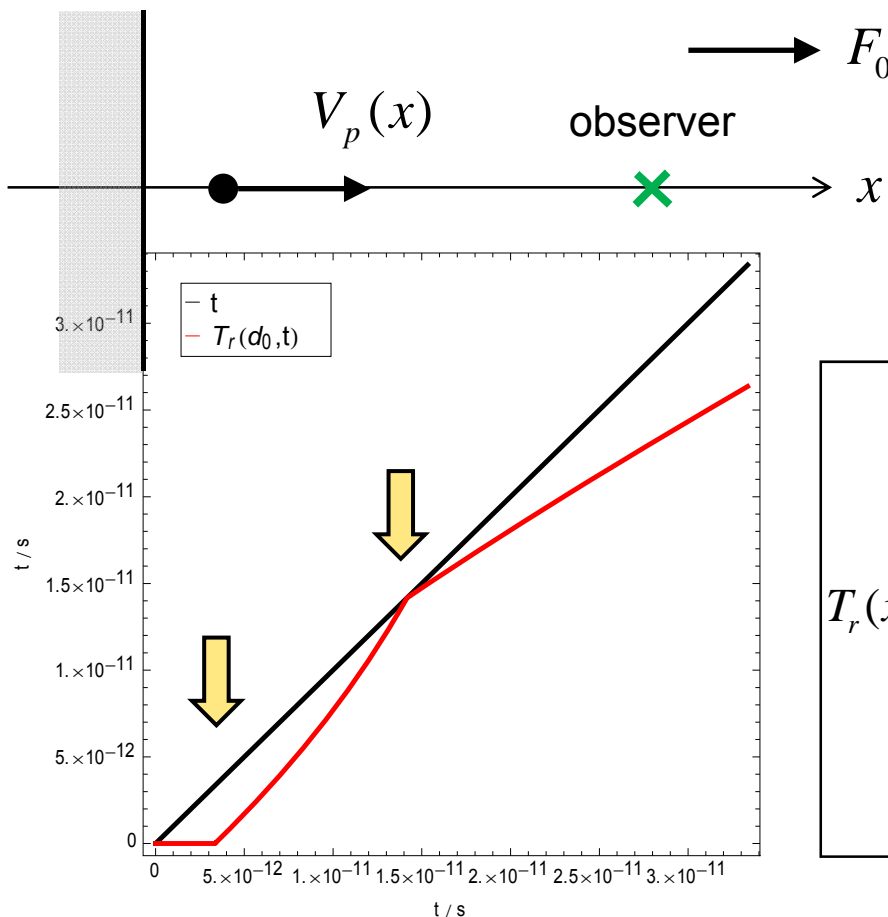
$$\begin{aligned}
 m &= m_0 \\
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 F_0 &= e_0 E_0 \\
 E_0 &= 60 \text{ MV/m}
 \end{aligned}$$

$$\frac{m v(x) v'(x)}{\left(1 - \frac{v(x)^2}{c^2}\right)^{3/2}} = F_0, \quad v(0) = 0$$

$$\Rightarrow X_p(t) = \frac{c(\sqrt{c^2 m^2 + F_0^2 t^2} - cm)}{F_0}$$

# Emittance studies for the PITZ injector

## Effect of retardation for single electron fields



Retardation time at arbitrary observer position as a function of time:

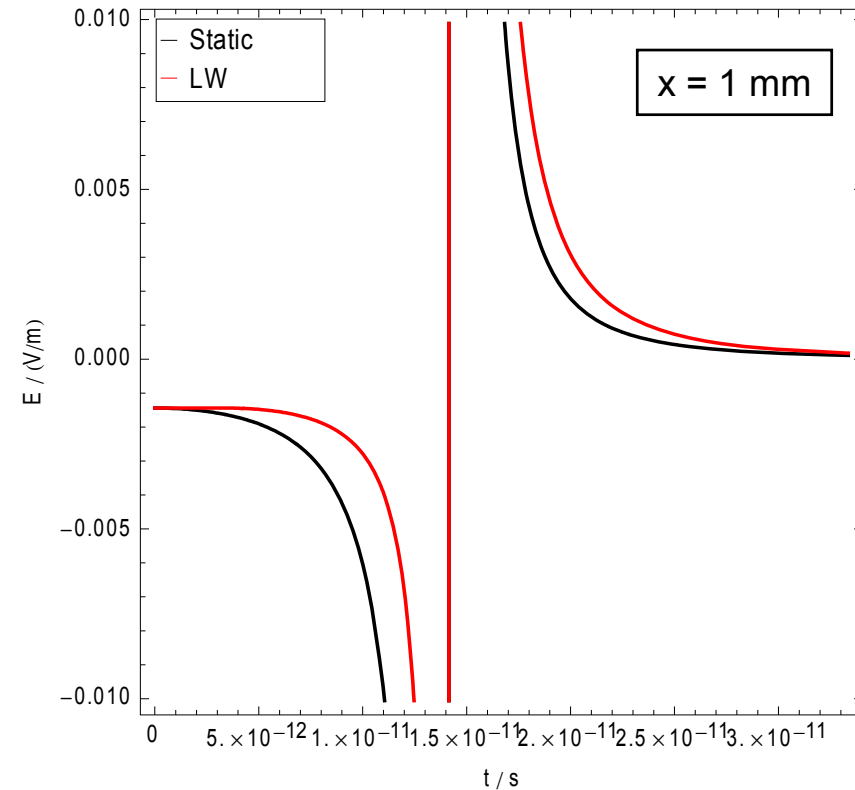
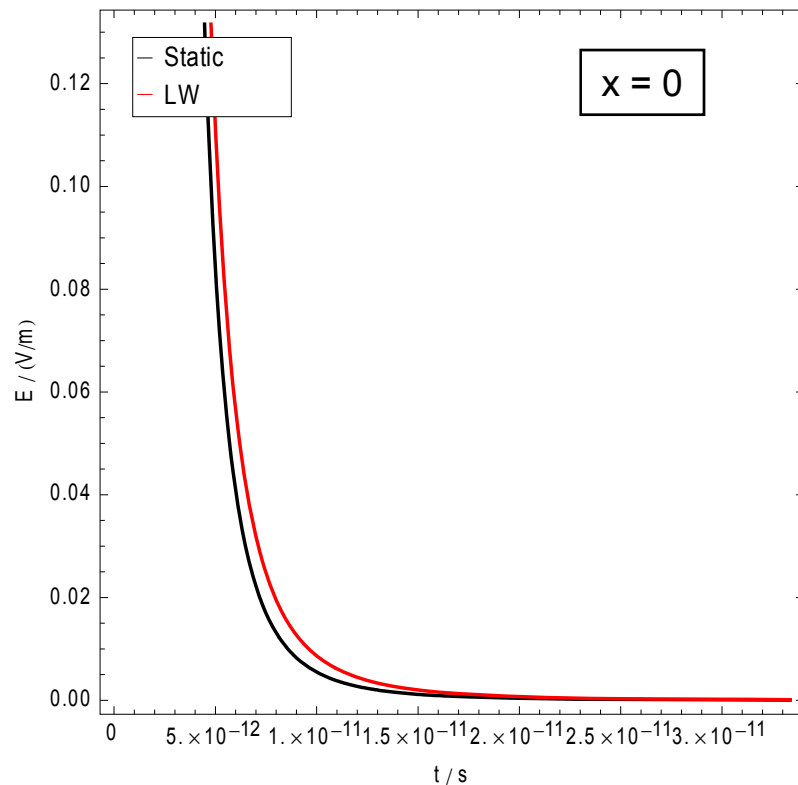
$$T_r(x, t) = t - \frac{|x - X_p[T_r(x, t)]|}{c}$$

$$T_r(x, t) = \begin{cases} \frac{(ct - x) [2c^2 m + F_0(x - ct)]}{2c [c^2 m + F_0(x - ct)]}, & ct > x > X_p(t) \\ \frac{(ct + x) [2c^2 m + F_0(x + ct)]}{2c^3 m + 2c F_0(ct + x)}, & x < X_p(t) \\ 0, & x > ct \end{cases}$$

# Emittance studies for the PITZ injector

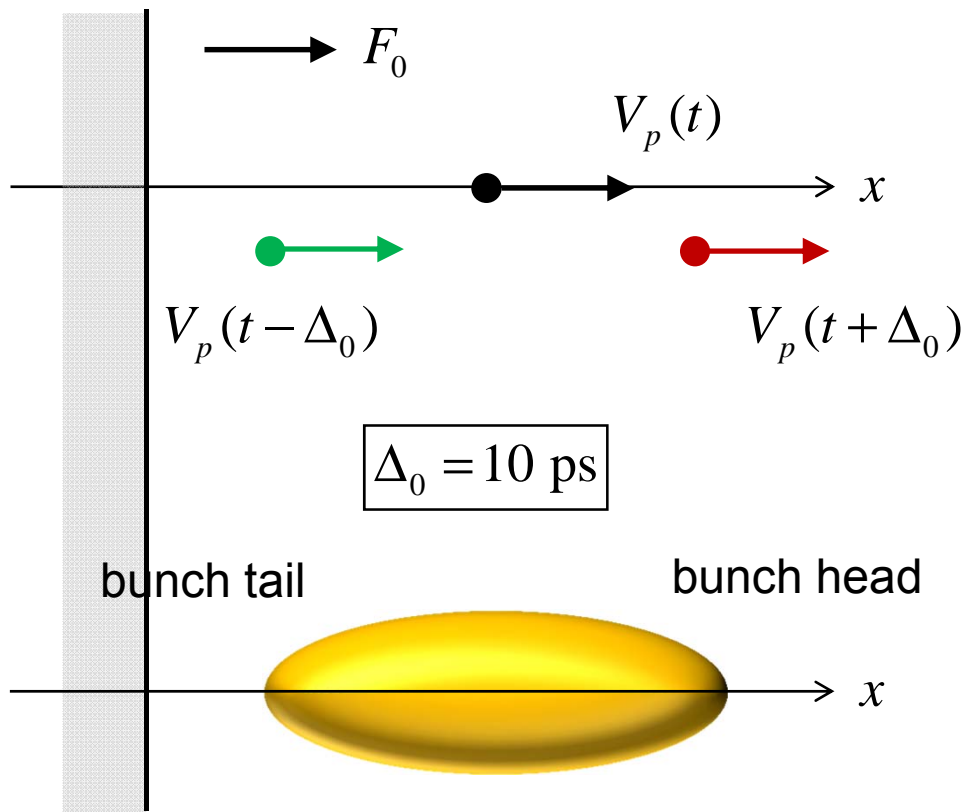
## Effect of retardation for single electron fields

Electron fields at fixed positions

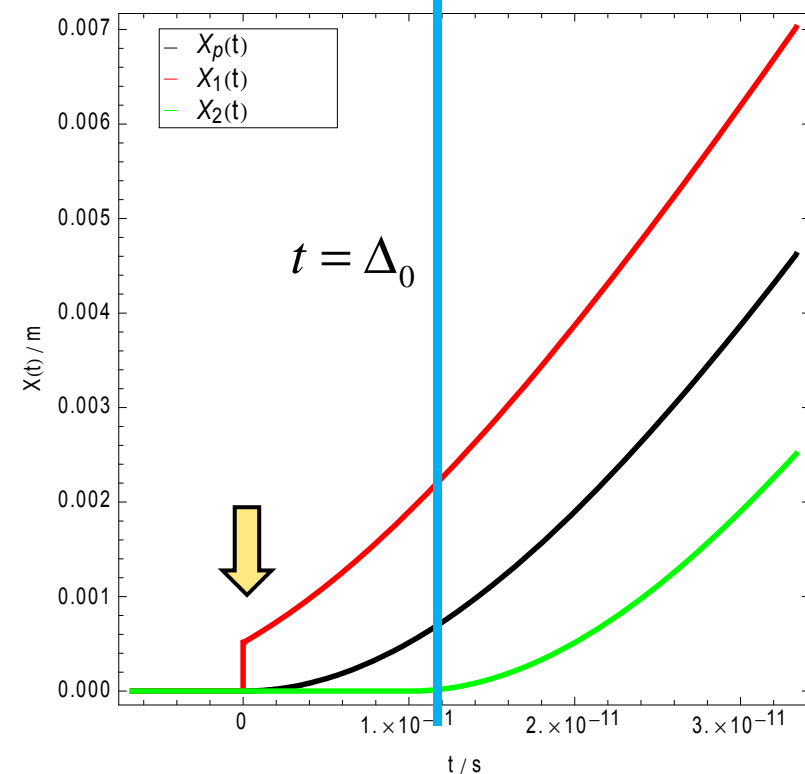


# Emittance studies for the PITZ injector

## Effect of retardation for single electron fields



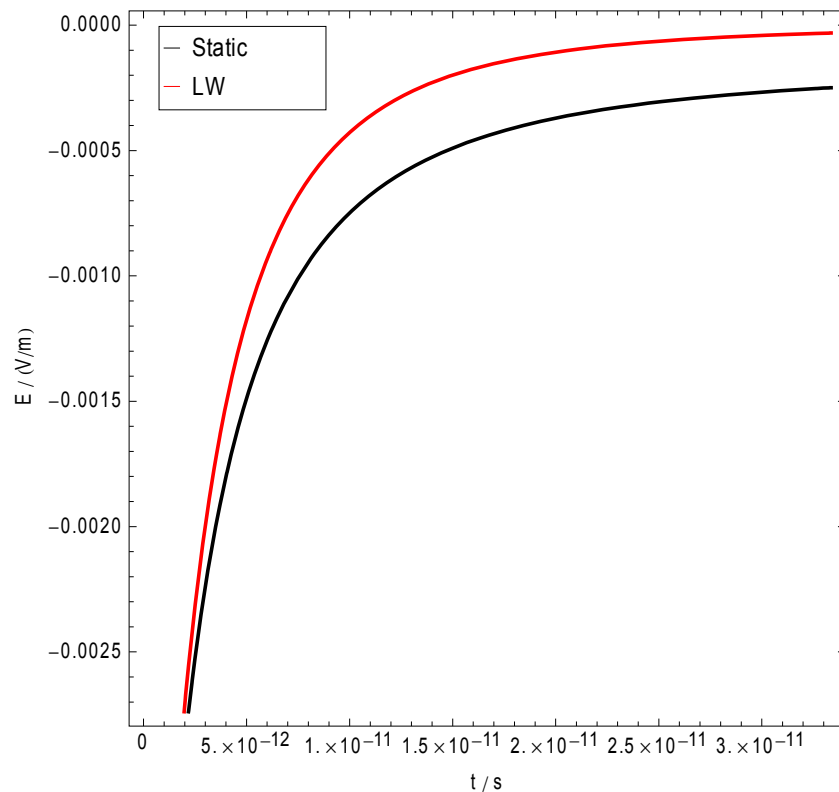
Driver and test particle trajectories



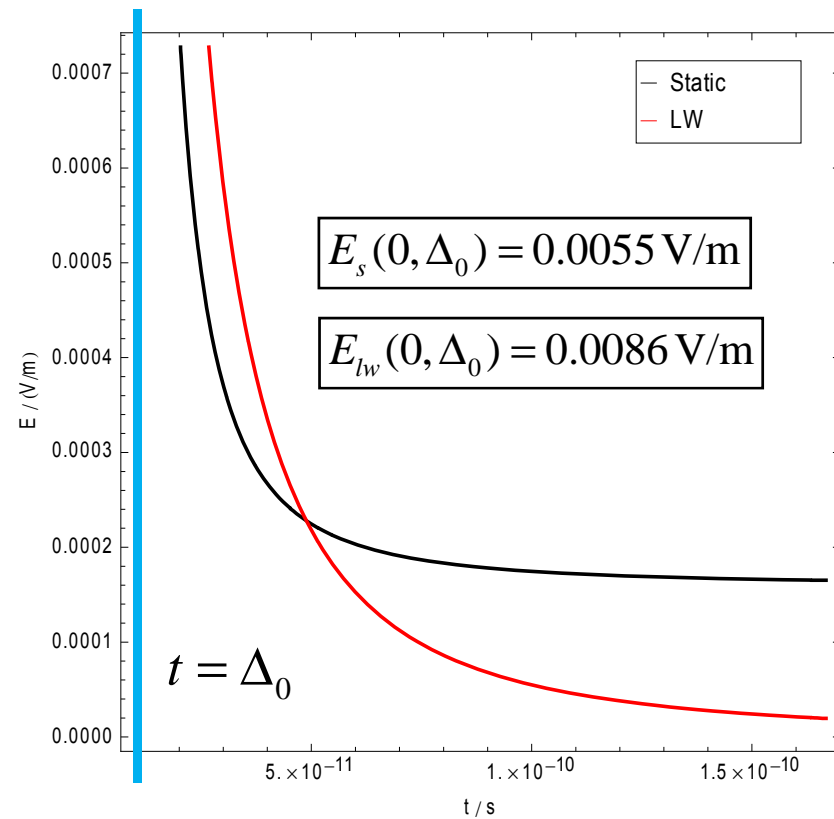
# Emittance studies for the PITZ injector

## Effect of retardation for single electron fields

Fields at head particle



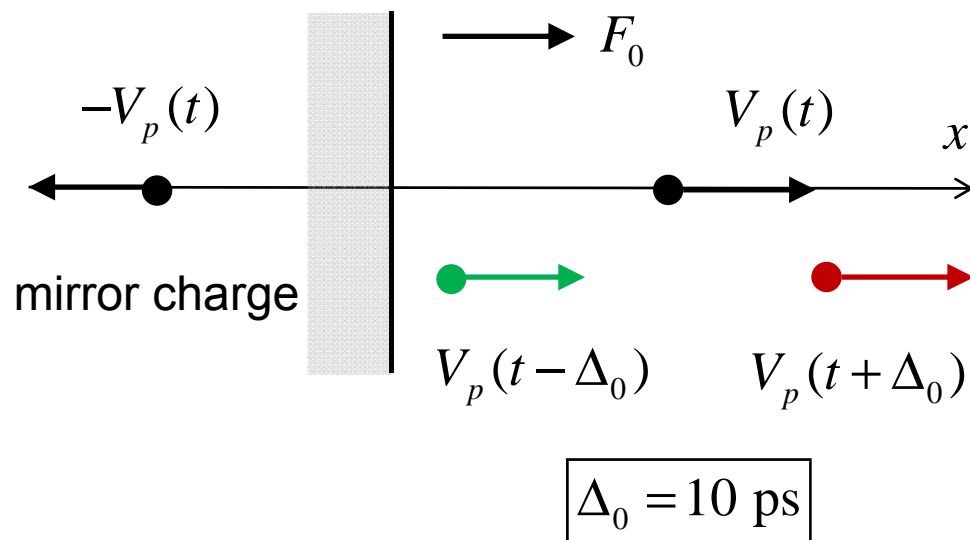
Fields at tail particle



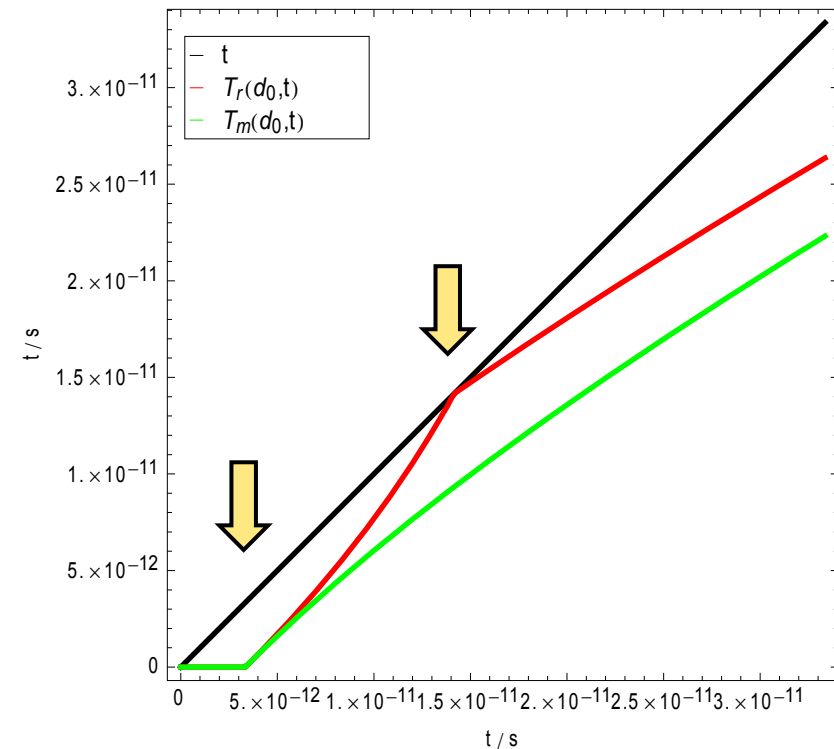
# Emittance studies for the PITZ injector

