Torsion and (soft) SUSY breaking
Albion Lawrence,
Brandeis University

I. Introduction

II. Worldsheet computation

III. $N=2$ aux fields for complex structure moduli in CY,
    A. J. T. McGreevy, hep-th/0401034
    0401233

IV. Kähler moduli for $II^*$ on CY - aux fields
    A. J. T. Sander (Brandeis), M. Schult (U Iowa),
    B. Wecht (M T) - to appear

V. Worldsheet instanton effects

Basic motivations
1. Learn how to organize/compute SUSY effects
2. Develop a "microscopic" (worldsheet) point of view (not important)

We will find this a complementary to
   and in some sense a repackaging of
   the work of Groß, Louis & Woldram, Paris group
   NB work with local (noncompact) models

Seminar
   given at DESY workshop on
   generalized geometries and flux compactifications
   Feb. 23, 2007

Notes re-written March 9, 2007
I. Introduction

A. Aux fields and (soft) SUSY breaking

*Girandello & Grisaru* - couplings in $N=1$ superfield action $= \lambda$ aux superfields

$$g \rightarrow \tilde{g} = g + \mathcal{O}(\tilde{g})$$

Expectation value breaks SUSY

Example: gaugino mass

$$\mathcal{L}_{\text{aux}} = \int d^4 \theta \bar{\psi} W \psi W^\dagger + \text{h.c.}$$

$$W = \lambda + \mathcal{O}(\tilde{g})$$

$$\tilde{g} \rightarrow \tilde{g} + \mathcal{O}(\tilde{g})$$

(*theta angle
not Grassmann
variable*)

$$\tilde{g} i = \tilde{g}_{\text{susy}} + (\tilde{g}_{\text{aux}})$$

Efficient way to encode soft SUSY terms (a MSSM)

* Reflects the scenario in which SUSY orches in "hidden sector"
  $$g \rightarrow g (\tilde{g})$$
  hidden sector fields (by integrating out messengers), moduli, etc.

* Wish to identify $\tilde{g}_{\text{susy}}$ in string models
  (as corresponding to features of string background - e.g. fluxes, etc.)
3. N=2 superfields

(1) Motivation: for type II models of D-branes and fluxes, open string couplings $g_1, g_2$ includes closed string modes.

"Often, $\mathcal{E}$ for closed string modes descends from $N=2$ multiplets.

$\Rightarrow$ study $N=2$ auxiliary fields. Some compatible with breaking $N=2 \to N=1$; others correspond to superpotential terms breaking $N=1 \to N=0$.

(2) String theory preliminaries

- $\cal M_0, \cal M_6 \cong \mathbb{R}^2 \times \mathbb{R}$
- Perturbative $N=(2,2)$ d=4 Minkowski vacua
- $N=(3,3)$ worldsheet SCFTs

\begin{align*}
\Omega_{\cal M_0}(\mathbb{R}) & \quad \text{N=1 SUSY current for linear} \quad \mathbb{R}^3
\end{align*}

- CY case: SU(3) holonomy, $Q_3 = 0$
- Complex structure, $\nabla_3 = 0$
- More general $(2,2)$ vacua: $SU(3) \times SU(3)$ structure, 2 globally defined spinors on $M_6$
- Almost complex structures $J \oplus (721) J^\perp = 0$
- $J^-$ generates $O(3)$, for linear\;
- $J^+$ is imaginary\;

\[ \frac{\mathbb{R}^3}{\mathbb{R}^3} \]

- Banks, Dixon, Friedan, Martinec
- Banks, Dixon
(c) N=2 auxiliary fields

- Bont Levi & Siegel: worldsheet gives
  `nabla`-decomposition of N=2 \( \mathfrak{su}(2)_R \)
doublet of N=1 supercharges

- use \( \mathfrak{su}(2)_R \) doublet of \( N=1 \) superspace
  variables \((\Theta, \bar{\Theta})\)
  
  \[
  \Theta = \Theta_X \\
  \bar{\Theta} = \bar{\Theta}_X
  \]

- same as developed by
  Grimm, Schwartz, West
  de Wit & van Holten
  de Roo et al.

