Quark Masses and α_s from the total hadronic cross section in e^+e^- annihilation

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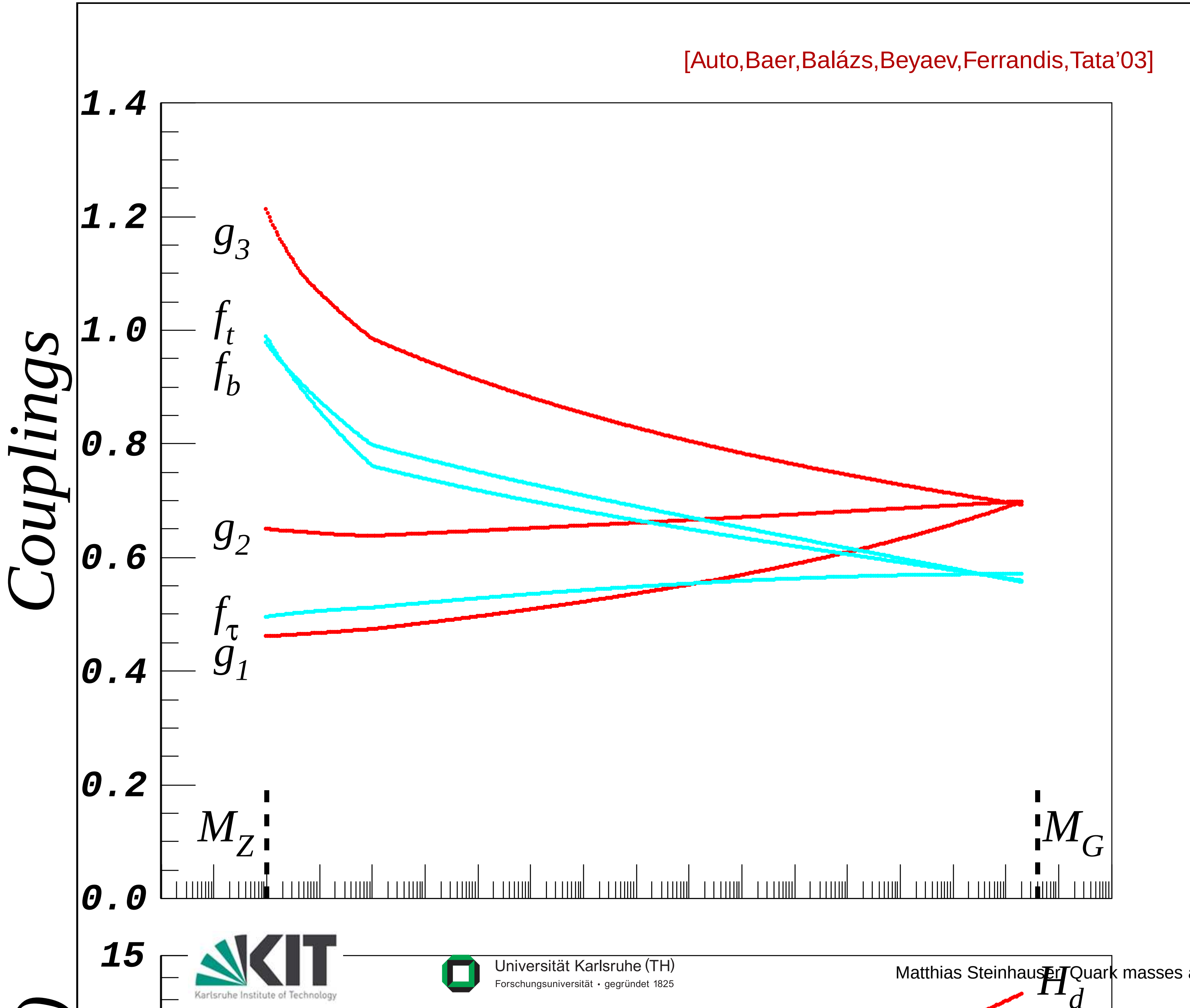
Outline

1. Introduction
2. α_s
3. m_b

Introduction

- Quark masses
  - B decays: $\Gamma \sim m_b^5 \ldots$
  - Spectroscopy
  - Higgs decay ➪ ILC
    $\Gamma(H \to b\bar{b}) = \frac{G_F M_H}{4\sqrt{2}\pi} m_b^2 \left(1 + \mathcal{O}(\alpha_s) + \ldots\right)$
  - Yukawa unification

Introduction



➪ needed:

$$\frac{\delta m_t}{m_t} \approx \frac{\delta m_b}{m_b}$$

$\delta m_t \approx 1$ GeV

➪

$\delta m_b \approx 25$ MeV necessary

Introduction

- Quark masses
  - B decays: $\Gamma \sim m_b^5 \ldots$
  - Spectroscopy
  - Higgs decay ➪ ILC
    $\Gamma(H \to b\bar{b}) = \frac{G_F M_H}{4\sqrt{2}\pi} m_b^2 \left(1 + \mathcal{O}(\alpha_s) + \ldots\right)$
  - Yukawa unification
- Strong coupling α_s and quark masses

➪ Fundamental parameters of QCD/SM

Quark mass definitions

$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{4} G^2_{\mu\nu} + \sum_q \bar{\psi}_q \left(\not{D} - m_q \right) \psi_q$$

- pole mass
- $\overline{\mathrm{MS}}$ mass
- kinetic mass [Bigi,Shifman,Uraltsev,Vainshtein'97]
- 1S mass [Hoang,Smith,Stelzer,Willenbrock'99]
- PS mass [Beneke'98]
- RS mass [Pineda'01]
- ...

Light quark masses, top quark mass

PDG:

$$
\begin{aligned}
m_u &= 1.5 \ldots 3.0 \text{ MeV} \\
m_d &= 3 \ldots 7 \text{ MeV} \\
\overline{m} = \frac{m_u + m_d}{2} &= 2.5 \ldots 5.5 \text{ MeV} \\
m_s &= 95 \pm 25 \text{ MeV}
\end{aligned}
$$

[Chetyrkin et al., Jamin et al., Lattice, . . .]

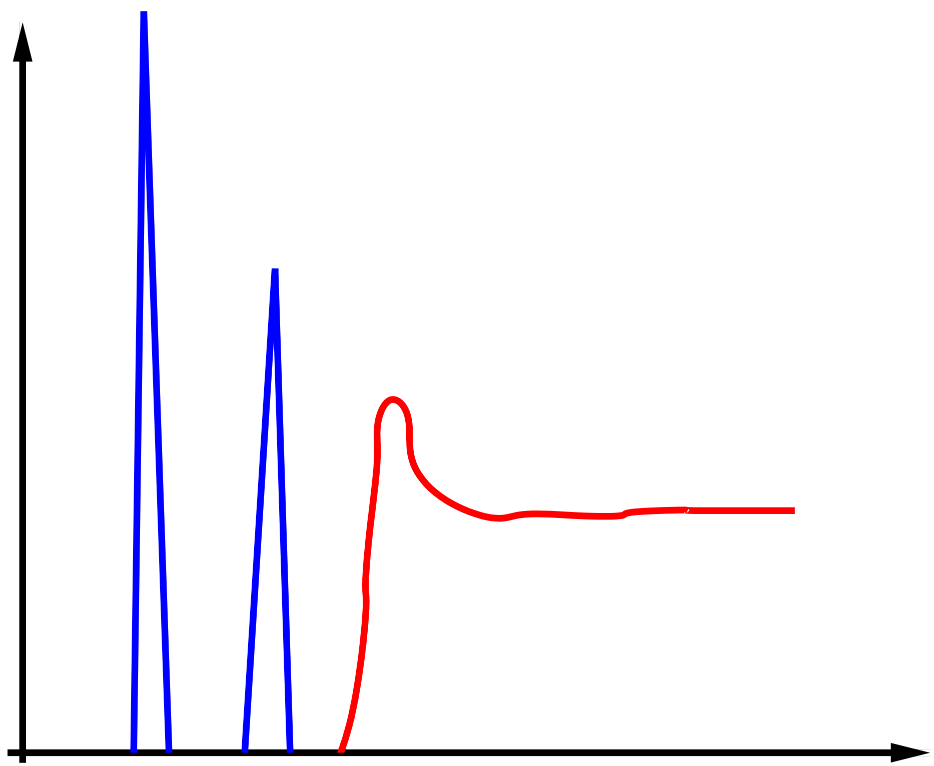
➪ less accurately known than heavy quark masses

$$m_t = 170.9 \pm 1.8 \text{ GeV}$$

[CDF,D0]

Charm/Bottom

- Consider $\sigma(e^+e^- \to \text{hadrons})$



- m_c, m_b ➩ sum rules, (“SVZ” sum rules) [Novikov et al.’78]