## Effect of retardation for single electron fields

- Add mirror charge at conducting surface



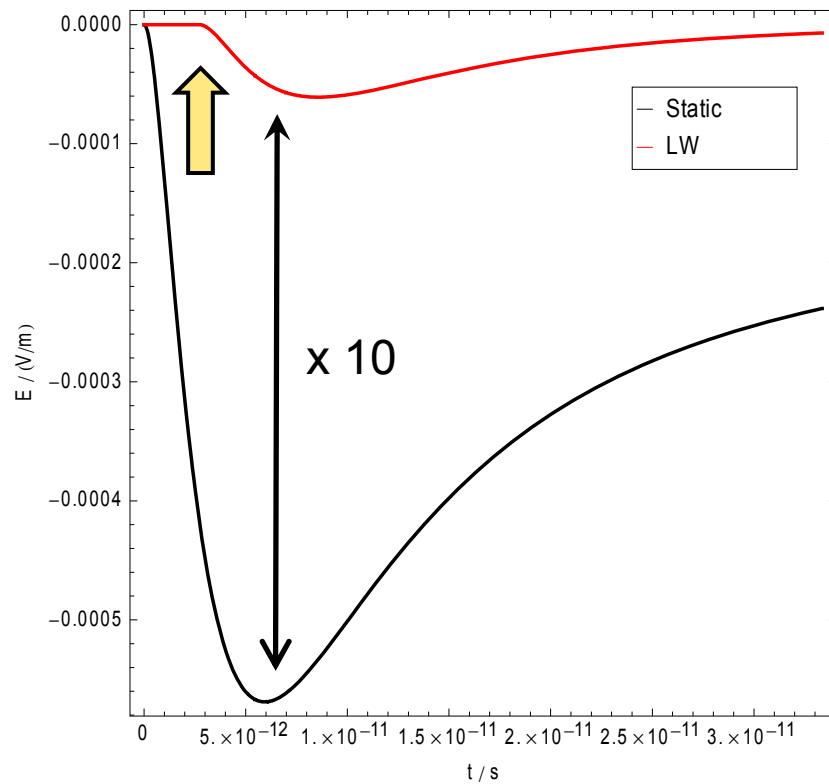
Retardation times of particle and its mirror



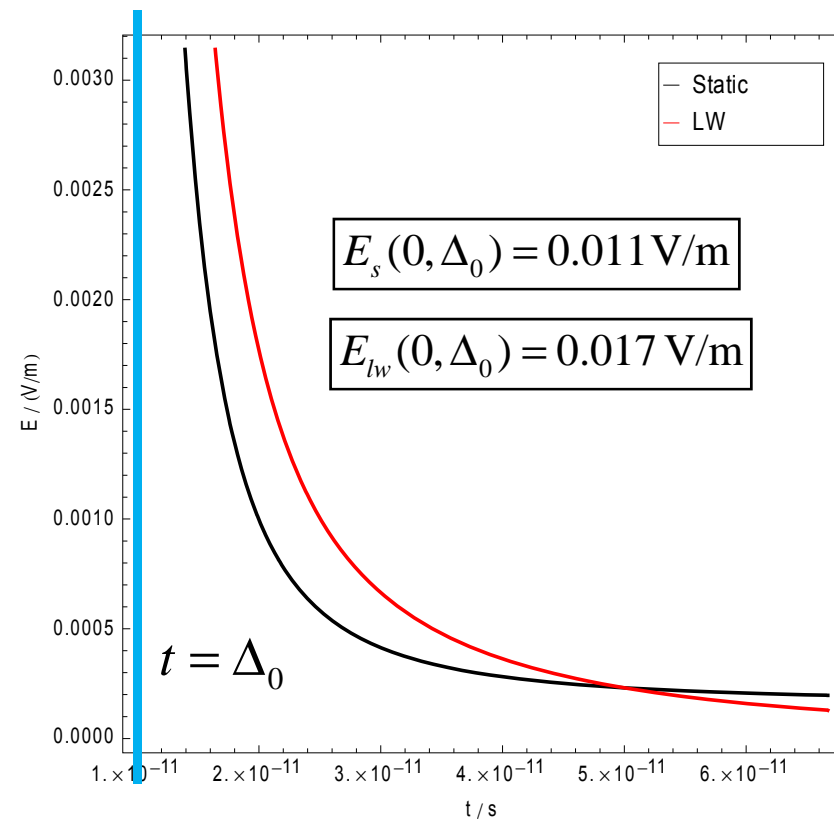
# Emittance studies for the PITZ injector

## Effect of retardation on single electron dynamics

Total fields at head particle

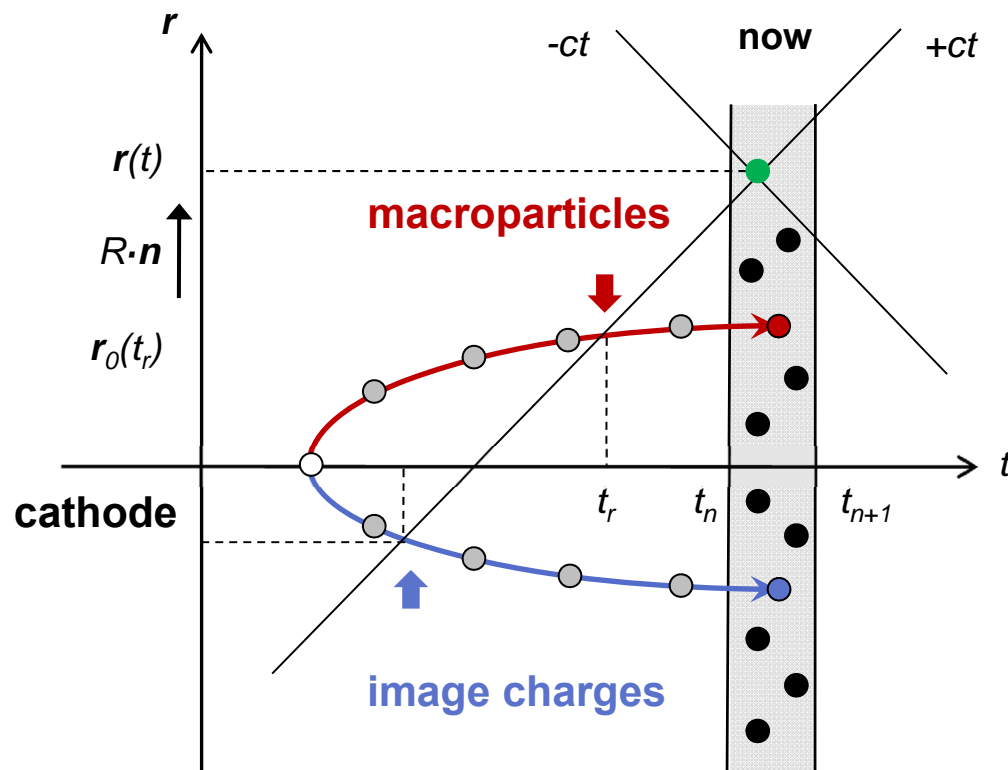


Total fields at tail particle



# Emittance studies for the PITZ injector

## Lienard-Wiechert particle-particle approach



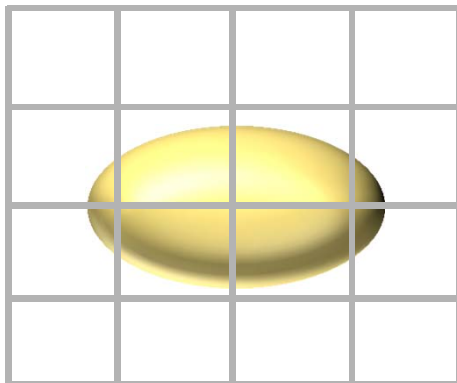
- Store full particle history – huge memory
- Search at every time step retarded times and positions for all particle positions -  $N_p^2 \cdot N_t$  operations
- Full physics
- Accuracy depends only on
  - Time step
  - Number of particles



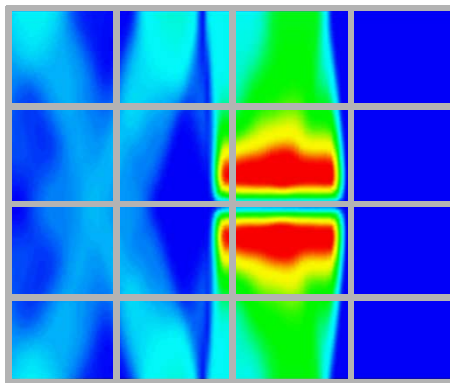
# Emittance studies for the PITZ injector

## The PIC approach

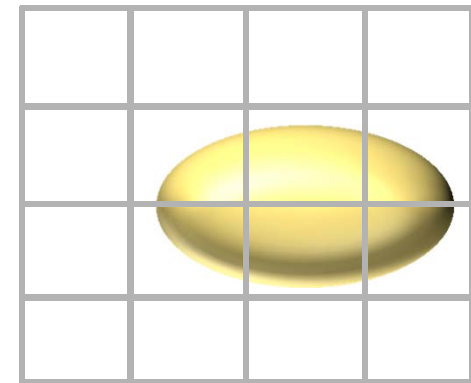
Bring particles  
to computational grid



Solve full set of Maxwell  
equations on the grid



Push particle positions and  
momenta

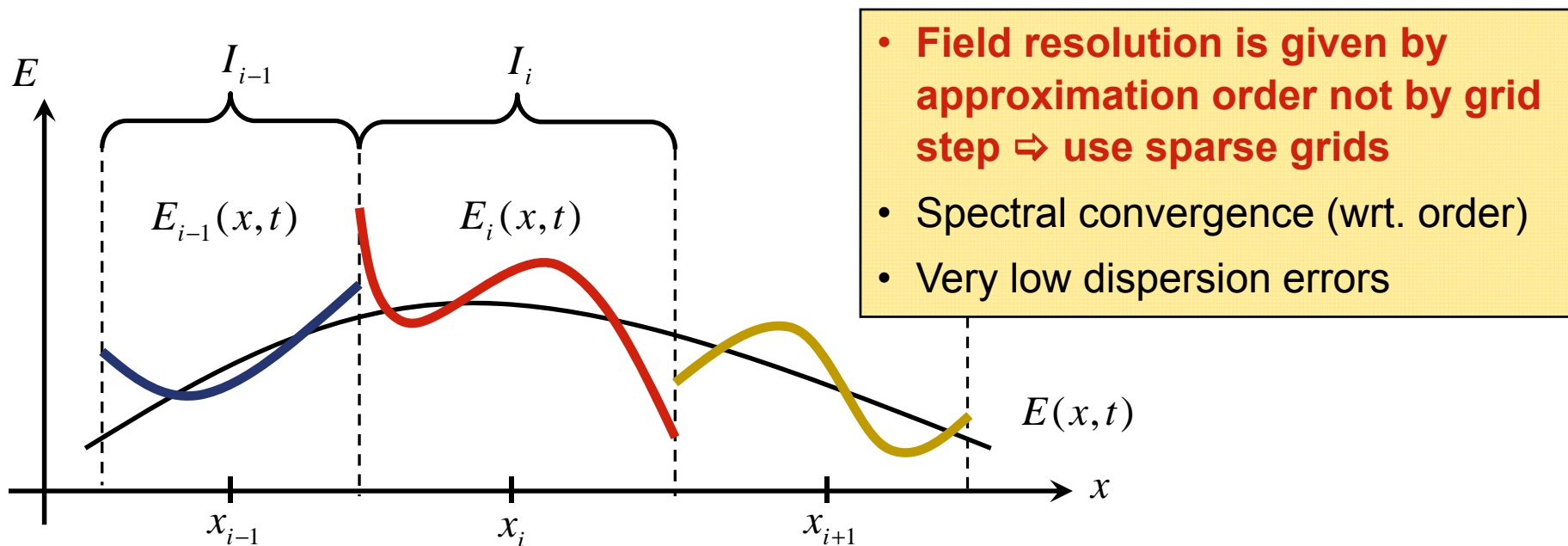


- Includes all effects; can handle arbitrary geometry
- Numerically more efficient than LW approach
- Accuracy depends on: accuracy of field solution, number of particles, current smoothing, time step,...

# Emittance studies for the PITZ injector

## High order DG-FEM method

- Very accurate field solutions using piecewise high order polynomial approximation with grid cells (DG-FEM)\*



\* E. Gjonaj et al., *New J. of Phys.*, Vol. 8, (2006), pp. 1-21

# Emittance studies for the PITZ injector

## Charge conserving current interpolation

- Macroparticle current density:

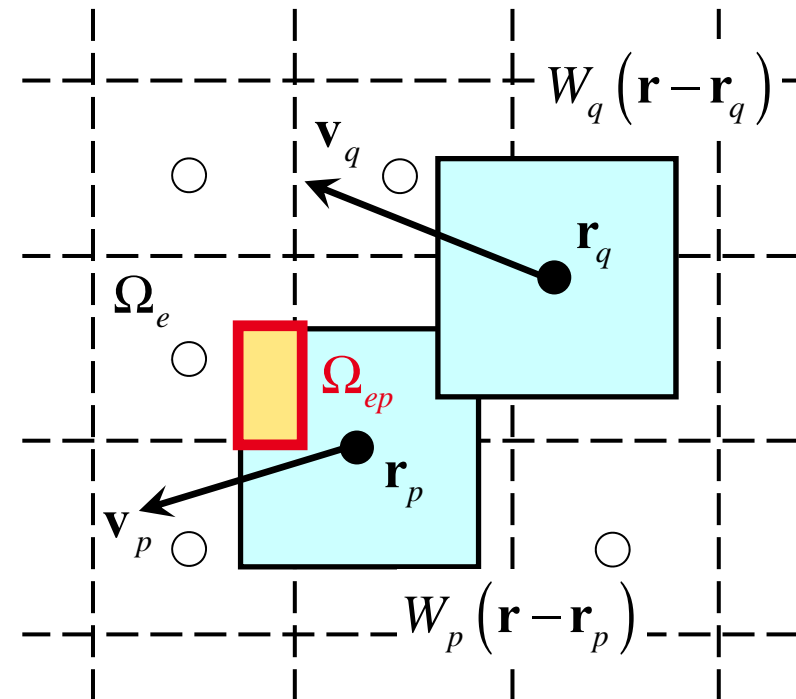
$$\mathbf{j}(\mathbf{r}, t) = \sum_p Q_p \mathbf{v}_p(t) W_p[\mathbf{r} - \mathbf{r}_p(t)]$$

- DG-FEM current projection:

$$\mathbf{j}_i^e(t) = \sum_p Q_p \mathbf{v}_p(t) \int_{\Omega_{ep}(t)} d^3 \mathbf{r} W_p(\mathbf{r}, t) \phi_i^e(\mathbf{r})$$

- Discrete current / time step / cell:

$$\mathbf{J}_i^e(t^n, t^{n+1}) = \int_{t^n}^{t^{n+1}} dt \mathbf{j}_i^e(t)$$

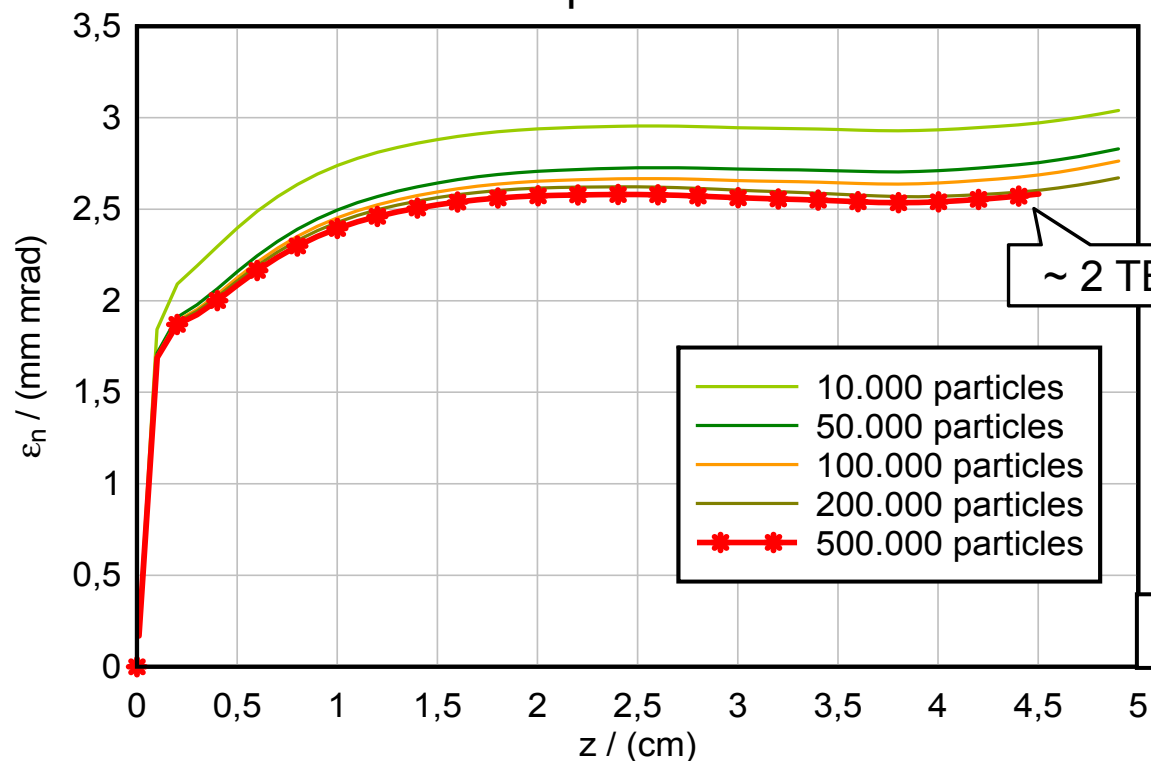


- Choice of particle size most critical parameter for simulation accuracy

# Emittance studies for the PITZ injector

## Beam dynamics in the gun: LW simulations

Projected transverse emittance for different particle numbers



$Q = 1 \text{ nC}$   
 $XY_{\text{rms}} = 0.75 \text{ mm}$   
non-optimized setup  
(data from 2010)

