Moduli Stabilization in Type IIB Orientifolds

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Based on Work:


Outline

1. Type $IIB$ Calabi–Yau orientifolds with $D3/D7$–branes

2. Stability of moduli stabilization in KKLT scenarios

3. Resolved toroidal orbifolds and their orientifolds

4. Fixing all moduli in resolved toroidal orbifolds

5. Concluding remarks and open questions
**Type IIB Orientifolds with D3/D7–Branes**

- Consider Type IIB Compactification on Calabi–Yau Manifold $Y_6 \implies N=2, D=4$

- Include orientifold action $\mathcal{O} = (-1)^{F_L} \Omega \, \sigma$ allowing for $O3/O7$–planes (Type IIB on CY orientifold $X_6$) $\implies N=1, D=4$

- Add set of $D3/D7$–branes to cancel tadpoles

Here:

\[
\begin{align*}
\sigma \, \Omega &= -\Omega \\
\sigma \, J &= J
\end{align*}
\]
Moduli of Calabi–Yau Orientifold $X_6$

$\sigma$ acts holomorphically $\implies$ split of cohomology of $X_6$

\[ H^{(p,q)}(X_6) = H_{+}^{(p,q)}(X_6) \oplus H_{-}^{(p,q)}(X_6) \]

\[ J = \sum_{k=1}^{h_{(1,1)}^{(+)}(X_6)} t^k \omega_k \quad , \quad \Omega = \sum_{\lambda=0}^{h_{(2,1)}^{-}(X_6)} X^\lambda \alpha_\lambda - F_\lambda \beta^\lambda \]

\[ B_2 = \sum_{a=1}^{h_{(1,1)}^{-}(X_6)} b^a \omega_a \quad , \quad C_2 = \sum_{a=1}^{h_{(1,1)}^{-}(X_6)} c^a \omega_a \]

\[ \text{Vol}(X_6) = \frac{1}{6} \int_{X_6} J \wedge J \wedge J = \frac{1}{6} \sum_{i,j,k=1}^{h_{(1,1)}^{(+)}(X_6)} K_{ijkl} t^i t^j t^k \]
### Moduli of Calabi–Yau Orientifold $X_6$

<table>
<thead>
<tr>
<th></th>
<th>CS moduli</th>
<th>Kähler moduli</th>
<th>add. moduli</th>
<th>chiral multiplet</th>
<th>$\int_{X_6} \Omega \wedge G_3$</th>
<th>TV/GVW</th>
</tr>
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<tbody>
<tr>
<td>$h_{(2,1)}^(-)(X_6)$</td>
<td>dilaton</td>
<td>$S$</td>
<td>$u^\lambda$</td>
<td>chiral multiplets</td>
<td>$\int_{X_6} \Omega \wedge G_3$</td>
<td>TV/GVW</td>
</tr>
<tr>
<td>$h_{(1,1)}^+(X_6)$</td>
<td>$t^k$</td>
<td>$b^a, c^a$</td>
<td>chiral/linear multiplets</td>
<td>$e^{-T}$</td>
<td>$\int_{C_4} J \wedge B_2$</td>
<td>KKLT</td>
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<tr>
<td>$h_{(1,1)}^-(X_6)$</td>
<td>add. vectors</td>
<td>$V_{\mu j}^j$</td>
<td>vector multiplets</td>
<td>-</td>
<td>-</td>
<td>Calibration</td>
</tr>
</tbody>
</table>

In fact (Grimm, Louis 0403067):

$$T^j = \frac{3}{4} K_{jkl} t^k t^l + \frac{3}{8} e^{\phi_{10}} K_{jbc} G^b (G + \overline{G})^c + \frac{3}{2} i C^j \quad , \quad j = 1, \ldots, h_{(1,1)}^+(X_6)$$

$$G^a = i c^a + S b^a \quad , \quad a = 1, \ldots, h_{(1,1)}^-(X_6)$$
Low–energy effective action

Lowest $\alpha'$–expansion at tree–level (supergravity approximation):

Full Kähler potential:

$$K = -\ln(S + \overline{S}) - \ln \int_{X_6} \Omega \wedge \Omega - 2 \ln \frac{1}{6} \int_{X_6} J \wedge J \wedge J$$

Gauge kinetic function ($D7$–brane wrapped on 4–cycle $C^j$):

$$f_j = T^j$$

Superpotential:

$$W = W_0(S, U) + \sum_{j=1}^{h_{(1,1)}^{(+)}(X_6)} \gamma_j(S, U) e^{a_j} T^j$$

*tree-level flux* *non-perturbative (KKLT)*
**KKLT setup**

Instanton effects yield $T$–dependence of $W \sim \gamma_j(S,U) \ e^{a_j} T^j$

- $D3$–brane wrapped on $4$–cycle $C_4$: $T \simeq$ volume of $C_4$, instanlon effect $\sim e^{-2\pi T} \Rightarrow a_j = -2\pi$

