NLO and Relativistic corrections to J/ψ exclusive and inclusive double charm production at B factories

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NLO and Relativistic corrections to J/ψ exclusive and inclusive double charm production at B factories (*Outline*)

based on Yu-Jie Zhang, Ying-Jia Gao, Kuang-Ta Chao, PPL96:092001(2006), Yu-Jie Zhang, Kuang-Ta Chao, PRL98:092003(2007), Zhi-Guo He, Ying Fan (Peking U.) , Kuang-Ta Chao PRD75:074011(2007).

1. Introduction

- Quarkonium is hot topic of theoretical physics. Bodwin, Braaten, and Lepage presented an effective theory, nonrelativistic QCD (NRQCD)[*], to describe heavy quarkonium production and annihilation.

- Some Successes of NRQCD Factorization Approach

  ⋆ Quarkonium Production at the Tevatron;

  ⋆ $\gamma\gamma \to J/\psi$ at LEP;

  ⋆ Quarkonium production in DIS at HERA.

- Some Challenging Problem of NRQCD Factorization Approach

  ⋆ Polarization of quarkonium at the Tevatron;

  ⋆ Inelastic quarkonium production at HERA;

  ⋆ Double $c\bar{c}$ production at B Factory.

[*]G. T. Bodwin, E. Braaten and G. P. Lepage, Phys. Rev. D **51**, 1125 (1995) [Erratum-ibid. D **55**, 5853 (1997)]

$\sigma[e^+e^- \to J/\psi c\bar{c}]$ at $\sqrt{s} = 10.6$GeV was measured by Belle in 2002 [*]:

$$\sigma[e^+e^- \to J/\psi + c\bar{c} + X] \;=\; \left(0.87^{+0.21}_{-0.19} \pm 0.17\right) \text{ pb} \,, \tag{1}$$

which is about an order of magnitude or at least a factor of 5 higher than theoretical predictions including both the color-singlet[**] and color-octet[***] $c\bar{c}$ contributions in the leading order of α_s ,

$$\sigma[e^+e^- \to J/\psi + c\bar{c} + X] = 0.10 \sim 0.15 \text{ pb}. \tag{2}$$

[*] K. Abe *et al.* [BELLE Collaboration], Phys. Rev. Lett. **89**, 142001 (2002).

[**]P. L. Cho and A. K. Leibovich, Phys. Rev. D **54**, 6690 (1996) ;F. Yuan, C. F. Qiao and K. T. Chao, Phys. Rev. D **56**, 321 (1997) ;Phys. Rev. D **56**, 1663 (1997) ;S. Baek, P. Ko, J. Lee and H. S. Song, J. Korean Phys. Soc. **33**, 97 (1998); V. V. Kiselev, A. K. Likhoded and M. V. Shevlyagin, Phys. Lett. B **332**, 411 (1994) ; K. Y. Liu, Z. G. He and K. T. Chao, Phys. Rev. D **68**, 031501 (2003).

[***] K. Y. Liu, Z. G. He and K. T. Chao, Phys. Rev. D **69**, 094027 (2004).

- $\sigma(e^+e^- \to J/\psi + X) = (1.47 \pm 0.10 \pm 0.11)$ pb was also measured by Belle [**], then Belle got [*]:

$$\frac{\sigma(e^+e^- \to J/\psi + c\bar{c} + X)}{\sigma(e^+e^- \to J/\psi + X)} = 0.59^{+0.15}_{-0.13} \pm 0.12 \tag{3}$$

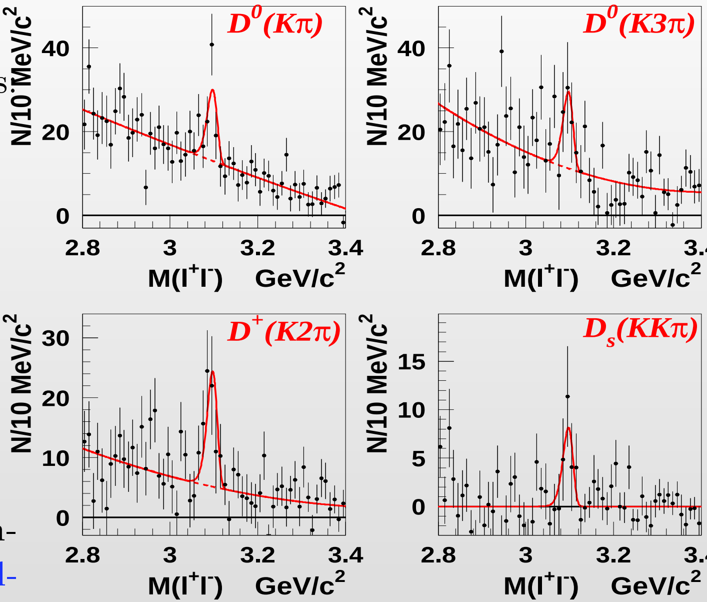
which is larger than the theoretical prediction 0.1 [***].

- In EPS'2003 Belle's result:

$$\frac{\sigma(e^+e^- \to J/\psi + c\bar{c} + X)}{\sigma(e^+e^- \to J/\psi + X)} = 0.82 \pm 0.15 \pm 0.14 \tag{4}$$

[*] K. Abe *et al.* [BELLE Collaboration], Phys. Rev. Lett. **89**, 142001 (2002).
[**] K. Abe *et al.* [BELLE Collaboration], Phys. Rev. Lett. **88**, 052001 (2002).
[***]P. L. Cho and A. K. Leibovich, Phys. Rev. D **54**, 6690 (1996) ;F. Yuan, C. F. Qiao and K. T. Chao, Phys. Rev. D **56**, 321 (1997) ;Phys. Rev. D **56**, 1663 (1997) ;S. Baek, P. Ko, J. Lee and H. S. Song, J. Korean Phys. Soc. **33**, 97 (1998); V. V. Kiselev, A. K. Likhoded and M. V. Shevlyagin, Phys. Lett. B **332**, 411 (1994) ; K. Y. Liu, Z. G. He and K. T. Chao, Phys. Rev. D **68**, 031501 (2003).

- $\mathcal{L} = 101\,\mathrm{fb}^{-1}$
- Reconstruct all charm ground states: D^0, D^+, D_s^+, Λ_c
- Fit $D(\Lambda_c)$ signals in $M_{\ell^+\ell^-}$ bins
- Fit J/ψ to yields distributions:
  $N_{D^0\to K\pi} = 49.6 \pm 13.3\ (3.7\sigma)$
  $N_{D^0\to K3\pi} = 53.0 \pm 21.2\ (2.5\sigma)$
  $N_{D^+\to K2\pi} = 56.2 \pm 15.4\ (3.6\sigma)$
  $N_{D_s^+\to KK\pi} = 23.8 \pm 9.4\ (2.6\sigma)$
  $N_{\Lambda_c\to Kp\pi} = 3.0 \pm 4.2$
- All $c\bar{c}$ final states except for Ξ_c reconstructed $\Rightarrow$ Do not need model to calculate $2(c\bar{c})$ X-section!



$$\left.\frac{\sigma(e^+e^-\to J/\psi\, c\bar{c})}{\sigma(e^+e^-\to J/\psi\, X)}\right|_{P_{J/\psi}>2.0\,\mathrm{GeV}/c} = \frac{0.5(N_{D^0}+N_{D^+}+N_{D_s^+}+N_{\Lambda_c})+N_{(c\bar{c})res}}{N_{J/\psi}} = 0.82 \pm 0.15 \pm 0.14$$

Belle's result of $J/\psi c\bar{c}$ in EPS'2003.

- Even more seriously, the cross section of $e^+e^- \to J/\psi + \eta_c$ measured by Belle in 2002 [*]:

$$\sigma[J/\psi + \eta_c] \times B^{\eta_c}[> 2] \ = \ \left(33^{+7}_{-6} \pm 9\right) \ \text{fb} \tag{5}$$

where $B^{\eta_c}[> 2]$ is the branching fraction for the η_c decay into more than 2 charged tracks, so Eq. (5) give the lower bound for the cross section.

- Braaten-Lee [**] and Liu-He-Chao [***] gave the theoretical result at leading order of α_s and the charm quark relative velocity v, $\sigma[e^+e^- \to J/\psi\eta_c] = 3.8 \sim 5.5\text{fb}$ (depending on the used parameters m_c,α_s, and long-distance matrix element).

- Compared with Eq. (5), there was a large discrepancy of about a factor of 10 between theory and experiment. It is a challenge to our current understanding of charmonium production based on NRQCD and perturbative QCD.