~ 2 TB

$\Delta t = 0.1 \text{ ps}$

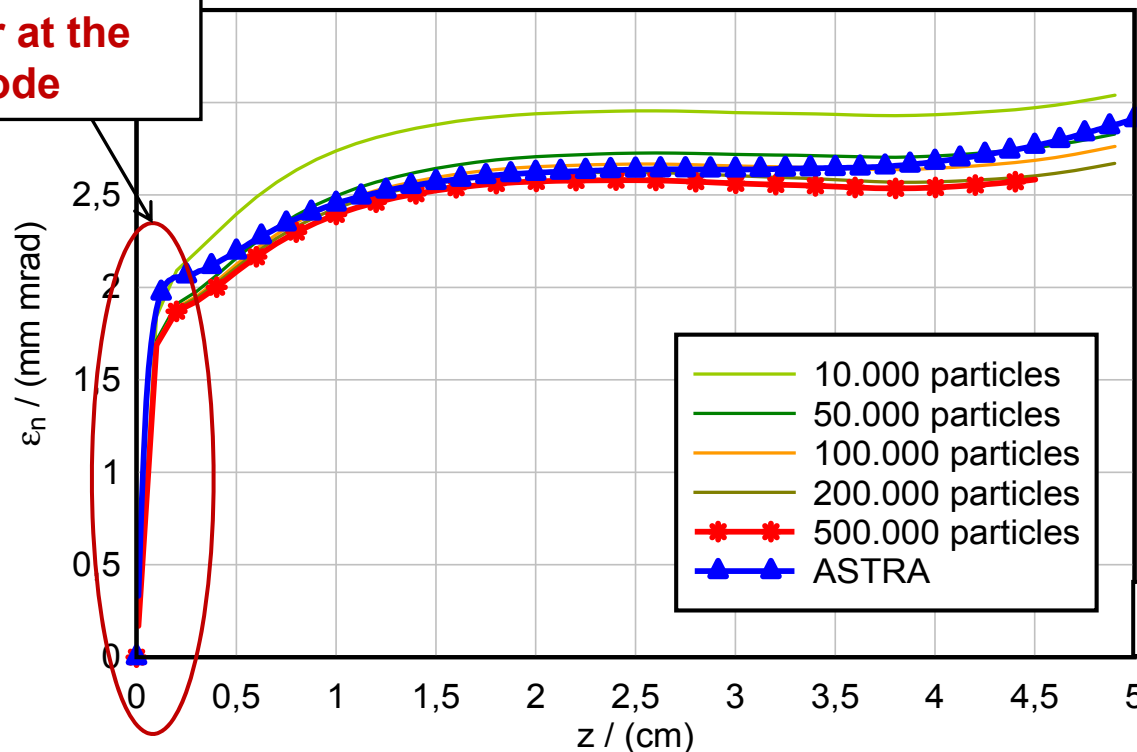
# Emittance studies for the PITZ injector

## Beam dynamics in the gun: LW simulations

Projected transverse emittance:  
comparison with ASTRA

$Q = 1 \text{ nC}$   
 $XY_{\text{rms}} = 0.75 \text{ mm}$   
non-optimized setup  
(data from 2010)

systematic different  
behavior at the  
cathode



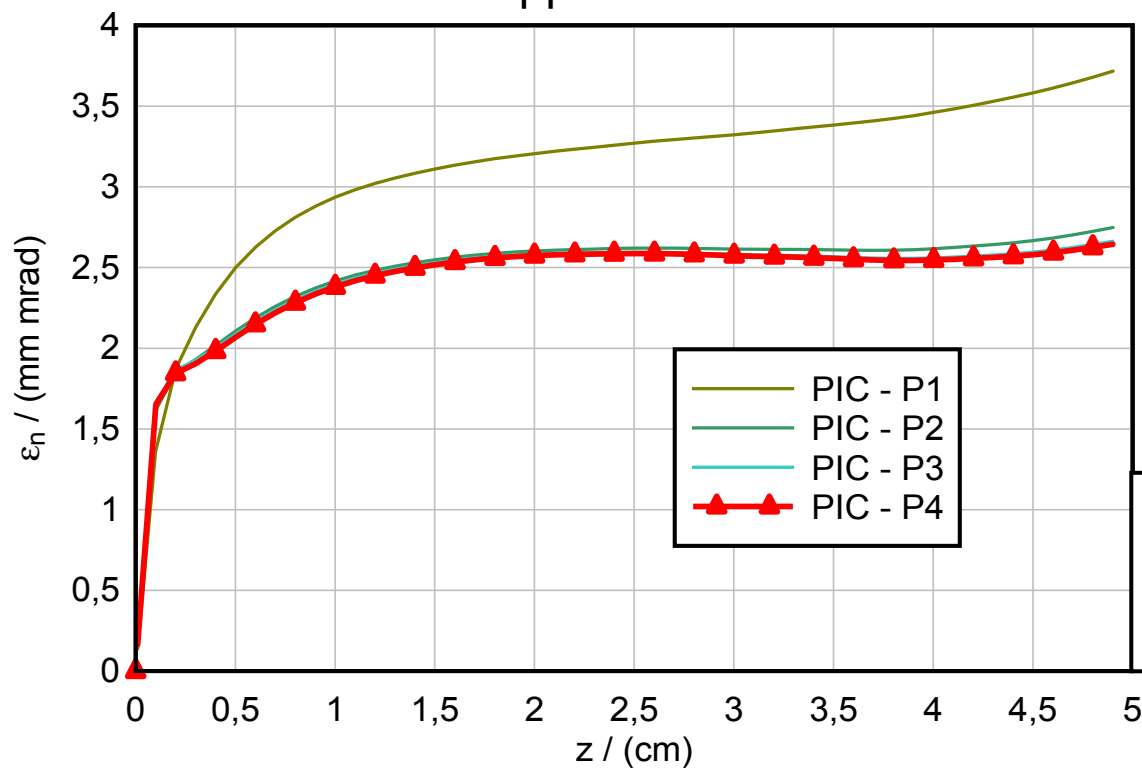
$\Delta t = 0.1 \text{ ps}$

# Emittance studies for the PITZ injector

## Beam dynamics in the gun: PIC simulations

$Q = 1 \text{ nC}$   
 $XY_{\text{rms}} = 0.75 \text{ mm}$   
non-optimized setup  
(data from 2010)

Projected transverse emittance for different approximation orders

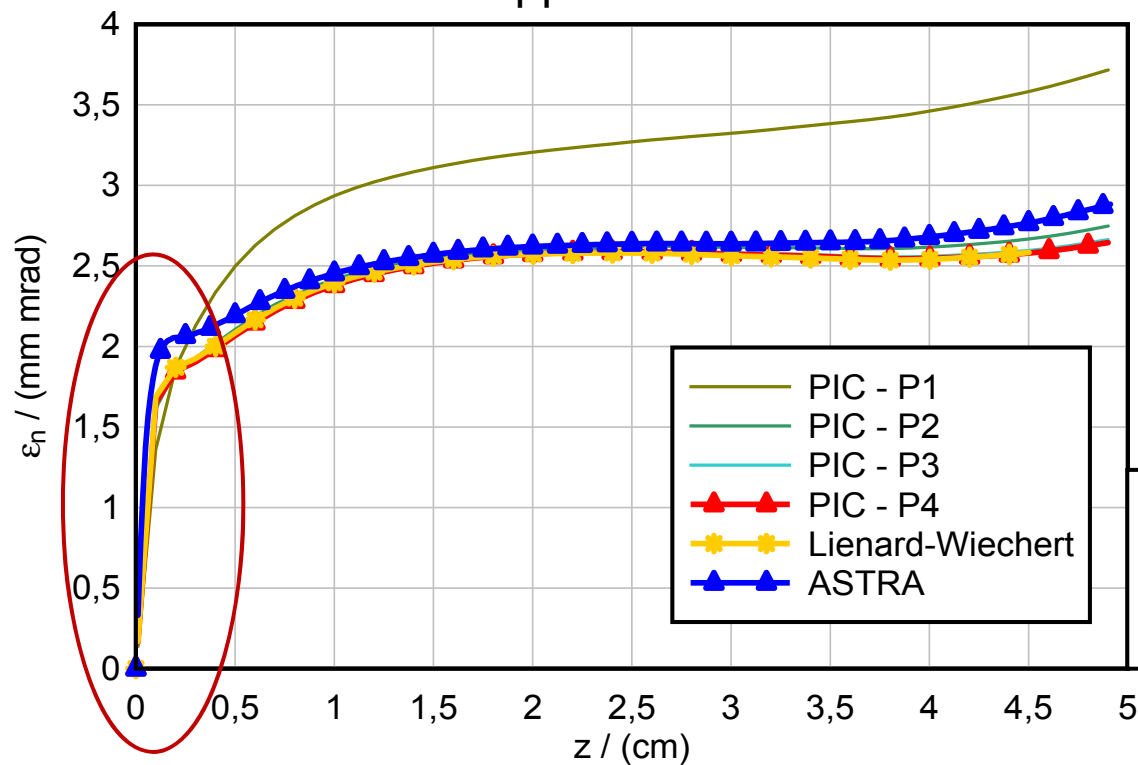


$\Delta z = 1 \text{ mm}$   
 $\Delta s = 10 \mu\text{m}$   
 $N_p = 50.000$

# Emittance studies for the PITZ injector

## Beam dynamics in the gun: PIC simulations

Projected transverse emittance for different approximation orders



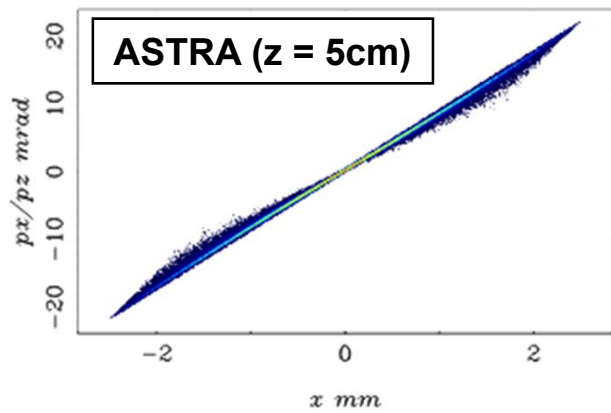
$Q = 1 \text{ nC}$   
 $XY_{\text{rms}} = 0.75 \text{ mm}$   
non-optimized setup  
(data from 2010)

$\Delta z = 1 \text{ mm}$   
 $\Delta s = 20 \mu\text{m}$   
 $N_p = 50.000$

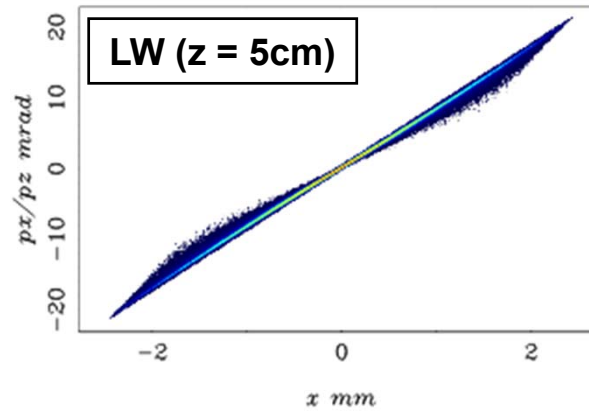
# Emittance studies for the PITZ injector

## Beam dynamics in the gun

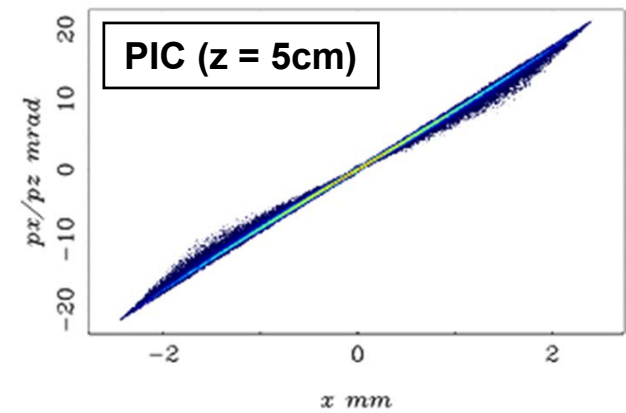
$Q = 1 \text{ nC}$   
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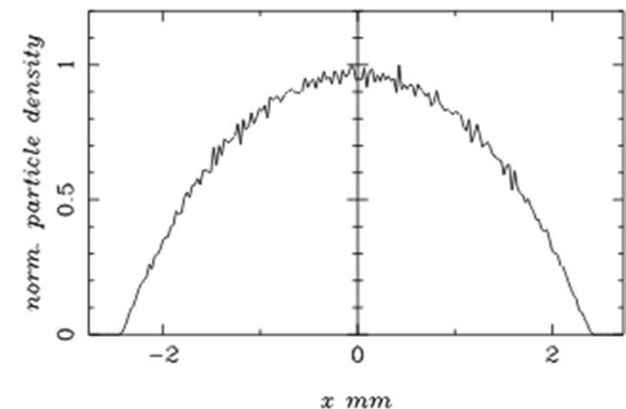
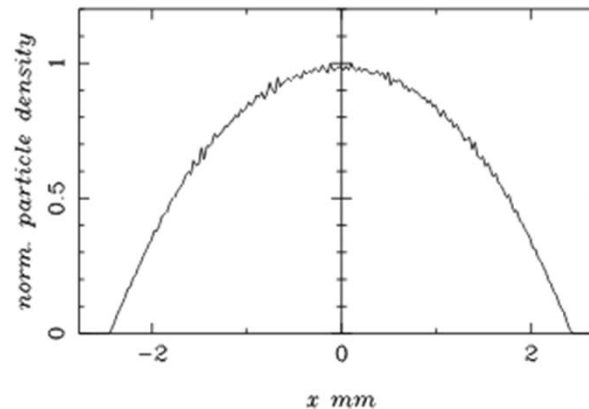
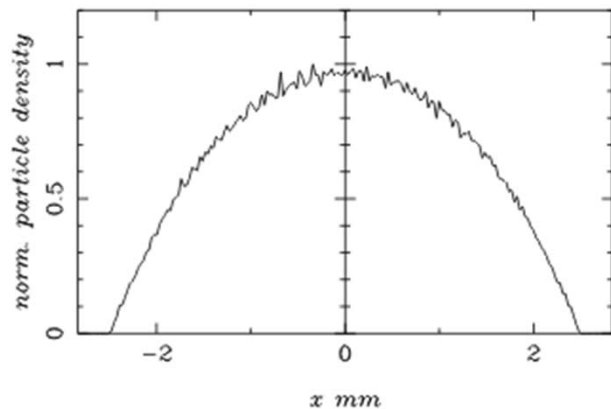
Transverse Distribution



Transverse Distribution



Transverse Distribution

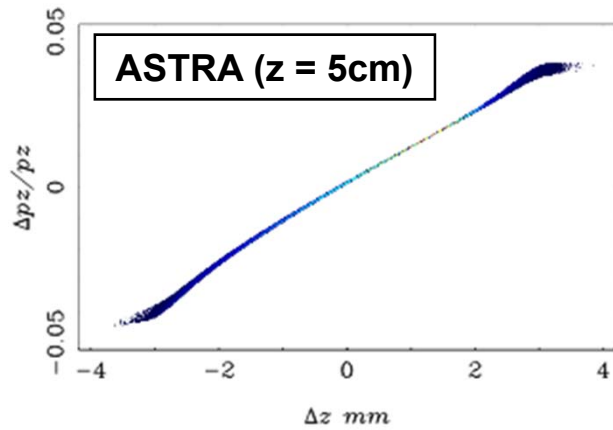




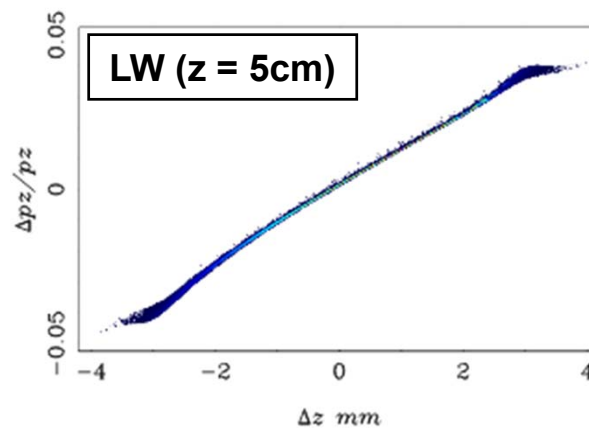
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## Beam dynamics in the gun

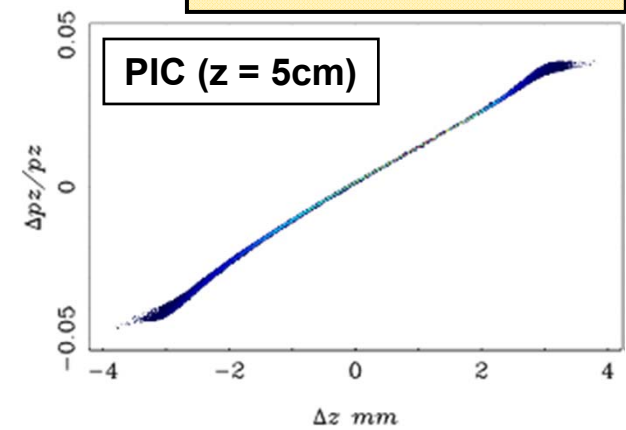
$Q = 1 \text{ nC}$   
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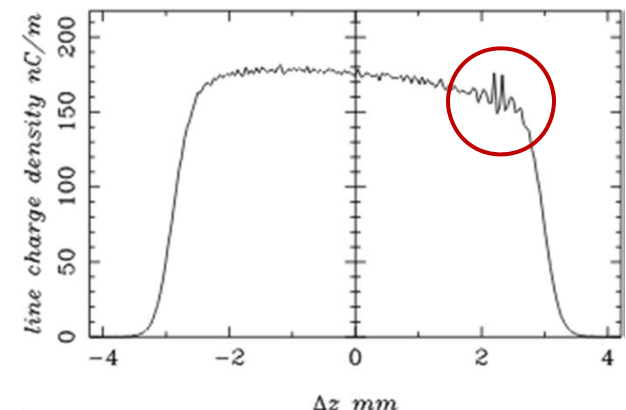
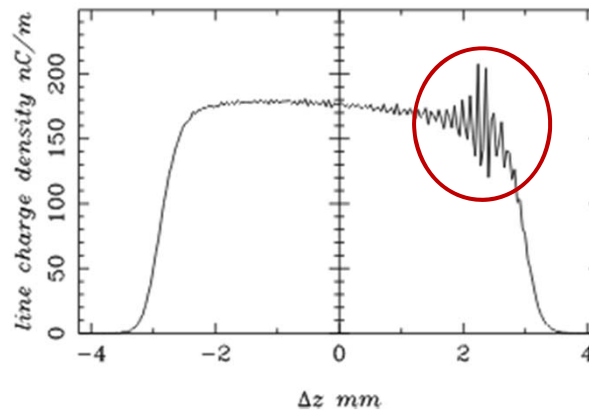
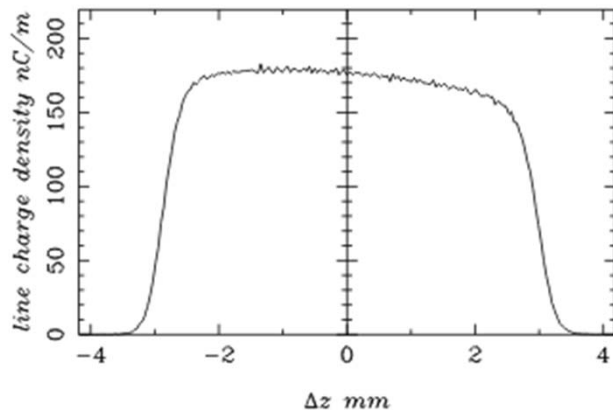
Longitudinal Distribution