- Gauge theory on $D7$–brane: gauge coupling $g_a^{-2} \sim \text{Re}(T)$, gaugino condensation $\sim e^{-T/b_a} \ b_{SU(M)} = \frac{M}{2\pi} \Rightarrow a_j = -\frac{2\pi}{M}$

On $D7$ $\gamma_j(S,U)$ may comprise one–loop effects and further instanton effects (from $D(-1)$–branes):

- one–loop corrections to the gauge coupling $\gamma_j \sim \eta(U)^{-2/b_a}$

- additional instantons in the $D7$–gauge theory $\gamma_j \sim e^{-S/b_a \int_{C_4} F \wedge F}$
**Moduli stabilization**

Supersymmetric $AdS$ vacuum solutions:

\[
F^M = 0, \quad M = S, U^\lambda, T^j
\]

give extremal points $M$ in the scalar potential $V(M, \overline{M})$

- In $AdS$ vacua stability is already guaranteed, if all scalar masses fulfill the *Breitenlohner–Freedman bound*

- At *any* $N=1$ supersymmetric $AdS$ minimum in supergravity theories *all* scalar masses are *above* this bound

However we need positive definite mass matrix $\mathcal{M}$ to obtain a stable uplift to $dS$, i.e.:

\[
\mathcal{M} > 0
\]
On the stability of moduli stabilization in KKLT scenarios

Is $\mathcal{M} > 0$ always possible? No!

Consider the case $h_{(2,1)}(X_6) = 0$, $n := h_{(1,1)}(X_6) \geq 1$

$$W(S, T^j) = B + A S + \sum_{i=1}^{h_{(1,1)}(X_6)} \gamma_i e^{a_i T_i}, \quad a_i \in \mathbb{R}^-, \quad \gamma_i \in \mathbb{C}$$

with a general Kähler potential: $K = -\ln(S + \overline{S}) - K(T^1 + \overline{T}^1, \ldots, T^n + \overline{T}^n)$

Task: Compute scalar potential $V$ and scalar masses $\mathcal{M}$ and check positivity of the scalar mass matrix $\mathcal{M}$
On the stability of moduli stabilization in KKLT scenarios

In fact $\mathcal{M}$ decomposes into $\mathcal{M} = \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix}$ w.r.t. $s = s_1 + is_2$ and $t^j = t^j_1 + it^j_2$

**Matrix Analysis:** Analyze principal submatrices and minors of $\mathcal{M}$

**Result:** Mass matrix $\mathcal{M}$ fails to be positive definite, if:

$$\forall i=1,\ldots,n \quad K_{Ti} < 0$$

Go back to original CY moduli $t^j$ (volumina of 2–cycles, $J = \sum_{j=1}^{n} t^j \omega_j$):

$$\forall i=1,\ldots,n \quad t^i > 0$$

We also find:

$$\exists i \in \{1,\ldots,n\} \quad t^i > 0 \quad \land \quad \mathcal{K}_{ii} \geq 0$$
On the stability of moduli stabilization in KKLT scenarios

Examples: (i) $K = - \ln(T^1 + \overline{T}^1) - \ln(T^2 + \overline{T}^2) - \ln(T^3 + \overline{T}^3)$ has $K_T^1 < 0$, $K_T^2 < 0$, $K_T^3 < 0$ \(\implies\) no stable uplift possible.

(ii) $K = 2 \ln \left[ (T^2 + \overline{T}^2)^{3/2} - (T^1 + \overline{T}^1)^{3/2} \right]$ has $t^2 > 0$, $\mathcal{K}_{22} = \frac{32}{3} t^2 > 0$ \(\implies\) no stable uplift possible.

This result generalizes work of Choi, Falkowski, Nilles, Olechowski, Pokorski for $n = 1$.