[*] K. Abe *et al.* [BELLE Collaboration], Phys. Rev. Lett. **89**, 142001 (2002).
[**]E. Braaten and J. Lee, Phys. Rev. D **67**, 054007 (2003)
[***]K. Y. Liu, Z. G. He and K. T. Chao, Phys. Lett. B **557**, 45 (2003)

PRELIMINARY!

$$\sigma\left(e^+e^- \to (\bar{c}c)_1\,(c\bar{c})_2\right) \times \mathcal{B}\left((c\bar{c})_2 \to > 2\ \ charged\right) \qquad \textbf{(fb)}$$

RECOIL CHARMONIUM	RECONSTRUCTED CHARMONIUM: J/ψ	χ_{c1}	χ_{c2}	$\psi(2S)$
η_c	$46 \pm 6^{+7}_{-9}$ (2.3)	< 21 $(1.3 \cdot 10^{-3})$	$< 38\,(0.5 \cdot 10^{-3})$	$18 \pm 8 \pm 7\,(0.9)$
J/ψ	< 8 (8.7)	< 21	< 38	< 64 (7.2)
χ_{c0}	$16 \pm 5 \pm 4$	< 21	< 38	$17 \pm 8 \pm 7$
χ_{c1}	< 8	< 21	< 38	< 24
χ_{c2}	< 8	< 21	< 38	< 24
$\eta_c(2S)$	$25 \pm 6 \pm 6$ (0.9)	$< 21(0.5 \pm \cdot 10^{-3})$	$< 38(0.2 \cdot 10^{-3})$	$31 \pm 9 \pm 10$ (0.4)
$\psi(2S)$	$< 16(7.2)$	< 21	< 38	$< 18(1.5)$

Belle's result of double charmonium in EPS'2003.

- Belle gave the new data in 2004[*]:

$$\sigma[J/\psi + \eta_c] \times B^{\eta_c}[> 2] \;=\; (25.6 \pm 2.8 \pm 3.4)\;\text{fb}, \tag{6}$$

- BaBar gave their data at 2005[**]:

$$\sigma[J/\psi + \eta_c] \times B^{\eta_c}[> 2] \;=\; \left(17.6 \pm 2.8^{+1.5}_{-2.1}\right)\;\text{fb}, \tag{7}$$

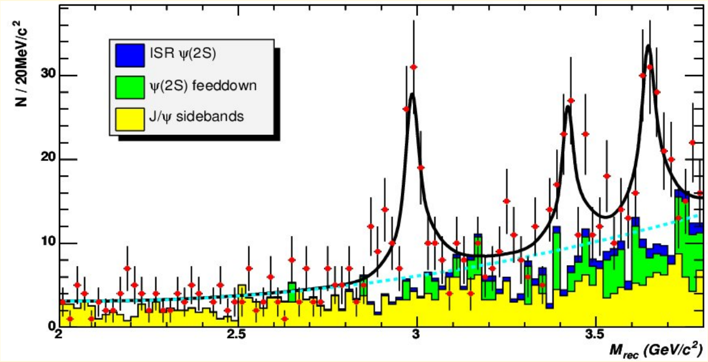
where $B^{\eta_c}[> 2]$ is the branching fraction for the η_c decay into more than 2 charged tracks, so Eq. (6) and Eq. (7) give the lower bound for the cross section.

- The result from BaBar and Belle could be larger than theoretical predictions by a factor of 5 .

[*] K. Abe *et al.* [BELLE Collaboration], Phys. Rev. Lett. **89**, 142001 (2002). P. Pakhlov [Belle Collaboration], arXiv:hep-ex/0412041.

[**] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. D **72**, 031101 (2005)

Double $c\bar{c}$ production



124 fb^{-1}, preliminary, hep-ex in preparation.

	J/ψ $c\bar{c}$	η_c	χ_{c0}	$\eta_c(2S)$
Expt	$\sigma \times \mathcal{B}_{>2}$ (BaBar)	$17.6 \pm 2.8 \pm 2.1$	$10.3 \pm 2.5 \pm 1.8$	$16.4 \pm 3.7 \pm 3.0$
	$\sigma \times \mathcal{B}_{>2}$ (BELLE)	$25.6 \pm 2.8 \pm 3.4$	$6.4 \pm 1.7 \pm 1.0$	$16.5 \pm 3.0 \pm 2.4$
Th.	Braaten Lee PRD 67 054007(2003)	2.31 ± 1.09	2.28 ± 1.03	0.96 ± 0.45
	Liu, He, Chao hep-ph/0408141	5.5	6.9	3.7

Applicability of NRQCD : Bondar, Chernyak, hep-ph/0412335

BaBar's result of double charmonium production

Some theoretical studies have been suggested in order to resolve this large discrepancies.

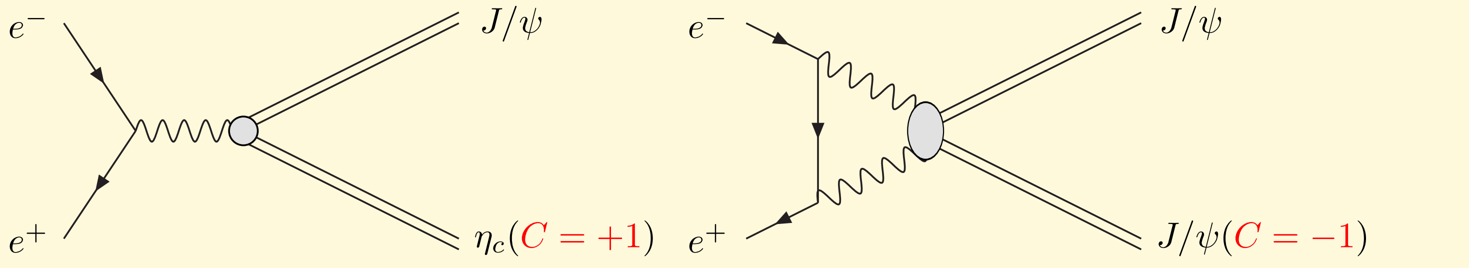
To the $e^+e^- \to J/\psi + c\bar{c}$,

- Liu, He, and Chao calculated the color-octet contribution and $J/\psi + c\bar{c}$ production via two photons in the NRQCD factorization formulas. But it can not make up the large discrepancy[*].
- Hagiwara, *et al.* introduced a large renormalization K factor $(K \sim 4)$[**].
- Kaidalov introduced the nonperturbative quark-gluon-string model [***].
- Kang, Lee, and Lee get $\sigma(e^+e^- \to J/\psi + c\bar{c} + X)/\sigma(e^+e^- \to J/\psi + X) = 0.049$ in color-evaporation-model[****].
- Berezhnoy calculate the cross section with the light cone wave function with massive charm quark. He find the dependence on the wave function shape can be neglected for the process $e^+e^- \to J/\psi c\bar{c}$. His result is consistent with our relativistic corrections result [*****].

[*] K. Y. Liu, Z. G. He and K. T. Chao, Phys. Rev. D **69**, 094027 (2004) [arXiv:hep-ph/0301218].
K. Y. Liu, Z. G. He and K. T. Chao, Phys. Rev. D **68**, 031501 (2003) [arXiv:hep-ph/0305084].
[**] K. Hagiwara, *et al.*, Phys. Rev. D **70**, 034013 (2004) [arXiv:hep-ph/0401246].
V. V. Braguta, arXiv:0709.3885 [hep-ph].
[***] A. B. Kaidalov, JETP Lett. **77**, 349 (2003) [arXiv:hep-ph/0301246].
[****] D. Kang, *et al.*, Phys. Rev. D **71**, 094019 (2005) [arXiv:hep-ph/0412381];
[*****] A. V. Berezhnoy, arXiv:hep-ph/0703143.

To the $e^+e^- \to J/\psi + \eta_c$

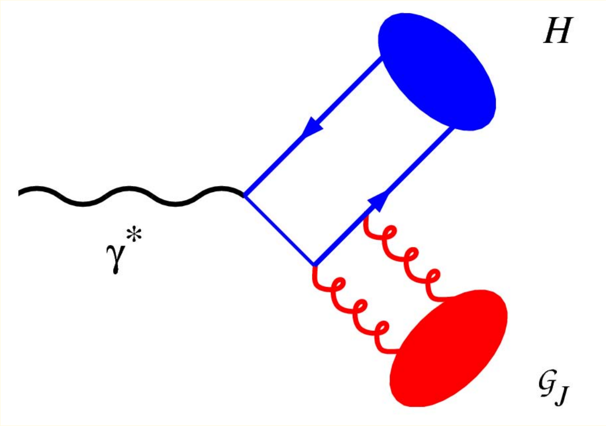
- Bodwin, Braaten, and Lee proposed that processes proceeding via two virtual photons may be important, and Belle data for $J/\psi + \eta_c$ might essentially include the $J/\psi + J/\psi$ events which were produced via two photons [*]. They got that, the cross section of $J/\psi + J/\psi$ is larger than that of $J/\psi + \eta_c$ by about a factor of 3.7.



One photon and two photons process

[*]G. T. Bodwin, J. Lee and E. Braaten, Phys. Rev. Lett. **90**, 162001 (2003).Phys. Rev. D **67**, 054023 (2003).