Longitudinal Distribution



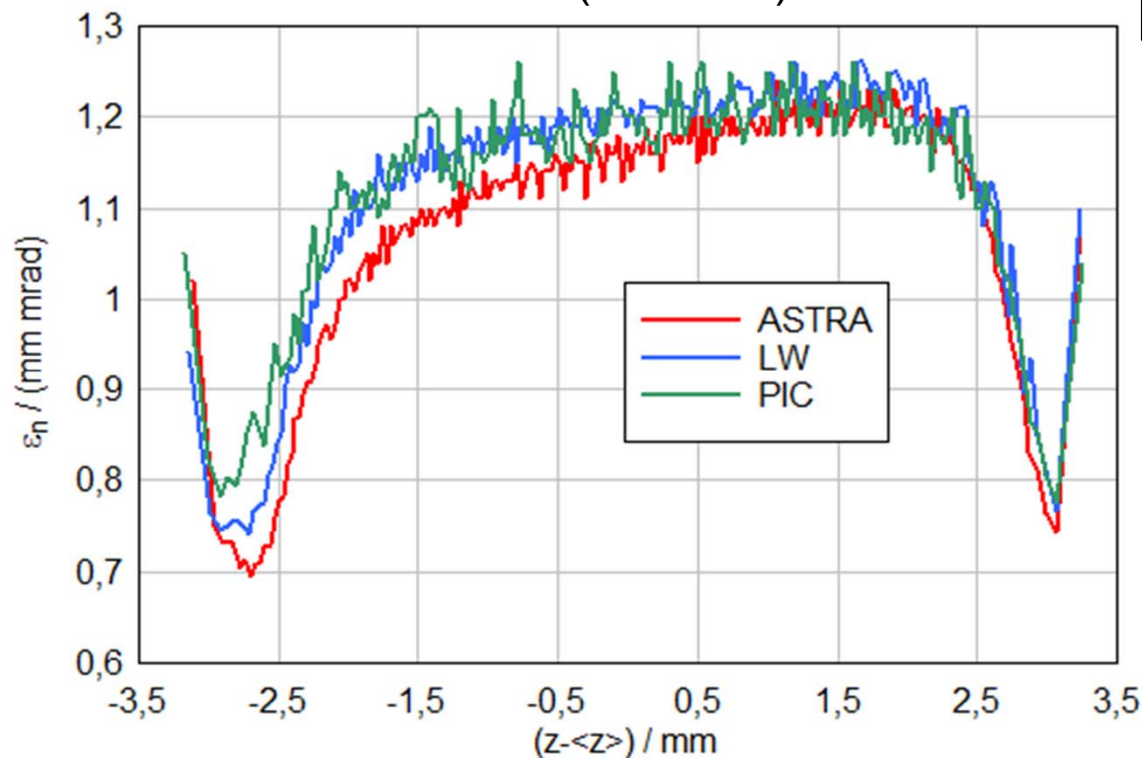
Longitudinal Distribution



# Emittance studies for the PITZ injector

## Beam dynamics in the gun

Slice emittances for the different codes (z = 5 cm)



Q = 1 nC  
XY\_rms = 0.75 mm  
non-optimized setup  
(data from 2010)

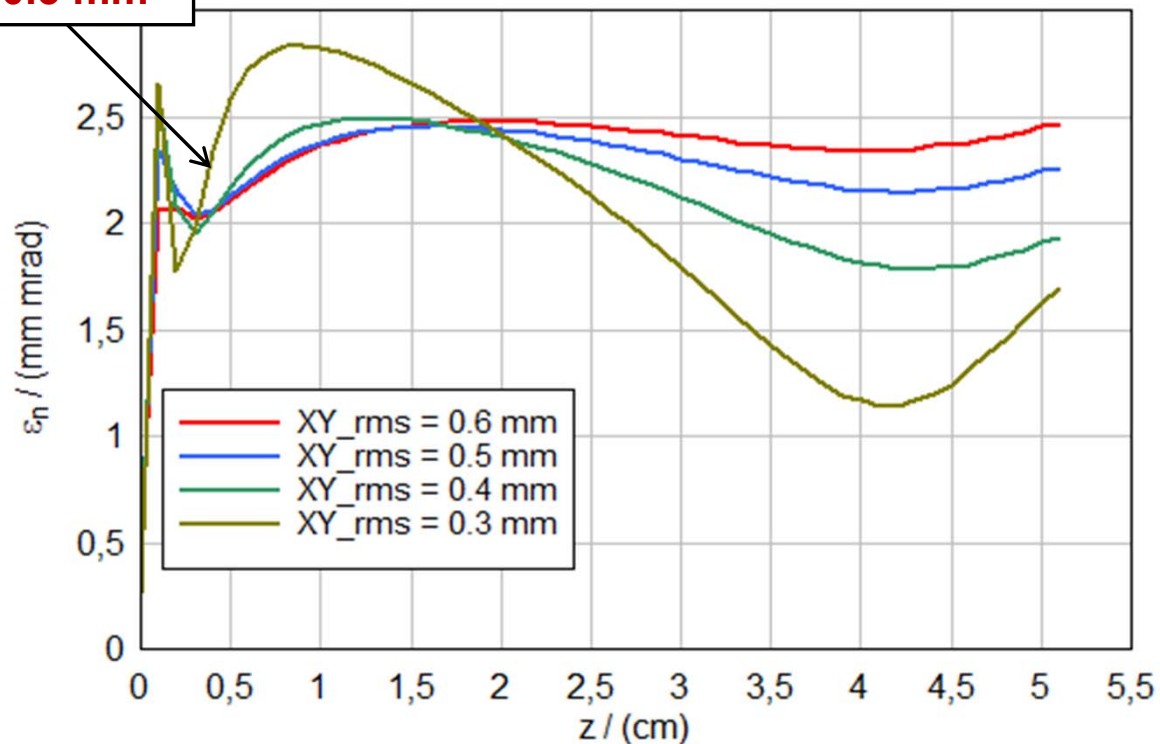
# Emittance studies for the PITZ injector

## Beam dynamics for PITZ-1.8 setup

$Q = 1 \text{ nC}$   
PITZ-1.8 setup  
(M. Krasilnikov, 2011)

no SPCH limiting at  
 $XY_{\text{rms}} = 0.3 \text{ mm}$

Projected transverse emittance for  
different  $XY_{\text{rms}}$  sizes



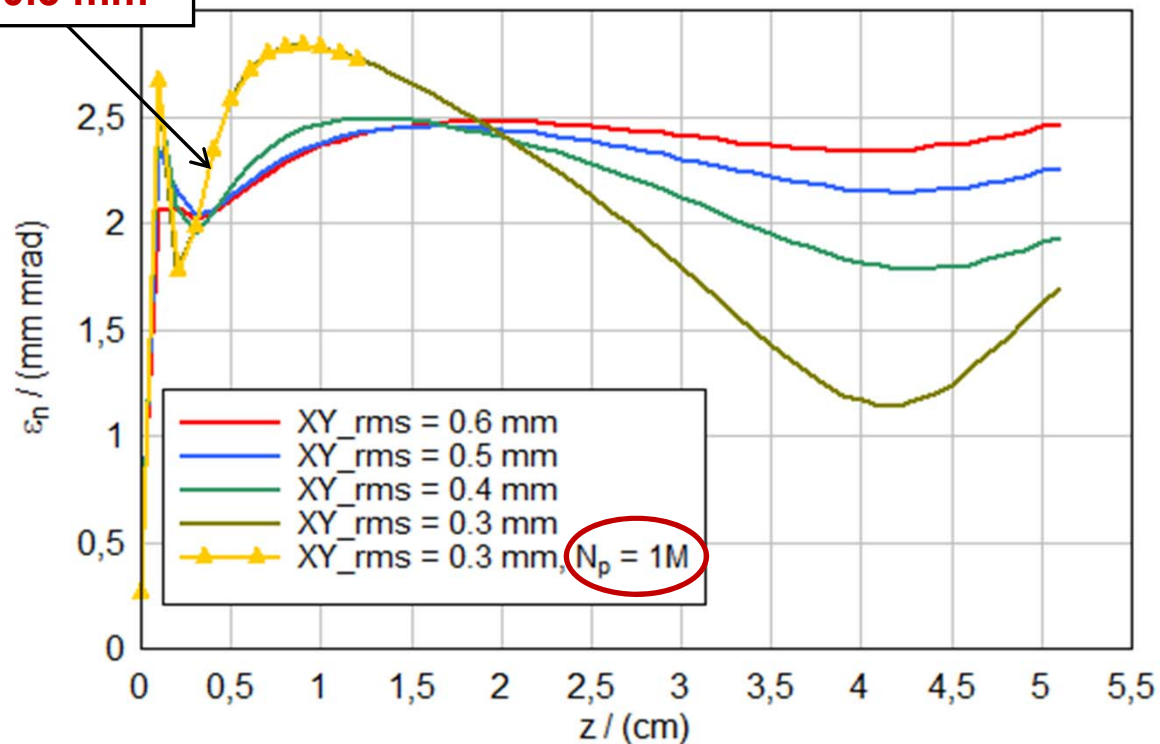
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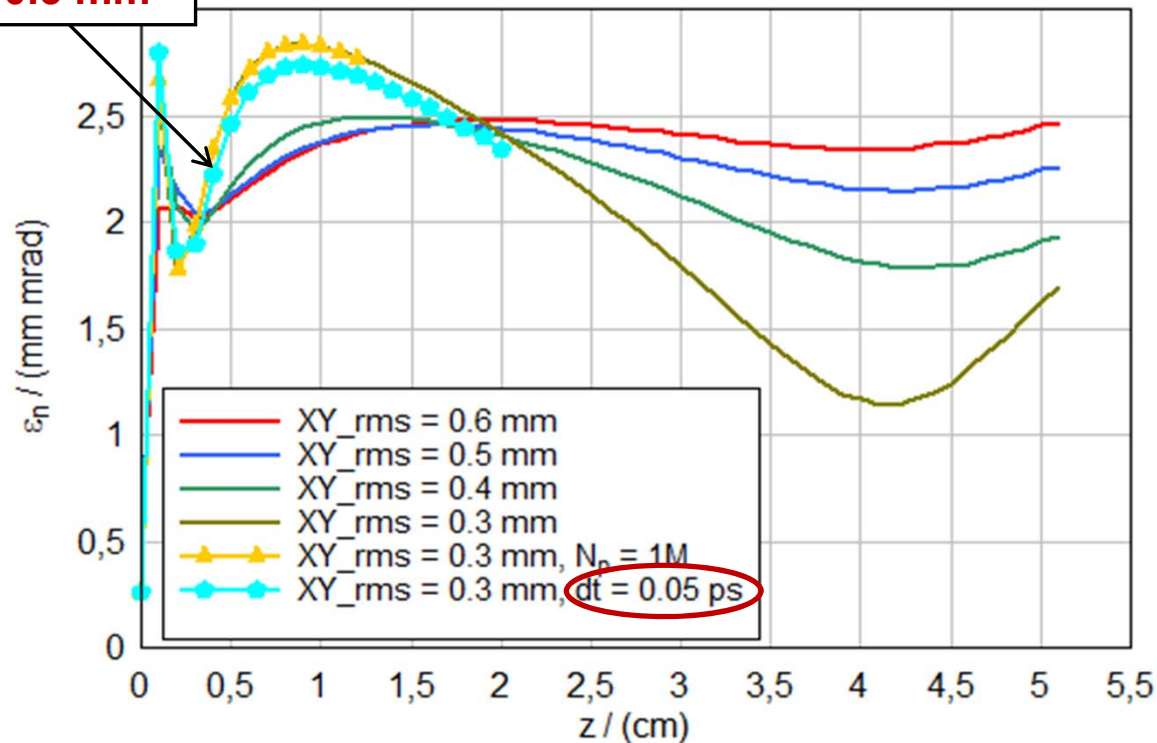
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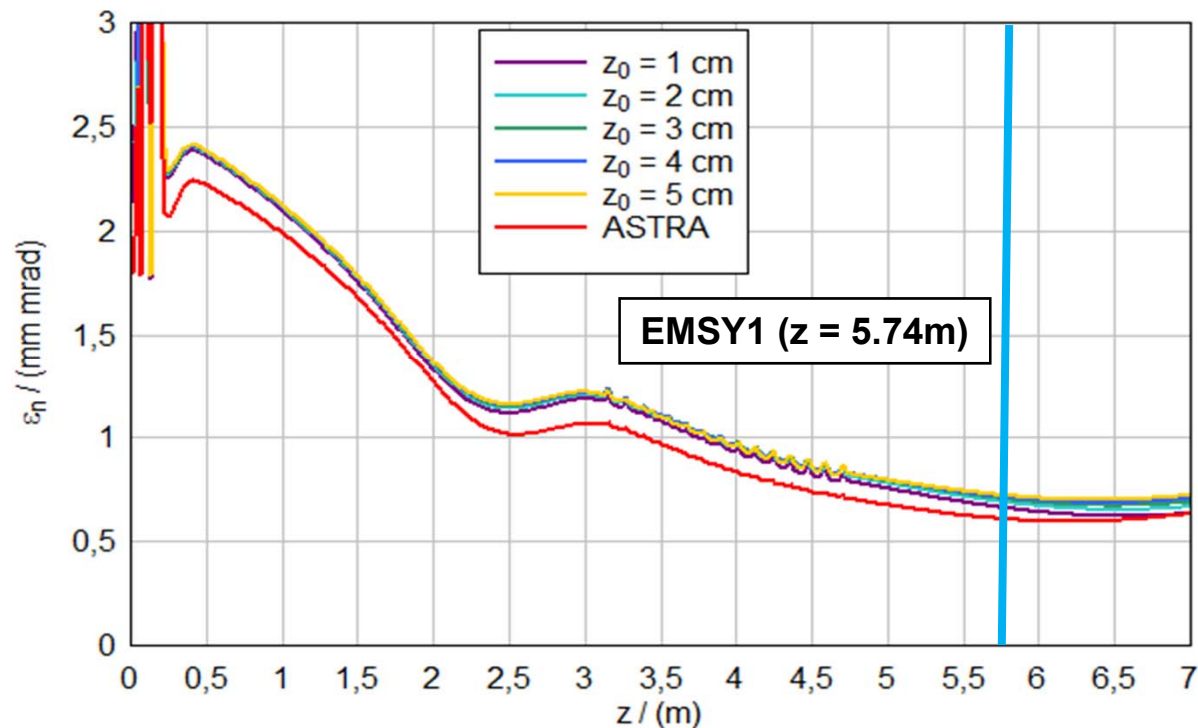


# Emittance studies for the PITZ injector

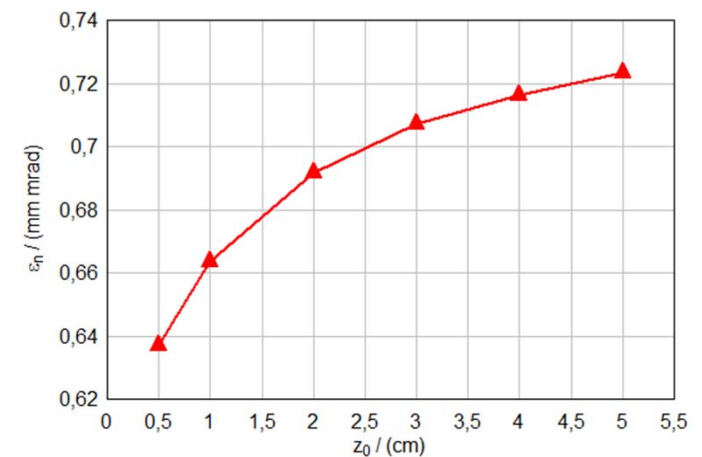
## Beam dynamics for PITZ-1.8 setup

- Restarted ASTRA simulations with LW bunch as initial distribution

$Q = 1 \text{ nC}$   
 $XY_{\text{rms}} = 0.4 \text{ mm}$   
PITZ-1.8 setup  
(M. Krasilnikov, 2011)



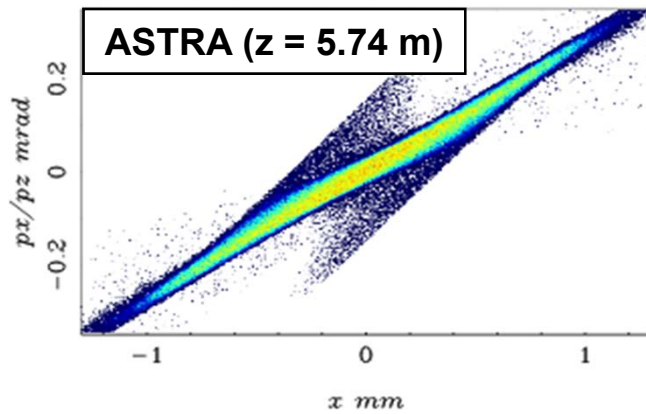
Emittance at EMSY1 vs.  $z_0$



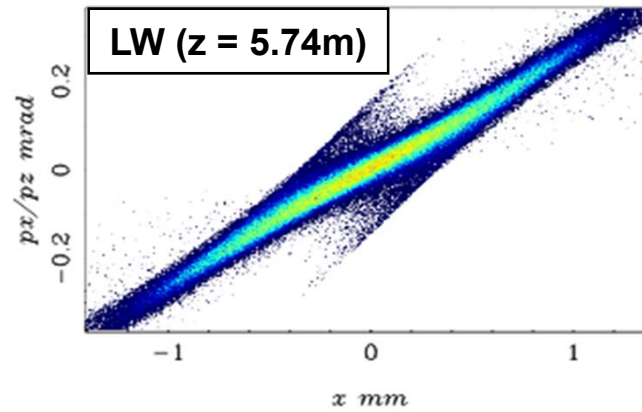
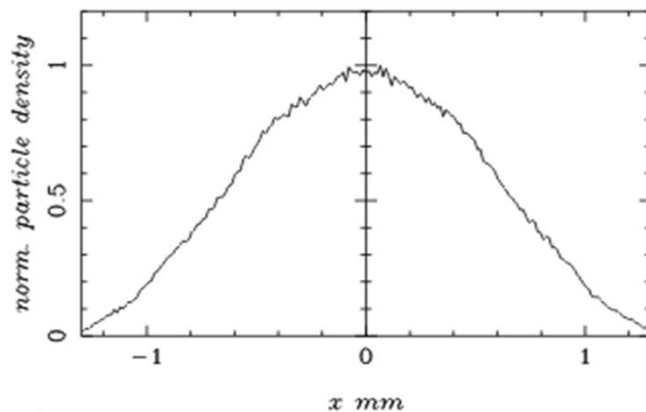
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## Beam dynamics for PITZ-1.8 setup