Moreover: No stable uplift possible for toroidal orbifolds without complex structure moduli, i.e. $h_{(2,1)}^- (X_6) = 0$! This concerns: $\mathbb{Z}_3, \mathbb{Z}_7, \mathbb{Z}_3 \times \mathbb{Z}_3, \mathbb{Z}_4 \times \mathbb{Z}_4, \mathbb{Z}_6 \times \mathbb{Z}_6, \mathbb{Z}_2 \times \mathbb{Z}_6'$ – both at the orbifold point and away from it.
Stabilization of Kaehler moduli from $H^{(1,1)}_-(X_6)$

Recall: We have $h^{(-)}_{(1,1)}(X_6)$ moduli $G^a = i \ c^a + S \ b^a$, but $\partial G^a W = 0$. In the covering space $Y_6$ of the orientifold theory $X_6$ some divisors $E$ (or divisor orbits under the orbifold group) are not invariant under the orientifold action $\sigma$: 

$$\sigma \ E = \tilde{E}$$

Construct invariant combinations:

$$E_i = \frac{1}{2} \ (E + \tilde{E}), \quad E_a = \frac{1}{2} \ (E - \tilde{E})$$

such that $\sigma \ E_i = E_i$ and $\sigma \ E_a = -E_a$, i.e. $\omega_i \in H_{+}^{(1,1)}(X_6)$ and $\omega_a \in H_{-}^{(1,1)}(X_6)$.

Consider $D3/D7$ wrapped around 4–cycle $C_i \simeq E_i$:

There is a calibration condition for $C_i$: $B_2 \wedge J = 0$

Marino, Minasian, Moore, Strominger
Stabilization of Kaehler moduli from $H_{-1,1}^{(1,1)}(X_6)$

With $B_2 = \sum_{\tilde{a}=1}^{h_{-1,1}^{(1,1)}(X_6)} b^{\tilde{a}} \omega_{\tilde{a}}$, integrate over odd 4–cycle $C_a$:

$$\int_{C_a} B_2 \wedge J = \mathcal{K}_{a\tilde{a}} b^{\tilde{a}} = 0 \quad \implies \quad \mathcal{K}_{a\tilde{a}} b^{\tilde{a}} = 0$$

[ $h_{-1,1}^{(1,1)}(X_6)$ equations in $h_{-1,1}^{(1,1)}(X_6)$ variables $b^a$ ]

Intersection form $\mathcal{K}_{a\tilde{a}}$ has generically $\det(\mathcal{K}_{a\tilde{a}}) \neq 0 \implies \quad b^a = 0$

In addition, this solution fulfills $F^{G^a} = 0$ and $F$–term equations for $F^S$, $F^{T^j}$, $F^{U^\lambda}$ decouple!
Stabilization of Kaehler moduli from \( H_{(1,1)}^{(1,1)}(X_6) \)

How to stabilize \( c^a \) of \( C_2 = \sum_{\tilde{a}=1}^{h_{(1,1)}(X_6)} c^{\tilde{a}} \omega_{\tilde{a}} \)?

Consider \( U(1) \) gauge factor \( A_\mu \) of \( D7 \) brane wrapping 4–cycle \( C_i \)

Then:

\[
D_\mu G^a = \partial_\mu G^a - 4i \kappa_4^2 \mu_7 (2\pi \alpha') A_\mu ,
\]

\[
D_\mu T^i = \partial_\mu T^i , \quad \text{(Jockers, Louis)}
\]

\( \implies \) massive \( U(1) \) vector \( A_\mu \) fixing \( \text{Im}(G^a) \) (Stückelberg mass term)

Summarize:

(i) Calibration condition fixes \( b^a \)

(ii) Gauging of \( G^a \) fixes \( c^a \)
Resolved toroidal $\mathbb{Z}_N$ Orbifolds

We discuss orientifolds $X_6$ of the resolved toroidal orbifolds $Y_6 = \frac{T^6}{\mathbb{Z}_N}$, $Y_6 = \frac{T^6}{\mathbb{Z}_N \times \mathbb{Z}_M}$.

$\mathbb{Z}_N$ acts crystalliographically on $T^6$: $\theta : (z^1, z^2, z^3) \longrightarrow (e^{2\pi i v^1} z^1, e^{2\pi i v^2} z^2, e^{2\pi i v^3} z^3)$, with $\pm v^1 \pm v^2 \pm v^2 = 0$ for $SU(3)$ holonomy ($\theta \Omega = \Omega$).