Charm/Bottom

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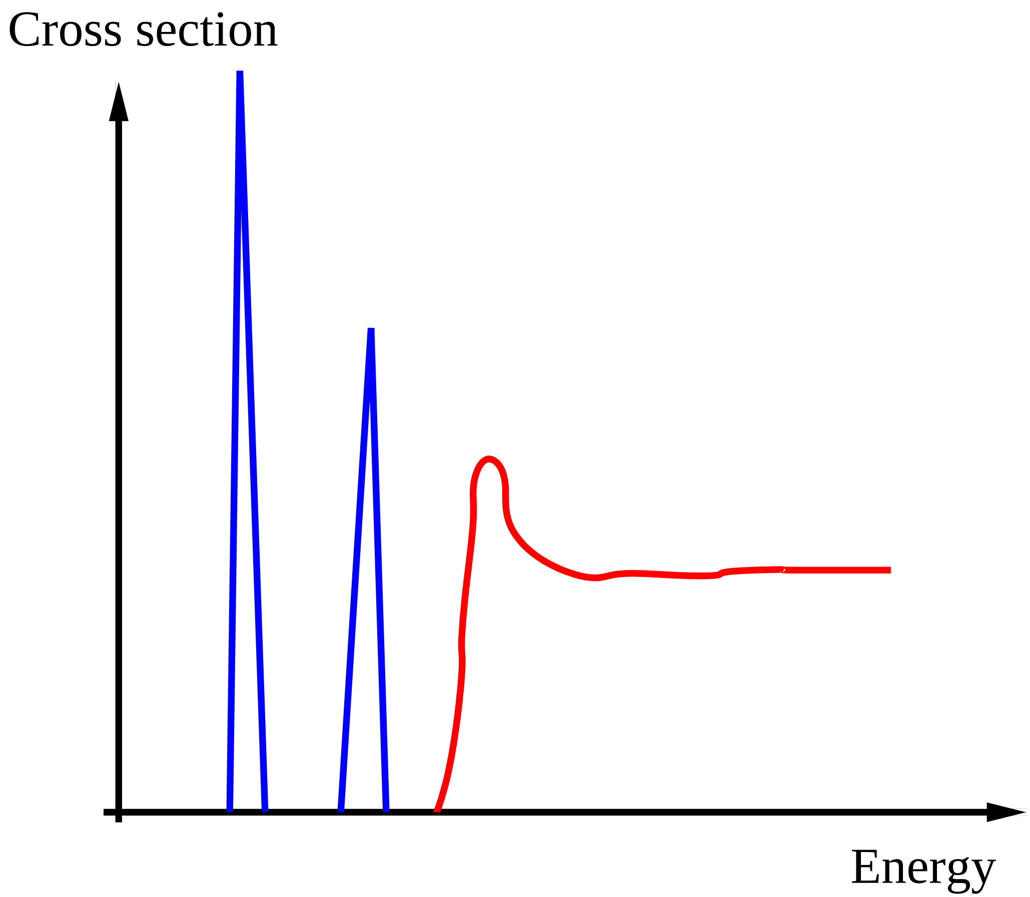
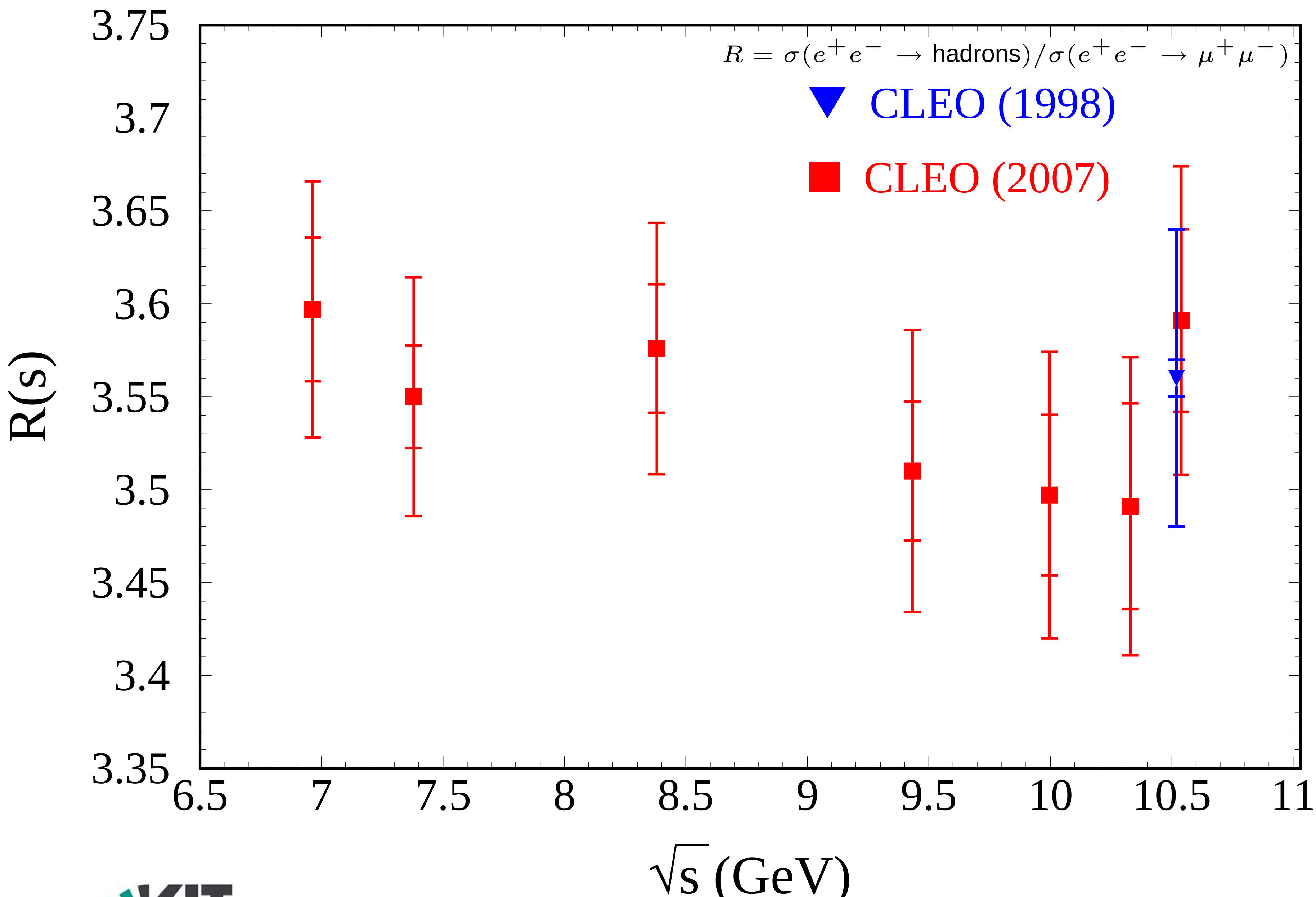


- m_c, m_b ➪ sum rules, (“SVZ” sum rules) [Novikov et al.’78]
- α_s from continuum

R measurement



α_s and R

basic idea: $R^{\text{exp}} = R^{\text{th}}(\alpha_s, m_q) \Rightarrow \alpha_s$

(weak dependence on variation of m_q)

$R^{\text{th}}(s)$:

rhad: [Harlander,MS'02]

- full quark mass dependence up to $\mathcal{O}(\alpha_s^2)$
- $\mathcal{O}(\alpha_s^3)$: $(m_q^2/s)^0$, $(m_q^2/s)^1$, $(m_q^2/s)^2$
- ...
- consistent running and decoupling of α_s

[v. Ritbergen,Larin,Vermaseren'97,Czakon'05]

[Chetyrkin,Kniehl,MS'97]

α_s and R

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(weak dependence on variation of m_q)

- $R^{\rm exp}(s) \Rightarrow \alpha_s^{(4)}(s) \quad (n_f = 4)$

$\sqrt{s}$ (GeV)	$\alpha_s^{(4)}(s)$	$\delta\alpha_s^{\rm stat}$	$\delta\alpha_s^{\rm sys,cor}$	$\delta\alpha_s^{\rm sys,uncor}$	$\alpha_s^{(4)}(s)\vert_{\rm CLEO}$
10.538	0.2113	0.0026	0.0618	0.0444	0.232
10.330	0.1280	0.0048	0.0469	0.0445	0.142
9.996	0.1321	0.0032	0.0516	0.0344	0.147
9.432	0.1408	0.0039	0.0526	0.0291	0.159
8.380	0.1868	0.0187	0.0461	0.0195	0.218
7.380	0.1604	0.0131	0.0404	0.0138	0.195
6.964	0.1881	0.0221	0.0386	0.0134	0.237

$\Uparrow$

massless approx.!!!