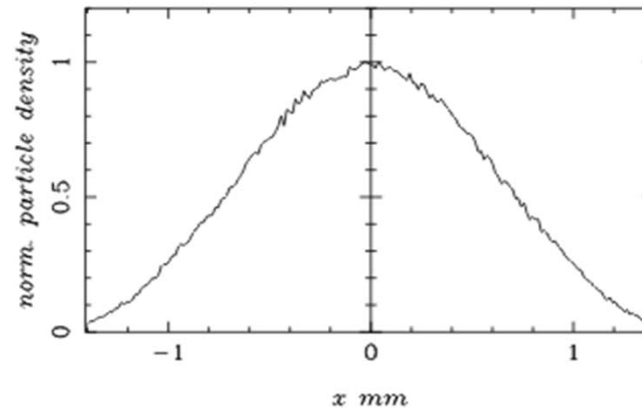
$Q = 1 \text{ nC}$   
 **$XY_{\text{rms}} = 0.4 \text{ mm}$**   
PITZ-1.8 setup  
(M. Krasilnikov, 2011)



Transverse Distribution



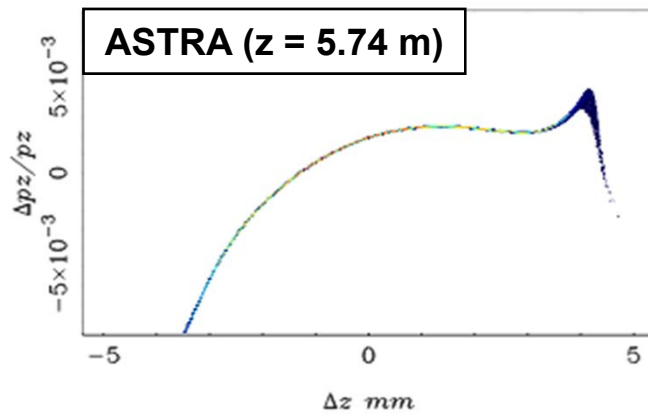
Transverse Distribution



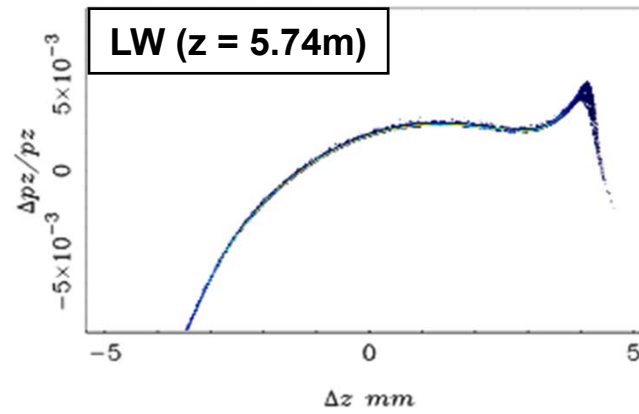
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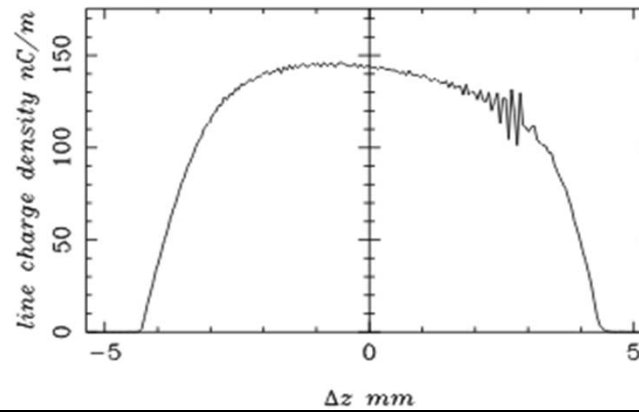
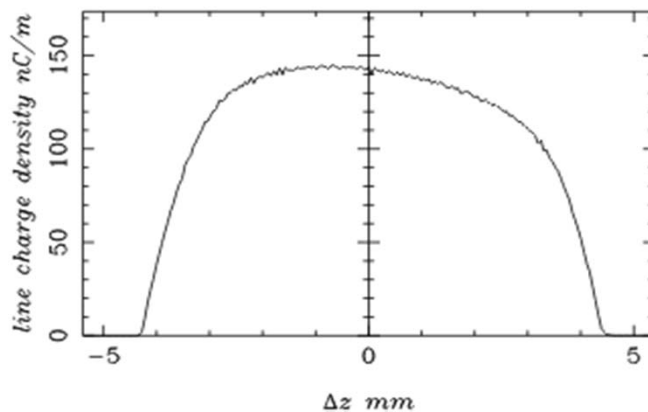
$Q = 1 \text{ nC}$   
 **$XY_{\text{rms}} = 0.4 \text{ mm}$**   
PITZ-1.8 setup  
(M. Krasilnikov, 2011)



Longitudinal Distribution



Longitudinal Distribution

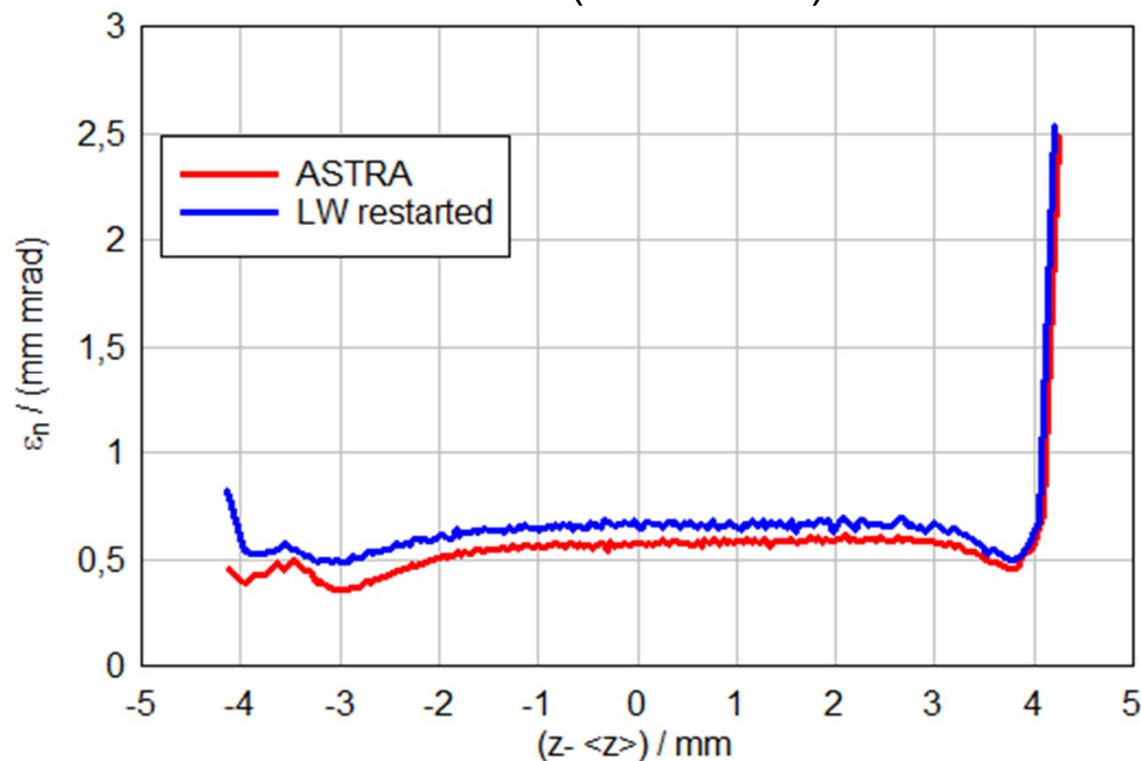




# Emittance studies for the PITZ injector

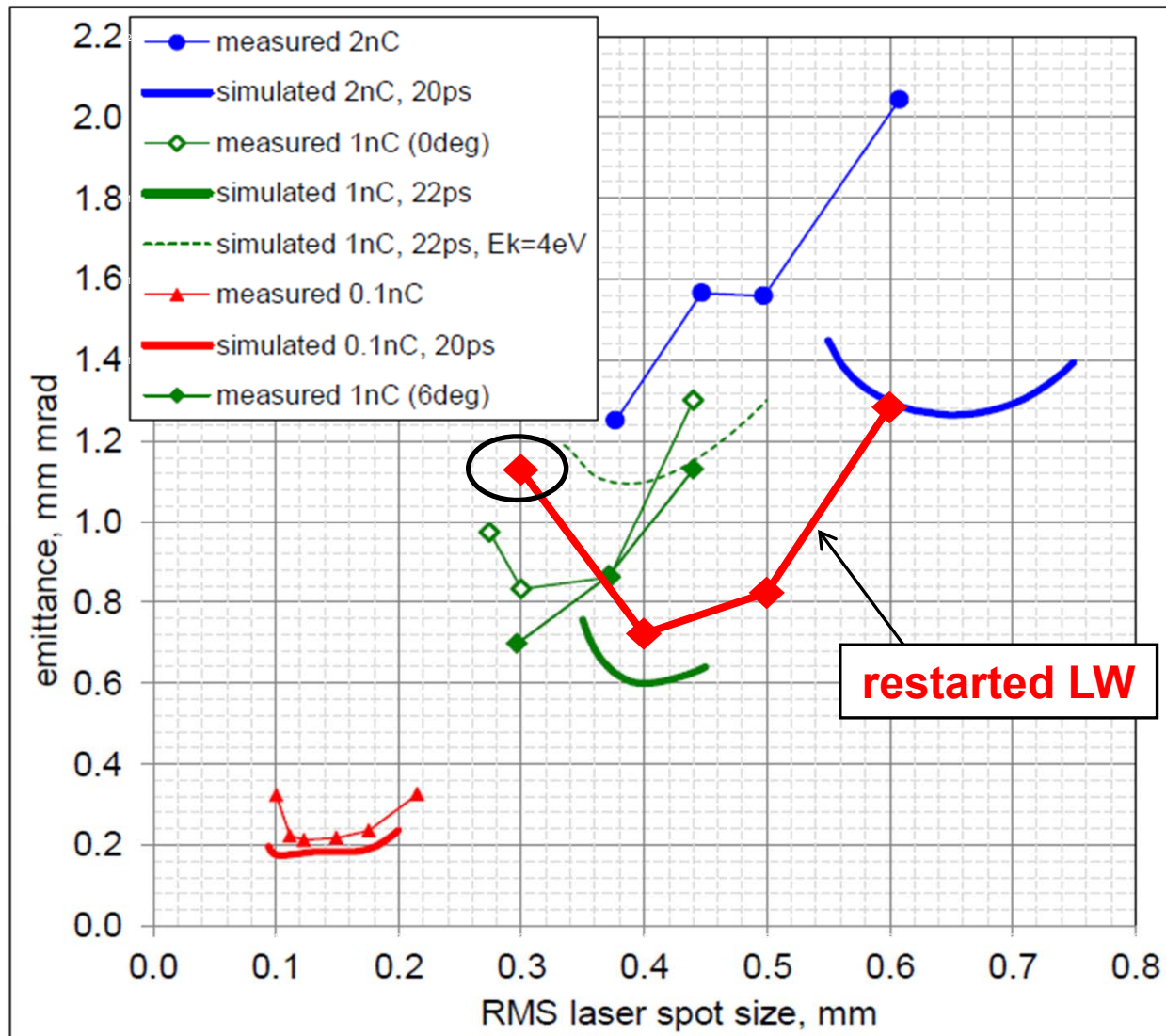
## Beam dynamics for PITZ-1.8 setup

Slice emittances for the different codes (z = 5.74 m)



Q = 1 nC  
**XY\_rms = 0.4 mm**  
PITZ-1.8 setup  
(M. Krasilnikov, 2011)

# Emittance studies for the PITZ injector



Q = 1 nC  
PITZ-1.8 setup  
(M. Krasilnikov, 2011)

# Emittance studies for the PITZ injector

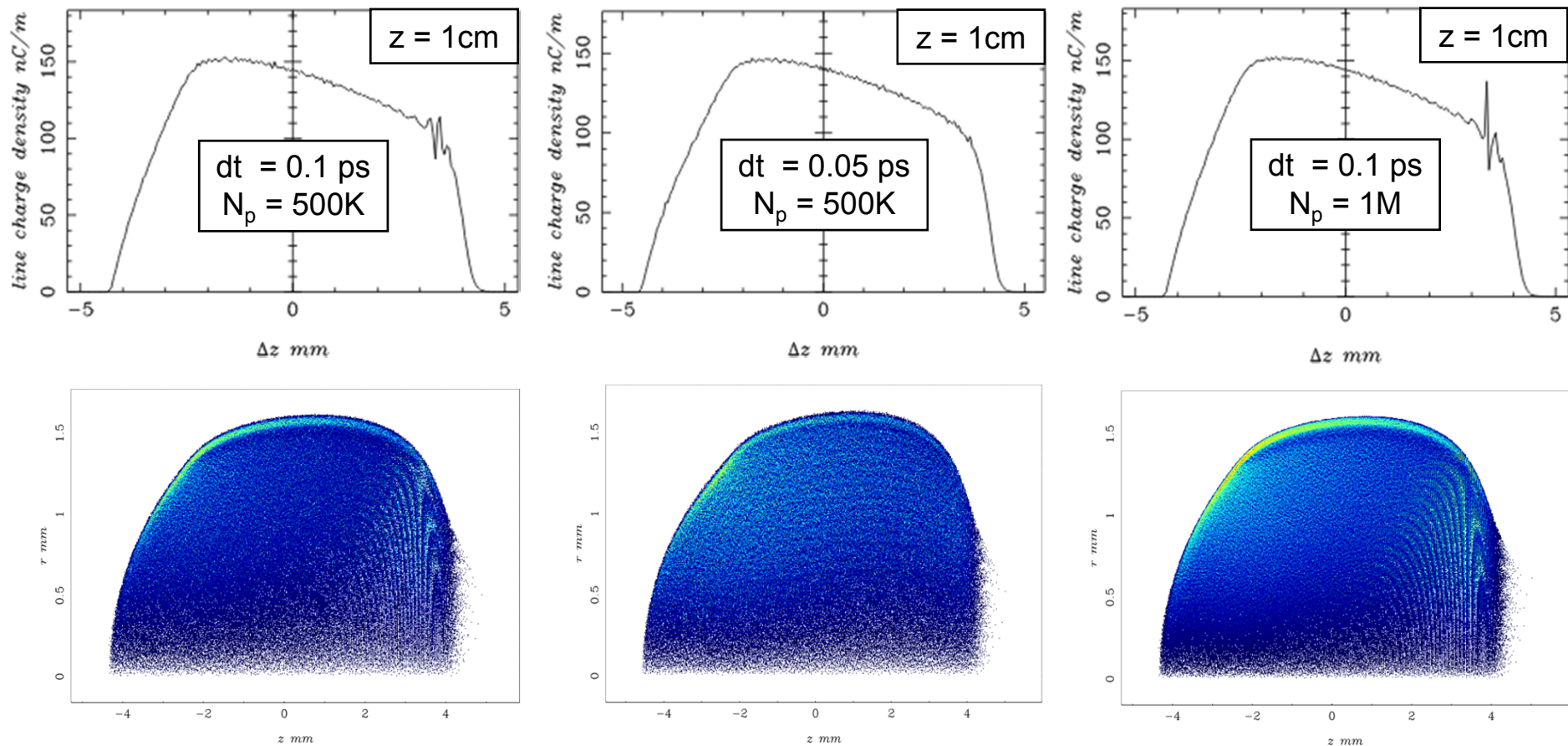
## Beam dynamics for PITZ-1.8 setup

- Still no good agreement with experiment
- Could extract full bunch charge at  $XY_{\text{rms}} = 0.3$  mm
- Slightly larger emittance than ASTRA
- Emittance minimum and curve pattern same as ASTRA
- **Are other effects responsible for the discrepancy (wakefields?)**
  
- Restart positions too close to cathode backplane
- Longitudinal density oscillations not fully understood
- Convergence of LW vs. time step is unclear

# Emittance studies for the PITZ injector

## Discussion on space charge at the cathode

$Q = 1 \text{ nC}$   
 $XY_{\text{rms}} = 0.3 \text{ mm}$



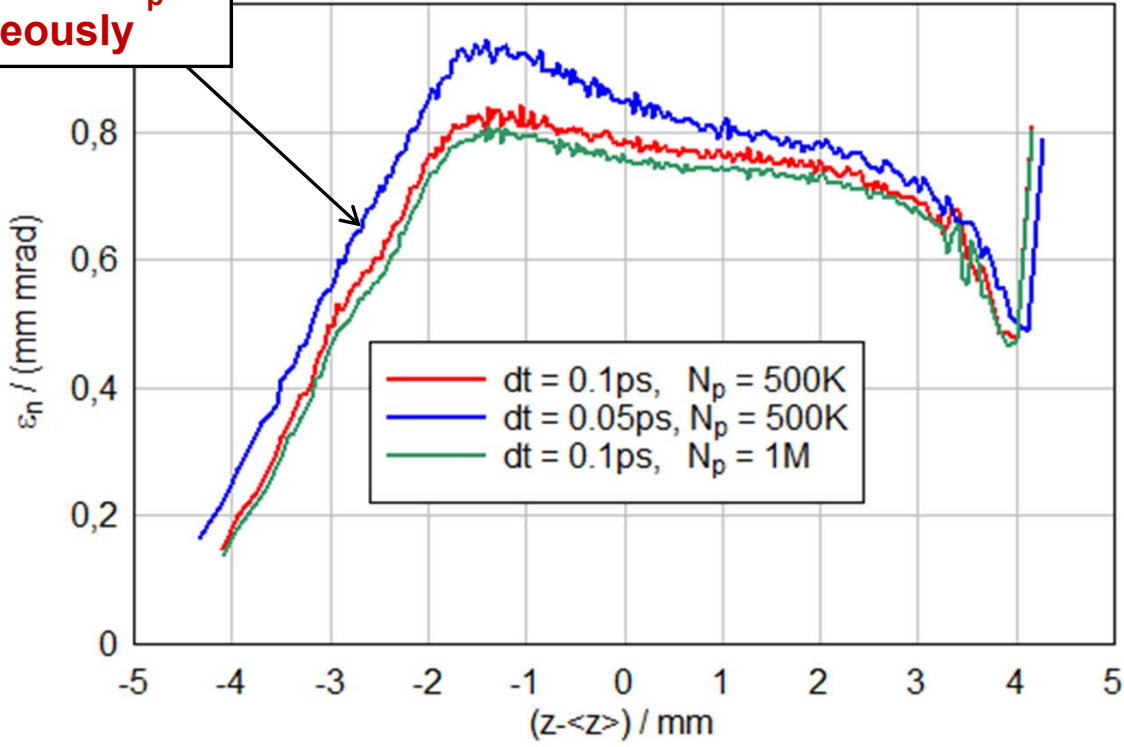
# Emittance studies for the PITZ injector

## Discussion on space charge at the cathode

$Q = 1 \text{ nC}$   
 $XY_{\text{rms}} = 0.3 \text{ mm}$   
PITZ-1.8 setup  
(M. Krasilnikov, 2011)