$\Longrightarrow$ twist $\theta$ produces conical singularities,

$\Longrightarrow$ resolve singularities and introduce orientifold action $\Longrightarrow X_6$

<table>
<thead>
<tr>
<th>Point group $\theta$</th>
<th>$v^i = \frac{1}{N}(n_1, n_2, n_3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathbb{Z}_3$</td>
<td>$\frac{1}{3}(1, 1, -2)$</td>
</tr>
<tr>
<td>$\mathbb{Z}_4$</td>
<td>$\frac{1}{4}(1, 1, -2)$</td>
</tr>
<tr>
<td>$\mathbb{Z}_6-I$</td>
<td>$\frac{1}{6}(1, 1, -2)$</td>
</tr>
<tr>
<td>$\mathbb{Z}_6-II$</td>
<td>$\frac{1}{6}(1, 2, -3)$</td>
</tr>
<tr>
<td>$\mathbb{Z}_7$</td>
<td>$\frac{1}{7}(1, 2, -3)$</td>
</tr>
<tr>
<td>$\mathbb{Z}_8-I$</td>
<td>$\frac{1}{8}(1, 2, -3)$</td>
</tr>
<tr>
<td>$\mathbb{Z}_8-II$</td>
<td>$\frac{1}{8}(1, 3, -4)$</td>
</tr>
<tr>
<td>$\mathbb{Z}_{12-I}$</td>
<td>$\frac{1}{12}(1, 4, -5)$</td>
</tr>
<tr>
<td>$\mathbb{Z}_{12-II}$</td>
<td>$\frac{1}{12}(1, 5, -6)$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Point group $\theta \times \omega$</th>
<th>$v^i = \frac{1}{N}(n_1, n_2, n_3)$</th>
<th>$w^i = \frac{1}{M}(m_1, m_2, m_3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathbb{Z}_2 \times \mathbb{Z}_2$</td>
<td>$\frac{1}{2}(1, 0, -1)$</td>
<td>$\frac{1}{2}(0, 1, -1)$</td>
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<tr>
<td>$\mathbb{Z}_2 \times \mathbb{Z}_4$</td>
<td>$\frac{1}{4}(1, 0, -1)$</td>
<td>$\frac{1}{4}(0, 1, -1)$</td>
</tr>
<tr>
<td>$\mathbb{Z}_2 \times \mathbb{Z}_6$</td>
<td>$\frac{1}{3}(1, 0, -1)$</td>
<td>$\frac{1}{6}(0, 1, -1)$</td>
</tr>
<tr>
<td>$\mathbb{Z}_2 \times \mathbb{Z}_6'$</td>
<td>$\frac{1}{2}(1, 0, -1)$</td>
<td>$\frac{1}{2}(0, 1, -1)$</td>
</tr>
<tr>
<td>$\mathbb{Z}_3 \times \mathbb{Z}_3$</td>
<td>$\frac{1}{3}(1, 0, -1)$</td>
<td>$\frac{1}{3}(0, 1, -1)$</td>
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<td>$\mathbb{Z}_3 \times \mathbb{Z}_6$</td>
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<td>$\mathbb{Z}_3 \times \mathbb{Z}_6'$</td>
<td>$\frac{1}{3}(1, 0, -1)$</td>
<td>$\frac{1}{3}(0, 1, -1)$</td>
</tr>
<tr>
<td>$\mathbb{Z}_4 \times \mathbb{Z}_4$</td>
<td>$\frac{1}{4}(1, 0, -1)$</td>
<td>$\frac{1}{4}(0, 1, -1)$</td>
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<tr>
<td>$\mathbb{Z}_4 \times \mathbb{Z}_4'$</td>
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<td>$\frac{1}{4}(0, 1, -1)$</td>
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<tr>
<td>$\mathbb{Z}_6 \times \mathbb{Z}_6$</td>
<td>$\frac{1}{6}(1, 0, -1)$</td>
<td>$\frac{1}{6}(0, 1, -1)$</td>
</tr>
</tbody>
</table>
Resolved toroidal $\mathbb{Z}_N \times \mathbb{Z}_M$ Orbifolds

Orbifold group $\Gamma \simeq \mathbb{Z}_N$ and twist $\theta^n \in \Gamma$

Fixpoint (singularity) $f^{(n)}$ under $\theta^n$, $n = 1, \ldots, N - 1$:

$$\theta^n \cdot f^{(n)} = f^{(n)} + \lambda, \quad \lambda \in \Lambda$$

In a small neighbourhood around singularity space looks like:

- $\mathbb{C}^3/\Gamma$, isolated singularity
- $\mathbb{C}^2/\Gamma^{(2)} \times \mathbb{C}$, non-isolated singularity

- locally resolve the singularities with the methods of toric geometry
- put together local patches according to the fixed set configuration
Example: $T^6/Z_{6-II}$ on $SU(2) \times SU(6)$