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- $R^{\mathrm{exp}}(s) \Rightarrow \alpha_s^{(4)}(s)$ $\quad$ ($n_f = 4$)
- Evolve to common scale and combine
  $\Rightarrow \alpha_s^{(4)}(9\ \mathrm{GeV}) = 0.160 \pm 0.024 \pm 0.024$

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- $\alpha_s^{(4)}(9\ \text{GeV}) \to \alpha_s^{(4)}(\mu_b^{\text{dec}}) \to \alpha_s^{(5)}(\mu_b^{\text{dec}}) \to \alpha_s^{(5)}(M_Z)$
  (practically) independent from μ_b^{dec} (4-loop running and 3-loop decoupling)

  RunDec: [Chetyrkin,Kühn,MS'00]

  $\Rightarrow \alpha_s^{(5)}(M_Z) = 0.110^{+0.010+0.010}_{-0.012-0.011} = 0.110^{+0.014}_{-0.017}$

[Kühn,MS,Teubner'07]

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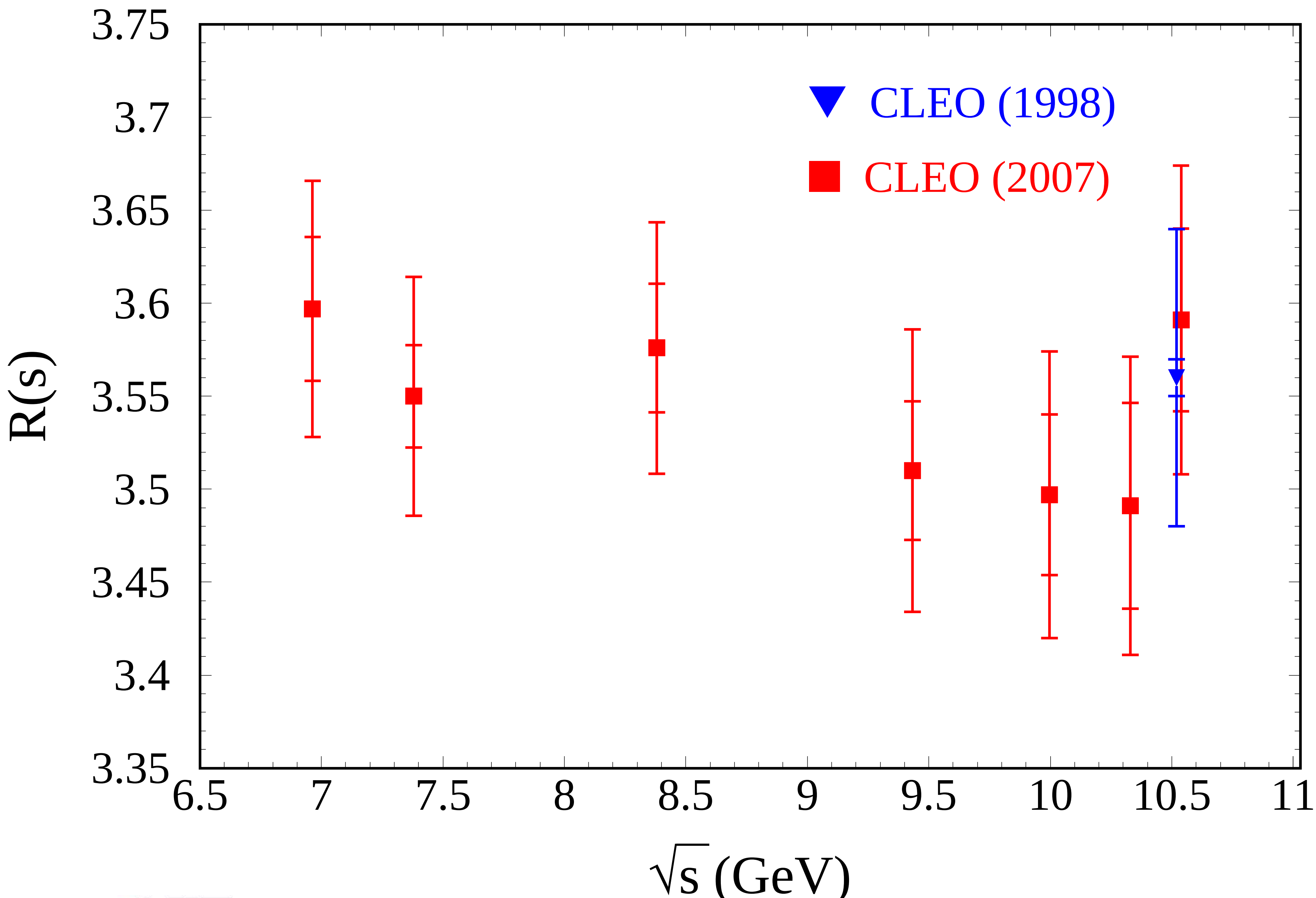
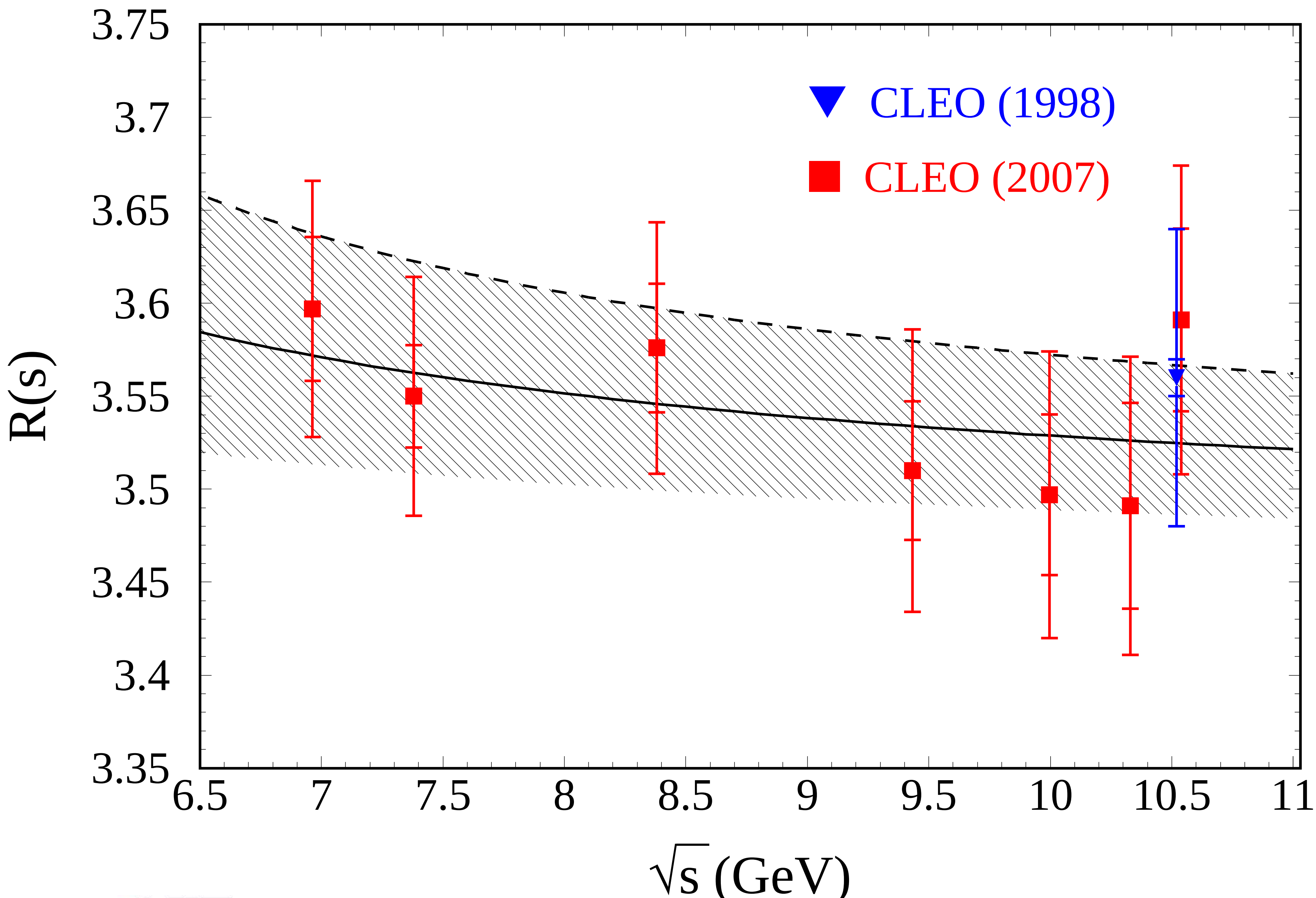
  [Kühn,MS,Teubner'07]
- CLEO analysis: $\alpha_s^{(5)}(M_Z^2)|_{\rm CLEO} = 0.126 \pm 0.005^{+0.015}_{-0.011}$
  - massless approximation for $R(s)$
  - no decoupling of α_s