a) Vector multiplets: \((\chi_{\mu}, \chi_{\mu})\) w/ \((Q_{\mu}, G_{\mu})\)

\[
V^\mu = W^\mu : \Theta^{\alpha} \Theta^{\beta} \gamma^{\mu} D_{\alpha} + \bar{\Theta}^{\dot{\alpha}} (\gamma^{\mu} D_{\dot{\alpha}} + \bar{g}^{\mu} F_{\nu}^{\dot{\nu}})
\]

\(-1\) scalar \( \Theta D - \bar{\Theta} D + \) \( \gamma_{\mu} \) \( \bar{g}^{\mu} F_{\nu}^{\dot{\nu}} \)

\( D\nu = \mathfrak{su}(2) \) triplet of auxiliary fields

\[\text{(Note: } Q_{\mu}\text{ takes } N=2 \text{ R, via spectral flow)}\]

\( Q_{\mu} \quad \text{NS} \rightarrow \text{NS} (R \rightarrow R)\)

\( Q_{\nu} \quad \text{NS, NS} \rightarrow \text{NS, NS} (R \rightarrow R)\)

\[
\Rightarrow \text{Def: NS-NS} \\
\text{Def: NS-R-R}
\]

b) "Hypermultiplets" \((\chi_{\mu}, \psi_{\mu})\)

\(\text{(actually: tensor multiplets)}\)

\[
H^L_{\mu} = \gamma^2 \Theta^{\alpha} \Theta^{\beta} \gamma^\mu + \bar{\Theta}^{\dot{\alpha}} \bar{\Theta}^{\dot{\beta}} \gamma^\mu \gamma^\nu \bar{g}_{\nu}^{\dot{\nu}} F_{\mu}^{\dot{\nu}}
\]

\(-1\) scalar \( \Theta D - \bar{\Theta} D + \) \( \gamma_{\mu} \) \( \bar{g}^{\mu} F_{\nu}^{\dot{\nu}} \)

\( \text{NS-NS \& \&} \) fields

\( \text{NS-R-R} \) scalar

\( \text{Scalar} \)
Our goal: identify $D, y, \tilde{y}$ microscopically (in terms of string backgrounds)

Note: certain combinations of aux fields break $N=2 \to N=1$

ex: $<D, y, \tilde{y}> \to 0, \tilde{y} \neq 0$

$Q_{y, \tilde{y}}(3)$ broken  ($\varepsilon$ shift broken)
$Q_{y, \tilde{y}}(1)$ preserved  $S_0 \cdot S_{D, y}$ etc.

Induces, for example, $N=5$ superpotential (Via Taylor-Vafa)
II. Worldsheet computations

For NS, NS aux fields:
vertex operators for scalar fields in \((-1,0)\) picture

\[ V_{\alpha\beta} \sim (\varphi, \overline{\varphi}) \]

renormalized super-conformal ghost

\[ V_{\alpha\beta} \sim \frac{1}{2} \omega \{ (\varphi - \overline{\varphi}) \otimes \overline{\varphi} \} \]

geometric spectral flow
NS \rightarrow NS

\[ \text{CY 5 - model:} \]
\[ \mathbb{R} \times \mathbb{C}^3 \otimes (x_1 \otimes x_2 \otimes x_3) \]

NR: \( C_{D+} \rightarrow \mathbb{R}, D_+ = 0 \) breaks \( N = 2 \rightarrow N = 1 \)
by breaking SU(2) \( \gamma \) by \( G_{\mathbb{R}} \)
breaks \( N = (2,2) \rightarrow N = (2,1) \)
IV. Aux fields $w$ Kähler moduli $e^{1/4}$ CY

$$w^i \equiv \Psi^i \xi^j \xi^k \xi^l \xi^m$$

$V_g \cdot e^{1/4} \equiv \xi^j \xi^k \xi^l \xi^m$ Note: $V_g$ should be minor to $\mathbf{2a} - \{\mathbf{suc}\}$ structure & NS-NS flux, of type $H_{1,1} + \mathbf{w}_+$

We break $\to (1,2) : \{\mathbf{suc}\}$ with $\mathbf{suc}$ & expect $\mathbf{suc}(1) \times \mathbf{suc}(1) / \mathbf{suc}(1)$ structure