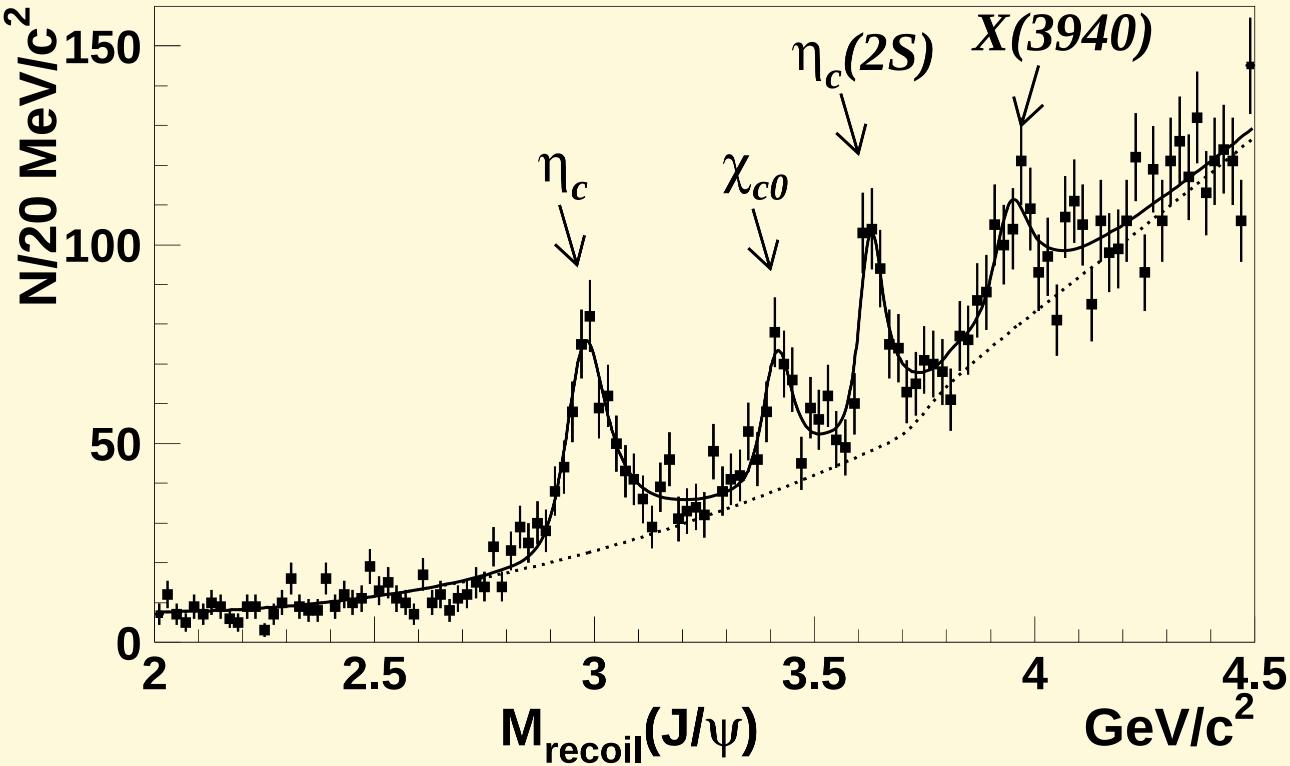
Brodsky, Goldhaber, and Lee suggested that the final states observed by Belle might contain J/ψ and a $M \sim 3\text{GeV}$ spin-J glueball $\mathcal{G}_J$ ($J = 0, 2$)[*].



Feynman diagram of glueball production

[*]S. J. Brodsky, A. S. Goldhaber and J. Lee, Phys. Rev. Lett. **91**, 112001 (2003).

Motivated by these proposals, the Belle Collaboration presented an updated analysis, and ruled out the $J/\psi + J/\psi$ and spin-0 glueball scenarios[*]. η_c, χ_{c0}, $\eta_c(2S)$, and $X(3940)$ was seen, and no evidence for J/ψ, χ_{c1}, and $\psi(2S)$.



The mass of the system recoiling against the reconstructed J/ψ in Belle

[*]K. Abe *et al.,*(Belle Collaboration), Phys.Rev. D70 (2004) 071102.K. Abe *et al.*, arXiv:hep-ex/0507019.

- Ma and Si studied this process by using light-cone distribution amplitudes to parameterize nonperturbative effects [*].
- Similar approaches were also considered by Bondar and Chernyak [**]. But the cross section is sensitive to the specific form of quark distributions.
- Bodwin, Kang, and Lee compute light-cone distribution functions by the Cornell potential model . Their resulting light-cone cross section is similar in magnitude to the NRQCD factorization cross sections and [***].
- Berezhnoy calculate the cross section with the non-zero value of charm quark mass. He find $e^+e^- \to J/\psi\eta_c$ at 10.6 GeV can be increased by factor 1.2-3 depending on the c quark mass value and the wave function shape. his result is consistent with relativistic corrections result [****].

[*]J.P. Ma and Z.G. Si, Phys.Rev. D70 (2004) 074007.
[**] A. E. Bondar and V. L. Chernyak, Phys. Lett. B **612**, 215 (2005)
[***]G. T. Bodwin, D. Kang and J. Lee, arXiv:hep-ph/0603185.
[****] A. V. Berezhnoy, arXiv:hep-ph/0703143.

- Ebert and Martynenko have calculated the relativistic effects, they obtain a growth of the cross section by a factor $2 \sim 2.5$ in the range of the center-of-mass energy $\sqrt{s} = 6 \sim 12$ GeV[*].
- Bodwin, Lee, and Yu present a new calculation, in the NRQCD factorization formalism, of the relativistic corrections to the double-charmonium cross section $\sigma[e^+e^- \to J/\psi + \eta_c]$. The coming talk[**].
- Choi and Ji analyze the exclusive charmonium $J/\psi + \eta_c$ pair production in e^+e^- annihilation using the nonfactorized perturbative QCD and the light-front quark model(LFQM) that goes beyond the peaking approximation. Their nonfactorized result enhances the NRQCD result by a factor of $3 \sim 4$ at $\sqrt{s} = 10.6$ GeV [***].
- Hagiwara, Kou and Qiao obtained a result consistent with Braaten and Liu, and conjectured that higher-order corrections in α_s may be huge [****].

[*] D. Ebert and A. P. Martynenko, arXiv:hep-ph/0605230.
[**] G. T. Bodwin, J. Lee and C. Yu, arXiv:0710.0995 [hep-ph].
[***]H. M. Choi and C. R. Ji, arXiv:0707.1173 [hep-ph].
[****]K. Hagiwara, E. Kou and C. F. Qiao, Phys. Lett. B **570**, 39 (2003).

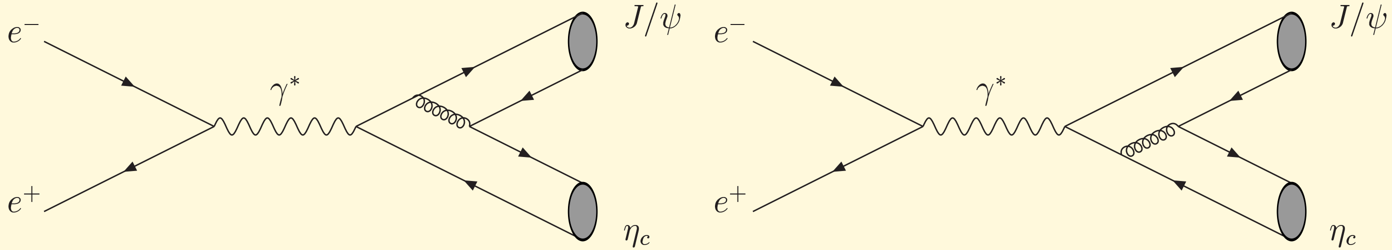
- It was claimed by Quarkonium Working Group that ”The discrepancies between theory and experiment in these measurements are among the largest in the standard model”[*].
- There are also other suggestions to resolve these problems can be found in the CERN yellow report issued by Quarkonium Working Group [*].

[*]N. Brambilla *et al.*, CERN Yellow Report, CERN-2005-005

2. $e^+e^- \to J/\psi + \eta_c$

2.1. Leading Order Calculation of $e^+e^- \to J/\psi + \eta_c$

In leading order in strong coupling constant α_s, $J/\psi + \eta_c$ can be produced at order $\alpha^2\alpha_s^2$.



$+2$ flipped diagrams

Born diagrams for $e^-(k_1)e^+(k_2) \to J/\psi(2p_1)\eta_c(2p_2)$**.**

Using the NRQCD factorization formalism[*] and selecting feynman gauge, then we get the amplitude of born diagrams

$$i\mathcal{M}_{Born} = \frac{4096\pi e_c \alpha \alpha_s m |R_s(0)|^2}{3s^3} \epsilon_{\alpha\beta\nu\rho} p_1^{\alpha} p_2^{\beta} \varepsilon^{*\nu} \bar{v}_e(k_2) \gamma^{\rho} u_e(k_1), \tag{8}$$

$s = (k_1 + k_2)^2$, $e_c = \frac{2}{3}$ is the fractional electric charge of the charm quark.

