Supersymmetric AdS and Bubble Solutions

JPG, N. Kim, D. Waldram

JPG, O. Mac Conamhna, T. Mateos, D. Waldram
1. Classify most general supersymmetric AdS backgrounds of string/M-theory: most general dual SCFTs

2. Construct explicit solutions

- Interesting class of geometries.
  e.g. a-maximisation
- M-theory examples give novel SCFTs
- Rich sets of new explicit solutions
  e.g. \( Y^{p,q} \) Sasaki-Einstein (JPG, Martelli, Sparks, Waldram)

- Can deform CFT to get different dynamics in IR
- Can wick rotate \( \rightarrow \) “BPS bubbles” dual to smooth BPS states of SCFTs (LLM)
Consider warped products:

\[ ds^2 = e^{2A(y)}[ds^2(AdS) + ds^2(M)(y)] \]

with fluxes preserving isometries of \( AdS \)

Classify geometries using \( G \)-structures. Have classified most general:

**D=11:**
- \( AdS_5, \; N = 1, 2 \)
- \( AdS_4, \; N = 1, 2 \)
- \( AdS_3, \; N = (4, 0), (2, 0)_I, (2, 0)_{II} \)

**Type IIB**
- \( AdS_5, \; N = 1 \)

Consider them as special cases of Minkowski space solutions
Special Case I: Sasaki-Einstein

Type IIB sugra:

\[ ds^2 = ds^2(AdS_5) + ds^2(X_5) \]
\[ F_5 = (1 + \ast)Vol(X_5) \]

where \( X_5 \) is Sasaki-Einstein.
Dual to N=1 SCFT in D=4.

D=11 sugra:

\[ ds^2 = ds^2(AdS_4) + ds^2(X_7) \]
\[ G_4 = Vol(AdS_4) \]

where \( X_7 \) is Sasaki-Einstein.
Dual to N=2 SCFTs in D=3.
$X_{2n+1}$ is SE iff the cone metric
\[ dr^2 + r^2 ds^2(X_{2n+1}) \]
is Ricci-flat Kähler.

Locally, a SE metric can be written
\[ ds^2(X_{2n+1}) = (d\psi + P)^2 + ds^2(B_{2n}) \]
where $B_{2n}$ is Kähler and Einstein with positive curvature and $dP = \mathcal{R}$.

Explicit examples:

\( D = 5 \): $Y^{p,q}$, $L^{a,b,c}$.
Dual SCFTs identified. Toric geometry, quiver gauge theories, dimer models, a-maximisation.....

\( D = 7 \) examples: dual SCFTs obscure.

Constructions of SE metrics extend to all odd dimensions.
*Special Case II: Main Focus*

*AdS*$_3$ in Type IIB sugra:

\[
    ds^2 = e^{2A}[ds^2(AdS_3) + ds^2(Y_7)] \\
    F_5 = (1 + \ast)Vol(AdS_3) \wedge F_2
\]

Dual to N=(0,2) SCFTs in D=2.

*AdS*$_2$ in D=11 sugra:

\[
    ds^2 = e^{2A}[ds^2(AdS_2) + ds^2(Y_9)] \\
    G_4 = Vol(AdS_2) \wedge F_2
\]

Dual to N=2 SCQM.

*Locally,* metrics can be written *(Kim)*

\[
    ds^2(Y_7) = \frac{1}{4}(dz + P)^2 + e^{-4A}ds^2(B_6) \\
    ds^2(Y_9) = (dz + P)^2 + e^{-3A}ds^2(B_8)
\]

where $B_{2n}$ is Kähler and satisfies

\[
    \square R - \frac{1}{2}R^2 + R_{ij}R^{ij} = 0 \quad (*)
\]

with $dP = \mathcal{R}$ and $R > 0.$
1/8 BPS Bubbles:

Type IIB sugra:

\[ ds^2 = e^{2A} \left[ -\frac{1}{4} (dt + P)^2 + ds^2(S^3) + e^{-4A} ds^2(B_6) \right] \]

\[ F_5 = (1 + \ast) Vol(S^3) \wedge F_2 \]

D=11 sugra:

\[ ds^2 = e^{2A} \left[ -(dt + P)^2 + ds^2(S^2) + e^{-3A} ds^2(B_8) \right] \]

\[ G_4 = Vol(S^2) \wedge F_2 \]

Again \( B_{2n} \) is Kähler and satisfies

\[ \Box R - \frac{1}{2} R^2 + R_{ij} R^{ij} = 0 \quad (\ast) \]

with \( dP = \mathcal{R} \) and now \( R < 0 \).

Generalise the 1/2 BPS configurations of LLM.
3 constructions of Kähler manifolds satisfying (*). Focus on IIB examples.