<table>
<thead>
<tr>
<th>Group elements</th>
<th>Order</th>
<th>Fixed Set</th>
<th>Conj. Classes</th>
<th>Divisor</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathbb{Z}_6 \theta$</td>
<td>6</td>
<td>12 fixed points</td>
<td>12</td>
<td>$E_1, \beta \gamma$</td>
</tr>
<tr>
<td>$\mathbb{Z}_3 \theta^2$</td>
<td>3</td>
<td>3 fixed lines</td>
<td>3</td>
<td>$E_2, \beta, E_4, \beta$</td>
</tr>
<tr>
<td>$\mathbb{Z}_2 \theta^3$</td>
<td>2</td>
<td>4 fixed lines</td>
<td>4</td>
<td>$E_3, \gamma$</td>
</tr>
</tbody>
</table>

$\beta = 1, 2, 3$, $\gamma = 1, \ldots, 4$

$\implies$ In total $h^{\text{twisted}}_{(1,1)} = 12 \cdot 1 + 3 \cdot 2 + 4 \cdot 1 = 22$ exceptional divisors.
Intersection numbers (volume form): Example $T^6/Z_{6-II}$

After putting together all local patches (invoking schematic picture of fixed set orbits) we arrive at a full basis of $H^{(2)}(Y_6)$ and calculate the intersection ring:

$$J = \sum_{i=1}^{3} r_i R_i - \sum_{\beta, \gamma} t_{1,\beta\gamma} E_{1,\beta\gamma} - \sum_{\beta} (t_{2,\beta} E_{2,\beta} + t_{4,\beta} E_{4,\beta}) - \sum_{\gamma} t_{3,\gamma} E_{3,\gamma}$$

$$V = 6 \ r_1 \ r_2 \ r_3 + r_3 \sum_{\beta=1}^{3} t_{2,\beta} \ t_{4,\beta} - \sum_{\beta, \gamma} t_{1,\beta\gamma} t_{2,\beta} t_{4,\gamma} - r_2 \sum_{\gamma=1}^{4} t_{3,\gamma}$$

$$-r_3 \sum_{\beta=1}^{3} (t_{2,\beta}^2 + t_{4,\beta}^2) - \sum_{\beta, \gamma} t_{1,\beta\gamma}^3 + \sum_{\beta=1}^{3} t_{2,\beta}^2 t_{4,\beta}$$

$$-\frac{4}{3} \left[ \sum_{\beta=1}^{3} (t_{2,\beta}^3 + t_{4,\beta}^3) + \sum_{\gamma=1}^{4} t_{3,\gamma}^3 \right]$$

$$+ \sum_{\beta, \gamma} (t_{1,\beta\gamma} t_{2,\beta}^2 + t_{1,\beta\gamma} t_{3,\gamma}^2 + t_{1,\beta\gamma} t_{4,\beta}^2).$$
Introducing the orientifold projection

In the orbifold limit \( \sigma : z^i \rightarrow -z^i \) introduces 64 \( O3 \)-planes. Some of them may be identified under orbifold group \( \theta \) or may be grouped into orbits under the orbifold group.

In the smooth case the orientifold action has to be chosen on the local patches in terms of the local coordinates \( (z^i, y^i) \).

Those of the 64 \( O3 \)-planes on the cover, which are located away from the resolved patches (resulting from the global involution) remain the same in the resolved orientifold.

The \( O3 \)-plane solutions, which concide with the orbifold fixed sets are replaced by the solutions of the corresponding patch.
Introducing the orientifold projection: Example $T^6/Z_{6-II}$

$T^6 = \mathbb{Z}_6 \times \mathbb{Z}_6$.

$E_1,\beta \gamma , \ 4 \ E_3,\gamma$

$3 \ E_2,\gamma , \ 3 \ E_4,\beta$

$\beta=1,2,3$

$\gamma=1,\ldots,4$

$\sigma \ E_{1,1\gamma} = E_{1,1\gamma} , \sigma \ E_{3,\gamma} = E_{3,\gamma}$

$\sigma \ E_{2,1} = E_{2,1} , \sigma \ E_{4,1} = E_{4,1}$

$\sigma \ E_{1,2\gamma} = E_{1,3\gamma} , \sigma \ E_{1,3\gamma} = E_{1,2\gamma}$

$\sigma \ E_{2,2} = E_{2,3} , \sigma \ E_{2,3} = E_{2,2}$

$\sigma \ E_{4,2} = E_{4,3} , \sigma \ E_{4,3} = E_{4,2}$

Construct divisor pairs $(E, \tilde{E})$, with $\sigma E = E$ , $\sigma \tilde{E} = -\tilde{E}$:

$E_{1,\gamma} := \frac{1}{2} ( E_{1,2\gamma} + E_{1,3\gamma} )$ , $\tilde{E}_{1,\gamma} := \frac{1}{2} ( E_{1,2\gamma} - E_{1,3\gamma} )$