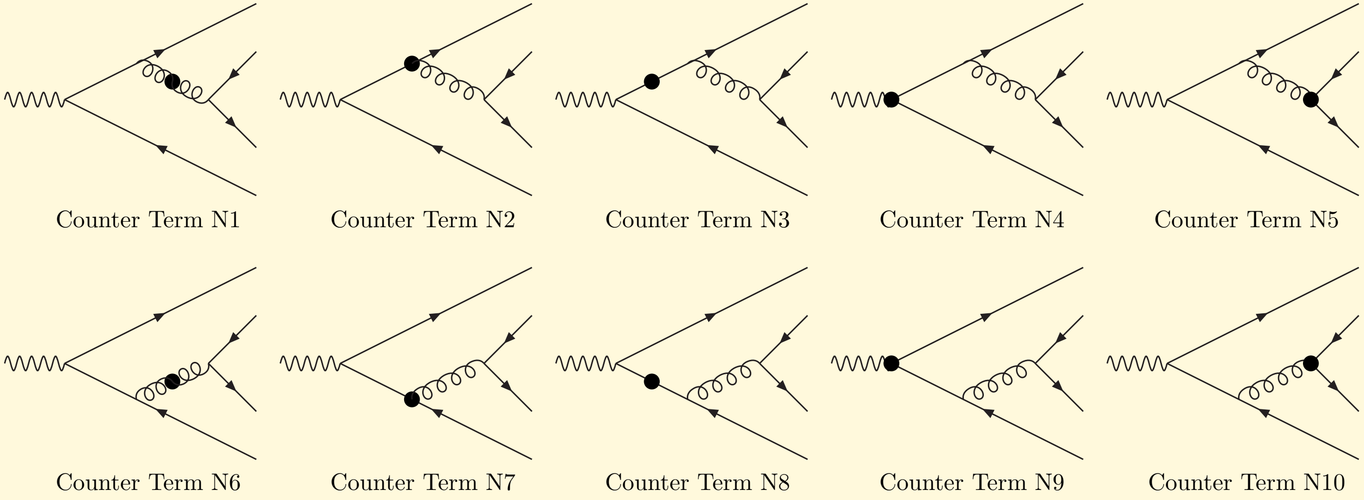
The vector indices ρ is for the virtual photon.

ε is the polarization vector of J/ψ. p_1 and p_2 are half momenta of J/ψ and η_c respectively.

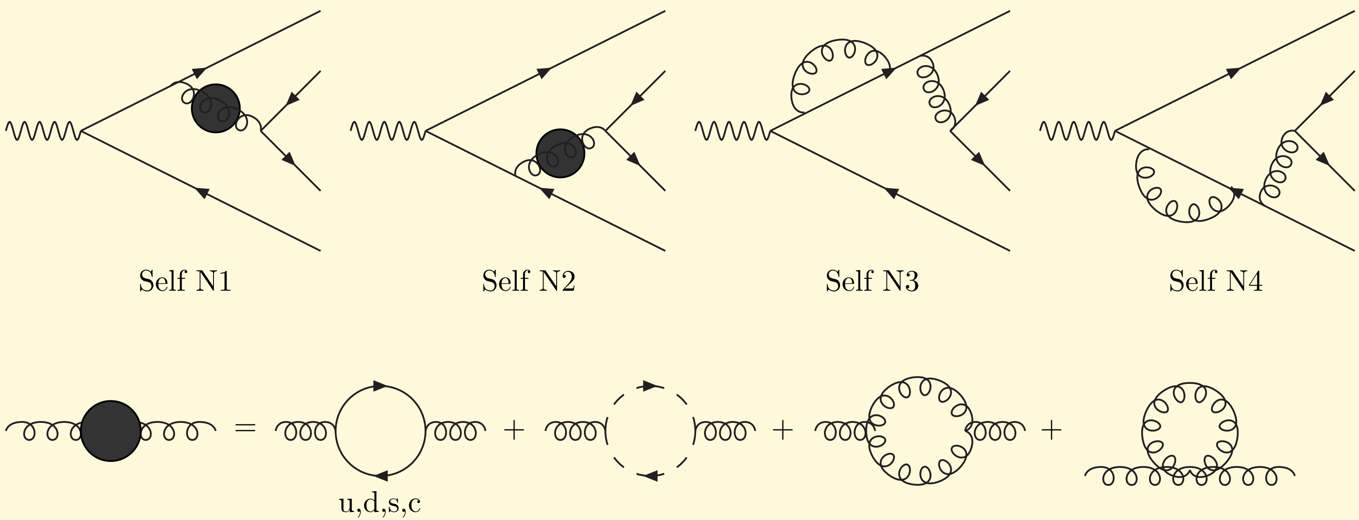
The coefficients $R_S(0)$ is the radial wave function at origin of the bound states.

[*] G. T. Bodwin, E. Braaten and G. P. Lepage, Phys. Rev. D **51**, 1125 (1995) [Erratum-ibid. D **55**, 5853 (1997)]

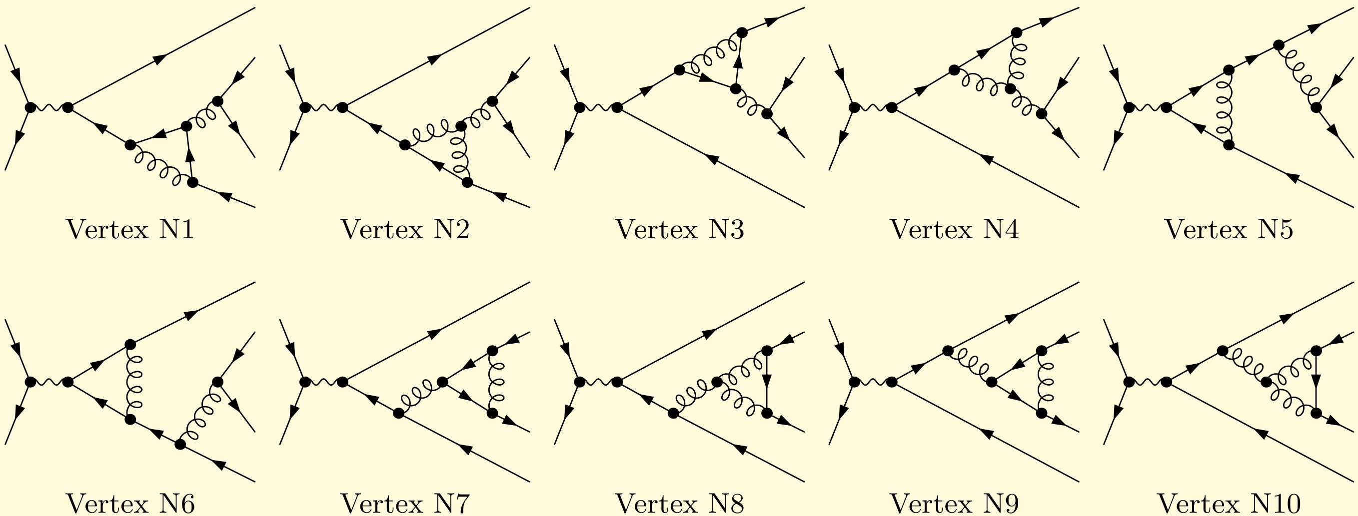
2.2. Virtual Correction of $e^+e^- \longrightarrow J/\psi + \eta_c$



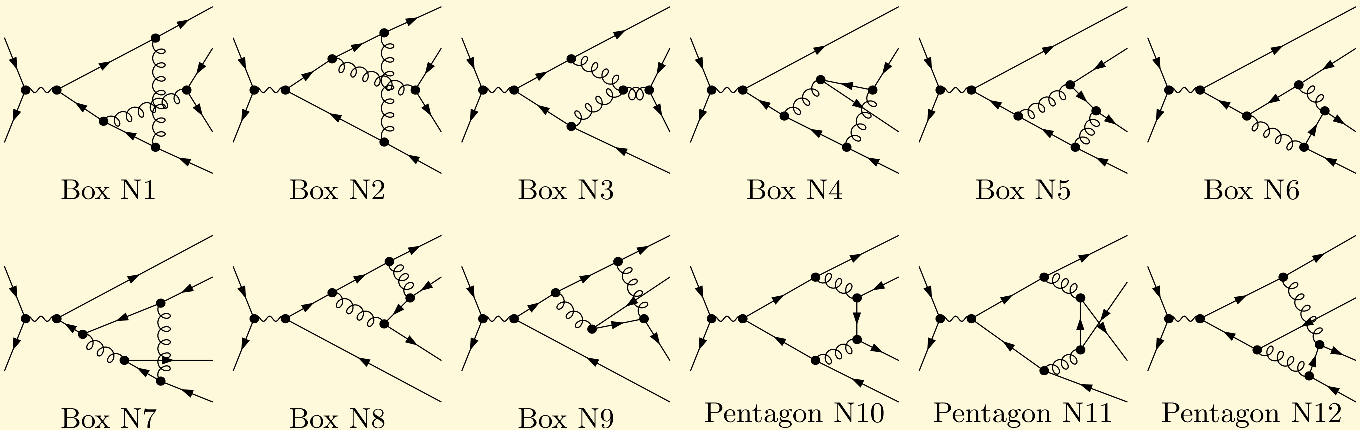
Half of counter term diagrams for $e^-(k_1)e^+(k_2) \rightarrow J/\psi(2p_1)\eta_c(2p_2)$**.**



Half of self-energy diagrams for $e^-(k_1)e^+(k_2) \to J/\psi(2p_1)\eta_c(2p_2)$**.**



Half of vertex diagrams for $e^-(k_1)e^+(k_2) \to J/\psi(2p_1)\eta_c(2p_2)$**.**



Half of box diagrams for $e^-(k_1)e^+(k_2) \to J/\psi(2p_1)\eta_c(2p_2)$**.**

There are three divergence :

- The self-energy and triangle diagrams are in general ultraviolet (UV) divergent
- while the triangle, box, and pentagon diagrams are in general infrared (IR) divergent
- Box N5 and N8 and Pentagon N10, which have a virtual gluon line connected with the $c\bar{c}$ in a meson, also contain the Coulomb singularities due to the exchange of longitudinal gluons between c and $\bar{c}$.