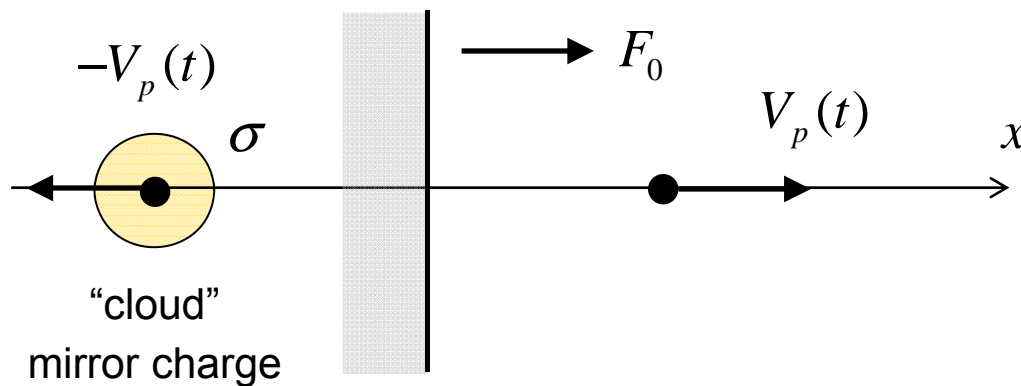
**No convergence: need to refine  $dt$  and  $N_p$  simultaneously**

LW slice emittances for different parameters ( $z = 1 \text{ cm}$ )



# Emittance studies for the PITZ injector

## Discussion on space charge at the cathode



$$\frac{m v(x) v'(x)}{\left(1 - \frac{v(x)^2}{c^2}\right)^{3/2}} = F_0 + F_m(x), \quad v(0) = 0$$

$$F_m(x) = \frac{k e^{-\frac{2x^2}{\sigma^2}}}{\sqrt{2\pi} x \sigma} - \frac{k \operatorname{Erf}\left(\frac{\sqrt{2}x}{\sigma}\right)}{4x^2}$$

### Point charge emission from ideal surface not possible:

- Need a cloud mirror charge distribution

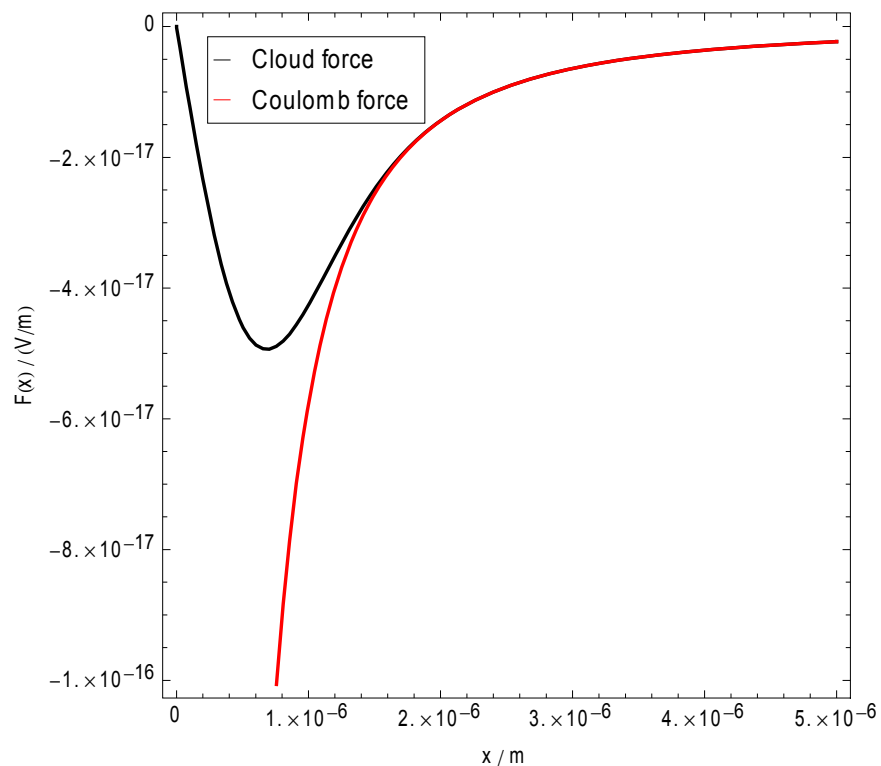
Main branch solution

$$v(x) = \sqrt{1 - \frac{16\pi^2 c^4 m^2 x^2 \sigma^2}{\left(\pi\sigma(4c^2 m x + k \operatorname{Erf}\left(\frac{\sqrt{2}x}{\sigma}\right) + 4F_0 x^2) - 2\sqrt{2\pi} k x\right)^2}}$$

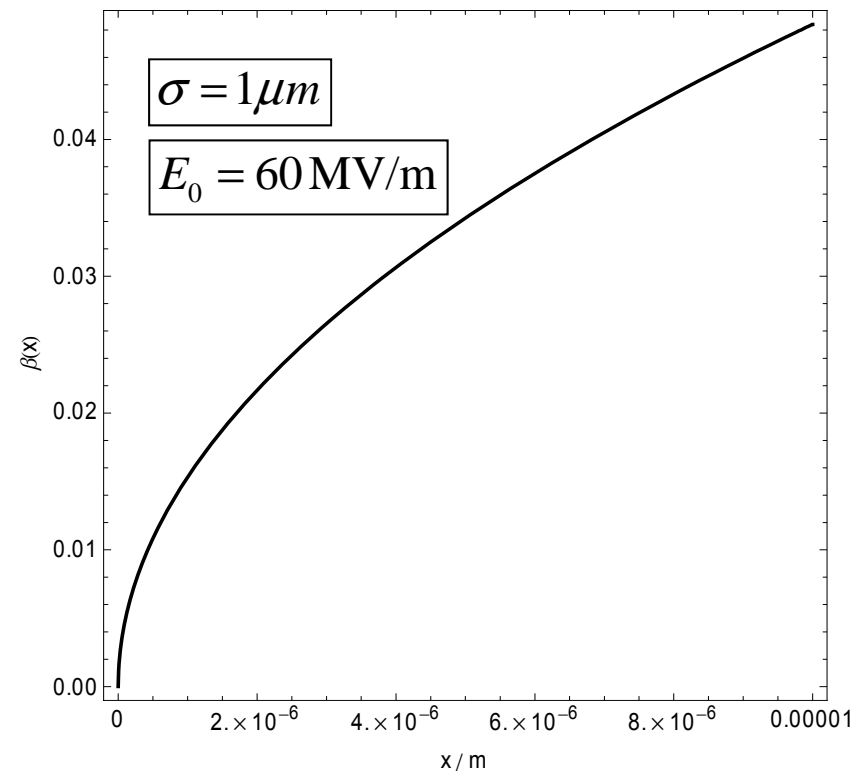
# Emittance studies for the PITZ injector

## Discussion on space charge at the cathode

Wall force modified by cloud charge



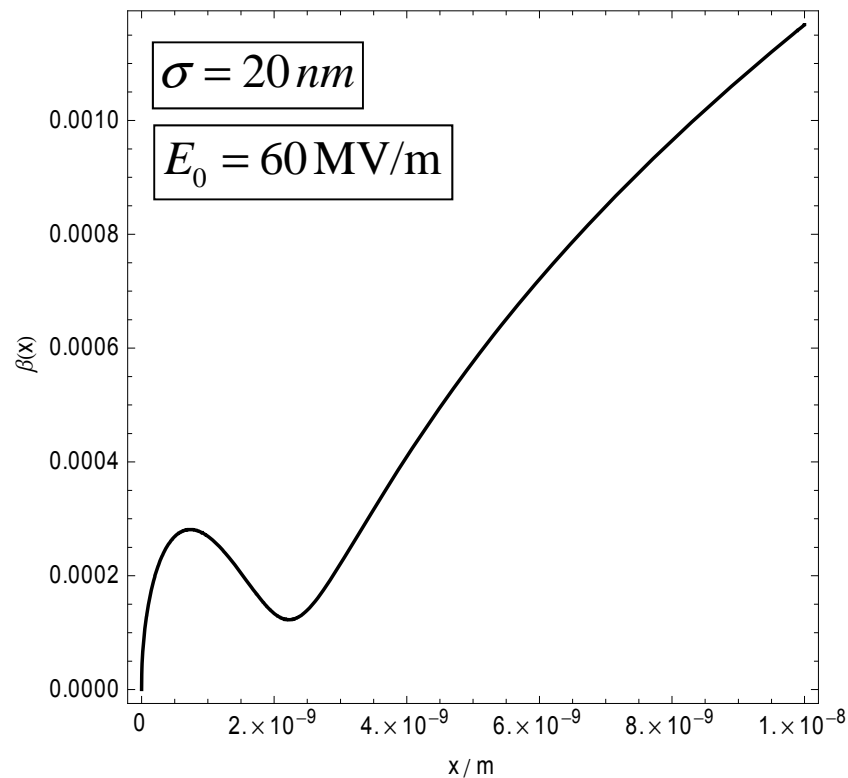
“Usual” trajectory: small wall interaction



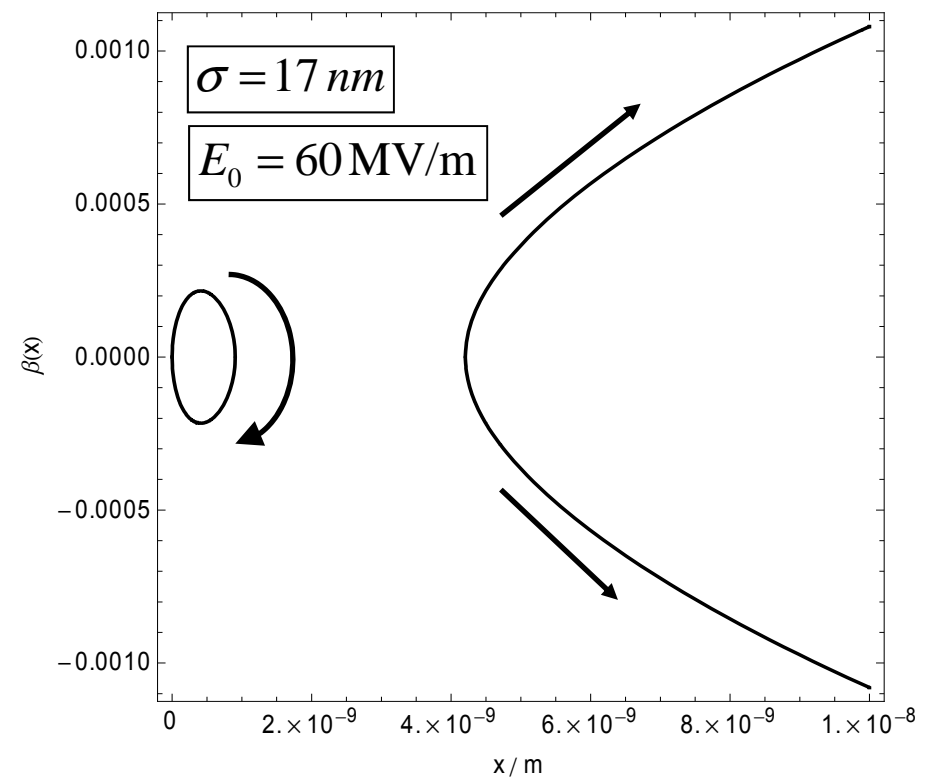
# Emittance studies for the PITZ injector

## Discussion on space charge at the cathode

Small size particle trajectory



SPCH limited trajectory

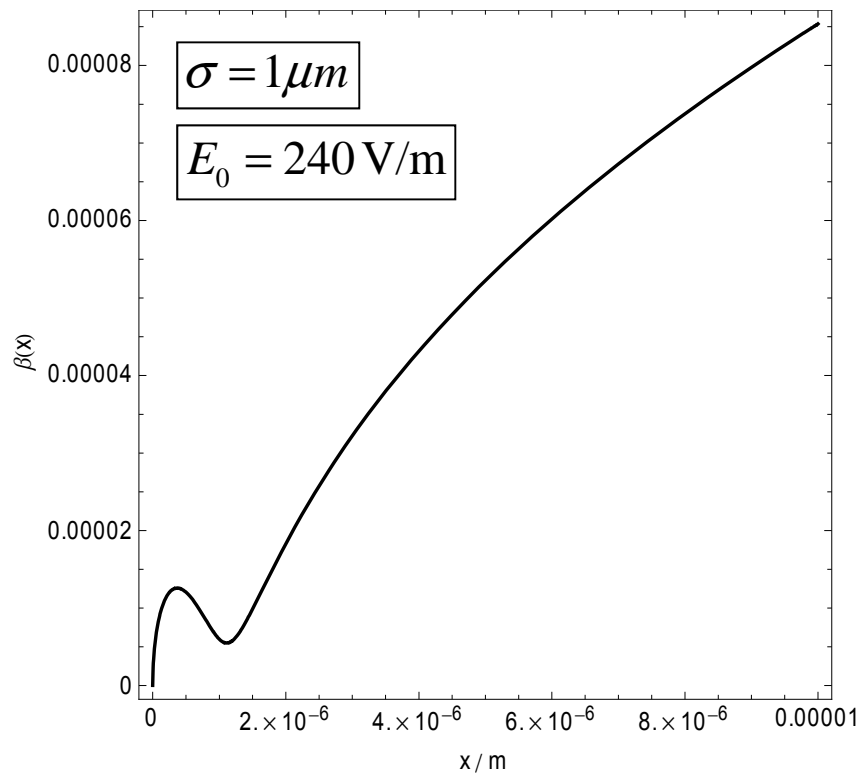




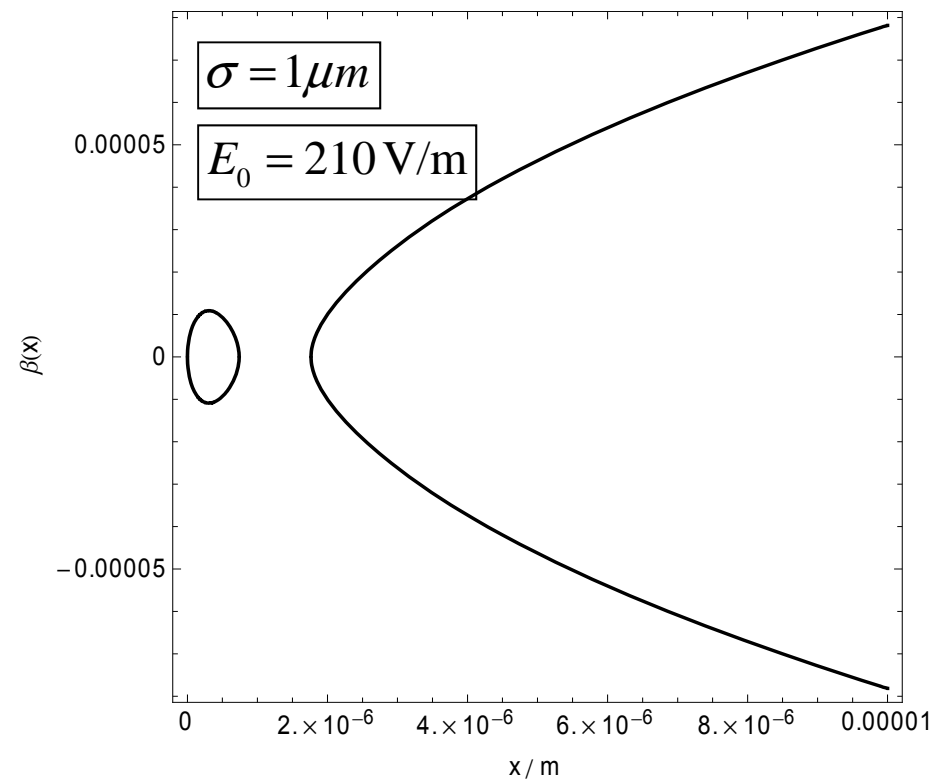
# Emittance studies for the PITZ injector

## Discussion on space charge at the cathode

Low field / large size particle trajectory



SPCH limited trajectory



# Emittance studies for the PITZ injector

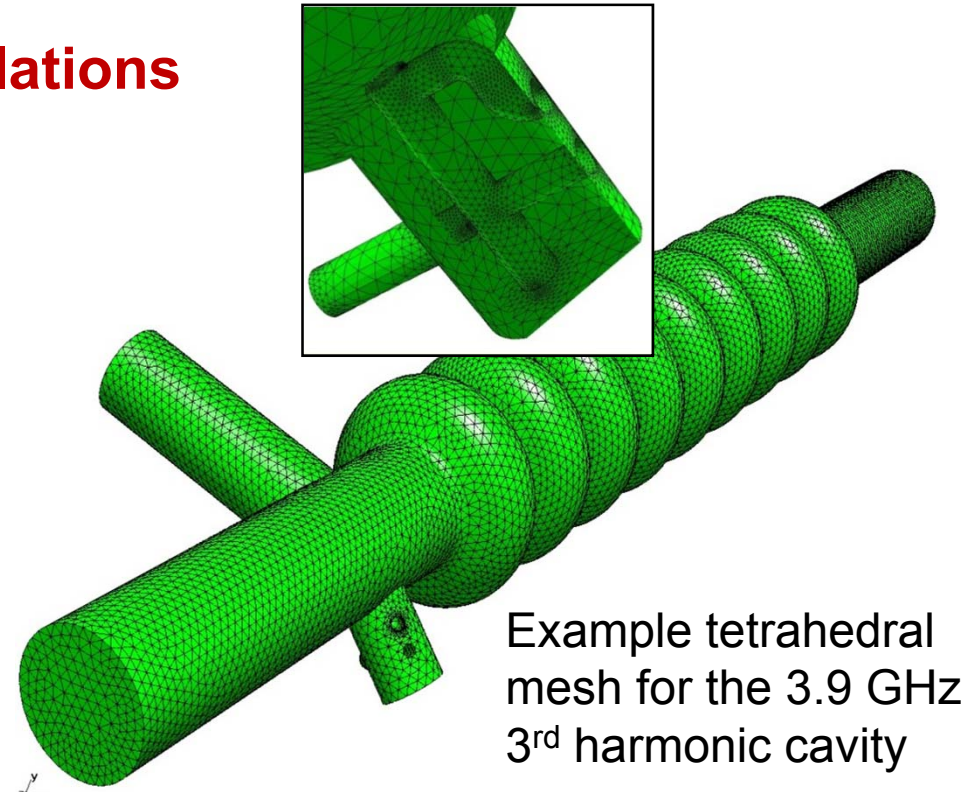
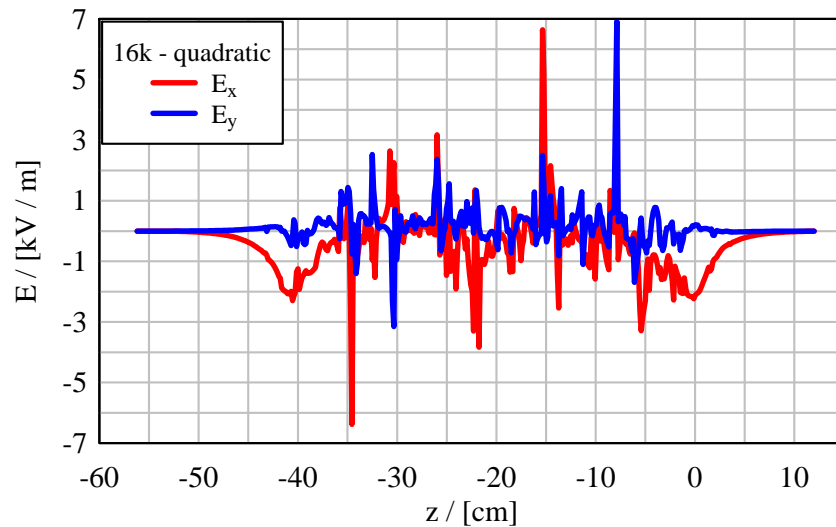
## Discussion on space charge at the cathode

- Minimum separation distance between particle and its mirror:
  - LW:  $\sim$ time step
  - PIC: macroparticle shape
  - ASTRA: longitudinal grid step (?)
- Needs to be reduced at high charge densities:  $\sigma < \lambda_D$
- but end up with individual particle interactions
  - No numerical convergence
- **Only cure, increase number of particles in the simulation**
  - Full charge extraction at 0.3 mm possible with ASTRA (?)