$E_{2} := \frac{1}{2} ( E_{2,2} + E_{2,3} )$ , $\tilde{E}_{2} := \frac{1}{2} ( E_{2,2} - E_{2,3} )$

$E_{4} := \frac{1}{2} ( E_{4,2} + E_{4,3} )$ , $\tilde{E}_{4} := \frac{1}{2} ( E_{4,2} - E_{4,3} )$  $\implies$ $h^{(-)}_{(1,1)}(X_6) = 6$
### Hodge numbers after the orientifold action

<table>
<thead>
<tr>
<th>$\mathbb{Z}_N$</th>
<th>Lattice $T^6$</th>
<th>$h_{(1,1)}^{(+)}$</th>
<th>$h_{(1,1)}^{(-)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathbb{Z}_3$</td>
<td>$SU(3)^3$</td>
<td>23</td>
<td>13</td>
</tr>
<tr>
<td>$\mathbb{Z}_4$</td>
<td>$SU(4)^2$</td>
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<td>0</td>
</tr>
<tr>
<td>$\mathbb{Z}_4$</td>
<td>$SU(2) \times SU(4) \times SO(5)$</td>
<td>27</td>
<td>0</td>
</tr>
<tr>
<td>$\mathbb{Z}_4$</td>
<td>$SU(2)^2 \times SO(5)^2$</td>
<td>31</td>
<td>0</td>
</tr>
<tr>
<td>$\mathbb{Z}_{6-I}$</td>
<td>$(G_2 \times SU(3)^2)^b$</td>
<td>19</td>
<td>6</td>
</tr>
<tr>
<td>$\mathbb{Z}_{6-I}$</td>
<td>$SU(3) \times G_2^2$</td>
<td>23</td>
<td>6</td>
</tr>
<tr>
<td>$\mathbb{Z}_{6-II}$</td>
<td>$SU(2) \times SU(6)$</td>
<td>19</td>
<td>6</td>
</tr>
<tr>
<td>$\mathbb{Z}_{6-II}$</td>
<td>$SU(3) \times SO(8)$</td>
<td>23</td>
<td>6</td>
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<tr>
<td>$\mathbb{Z}_{6-II}$</td>
<td>$(SU(2)^2 \times SU(3) \times SU(3))^d$</td>
<td>21</td>
<td>10</td>
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<tr>
<td>$\mathbb{Z}_{6-II}$</td>
<td>$SU(2)^2 \times SU(3) \times G_2$</td>
<td>25</td>
<td>10</td>
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<tr>
<td>$\mathbb{Z}_7$</td>
<td>$SU(7)$</td>
<td>15</td>
<td>9</td>
</tr>
<tr>
<td>$\mathbb{Z}_{8-I}$</td>
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<td>0</td>
</tr>
<tr>
<td>$\mathbb{Z}_{8-I}$</td>
<td>$SO(5) \times SO(9)$</td>
<td>27</td>
<td>0</td>
</tr>
<tr>
<td>$\mathbb{Z}_{8-II}$</td>
<td>$SU(2) \times SO(10)$</td>
<td>27</td>
<td>0</td>
</tr>
<tr>
<td>$\mathbb{Z}_{8-II}$</td>
<td>$SO(4) \times SO(9)$</td>
<td>31</td>
<td>0</td>
</tr>
<tr>
<td>$\mathbb{Z}_{12-I}$</td>
<td>$E_6$</td>
<td>18</td>
<td>7</td>
</tr>
<tr>
<td>$\mathbb{Z}_{12-I}$</td>
<td>$SU(3) \times F_4$</td>
<td>22</td>
<td>7</td>
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Kähler potential: Example $T^6/Z_{6-II}$

$$V = 3 \, r_1 \, r_2 \, r_3 + r_3 \sum_{\beta=1}^{2} t_{2,\beta} t_{4,\beta} - \sum_{\beta,\gamma} t_{1,\beta\gamma} t_{2,\beta} t_{4,\gamma} - \frac{1}{2} \, r_2 \, \sum_{\gamma=1}^{4} t_{3,\gamma}^2$$

$$- r_3 \sum_{\beta=1}^{2} \left( \frac{2}{3} t_{2,\beta}^2 + \frac{1}{2} t_{4,\beta}^2 \right) - \frac{1}{2} \sum_{\beta,\gamma} t_{1,\beta\gamma}^3 + 2 \sum_{\beta=1}^{2} t_{2,\beta}^2 t_{4,\beta}$$

$$- \frac{4}{3} \left[ \sum_{\beta=1}^{2} \left( \frac{4}{3} t_{2,\beta}^3 + \frac{1}{2} t_{4,\beta}^3 \right) + \frac{1}{2} \sum_{\gamma=1}^{4} t_{3,\gamma}^3 \right]$$

$$+ \sum_{\beta,\gamma} \left( t_{1,\beta\gamma} t_{2,\beta}^2 + \frac{1}{2} t_{1,\beta\gamma} t_{3,\gamma}^2 + \frac{1}{2} t_{1,\beta\gamma} t_{4,\beta}^2 \right)$$