For the Coulomb-singularity part of the virtual cross section, we find

$$\sigma \;=\; |R_S(0)|^4 \hat{\sigma}^{(0)} \left(1 + \frac{2\pi\alpha_s C_F}{v} + \frac{\alpha_s \hat{C}}{\pi} + \mathcal{O}(\alpha_s^2) \right), \tag{9}$$

Leading order of operators $\left\langle \mathcal{O}^{J/\psi}\left[{}^3S_1^{(1)}\right]\right\rangle$ and $\left\langle \mathcal{O}^{\eta_c}\left[{}^1S_0^{(1)}\right]\right\rangle$ are associated with $R_S(0)$, and next-to-leading order are $\pi\alpha_s C_F/v$ [*]. And two operators give a factor of 2 at $\mathcal{O}(\alpha_s)$, just the Coulomb-singularity term in Eq. (9).

Then the corresponding contribution of Coulomb-singularity has to be factored out and mapped into the wave functions of J/ψ and η_c:

$$\begin{aligned} \sigma \;&=\; |R_S(0)|^4 \left(1 + \frac{2\alpha_s}{\pi} \frac{C_F \pi^2}{v} \right) \hat{\sigma}^{(0)} \left[1 + \frac{\alpha_s}{\pi}\hat{C} + \mathcal{O}(\alpha_s^2) \right] \\ &\Rightarrow |R_S(0)|^4 \, \hat{\sigma}^{(0)} \left[1 + \frac{\alpha_s}{\pi}\hat{C} + \mathcal{O}(\alpha_s^2) \right] \quad . \end{aligned} \tag{10}$$

[*]G. T. Bodwin, E. Braaten and G. P. Lepage, Phys. Rev. D **51**, 1125 (1995) [Erratum-ibid. D **55**, 5853 (1997)]

We can consider the Coulomb-singularity in another picture. The next-to-leading order leptonic width of J/ψ is :

$$\Gamma(J/\psi \to e^+e^-) = |R_S(0)|^2 \frac{16\alpha^2}{9m_{J/\psi}^2} \left(1 + \frac{\pi\alpha_s C_F}{v} - 4C_F\alpha_s/\pi\right). \tag{11}$$

Then we have:

$$\begin{aligned} \sigma &= |R_S(0)|^4 \hat{\sigma}^{(0)} \left(1 + \frac{2\pi\alpha_s C_F}{v} + \frac{\alpha_s \hat{C}}{\pi} + \mathcal{O}(\alpha_s^2)\right) \\ &\Rightarrow \frac{\hat{\sigma}^{(0)} \left(1 + \frac{2\pi\alpha_s C_F}{v} + \frac{\alpha_s \hat{C}}{\pi}\right) \Gamma(J/\psi \to e^+e^-)^2}{\left(1 + \frac{\pi\alpha_s C_F}{v} - 4C_F\alpha_s/\pi\right)^2} \left(\frac{9m_{J/\psi}^2}{16\alpha^2}\right)^2 \\ &\Rightarrow \frac{\hat{\sigma}^{(0)} \left(1 + \frac{\alpha_s \hat{C}}{\pi}\right) \Gamma(J/\psi \to e^+e^-)^2}{(1 - 4C_F\alpha_s/\pi)^2} \left(\frac{9m_{J/\psi}^2}{16\alpha^2}\right)^2. \end{aligned} \tag{12}$$

The $|R_S(0)|^2$ is replaced by the experimental date $\Gamma(J/\psi \to e^+e^-)$. Certainly, we should use leading-order leptonic width of J/ψ at the leading order calculation in this picture.

- UV term is canceled by counter terms.
- Then the result is UV-, IR-, and Coulomb-finite.
- So it need not and can not introduce real correction for this exclusive process.

2.3. Relativistic Corrections of $e^+e^- \to J/\psi + \eta_c$

The amplitude of the process can be expanded in terms of the quark relative momentum in charmonium:

$$
\begin{aligned}
&M(e^+e^- \to (c\bar{c})_{^3S_1} + (c\bar{c})_{^1S_0}) = (\frac{m_c}{E_1}\frac{m_c}{E_2})^{1/2} A(q_\psi, q_{\eta_c}) = \\
&(\frac{m_c}{E_1}\frac{m_c}{E_2})^{1/2}(A(0,0) + q_\psi^\alpha \frac{\partial A}{\partial q_\psi^\alpha}\Big|_{q_\psi = q_{\eta c} = 0} + q_{\eta_c}^\alpha \frac{\partial A}{\partial q_{\eta_c}^\alpha}\Big|_{q_\psi = q_{\eta c} = 0} + \\
&\frac{1}{2} q_\psi^\alpha q_\psi^\beta \frac{\partial^2 A}{\partial q_\psi^\alpha \partial q_\psi^\beta}\Big|_{q_\psi = q_{\eta c} = 0} + \frac{1}{2} q_{\eta_c}^\alpha q_{\eta_c}^\beta \frac{\partial^2 A}{\partial q_{\eta_c}^\alpha \partial q_{\eta_c}^\beta}\Big|_{q_\psi = q_{\eta c} = 0} + \ldots),
\end{aligned}
\tag{13}
$$

and $A(q_\psi, q_{\eta_c})$ is expressed as

$$
\begin{aligned}
A(q_\psi, q_{\eta_c}) = &\sum_{\lambda_1\lambda_2\lambda_3\lambda_4} \sum_{ijkl} \langle \frac{1}{2}, \lambda_1; \frac{1}{2}, \lambda_2 | 1, S_z \rangle \langle \frac{1}{2}, \lambda_3; \frac{1}{2}, \lambda_4 | 0, 0 \rangle \langle 3, i; \bar{3}, j | 1 \rangle \langle 3, k; \bar{3}, l | 1 \rangle \\
&A(e^+e^- \to c_{\lambda 1, i}(\frac{P1}{2} + q_\psi) \bar{c}_{\lambda 2, j}(\frac{P1}{2} - q_\psi) + c_{\lambda 3, k}(\frac{P2}{2} + q_{\eta_c}) \bar{c}_{\lambda 4, l}(\frac{P2}{2} - q_{\eta_c})),
\end{aligned}
\tag{14}
$$

2.4. Numerical Result of $e^+e^- \to J/\psi + \eta_c$

To be consistent with the NLO result the matrix element should be extracted from $\Gamma[\eta_c \to \gamma\gamma]$ and $\Gamma[J/\psi \to e^+e^-]$ at NLO of α_s and v^2:

$$\Gamma[\eta_c \to \gamma\gamma] = 2e_c^4\pi\alpha^2\Big((1 - \frac{(20-\pi^2)\alpha_s}{3\pi})\frac{\langle 0|\mathcal{O}_1(^1S_0^{\eta_c})|0\rangle}{m_c^2} - \frac{4}{3}\frac{\langle 0|\mathcal{P}_1(^1S_0^{\eta_c})|0\rangle}{m_c^4}\Big), \tag{15a}$$

$$\Gamma[J/\psi \to e^+e^-] = \frac{2e_c^2\pi\alpha^2}{3}\Big((1 - \frac{16\alpha_s}{3\pi})\frac{\langle 0|\mathcal{O}_1(^3S_1^{\psi})|0\rangle/3}{m_c^2} - \frac{4}{3}\frac{\langle 0|\mathcal{P}_1(^3S_1^{\psi})|0\rangle/3}{m_c^4}\Big), \tag{15b}$$

$$\Gamma[J/\psi \to LH] = (\frac{20\alpha_s^3}{243}(\pi^2-9))\Big((1-2.55\frac{\alpha_s}{\pi})\frac{\langle 0|\mathcal{O}_1(^3S_1^{\psi})|0\rangle/3}{m_c^2} - \frac{19\pi^2-132}{12\pi^2-108}\frac{\langle 0|\mathcal{P}_1(^3S_1^{\psi})|0\rangle/3}{m_c^4}\Big). \tag{15c}$$