R: experiment + theory



R: experiment + theory



α_s from R

- $\alpha_s^{(5)}(M_Z) = 0.110^{+0.010+0.010}_{-0.012-0.011} = 0.110^{+0.014}_{-0.017}$ [Kühn,MS,Teubner'07]
- Combine with $\alpha_s^{(5)}(M_Z) = 0.124^{+0.011}_{-0.014}$ [Kühn,MS'01]
  R measurements between 2 and 10.5 GeV from BES'01, MD-1'96, CLEO'97

$\Rightarrow$ $\boxed{\alpha_s^{(5)}(M_Z) = 0.119^{+0.009}_{-0.011}}$

- Compare: $\alpha_s^{(5)}(M_Z) = 0.1189 \pm 0.0010$ [Bethke'06]

Sum rules

$$R_Q = \frac{\sigma(e^+e^- \to Q\bar{Q} + \ldots)}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

$$\mathcal{M}_n \equiv \int \frac{\mathrm{d}s}{s^{n+1}} R_Q(s)$$ (moments)

$$R_Q = 12\pi \text{Im}\left[\Pi_Q(q^2 = s + i\varepsilon)\right]$$

$$\mathcal{M}_n = \frac{12\pi^2}{n!} \left(\frac{\mathrm{d}}{\mathrm{d}q^2}\right)^n \Pi_Q(q^2)\Big|_{q^2=0}$$

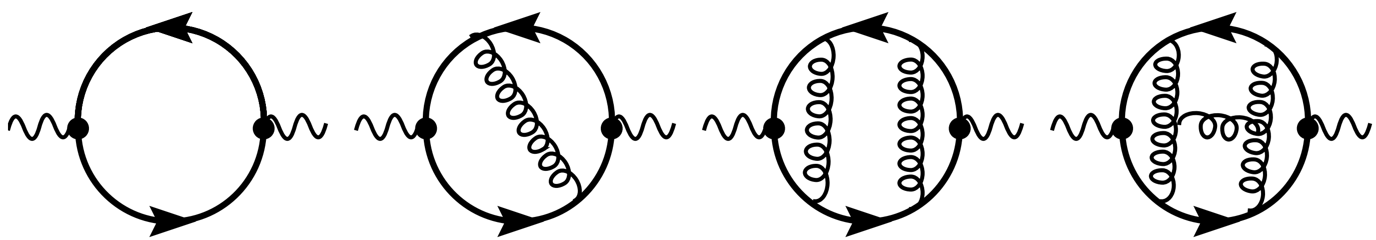
(dispersion relation)

$\mathcal{M}_n^{\mathrm{th}}$

$$\mathcal{M}_n = \frac{12\pi^2}{n!}\left(\frac{\mathrm{d}}{\mathrm{d}q^2}\right)^n \Pi_Q(q^2)\Bigg|_{q^2=0}$$

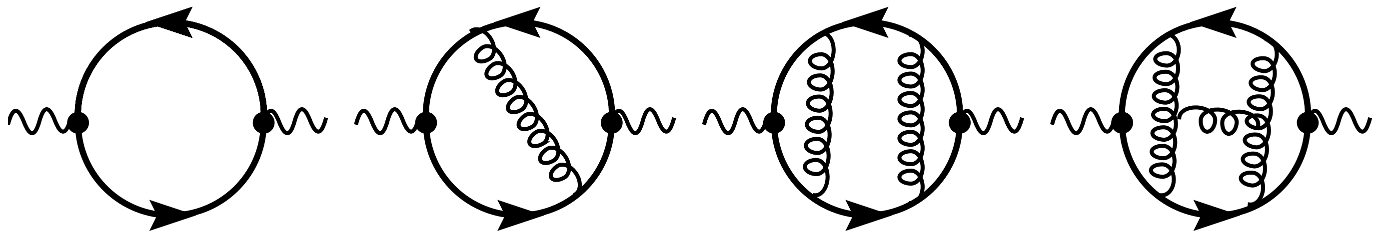
➪ compute Taylor expansion

$$\Pi_Q(q^2) = Q_Q^2 \frac{3}{16\pi^2}\sum_{n\geq 0} \bar{C}_n \left(\frac{q^2}{4m_Q^2}\right)^n$$



$$\mathcal{M}_n^{\mathrm{th}} = \frac{9}{4}Q_Q^2\left(\frac{1}{4m_Q^2}\right)^n \bar{C}_n \qquad \frac{\delta m_Q}{m_Q} = \frac{1}{2n}\frac{\delta\mathcal{M}_n}{\mathcal{M}_n}$$

C_n to 4 loops



- 1, 2 and 3 loops: `MATAD` [MS'96-'00]
- 4 loops:
  - method: 1. reduce to master integrals

    [Laporta,Remiddi'96; Laporta'01]

    2. compute masters
  - several Million equations; several GB tables
  - all steps cross-checked

    1. [Chetyrkin,Kühn,Sturm'06; Boughezal,Czakon,Schutzmeier'06]

    2. [Schröder,Vuorinen'05; Chetyrkin,Faisst,Sturm,Tentyukov'06],. . .

C_n to 4 loops

$$\bar{C}_n = \bar{C}_n^{(0)} + \frac{\alpha_s(\mu)}{\pi}\left(\bar{C}_n^{(10)} + \bar{C}_n^{(11)} l_{m_b}\right)$$
$$+\left(\frac{\alpha_s(\mu)}{\pi}\right)^2\left(\bar{C}_n^{(20)} + \bar{C}_n^{(21)} l_{m_b} + \bar{C}_n^{(22)} l_{m_b}^2\right)$$
$$+\left(\frac{\alpha_s(\mu)}{\pi}\right)^3\left(\bar{C}_n^{(30)} + \bar{C}_n^{(31)} l_{m_b} + \bar{C}_n^{(32)} l_{m_b}^2 + \bar{C}_n^{(33)} l_{m_b}^3\right)$$

$$l_{m_b} = \ln(m_b^2/\mu^2)$$

n	$\bar{C}_n^{(0)}$	$\bar{C}_n^{(10)}$	$\bar{C}_n^{(11)}$	$\bar{C}_n^{(20)}$	$\bar{C}_n^{(21)}$	$\bar{C}_n^{(22)}$	$\bar{C}_n^{(30)}$	$\bar{C}_n^{(31)}$	$\bar{C}_n^{(32)}$	$\bar{C}_n^{(33)}$
1	1.0667	2.5547	2.1333	3.1590	3.4425	0.0889	−7.7624	−0.0599	1.5851	−0.0543
2	0.4571	1.1096	1.8286	3.2319	5.0798	1.9048	—	4.0100	7.2551	0.1058
3	0.2709	0.5194	1.6254	2.0677	4.5815	3.3185	—	5.6496	13.4967	2.3967
4	0.1847	0.2031	1.4776	1.2204	3.4726	4.4945	—	3.9381	17.2292	6.2423

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$$-8.0 \le \bar{C}_2^{(30)} \le 9.5\,, -8.0 \le \bar{C}_3^{(30)} \le 8.3\,, -8.0 \le \bar{C}_4^{(30)} \le 7.4$$

$$\mathcal{M}^{\text{exp}} = \mathcal{M}^{\text{res}} + \mathcal{M}^{\text{thresh}} + \mathcal{M}^{\text{cont}}$$

- $\mathcal{M}^{\text{res}}$: $R^{\text{res}}(s) = \frac{9\pi M_R \Gamma_{ee}}{\alpha^2} \left(\frac{\alpha}{\alpha(s)}\right)^2 \delta(s - M_R^2)$

	$\Upsilon(1S)$	$\Upsilon(2S)$	$\Upsilon(3S)$	$\Upsilon(4S)$
M_Υ(GeV)	9.46030(26)	10.02326(31)	10.3552(5)	10.5794(12)
Γ_{ee}(keV)	1.340(18)	0.612(11)	0.443(8)	0.272(29)
$(\alpha/\alpha(M_\Upsilon))^2$	0.932069	0.93099	0.930811	0.930093

$$\mathcal{M}^{\rm exp} = \mathcal{M}^{\rm res} + \mathcal{M}^{\rm thresh} + \mathcal{M}^{\rm cont}$$

- $\mathcal{M}^{\rm res}$: $\quad R^{\rm res}(s) = \frac{9\pi M_R \Gamma_{ee}}{\alpha^2}\left(\frac{\alpha}{\alpha(s)}\right)^2 \delta(s - M_R^2)$
- $\mathcal{M}^{\rm thresh}$: CLEO data up to 11.24 GeV

$\mathcal{M}^{\rm exp} = \mathcal{M}^{\rm res} + \mathcal{M}^{\rm thresh} + \mathcal{M}^{\rm cont}$