A. $\mathbf{suc}(1)$ structure

$S(\mathbf{8}+\mathbf{3}) = \chi(\mathbf{8}+\mathbf{3}) + K$ $\to \mathbf{suc}(1)$ (cases 6 of cases)

$\mathbf{suc}(1)$ structure defn

We can show that for $w_{\mathbf{suc}}$ of $\mathbf{suc}(1)$, the corresponding auxiliary fields are

$\Psi^i = \mathbf{w}_x + \mathbf{w}_y \xi^j \xi^k \xi^l \xi^m$

and for $w_{\mathbf{suc}}$ of $\mathbf{suc}(1)$:

$\Psi^i = \mathbf{w}_x + \mathbf{w}_y \xi^j \xi^k \xi^l \xi^m$

These are compatible with the superpotential $W = \mathbf{W}(\mathbf{8}+\mathbf{3}) + \mathbf{suc}$

$Y^i \cdot D\mathbf{W}$

also compatible with $\mathbf{10}$ of $\mathbf{suc}(1)$ transformations.
Furthermore, consider "universal hypermultiplet" containing the axion-dilaton tower.

Using 10D SUSY transformations, we find:

\[ Y_6 = \overline{\psi}_6 i H_f \quad \xi_6 = \overline{\psi}_6 i H_f \]

In Fidanzio, Muriel, and Tonaniello, these are self-mirror. This makes sense, since \( \mathbb{Z}_2 \) in the universal hyper in both IIA and IIB.

Not with pure SU(3) structure, we cannot independently tune \( Y_6, \xi_6 \).

B. SU(3) x SU(3) structure

In progress, but note that in general SUSY implies torsion, and there are different spinors on \( M_6 \) for each of \( G_6, U_6 \)

In tuning the torsions for each spinor corresponds to SU(3) x SU(3) structure.
V. Worldsheet instantons

Why are we considering:

1. IIA "mirror" $\frac{\mathcal{F}}{\mathcal{G}} \to \mathcal{W} = (\mathcal{H} \times \mathcal{G}) \times S\mathcal{L}$

$\mathcal{G}$ structure stabilized in the interior of the space of $\mathcal{G}$ structures.

In IB, we expect the "mirror" to be stabilized in the interior of the space of Kähler structures (to the extent this statement makes sense -- for U(1).)

2. "Mirror" of $\mathcal{H}^e$ flux expected to be (typically) non-geometric
   \(\Rightarrow\) requires string-scale features.

We know little about worldsheet instantons for compactifications with $H_B$ structure:

We will consider pretend we can treat $\frac{\mathcal{F}}{\mathcal{G}}$ as a "small deformation". Expand around underlying $\mathcal{N}=2,2$ $\sigma$-model.
We are breaking \( N = 2 \to N = 1 \) by turning on \( \hat{g} \).

\( N = 1 \) superfield for Kähler moduli:
\[
\Phi = \Phi^* + i \Phi^* + \partial \Phi^* + \partial^* \Phi
\]

Terms breaking \( N = 2 \to N = 1 \) will include superpotential \( W(\Phi) \).

We claim that \( \mathfrak{sl}_2, W(\Phi) \) will generically receive instanton corrections.

\[
W(\Phi) = \lambda \Phi^2 + \cdots \quad \text{when} \quad \frac{2}{3} \neq 0 \quad \text{well}.
\]

\[
\lambda \Phi^2 + \frac{1}{2} \lambda \Phi^3 + \cdots
\]

\( N = (7,3) \) - single worldsheet instanton has \( \Phi \)

\( \Phi \) worldsheet fermion O-modes (Cimco et al.)
\[
2 \epsilon \Phi^i, \quad 2 \epsilon \Phi^i, \quad 2 \epsilon \Phi^i, \quad 2 \epsilon \Phi^i
\]

at \( \mathcal{O}(\Phi^2) \): not enough fermions in correlator to

\( \Phi \) break up O-modes of instanton

at \( \mathcal{O}(\Phi) \): exactly enough fermions to absorb

O-modes of instanton.