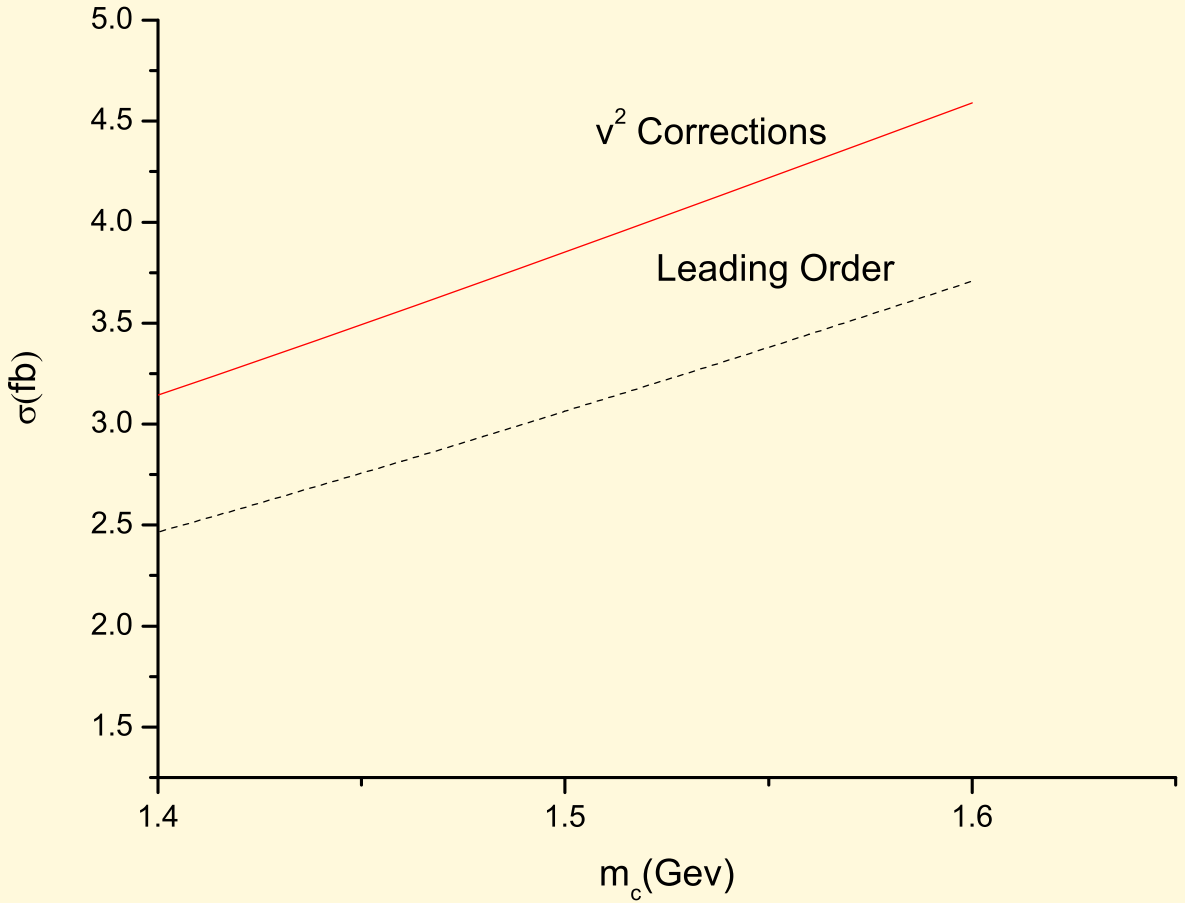
The experimental data of these decay rates can be found from PDG[*], and we choose their central values $\Gamma[J/\psi \to e^+e^-] = 5.55$KeV, $\Gamma[J/\psi \to LH] = 69.3$KeV, and $\Gamma[\eta_c \to \gamma\gamma] = 7.14$KeV.

[*] W. M. Yao *et al.* [Particle Data Group], J. Phys. G **33**, 1 (2006).

TABLE I: Experimental and calculated cross sections of $\sigma[e^+e^- \to J/\psi + \eta_c]$ with m_c =1.5 GeV and α_s=0.26. See text for the definitions of $\sigma_{LO(\alpha_s,v^2)}$, $\sigma_{NLO(\alpha_s)}$, $\sigma_{NLO(v^2)}$, and $\sigma_{NLO(\alpha_s,v^2)}$.

Experimental Result					
$\sigma_{Belle}[e^+e^- \to J/\psi + \eta_c] \times \mathcal{B}^{\eta_c}[\geq 2]$(fb)			$\sigma_{Babar}[e^+e^- \to J/\psi + \eta_c] \times \mathcal{B}^{\eta_c}[\geq 2]$(fb)		
$25.6 \pm 2.8 \pm 3.4$			$17.6 \pm 2.8 \pm 2.1$		
Theoretical Result					
$\langle 0\|\mathcal{O}_1(^1S_0^{\eta_c})\|0\rangle$	$\frac{\langle 0\|\mathcal{O}_1(^3S_1^{\psi})\|0\rangle}{3}$	$\frac{\langle 0\|\mathcal{P}_1(^1S_0^{\eta_c})\|0\rangle}{m_c^2}$	$\frac{\langle 0\|\mathcal{P}_1(^3S_1^{\psi})\|0\rangle}{3m_c^2}$	α_s	σ (fb)
0.243Gev^3	0.252Gev^3	0	0	$\alpha_s = 0.26$	$\sigma_{LO(\alpha_s,v^2)} = 2.26$
0.337Gev^3	0.450Gev^3	0	0	$\alpha_s = 0.26$	$\sigma_{NLO(\alpha_s)} = 10.92$
0.286Gev^3	0.295Gev^3	0.0321Gev^3	0.0321Gev^3	$\alpha_s = 0.26$	$\sigma_{NLO(v^2)} = 3.87$
0.432Gev^3	0.573Gev^3	0.0514Gev^3	0.0514Gev^3	$\alpha_s = 0.26$	$\sigma_{NLO(\alpha_s,v^2)} = 20.04$

$$\begin{aligned} K_{v^2} &= 1.71 \\ K_{\alpha_s} &= 4.83 \\ K_{v^2,\alpha_s} &= 8.67 \end{aligned} \qquad (16)$$



$e^+e^- \to J/\psi + \eta_c$ **cross sections with relativistic corrections to long-distance matrix elements extracted from charmonium decays (without NLO QCD radiative corrections). The lower line represents the LO result in v, and the upper line represents the result with v^2 corrections to the short-distance coefficients. Here the coupling constant is fixed as $\alpha_s = 0.26$.**

If we include the radiative corrections only, and the QED contributions is taken into account.

State	$\sigma_{BaBar}B[>2]$	$\sigma_{Belle}B[>2]$ ($B[>0]$)	$m_c=1.5$ GeV Tree	$m_c=1.5$ GeV Loop	$m_c=1.4$ GeV Tree	$m_c=1.4$ GeV Loop
$J/\psi\eta_c$	$17.6\pm2.8^{+1.5}_{-2.1}$	$25.6\pm2.8\pm3.4$	9.9	18.2	11.7	22.1
$\psi(2S)\eta_c$		$16.3\pm4.6\pm3.9$	6.3	11.5	7.4	13.9
$J\psi\eta_c(2S)$	$16.4\pm3.7^{+2.4}_{-3.0}$	$16.5\pm3.0\pm2.4$	6.3	11.5	7.4	13.9
$\psi(2S)\eta_c(2S)$		$16.0\pm5.1\pm3.8$	4.0	7.3	4.7	8.8
$J\psi\eta_c(3S)$			5.4	9.9	6.4	12.0

Comparison of theoretical predictions with experimental data in fb for $e^+e^-\to J/\psi(\psi(2S))+\eta_c(\eta_c(2S))$

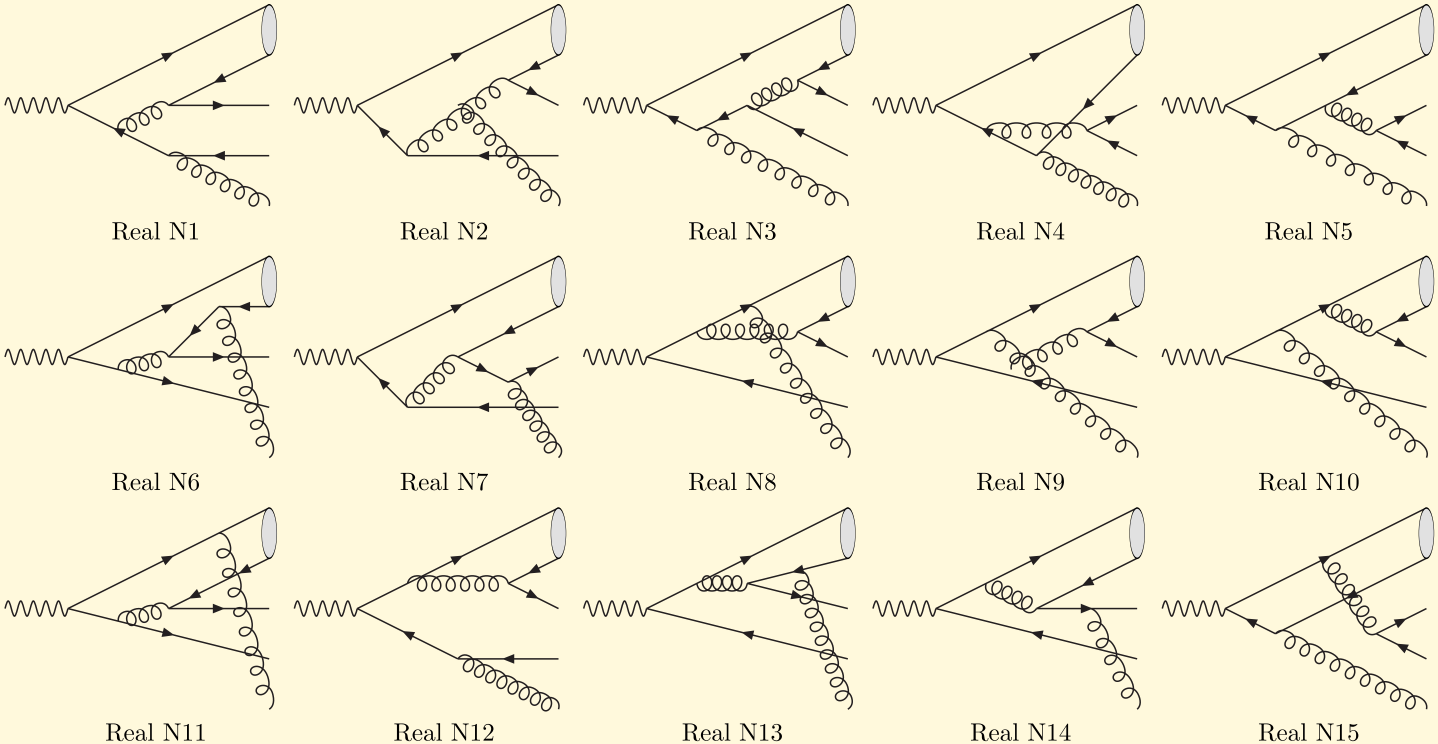
and $J/\psi+\eta_c(3S)$. Here $|R_S(0)|^2=1.01\mathrm{GeV}^3$, $|R_{2S}(0)|^2=0.543\mathrm{GeV}^3$,$|R_{3S}(0)|^2=0.410\mathrm{GeV}^3$, $\Lambda=0.338\mathrm{GeV}$, $\sqrt{s}=10.6\mathrm{GeV}$, and $\mu=2m_c$. $B[>2](B[>0])$ means the branching fraction for the $\eta_c(\eta_c(2S))$ to decay into more than 2 (at least 1) charged tracks. When the $\eta_c(\eta_c(2S))$ is recoiled with $\psi(2S)$, $B[>0]$ should be used.