# Field map calculations

## Field noise in eigenmode simulations

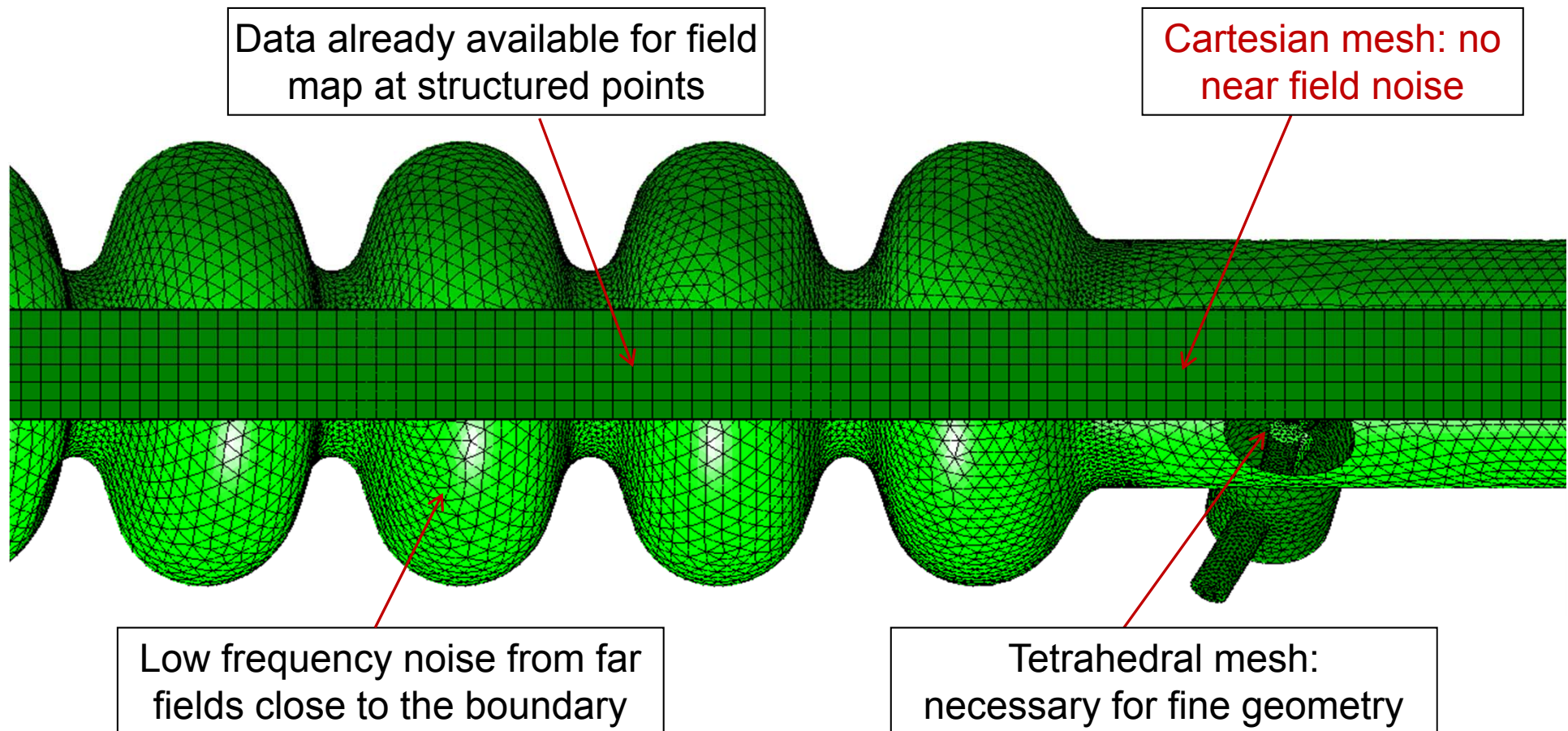
Typical field noise on-axis for computations with unstructured grids



- Need huge amount of DoFs to compensate for mesh asymmetry
- Special treatment for the coupler kicks (PAC 2009, DESY / TEMF 2010)
- **So far completely noise-free field maps only possible with Cartesian grids**