**Construction 1**

**Line bundles over Kähler-Einstein**

Inspired by construction of $Y^{p,q}$

$$ds^2(B_{2n+2}) = \frac{1}{x} \left[ \frac{dx^2}{4x^2U} + U(D\phi)^2 + ds^2(KE_{2n}^+) \right]$$

$ds^2(KE_{2n}^+)$ Kähler-Einstein positive curvature.

$D\phi = d\phi + B$ with $dB = 2J_{KE}$.

This is locally Kähler (Page, Pope).

Demand it is Einstein: $\rightarrow$ Sasaki-Einstein in all odd dimensions including $Y^{p,q}$ in $D = 5$ (JPG, Martelli, Sparks, Waldramm)
Demand that it solves (*): Second order ODE for $U(x)$. Polynomial solutions → infinite new classes of $AdS$ and bubble solutions.

$AdS$ Solutions ($R > 0$):

$AdS_3$ in Type IIB sugra:

$$ds^2 = e^{2A}[ds^2(AdS_3) + ds^2(Y_7)]$$

with globally defined

$$ds^2(Y_7) = \frac{1}{4}(dz + P)^2 + e^{-4A}ds^2(B_6)$$

Note: $B_6$ depends on $KE_4^+$ which must be $S^2 \times S^2$, $CP^2$ or $dP_k$, $k = 3, \ldots, 8$.

Isometries of $Y_7$:

e.g. for $KE_4 = CP^2$: $U(1) \times U(1) \times SU(3)$. Cohomogeneity one.
BPS Bubbles and superstars \((R < 0)\)

Type IIB sugra:

\[
ds^2 = e^{2A} \left[ -\frac{1}{4}(dt + P)^2 + ds^2(S^3) + e^{-4A}ds^2(B_6) \right]
\]

D=5 minimal gauged sugra: there exist singular, charged BPS solutions ("black holes") that asymptote to \(AdS_5\). These have an \(S^3\) factor.

Can be uplifted to solutions of type IIB sugra: the charge becomes \(SO(6)\) "angular momentum" on the \(S^5\): the "superstar" is a collection of giant gravitons rotating equally along three \(U(1)\)'s of \(S^5\).
Construction 2

Type IIB sugra:

Using $D = 5$ gauged sugra coupled to vector multiplets, can construct $D = 10$ superstar solutions with three different $U(1)$ charges.

View them as BPS bubbles: extract out Kähler geometry and use it to construct new $AdS_3$ geometries.

Explicit metric

$Y_7$ has 4 $U(1)$ isometries: co-homogeneity three.

Previous construction, for $KE_4 = CP^2$, is a special case: isometries were $U(1) \times U(1) \times SU(3)$. 
Construction 3

Kähler manifold is product of KE:

\[ ds^2(B_{2n+2}) = \sum_{i=1}^{n+1} ds^2(KE_2^{(i)}) \]

where \( KE_2^{(i)} \) can be

\( T^2: \ l_i = 0; \)

\( S^2: \ l_i = 1; \)

\( H^2/\Gamma: \ l_i = -1; \)

If two \( l_i \) equal can replace with \( KE_4 \)

\( (*) \) is solved if:

\[ \sum_{i=1}^{n+1} l_i^2 = \left( \sum_{i=1}^{n+1} l_i \right)^2 \]
Type IIB \( AdS_3 \) solutions:
One parameter family

Interesting special case: \( B_6 = (H^2/\Gamma) \times KE_4^+ \)

Solution is:
\[
ds^2 = ds^2(AdS_3) + \frac{3}{4} ds^2(H^2/\Gamma) \\
+ \frac{9}{4} \left[ ds^2(KE_4^+) + \frac{1}{9}(dz + P)^2 \right]
\]

If \( KE_4^+ = CP^2 \) this describes a D3-brane wrapping a holomorphic \( H^2/\Gamma \) in a \( CY_4 \) (Maldacena,Nunez; Naka)

Also BPS Bubbles.
Very rich class of supersymmetric $AdS_3$, $AdS_2$ and bubble solutions - much to explore.

Dual SCFTs:
Focus on type IIB $AdS_3$ solutions. Simplest solutions: flux quantisation implies that the solution depends on $p, q$ with $p|q = 1$. Also depends on a choice of $KE_4^+$. Central charge:

$$c = \frac{9pq^2(p + mq)}{3p^2 + 3mpq + m^2q^2} \frac{Mq}{m^2h^2} n^2,$$

where $M, m$ are fixed by choice of $KE_4^+$ eg for $CP^2$, $m = 3, M = 9$. Also, $h = h.c.f.(M/m^2, q)$.

Given the underlying geometry has some similarity with Sasaki-Einstein spaces ($Y^{p,q}$) perhaps the SCFTs are related?