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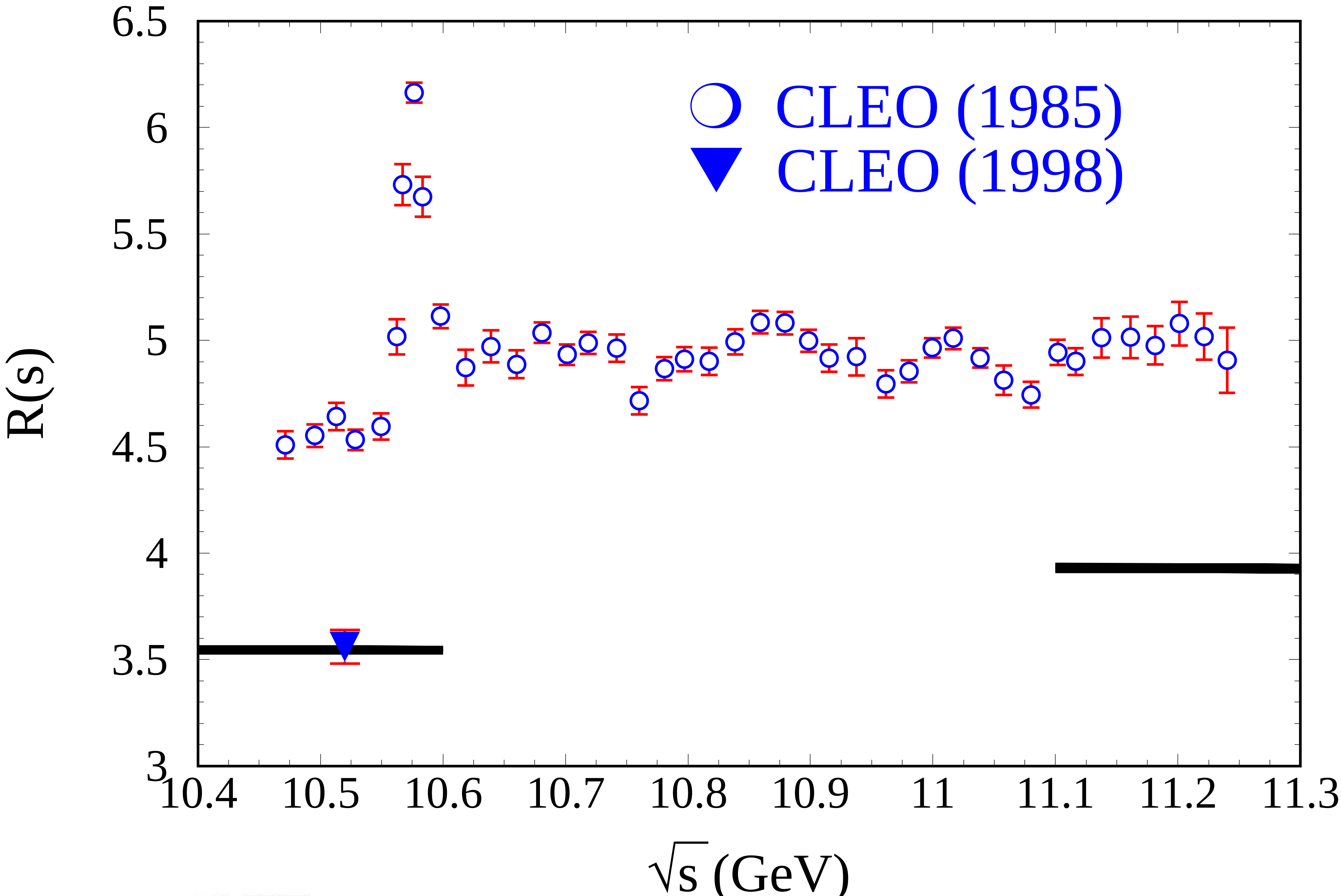
- $\mathcal{M}^{\rm thresh}$: CLEO data up to 11.24 GeV
- $\mathcal{M}^{\rm cont}$: $\sqrt{s} \geq 11.24$ GeV

  no data
  $R^{\rm theory}$ ⇨ full mass dependence up to $\mathcal{O}(\alpha_s^2)$ rhad: [Harlander,MS'02]

CLEO 1985 and 1998



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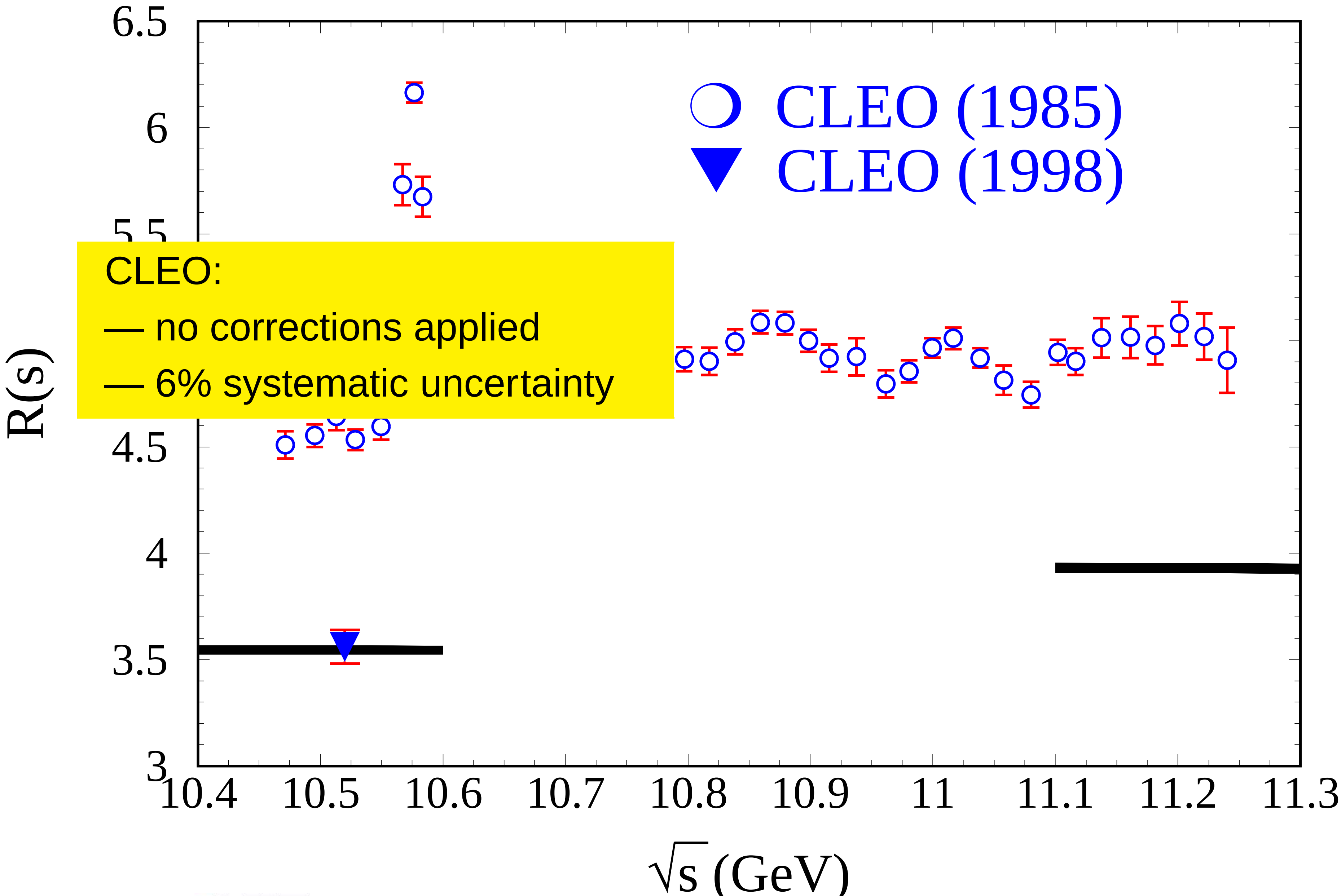
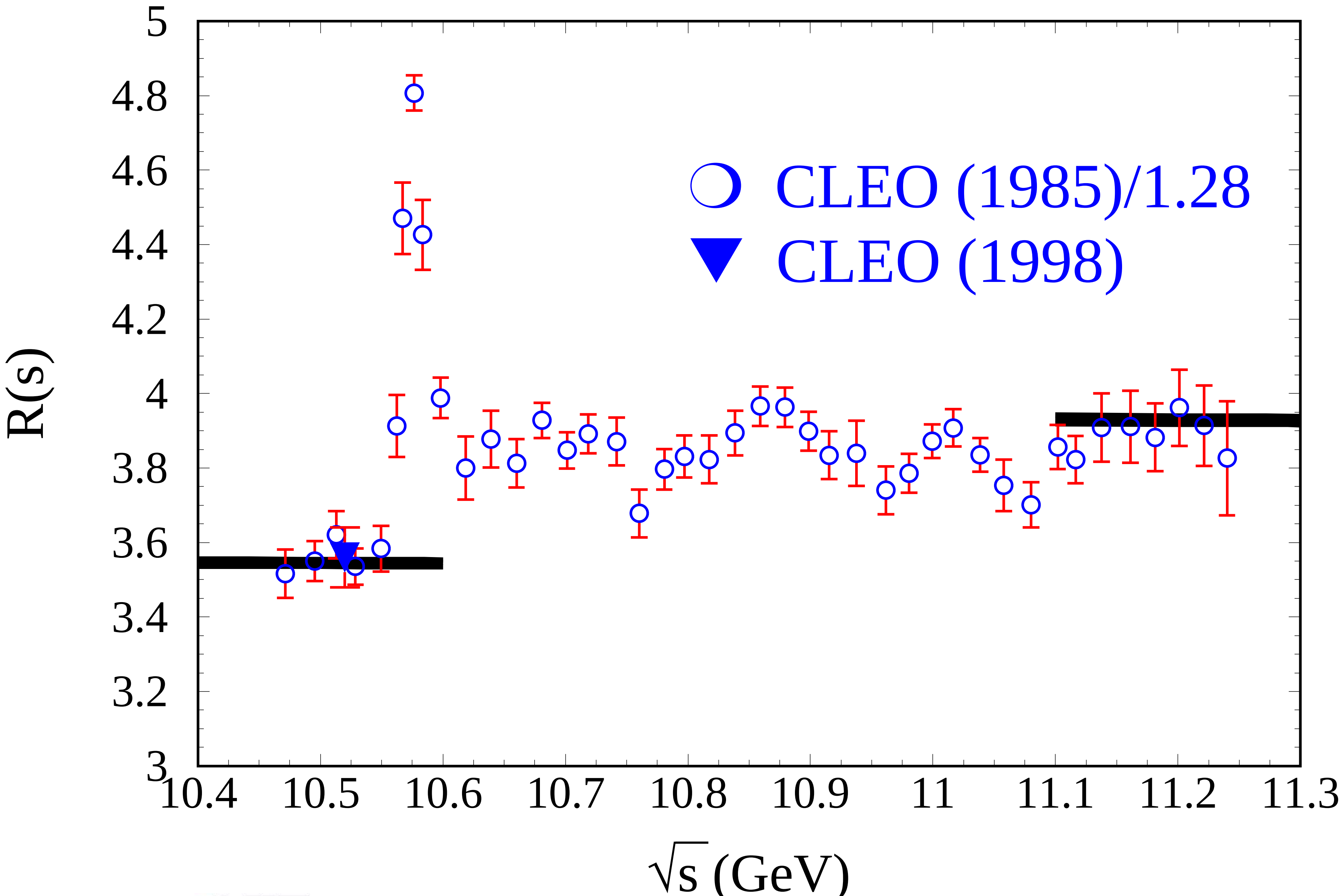