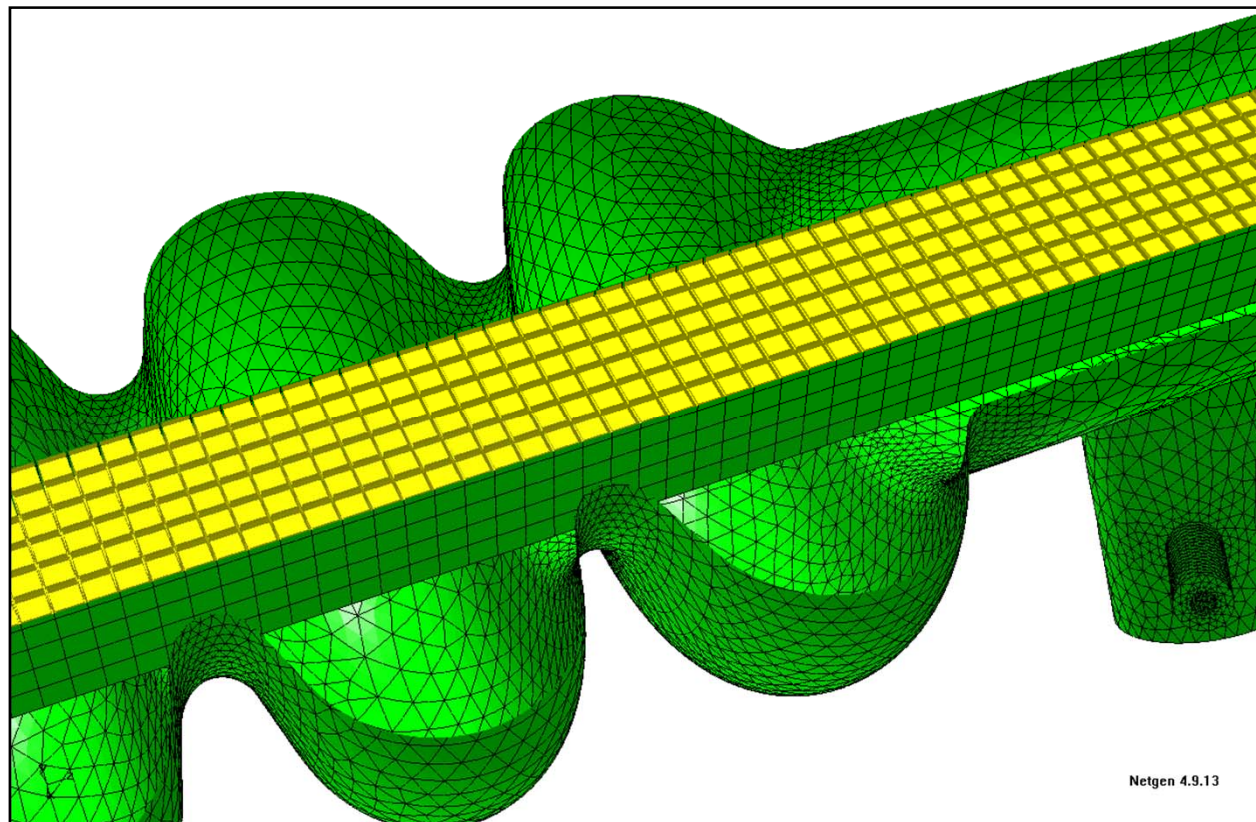
# Field map calculations

## Mixed mesh – high order FEM approach



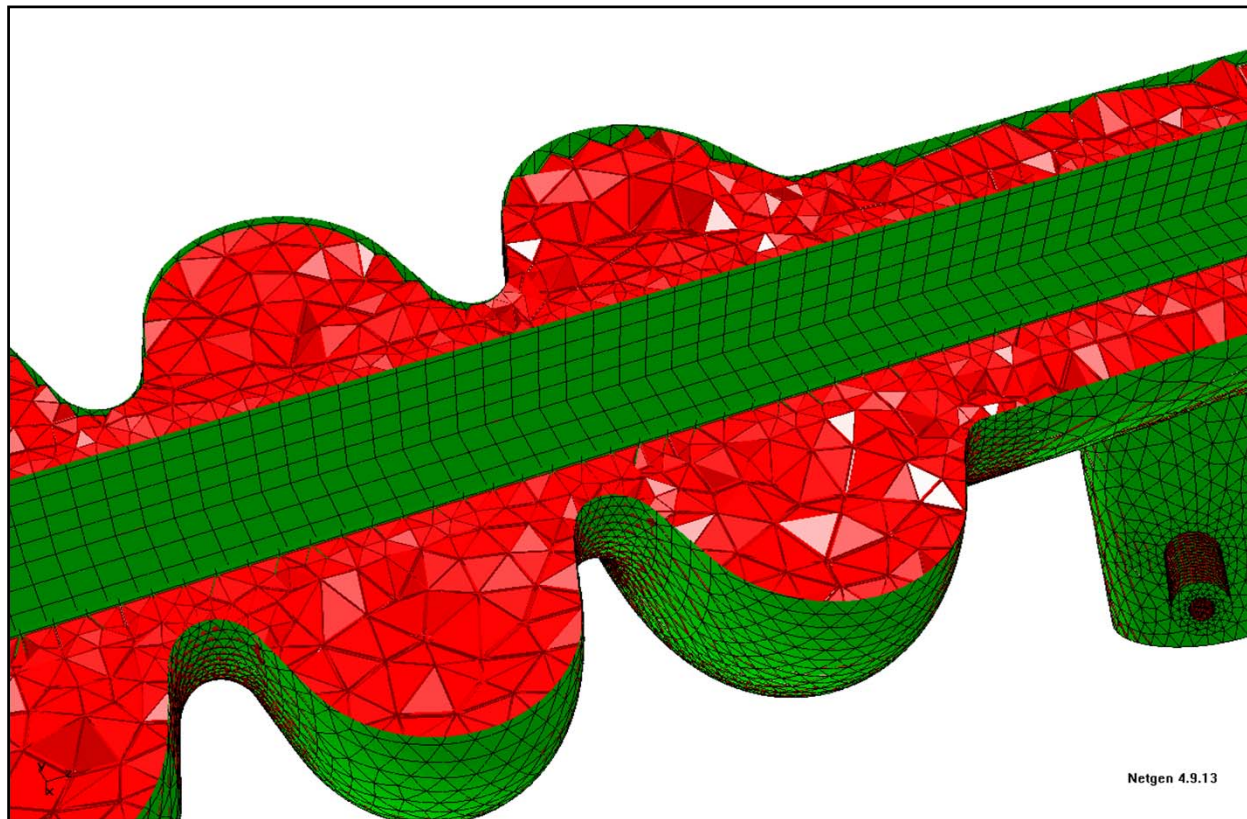
# Field map calculations

## Mixed mesh – high order FEM approach



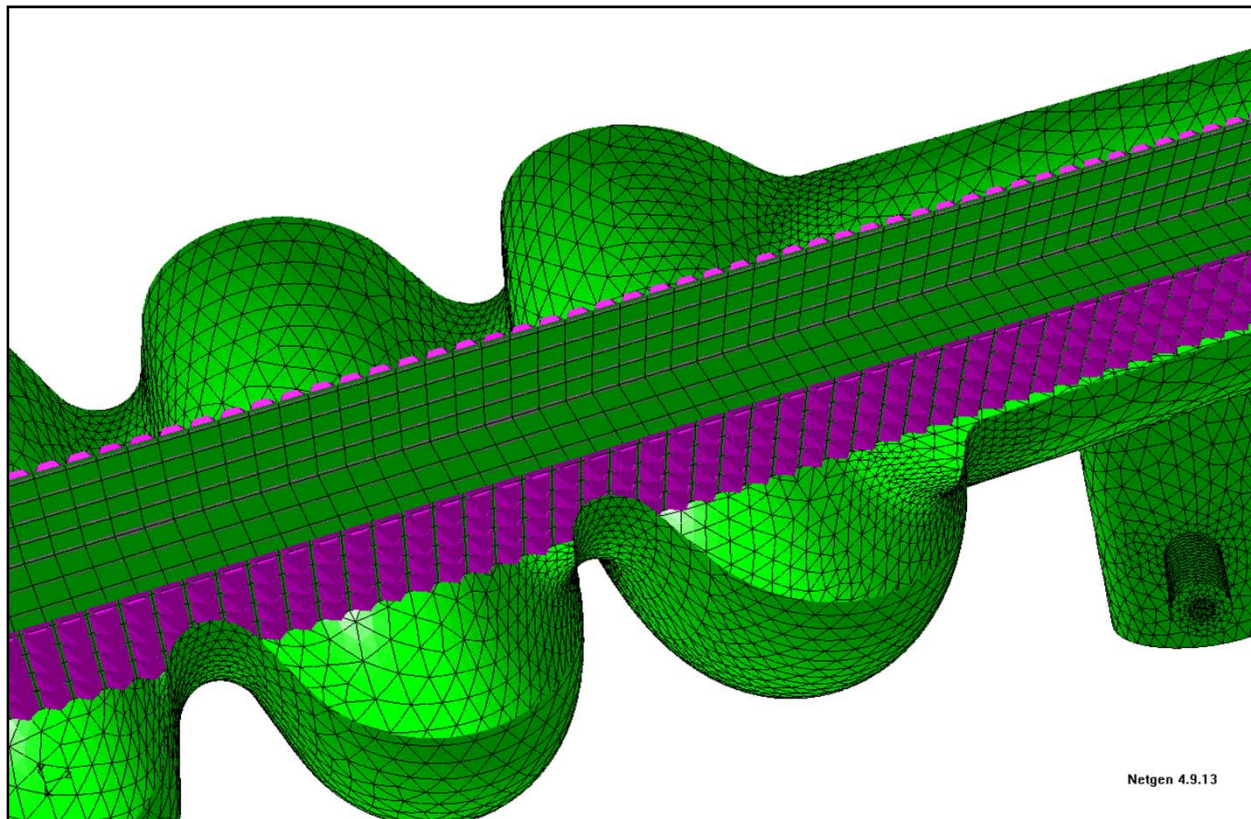
# Field map calculations

## Mixed mesh – high order FEM approach



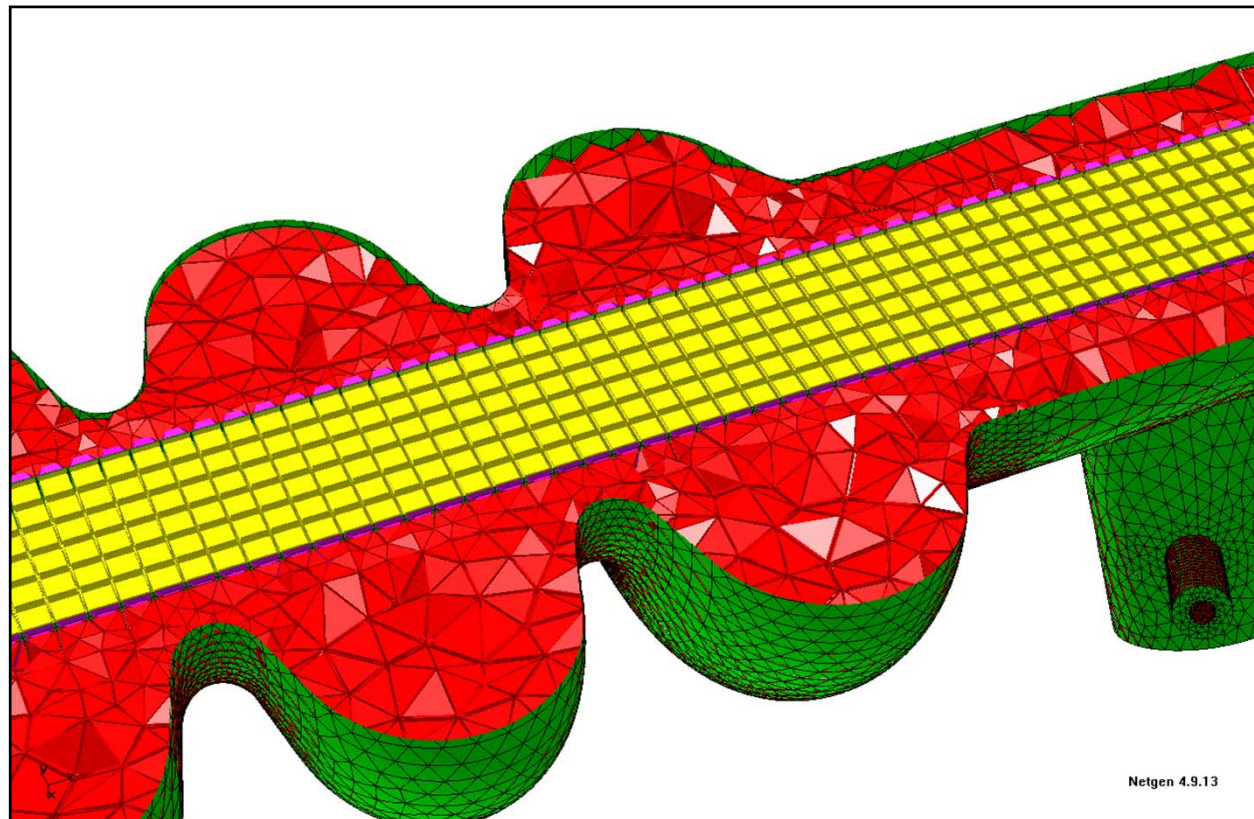
# Field map calculations

## Mixed mesh – high order FEM approach



# Field map calculations

## Mixed mesh – high order FEM approach



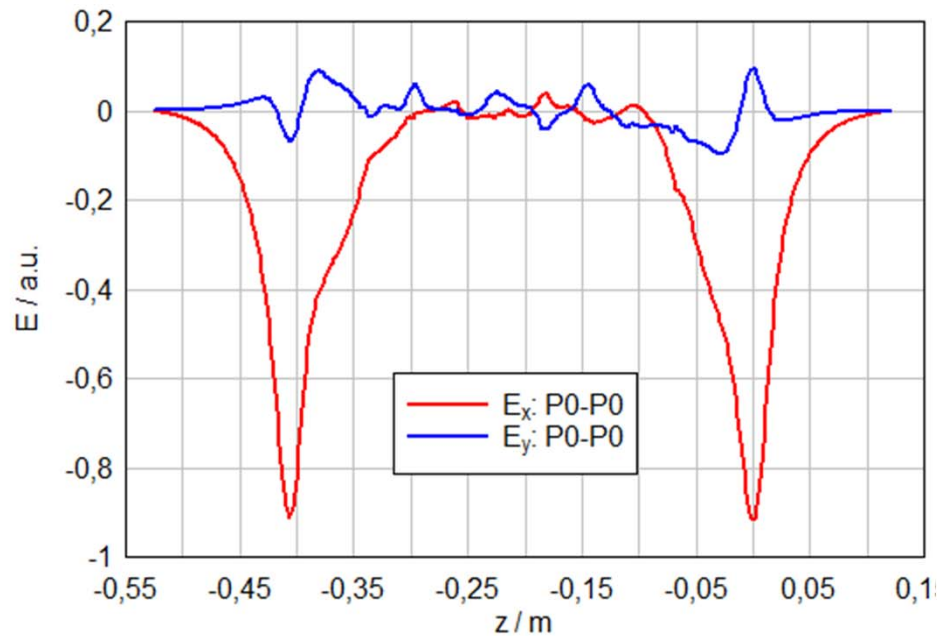


# Field map calculations

## Field quality study for the 3.9 GHz cavity

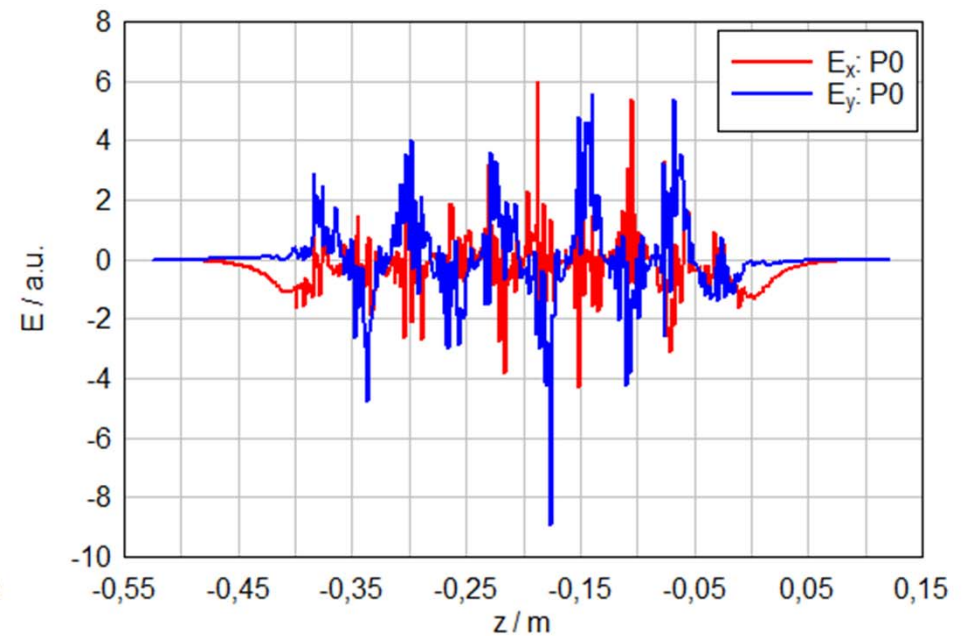
Fields on axis:

Mixed mesh with 350K elements



Fields on axis:

Tetrahedral mesh with 450K elements

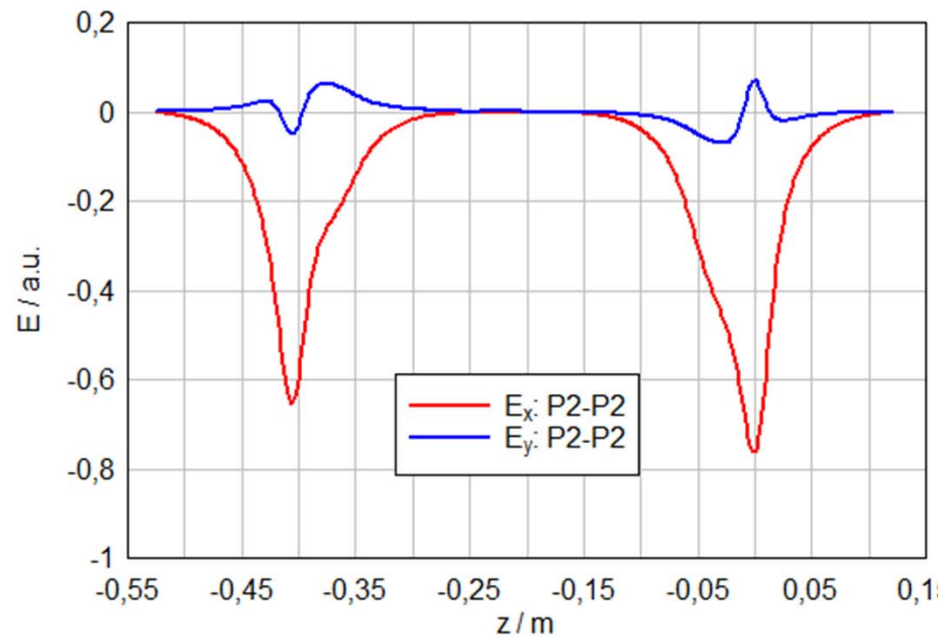


# Field map calculations

## Field quality study for the 3.9 GHz cavity

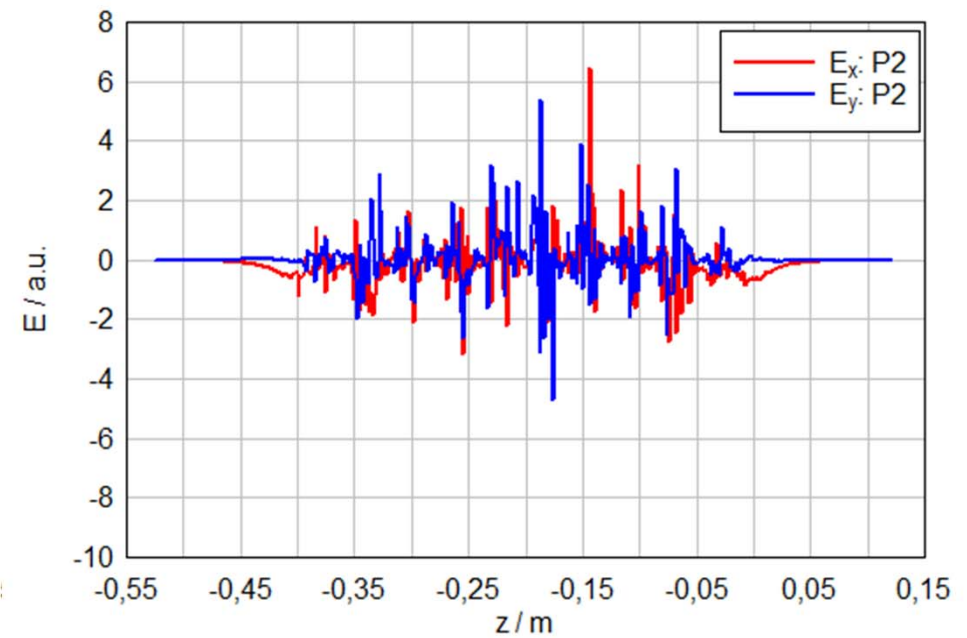
Fields on axis:

Mixed mesh with 350K elements



Fields on axis:

Tetrahedral mesh with 450K elements

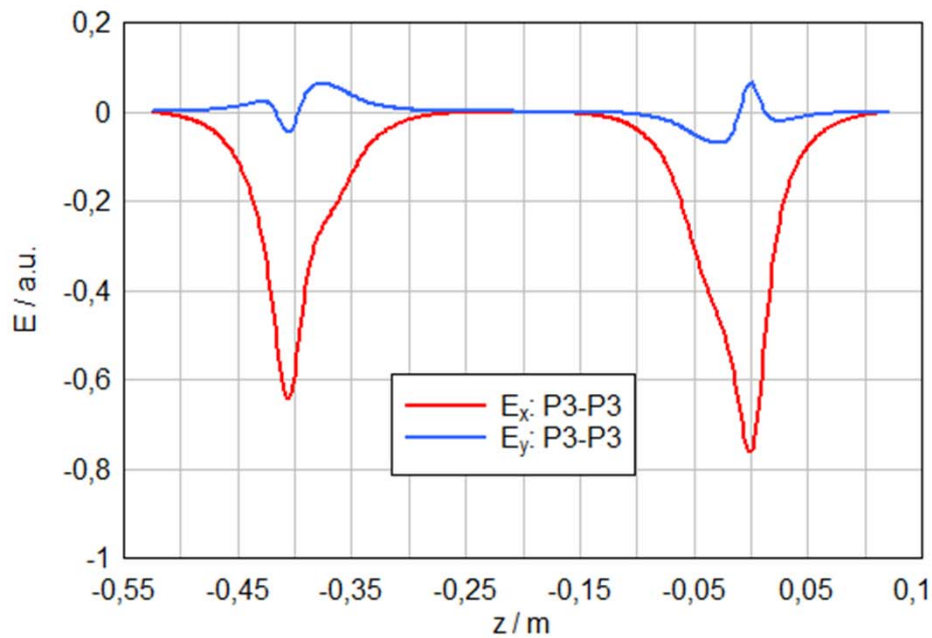


# Field map calculations

## Field quality study for the 3.9 GHz cavity

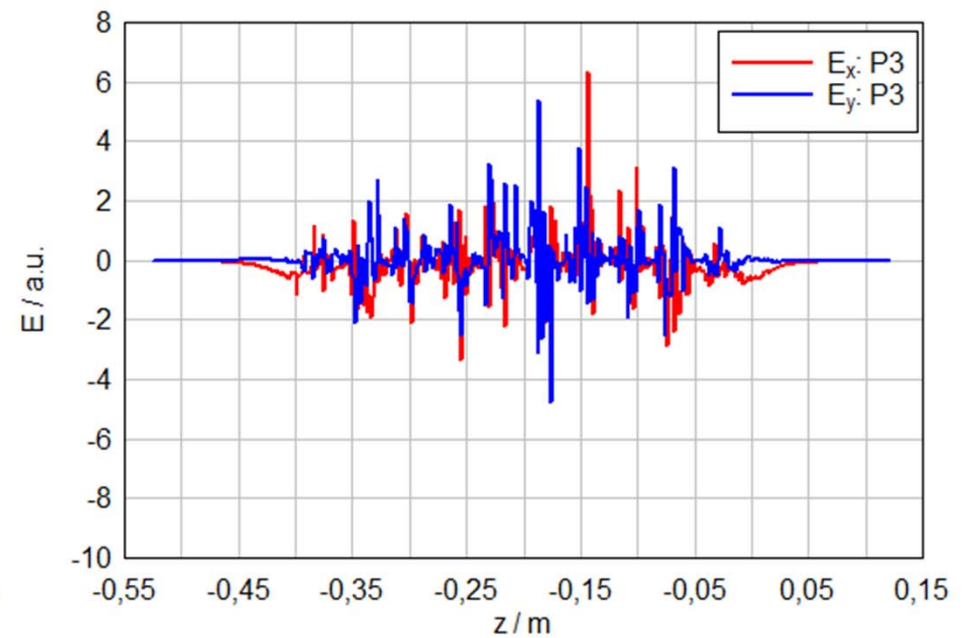
Fields on axis:

Mixed mesh with 350K elements



Fields on axis:

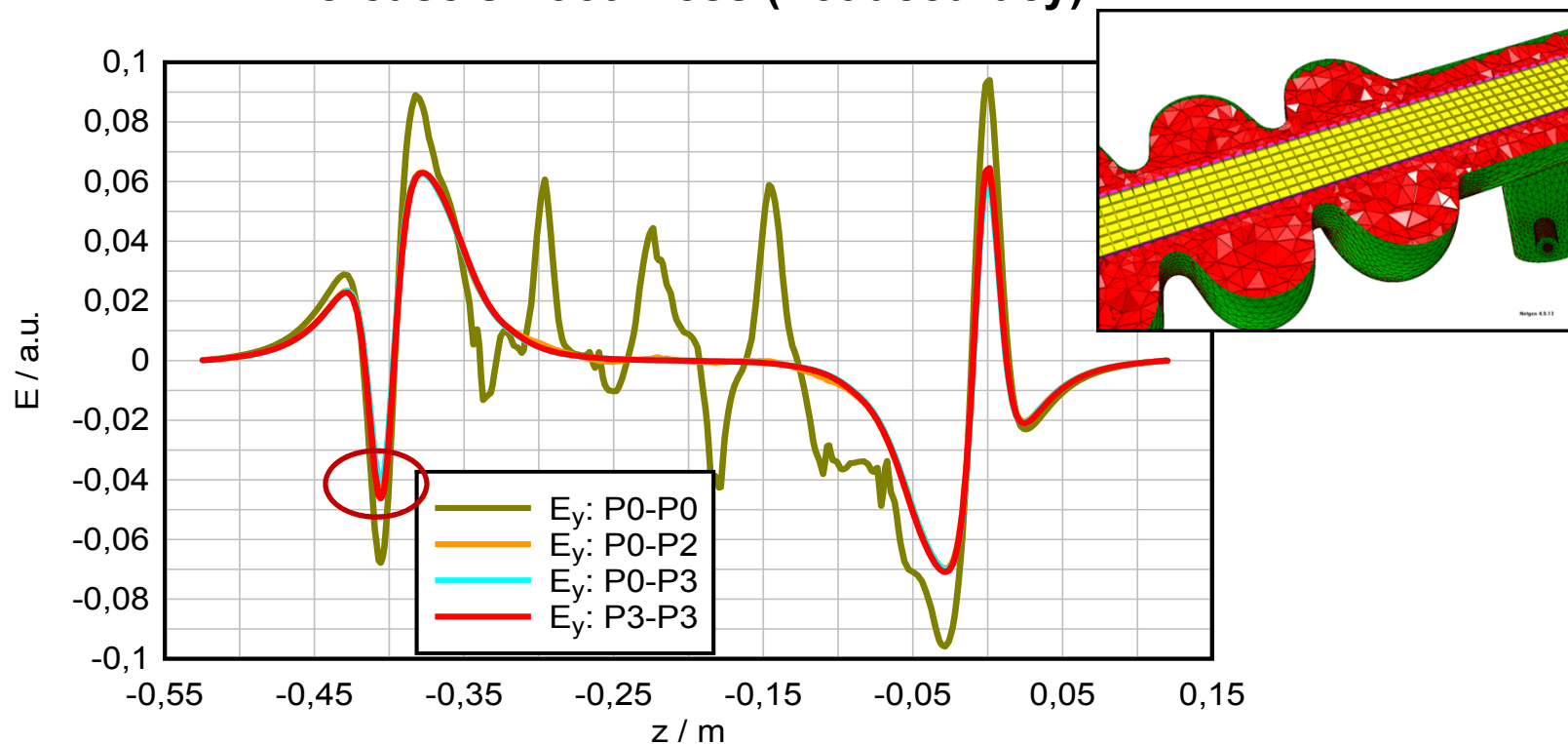
Tetrahedral mesh with 450K elements



# Field map calculations

## Field quality study for the 3.9 GHz cavity

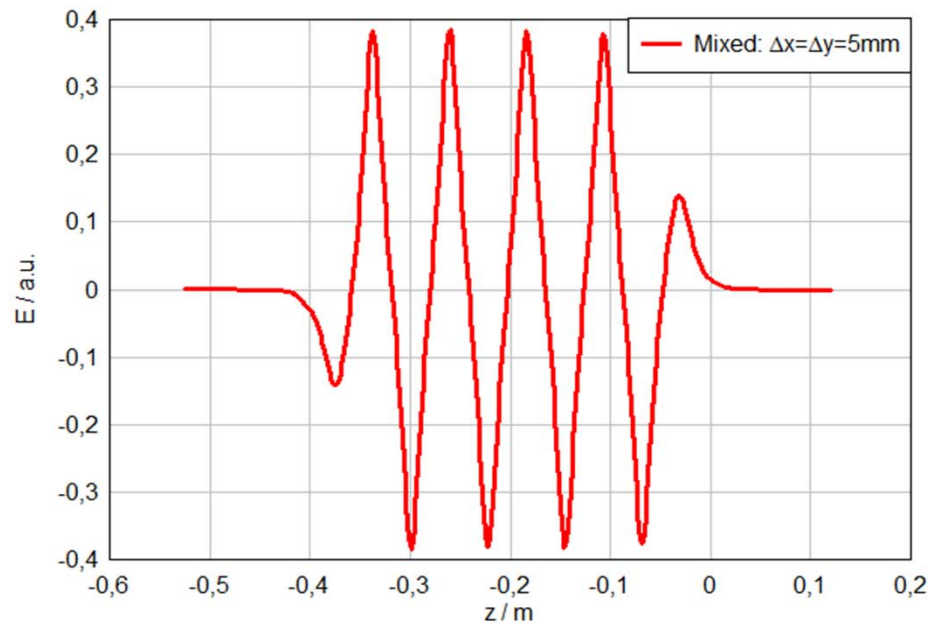
Use of different orders in different parts of the mesh to  
**increase smoothness (not accuracy)**



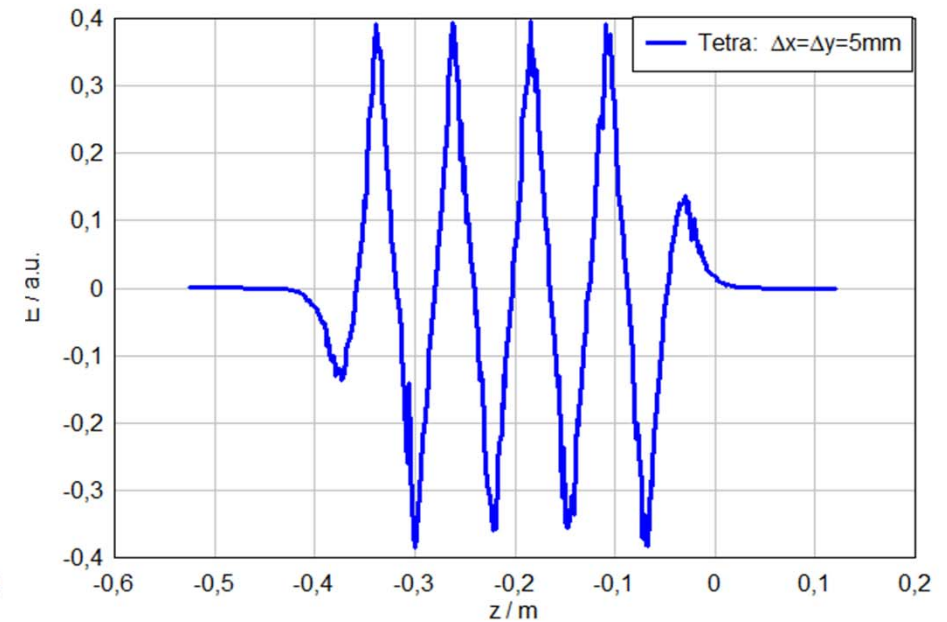
# Field map calculations

## Field quality study for the 3.9 GHz cavity

Transversal fields off axis:  
Mixed mesh with 350K elements



Transversal fields off axis:  
Tetrahedral mesh with 450K elements



Thank you for your  
attention