3. $e^+e^- \to J/\psi + c\bar{c}$

3.1. Leading Order and Virtual Correction of $e^+e^- \to J/\psi + c\bar{c}$

The leading order and virtual correction of $e^+e^- \to J/\psi + c\bar{c}$ is similar with $\boldsymbol{e^+e^- \to J/\psi + \eta_c}$. The feynman diagrams are same, so them were not showed here.

3.2. Real Correction of $e^+e^- \to J/\psi + c\bar{c}$



Fifteen of the thirty real correction diagrams.

- $\mathrm{BoxN3}$, $\mathrm{BoxN1}+\mathrm{BoxN4}$, $\mathrm{BoxN6}+\mathrm{BoxN7}+\mathrm{PentagonN12}$, $\mathrm{VertexN9}+\mathrm{BoxN9}$, $\mathrm{VertexN7}+\mathrm{BoxN2}+\mathrm{PentagonN11}$ are IR finite respectively.
- The IR term in counter term diagram and $\mathrm{BoxN5}+\mathrm{BoxN8}+\mathrm{PentagonN10}$ should be canceled by the real corrections.
- The IR term in $\mathrm{Real\ N1}$, $\mathrm{Real\ N7}$, $\mathrm{Real\ N12}$, and $\mathrm{Real\ N14}$ is just canceled by the IR term in counter term diagram and $\mathrm{BoxN5}+\mathrm{BoxN8}+\mathrm{PentagonN10}$.
- $\mathrm{Real\ N4}$, $\mathrm{Real\ N6}$, $\mathrm{Real\ N8}$, $\mathrm{Real\ N11}$, $\mathrm{Real\ N13}$, and $\mathrm{Real\ N15}$, which gluon connect the external charm and anti-charm in J/ψ , were canceled by themselves.
- The other five diagrams $\mathrm{Real\ N2}$, $\mathrm{Real\ N3}$, $\mathrm{Real\ N5}$, $\mathrm{Real\ N9}$, and $\mathrm{Real\ N10}$ are independent to IR singularity.
- Coulomb singularity terms in $\mathrm{BoxN8}+\mathrm{PentagonN10}$ should be canceled by next-to-leading order of operators $\left\langle \mathcal{O}^{J/\psi}\left[{}^1S_3^{(1)}\right]\right\rangle$.

- UV term is canceled by counter terms.
- IR term is canceled by virtual correction and real correction .
- Coulomb singularity is canceled by next-to-leading order of operators $\left\langle \mathcal{O}^{J/\psi}\left[{}^1S_3^{(1)}\right]\right\rangle$.
- Then the result is UV-, IR-, and Coulomb-finite.

3.3. Numerical Result of $e^+e^- \to J/\psi + c\bar{c}$

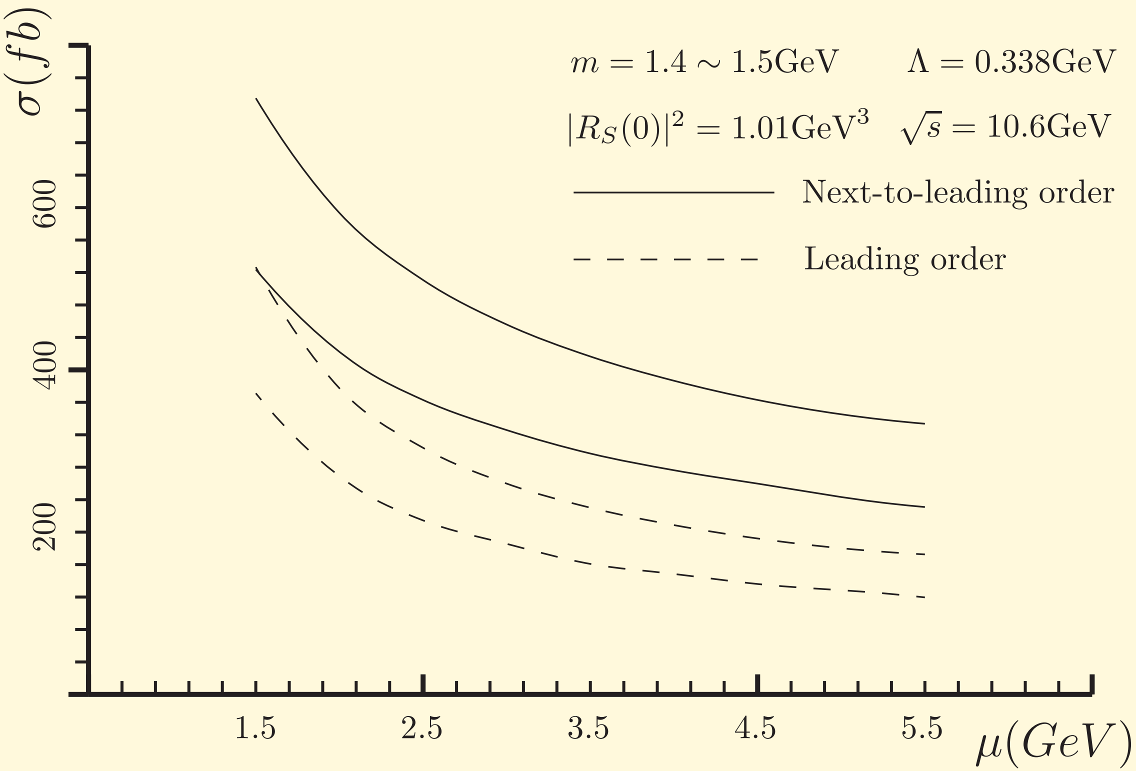
Using the experimental value $\Gamma(J/\psi \to e^+e^-) = 5.55 \pm 0.14 \pm 0.02$ KeV[*], we obtain $|R_S(0)|^2 = 1.01\text{GeV}^3$. Taking $\Lambda^{(4)}_{\overline{\text{MS}}} = 338\text{MeV}$, $m_{J/\psi} = m_{\eta_c} = 2m$ (in the nonrelativistic limit). If we set $m = 1.4\text{GeV}$ and $\mu = 2m$, the cross section at next-to-leading order of α_s is

$$\sigma(e^+ + e^- \to J/\psi + c\bar{c} + X) = 0.47 \text{ pb}. \tag{17}$$

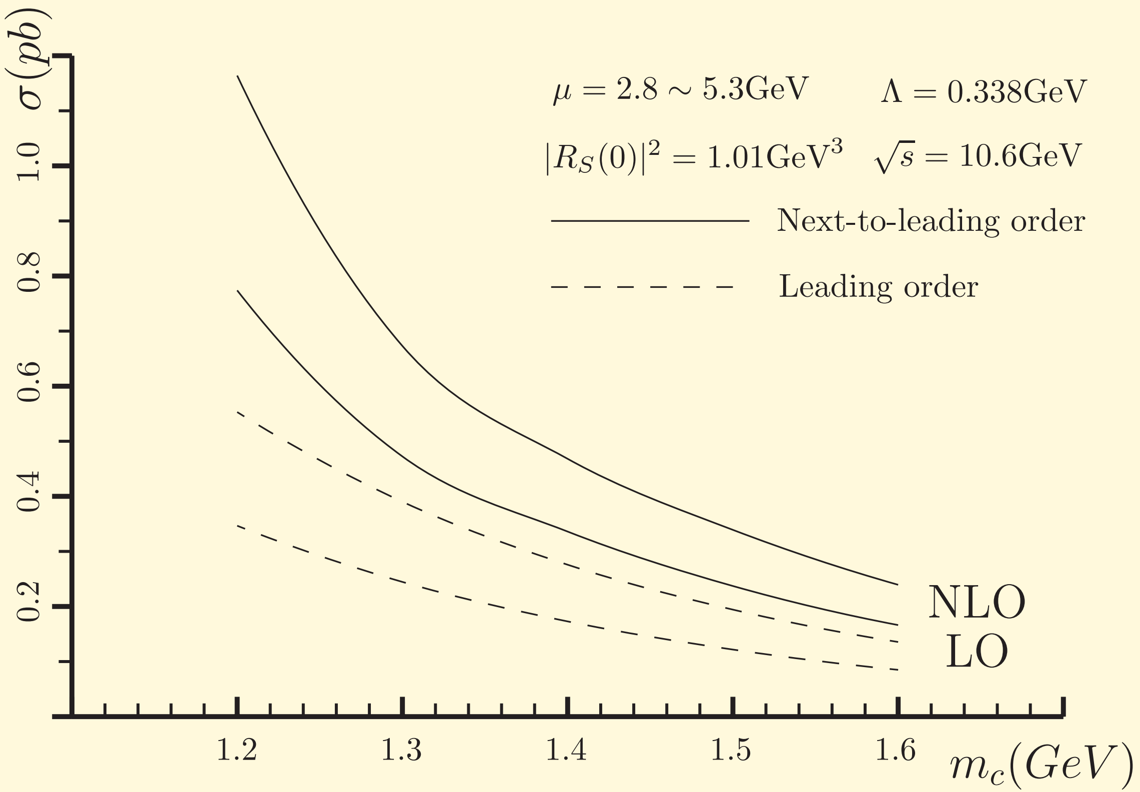
It is about a factor of 1.7 larger than leading order cross section 0.27 pb.