The total Kähler potential becomes:

$$K = - \ln(S + \overline{S}) - \ln(U^3 + \overline{U}^3) - 2 \ln V$$
Non-perturbative superpotential: $D3$–instantons

Non-perturbative superpotential from $D3$–instantons:

$$W = W_0 + \sum_{\text{contributing divisors } S_j} \gamma_j e^{a_j} \text{Vol}(S_j)$$

$D3$–instanton contribution, if there is a net number of two fermionic zero modes on $D3$:

$$\chi_{D3}(S_i) = \frac{1}{2} (N_+ - N_-) = 1$$

$$\begin{align*}
\epsilon_+ &= \phi |\Omega\rangle + \phi_\alpha \gamma^\alpha |\Omega\rangle + \phi_{\alpha\beta} \gamma^{\alpha\beta} |\Omega\rangle \\
\epsilon_- &= \phi_z \gamma^z |\Omega\rangle + \phi_{\alpha z} \gamma^{\alpha z} |\Omega\rangle + \phi_{\alpha\beta z} \gamma^{\alpha\beta z} |\Omega\rangle
\end{align*}$$

In local coordinate patch impose: (i) $\kappa$–symmetry fixing condition

(ii) orientifold projection
**Non-perturbative superpotential: D3–instantons**

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 surviving zero modes after $\kappa$–fixing & orientifolding

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(a) $O7$–plane wraps divisor $S_i$
(b) $O7$–plane intersects $S_i$ along one complex dimension
(c) $O7$–plane is parallel to $S_i$
Non-perturbative superpotential: gaugino condensation

Non-perturbative superpotential from gaugino condensation on $D7$–brane:

$$W = W_0 + \sum_{\text{contributing divisors } S_j} \gamma_j e^{a_j} \text{Vol}(S_j)$$

Gaugino condensation on $D7$, if there is from open strings:

(i) no adjoint matter & (ii) no bifundamental matter.

$$\implies$$ depends on: geometry and intersection properties of divisors $S_j$

Example $T^6/\mathbb{Z}_{6-II}$:

$$h_{(0,1)}(S_j) = 0 = h_{(0,2)}(S_j)$$

gaugino condensation on $D_1, D_3, E_2$
Complex structure and dilaton stabilization

Turn on 3–form flux $G_3$ and dynamically fix $S$ and $U^\lambda$: $W_0 = \int_{X_6} G_3 \wedge \Omega$

$$\frac{1}{(2\pi)^2 \alpha'} G_3 = \sum_{i=0}^{3} \left[ (a^i + iSc^i)\alpha_i + (b_i + iSd_i)\beta^i \right] + \sum_{j=1}^{6} \left[ (e^j + iSg^j)\gamma_j + (f_j + iSh_j)\delta^j \right]$$

20 real flux components $\alpha_i, \beta^i, \gamma_j, \delta^j$ constrained by the orbifold group $\mathbb{Z}_N$, with $(a^i, b_i, c^i, d_i) \in \mathbb{Z}^{20}$

$$N_{flux} = \frac{1}{(2\pi)^4 \alpha'^2} \int_{X_6} G_3 \wedge \ast G_3$$

Cancel $D3$–brane charge $Q_{3,tot}$ through flux, i.e. $N_{flux} = -2 \ Q_{3,tot}$ \left\{ \begin{align*}
\text{(no space – time filling)} \\
D3 – \text{branes}\end{align*} \right\}$

Example $T^6/\mathbb{Z}_{6-II}$: $h_{(2,1)}(X_6) = 1 \implies 8$ flux components $(a^0, b_0, c^0, d_0, a^1, b_1, c^1, d_1) \in \mathbb{Z}^8,$

$$N_{flux} = 44 \implies (a^0, b_0, c^0, d_0, a^1, b_1, c^1, d_1) = (5, 2, 0, -2, -3, 8, 1, 0)$$

$$\implies S = 3.16 - 0.167 \ i \ , \ U = 1.26 + 0.067 \ i \ , \ i.e. : g_{\text{string}} \sim 0.3$$
**Kähler moduli stabilization**