CLEO:

— no corrections applied

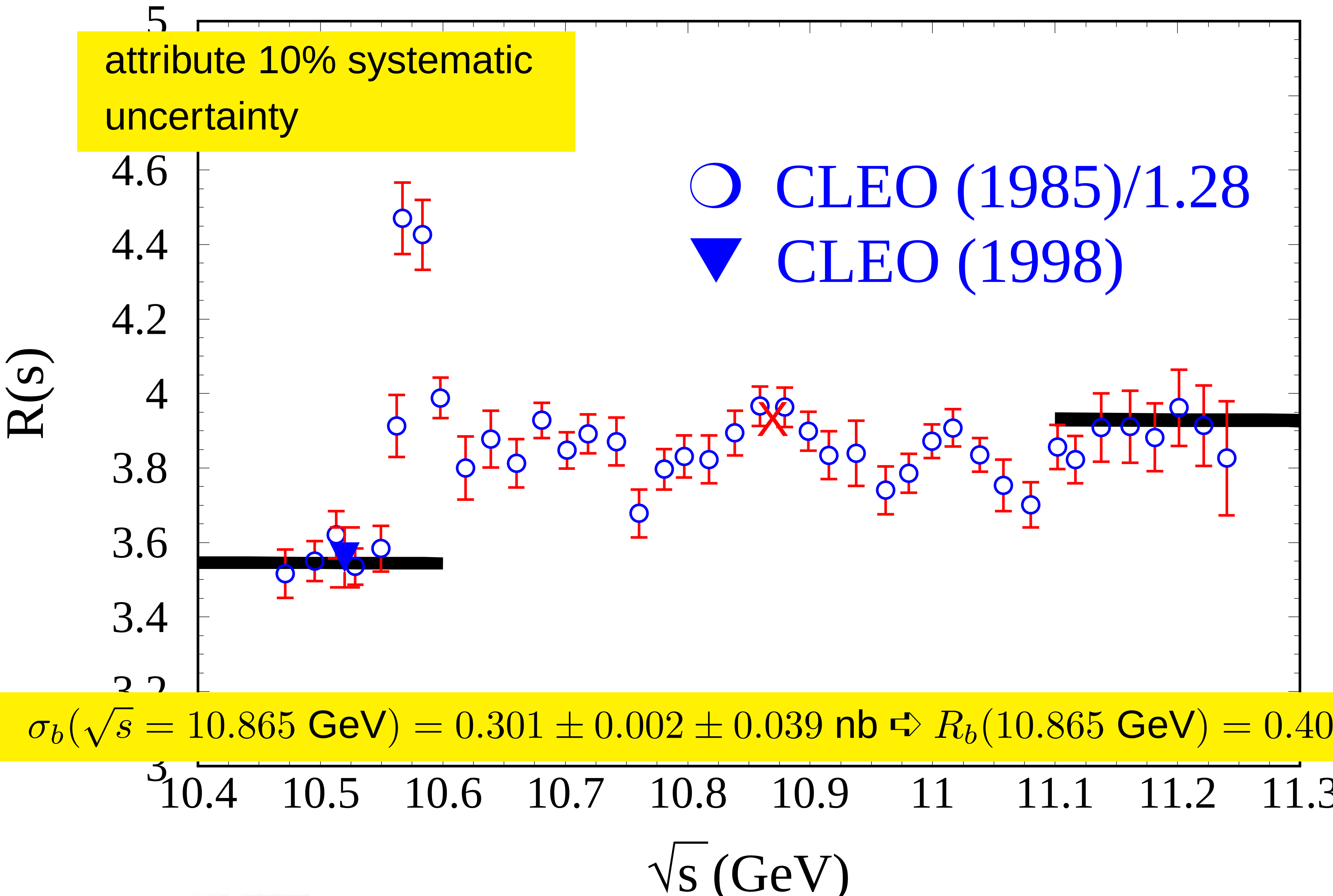
— 6% systematic uncertainty

CLEO 1985 and 1998



CLEO 1985 and 1998

attribute 10% systematic uncertainty



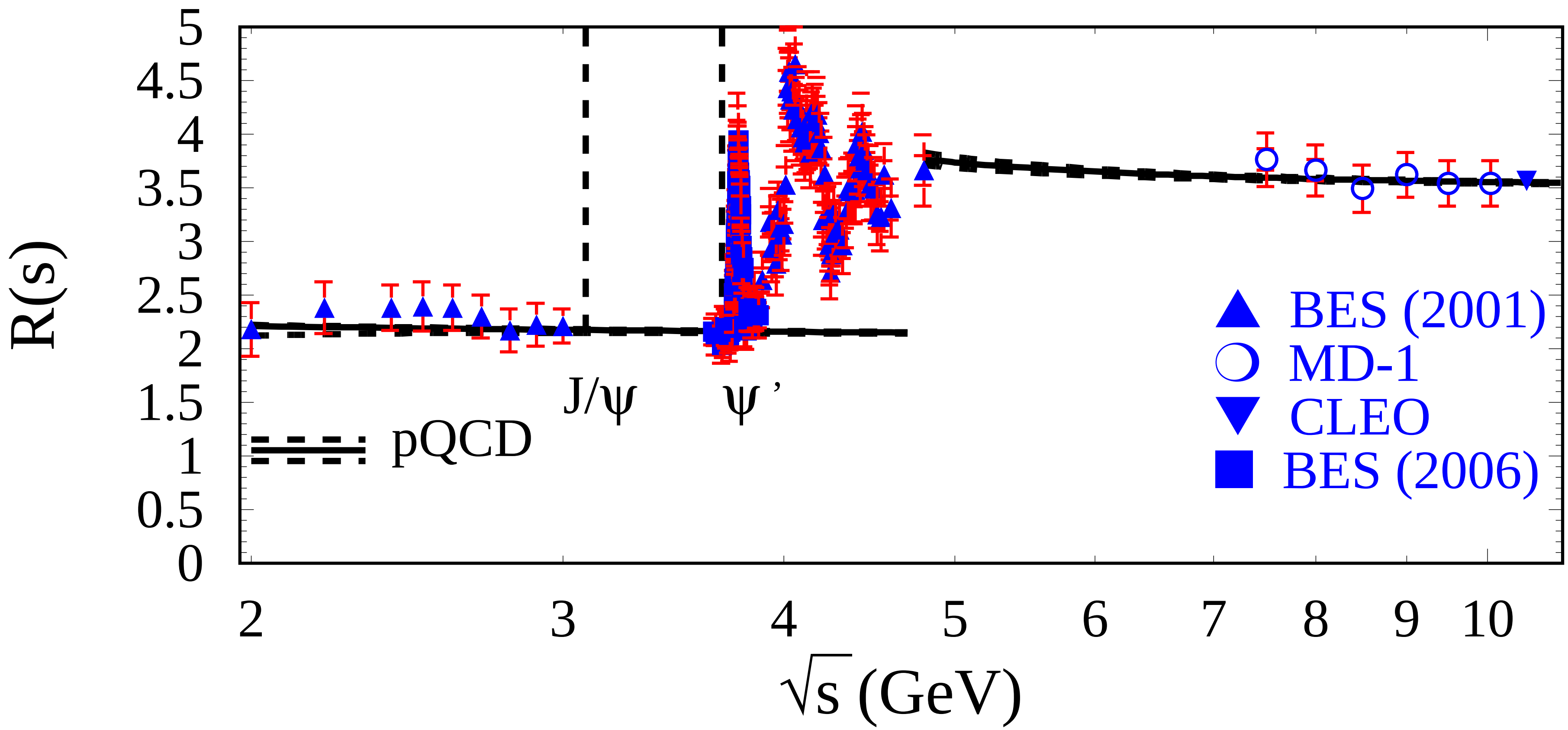
$\sigma_b(\sqrt{s} = 10.865$ GeV$) = 0.301 \pm 0.002 \pm 0.039$ nb ⇨ $R_b(10.865$ GeV$) = 0.409$ [CLEO'06

$\mathcal{M}^{\rm exp}$

n	$\mathcal{M}_n^{\rm res,(1S-4S)}$ $\times 10^{(2n+1)}$	$\mathcal{M}_n^{\rm thresh}$ $\times 10^{(2n+1)}$	$\mathcal{M}_n^{\rm cont}$ $\times 10^{(2n+1)}$	$\mathcal{M}_n^{\rm exp}$ $\times 10^{(2n+1)}$
1	1.394(23)	0.296(32)	2.911(18)	4.601(43)
2	1.459(23)	0.249(27)	1.173(11)	2.881(37)
3	1.538(24)	0.209(22)	0.624(7)	2.370(34)
4	1.630(25)	0.175(19)	0.372(5)	2.178(32)

$\mathcal{M}^{\rm exp} = \mathcal{M}^{\rm res} + \mathcal{M}^{\rm thresh} + \mathcal{M}^{\rm cont}$ (charm)

- $\mathcal{M}^{\rm res}$: J/Ψ, $\Psi(2S)$
- $\mathcal{M}^{\rm thresh}$: 3.73 GeV $\leq \sqrt{s} \leq 4.8$ GeV, BES01,06