The NLO relativistic correction is very weak. It gives an enhancement by a factor of 0.42%.

[*] W. M. Yao *et al.* [Particle Data Group], J. Phys. G **33**, 1 (2006).



Cross sections as functions of the renormalization scale μ. Here $|R_S(0)|^2 = 0.978\text{GeV}^3$, $\Lambda = 0.338\text{GeV}$, $\sqrt{s} = 10.6\text{GeV}$; NLO results are represented by solid lines and LO one by dashed lines; the upper line is for $m = 1.4\text{GeV}$ and the corresponding lower line is for $m = 1.5\text{GeV}$.

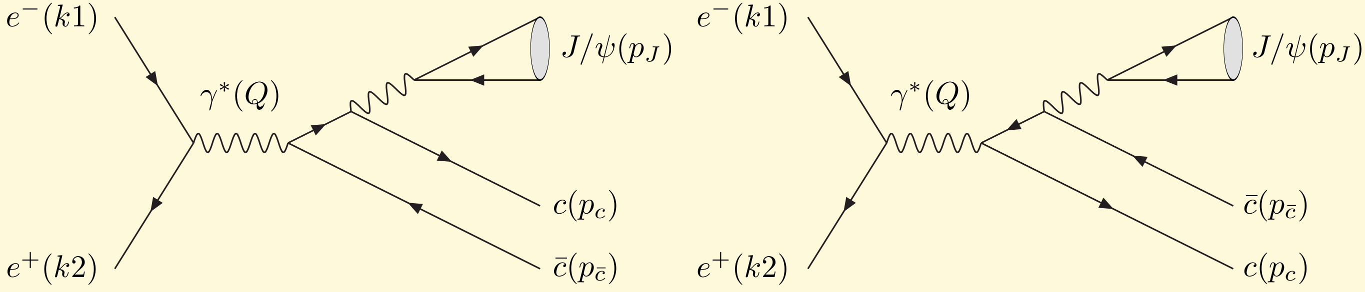


Cross sections as functions of the charm quark mass m_c. Here $|R_S(0)|^2 = 1.01\text{GeV}^3$, $\Lambda = 0.338\text{GeV}$, $\sqrt{s} = 10.6\text{GeV}$; NLO results are represented by solid lines and LO one by dashed lines; the upper line is for $\mu = 2.8\text{GeV}$ and the corresponding lower line is for $\mu = 5.3\text{GeV}$.

For the experiment date is the prompt $J/\psi + c\bar{c} + X$ cross section. We should consider the contribution of $e^+e^- \to 2\gamma^* \to J/\psi + c\bar{c}$, $e^+e^- \to \psi(2S) + c\bar{c} + X \to J/\psi + c\bar{c} + X$, $e^+e^- \to \chi_{cJ} + c\bar{c} \to J/\psi + c\bar{c} + X$, and so on. Then the cross section is:

$$\begin{aligned}\sigma_{prompt}(e^+e^- \to J/\psi + c\bar{c}X) = \sigma_{direct}(e^+e^- \to J/\psi + c\bar{c}X) \\ +\sigma_{direct}(e^+e^- \to \psi(2S) + c\bar{c}X) \times \mathrm{B}(\psi(2S) \to J/\psi X) \\ +\sum_J \sigma(e^+e^- \to \chi_{cJ} + c\bar{c}X) \times \mathrm{B}(\chi_{cJ} \to J/\psi X) + \cdots\end{aligned} \tag{18}$$

Two of six QED diagrams of $e^+e^- \to \gamma^* \to J/\psi + c\bar{c}$ were shown here. The correspond cross section is 8fb .



Two of six QED diagrams .

The cross sections of both color octet and singlet production of $e^+e^- \to \chi_{cJ} + c\bar{c} + X$ had been calculated by Liu [*]. Using Liu's result and the branching ratio for the $\chi_{cJ} \to J/\psi\gamma$ transition fraction $B = 1.31\%, 35.6\%, 20.2\%$ for $J = 0, 1, 2$ respectively[*]. The color octet J/ψ contribution also calculated by Liu [**] . Sum of them, we get their contributions are 14 fb.

[*] W. M. Yao *et al.* [Particle Data Group], J. Phys. G **33**, 1 (2006).

[**] K. Y. Liu, Z. G. He and K. T. Chao, Phys. Rev. D **69**, 094027 (2004).

$e^+e^- \to 2\gamma^* \to J/\psi c\bar{c}$ had been calculated by Liu in [*]. Using new parameters and Liu's result,

$$\sigma(e^+e^- \to 2\gamma^* \to J/\psi c\bar{c}) = 23 \times \frac{1.01}{0.810}\text{fb} = 29\text{fb}, \tag{19}$$

The factor $\frac{1.01}{0.810}$ is from the wave function at the origin.

In the framework of NRQCD, the difference between J/ψ and $\psi(2S)$ are from the wave function at origin. The contribution from $\psi(2S) \to J/\psi$ is enlarged the direct production of $J/\psi + c\bar{c} + X$ by a factor:

$$\frac{|R_{2S}(0)|^2}{|R_{1S}(0)|^2}\mathrm{B}(\psi(2S) \to J/\psi X) = \frac{M^2_{\psi(2S)}\Gamma_{\psi(2S)\to e^+e^-}}{M^2_{J/\psi}\Gamma_{J\psi\to e^+e^-}}\mathrm{B}(\psi(2S) \to J/\psi X) \tag{20}$$

By using $\Gamma(\psi(2S) \to e^+e^-) = 2.48 \times 10^{-6}\text{GeV}$ and the branching ratio for the $\psi(2S) \to J/\psi X$ transition fraction $B = 56.1\%$ in PDG2006, we can get the factor is 0.355.

[*] K. Y. Liu, Z. G. He and K. T. Chao, Phys. Rev. D **68**, 031501 (2003) [arXiv:hep-ph/0305084].

Combine those contributions, if we set $m = 1.5\text{GeV}$ and $\mu = 2m$, then the prompt cross section of $e^+e^- \to J/\psi + c\bar{c} + X$ at next-to-leading order of α_s is

$$\sigma_{prompt}(e^+ + e^- \to J/\psi + c\bar{c} + X) = 0.50 \text{ pb}. \tag{21}$$

It is 58% of the experiment date 0.87 pb in Eq. (1).

If we set $m = 1.4\text{GeV}$ and $\mu = 2m$, ignore the other difference of other contributions, then the prompt cross section of $e^+e^- \to J/\psi + c\bar{c} + X$ at next-to-leading order of α_s is

$$\sigma_{prompt}(e^+ + e^- \to J/\psi + c\bar{c} + X) = 0.71 \text{ pb}. \tag{22}$$

It is 81% of the experiment date 0.87 pb in Eq. (1).

4. Conclusions

We get the cross section of $e^+e^- \to J/\psi + \eta_c$ and $e^+e^- \to J/\psi + c\bar{c}$ at $\sqrt{s} = 10.6$ GeV at the NLO of α_s and v^2.

With $m = 1.5\text{GeV}$ and $\mu = 2m$, the cross section of $e^+e^- \to J/\psi + \eta_c$ is 2.26 fb at LO order, 3.87 fb include the NLO relativistic correction, 10.92 fb include the NLO radiative correction, 20.9 fb include the NLO both relativistic and radiative correction. The cross section include the NLO both relativistic and radiative correction reaches to the lower bound of experiment. We consider that the discrepancy between theory and experiment of $\sigma(e^+e^- \to J/\psi + \eta_c)$ has been resolved.

With $m = 1.4\text{GeV}$ and $\mu = 2m$, the prompt J/ψ production cross section of $e^+e^- \to J/\psi + c\bar{c}$ is estimated to be 0.71 fb, which is about 81% of the experiment date. The discrepancy between theory and experiment of $\sigma(e^+e^- \to J/\psi + c\bar{c})$ can be largely removed.

Thanks!