- **Compute Divisor Volumina** \( \text{Vol}(S_i) \sim \text{Re}(T^i) \): 
  \[ \text{Re}(T^i) = \frac{3}{4} \mathcal{K}_{ijk} t^j t^k = \frac{1}{4} \frac{\partial}{\partial t^i} \text{Vol}(X_6) \]

- **Stabilize the** \( h_{(+)}^{(1,1)}(X_6) \)** Kähler moduli through** \( W \)**

- **Determine supersymmetric** \( AdS \)** **Minimum** \( F_{T^j} = 0 \) **for** specific **\( W_0 \)**

**Example** \( T^6/\mathbb{Z}_{6-II} \): \( h_{(+)}^{(1,1)}(X_6) = 16 + 3 = 19 \) **and** \( h_{(-)}^{(1,1)}(X_6) = 6 \).  
With \( \bar{W}_0 = -e^{K_0/2}|W_0| = -0.35 \) we find:

\[
\begin{align*}
r_1 &= 3.04765 \quad , \quad r_2 = 2.91779 \quad , \quad r_3 = 4.53928 \quad , \\
t_{1,\gamma} &= 1.52711 \quad , \quad t_{1,1,\gamma} = 0.869367 \quad , \quad t_{3,\gamma} = 0.46524 \quad , \quad \gamma = 1, \ldots, 4 \quad , \\
t_{2,1} &= 0.443261 \quad , \quad t_2 = 0.663503 \quad , \quad t_4 = 0.967525 \quad , \quad t_{4,1} = 0.634432 \quad ,
\end{align*}
\]

\[ \implies \text{Vol}(X_6) = 115.94 \quad , \quad \det(\mathcal{K}_{ab}) = -91.25 \neq 0 \quad \implies \quad b^a = 0 \quad , \quad a = 1, \ldots, 6 \]
**AdS \rightarrow dS Uplift**

**Controled moduli stabilization**  
(closed string moduli $M$)  
**AdS minimum**

**SUSY broken**  
(open string moduli $C$)  
**dS minimum**

1. **KKLT:**  
Add anti-D3 \[ V_{D3} \sim \frac{1}{\text{Vol}(X_6)^2} \]  
\[ \Rightarrow V_{total} = V_{AdS} + V_{D3} \]  
SUSY broken explicitly

2. **Alternative:**  
Matter dominated SUSY breaking

$F$-terms of hidden matter fields: \[ F_C \neq 0 \]  
(Lbedev, Nilles, Ratz  
Lebedev, Löwen, Mambrini, Nilles, Ratz)

\[ F_T = 0 \quad , \quad F_C \sim m_{3/2} \quad , \quad m_T \gg m_C \sim m_{3/2} \]
3. Note: \( D \)-term uplift is problematic \( \text{(Choi, Falkowski, Nilles, Olechowski & Villadoro, Zwirner)} \)

a) \( D = \frac{1}{W} \sum_i \delta M_i \, D_i W = 0 \) for AdS minimum

b) \( D \sim m_{3/2}^2 \ll m_{3/2} \sim F \implies D^2 \sim m_{3/2}^4 \) compared to \( V_{AdS} \sim -3 \, m_{3/2}^2 \)
Concluding remarks and open questions

- We provided systematics of closed string moduli stabilization for flux vacua based on resolved toroidal $\mathbb{Z}_N$ & $\mathbb{Z}_N \times \mathbb{Z}_M$ orbifolds
  
  \textit{E.g.}: $\mathbb{Z}_{6-II}: 27$ moduli fixed \\
  $\mathbb{Z}_2 \times \mathbb{Z}_4: 63$ moduli fixed!

- Include open string moduli in all the discussions (brane positions, Wilson line moduli, bundle moduli) to improve the knowledge about moduli stabilization, in particular: matter dominated SUSY breaking
Concluding remarks and open questions

- Go beyond supergravity approximation, i.e. include $\alpha'$ and $g_{\text{string}}$ effects:
  
  *E.g.* Corrections in $g_{\text{string}}$ to Kähler potential $K$ in $\mathbf{N=1}$ CY orientifolds?

- Other instanton effects like $D(-1)$–instantons correct e.g. the Kähler potential $K$
  (discrete PQ symmetries not gauged by flux $G_3$)

  Complementarity: Fluxes protect gauged isometries from instanton corrections
  
  (Kashani–Poor, Tomasiello)
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<th>Lattice</th>
<th>(h_{\text{untw.}}^{(1,1)})</th>
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