- $\mathcal{M}^{\rm cont}$: $\sqrt{s} \geq 4.8$ GeV pQCD

m_b

$$\mathcal{M}_n^{\text{th}} \stackrel{!}{=} \mathcal{M}_n^{\text{exp}}$$

$$m_b(\mu) = \frac{1}{2}\left(\frac{\bar{C}_n}{\mathcal{M}_n^{\text{exp}}}\right)^{1/(2n)}$$

- 1. set $\mu = 10$ GeV ⇨ $m_b(10$ GeV$)$
  2. RGE ⇨ $m_b(m_b)$
- Uncertainties
  - $\delta\mathcal{M}_n^{\text{exp}}$
  - $\alpha_s(M_Z) = 0.1189 \pm 0.0020$ [Bethke'06]; $\delta\alpha_s \times 2$
  - $\mu = (10 \pm 5)$ GeV
  - $\delta\mathcal{M}_n^{\text{np}}$ (only charm)

m_b

$$\mathcal{M}_n^{\rm th} \stackrel{!}{=} \mathcal{M}_n^{\rm exp}$$

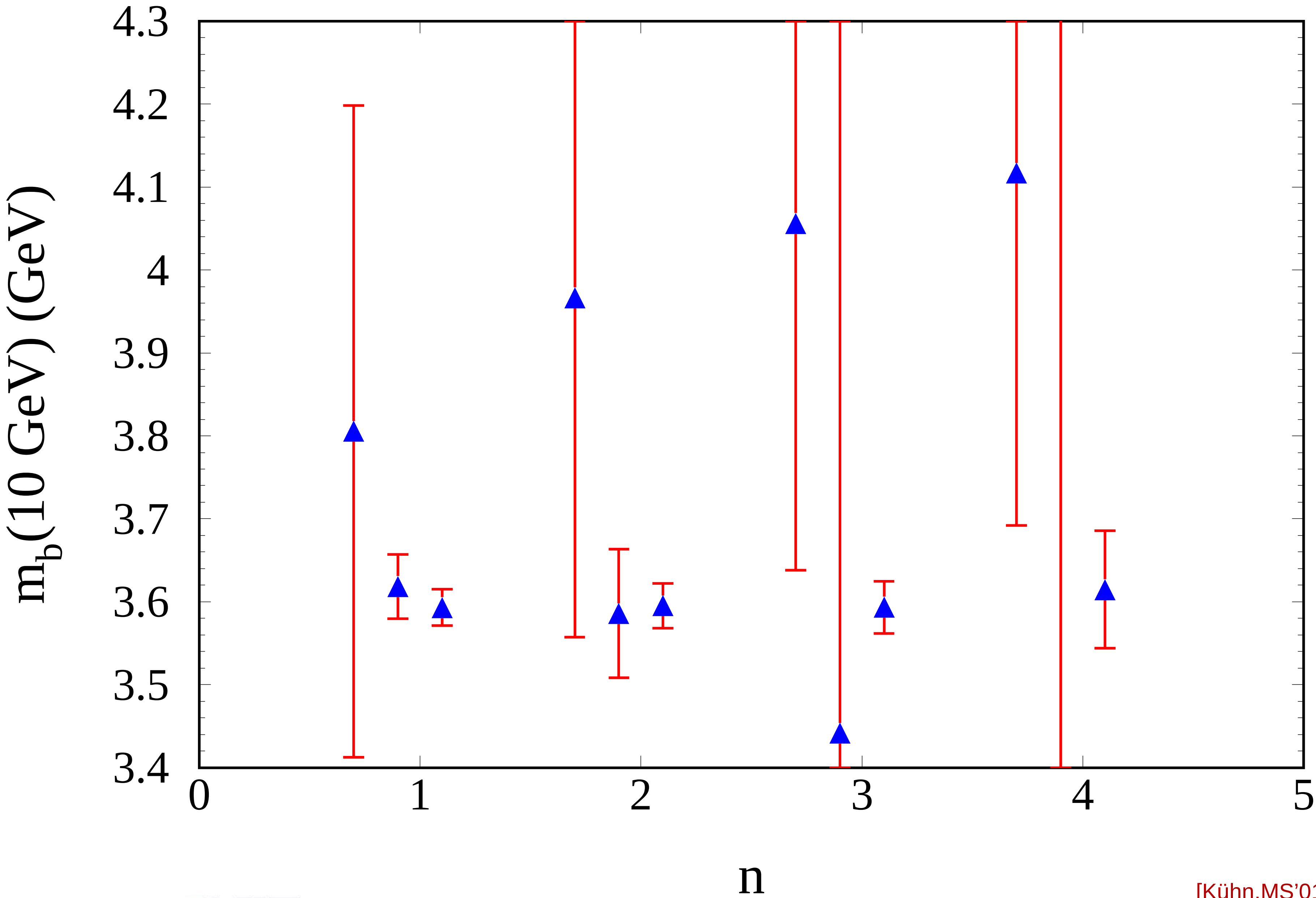
$$m_b(\mu) \quad = \quad \frac{1}{2}\left(\frac{\bar{C}_n}{\mathcal{M}_n^{\rm exp}}\right)^{1/(2n)}$$

n	$m_b(10$ GeV$)$	exp	α_s	μ	total	$\delta\bar{C}_n^{(30)}$	$m_b(m_b)$
1	3.593	0.020	0.007	0.002	0.021	—	4.149
2	3.609	0.014	0.012	0.003	0.019	0.006	4.164
3	3.618	0.010	0.014	0.006	0.019	0.008	4.173
4	3.631	0.008	0.015	0.021	0.027	0.012	4.185

$$\boxed{m_b(10\ \text{GeV}) = 3.609(25)\ \text{GeV}}$$

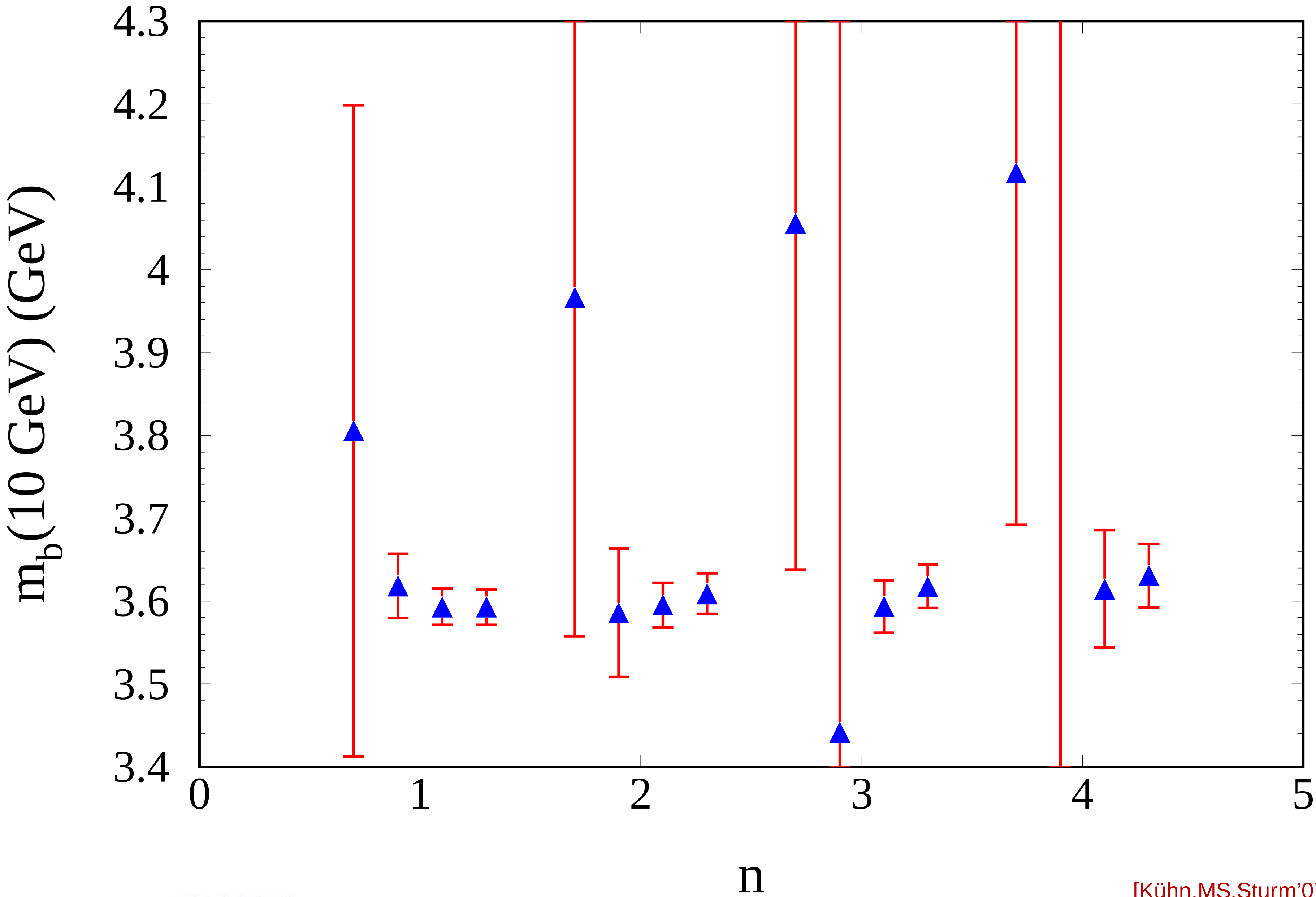
[Kühn,MS,Sturm'07]

$m_b(10\ \mathbf{GeV})$



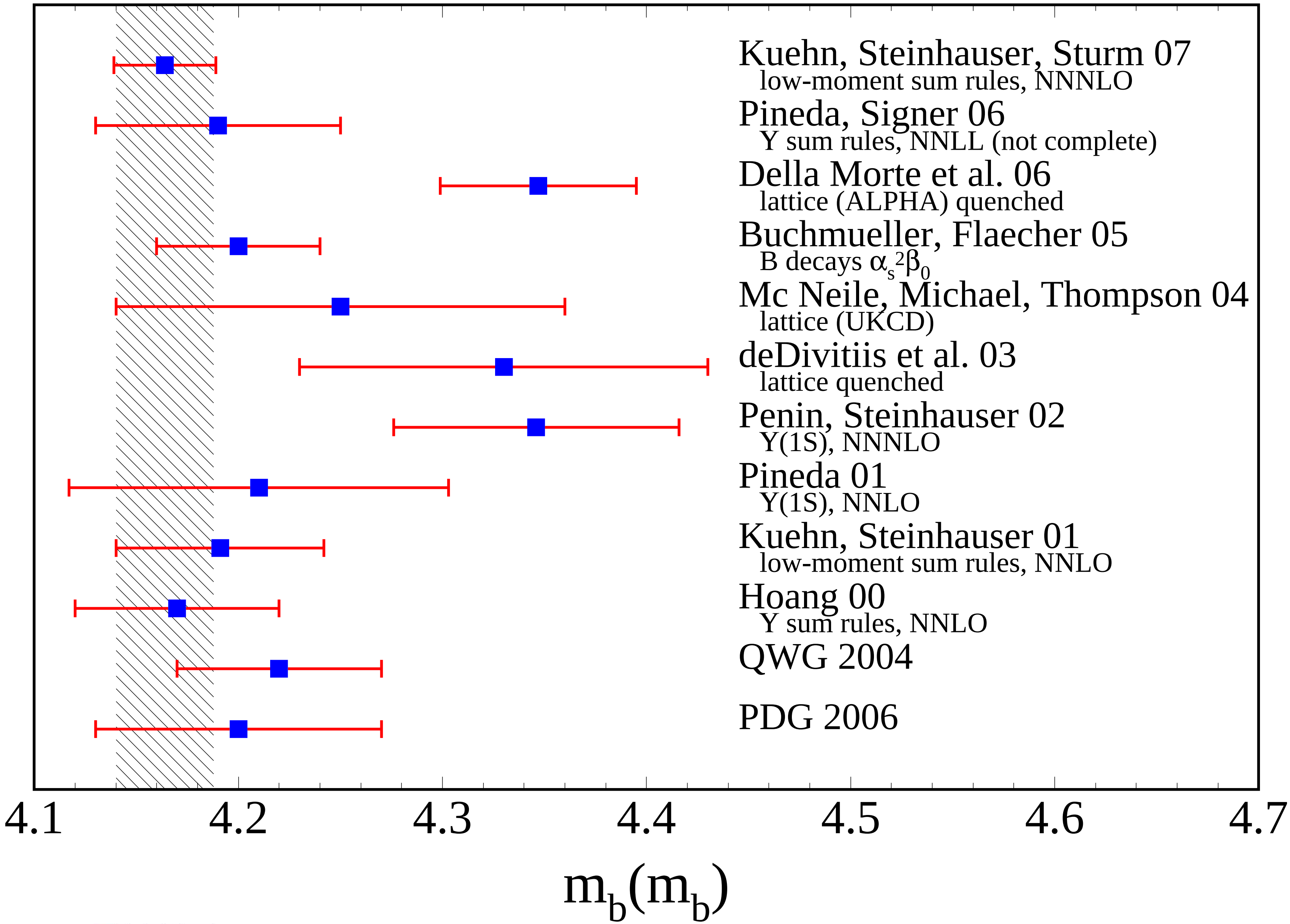
[Kühn,MS'01]

$m_b(10\ \mathbf{GeV})$



[Kühn,MS,Sturm'07]

Bottom — comparison



Conclusions

- Most precise values for m_c and m_b
  $m_c(m_c) = 1.286(13)$ GeV $\qquad m_b(m_b) = 4.164(25)$ GeV
- NNNLO analysis
- $\overline{\mathrm{MS}}$ mass
- Possible improvements: experimental measurements: $R(s)$, Γ_{ee}
- $\frac{\delta m_s}{m_s} \approx 10\%$
  $\frac{\delta m_c}{m_c} \approx 1\%$
  $\frac{\delta m_b}{m_b} \approx 0.6\%$
  $\frac{\delta m_t}{m_t} \approx 1\%$
- $R^{\mathrm{exp}} \Rightarrow \alpha_s^{(5)}(M_Z) = 0.119^{+0.009}_{-0.011}$