Renormalization of the chromomagnetic operator in HQET

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Quarkonium WG, Hamburg, October 2007

Introduction
The intermediate scheme
Application
Potentials

based on

Non-perturbative renormalization of the chromo-magnetic operator in Heavy Quark Effective Theory and the $B^* - B$ mass splitting.
D. Guazzini, H. Meyer & R.S., arXiv:0705.1809 [hep-lat], JHEP to appear.

The B-meson mass splitting from non-perturbative quenched lattice QCD.
A. G. Grozin, D. Guazzini, P. Marquard, H. B. Meyer, J. H. Piclum, R. S. and M. Steinhauser, PoS(LATTICE 2007)100, arXiv:0710.0578 [hep-lat]

HQET

HQET: in the rest frame of a B-meson

$$\overline{\psi}_{\rm b}[D_\mu\gamma_\mu + m_{\rm b}]\psi_{\rm b} \quad \rightarrow \quad \mathcal{L}_{\rm stat} + \mathcal{L}^{(1)} + {\rm O}(1/m_{\rm b}^2)\,,$$

$$\mathcal{L}_{\rm stat} \quad = \quad \overline{\psi}_{\rm h}[D_0 + m_{\rm b}]\psi_{\rm h} \qquad (*)$$

$$\tfrac{1}{2}(1+\gamma_0)\psi_{\rm h} \quad = \quad \psi_{\rm h}\,, \qquad \text{“large” components}$$

$$\mathcal{L}^{(1)} \quad = \quad \frac{1}{2m_{\rm b}}\overline{\psi}_{\rm h}(-\sigma\cdot\mathbf{B} - \tfrac{1}{2}\mathbf{D}^2)\psi_{\rm h}$$

(*) equivalent: $\mathcal{L}_{\rm stat} = \overline{\psi}_{\rm h} D_0 \psi_{\rm h}$ and $E_{\rm QCD} = E_{\rm stat} + m_{\rm b}$ (universal energy shift)

Spin splitting (continuum or lattice)

- in QCD

$$\Delta m^2 \equiv m_{\mathrm{B}^*}^2 - m_{\mathrm{B}}^2$$

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asymptotics:

$$4\lambda_2^{\mathrm{RGI}} = \lim_{m_\mathrm{b}\to\infty} \left\{ \left[\, 2b_0\bar{g}^2(m_\mathrm{b})\,\right]^{-\gamma_0/2b_0} \Delta m^2 \right\} = \mathrm{const.}\,,$$

$$\left(\, \gamma_0 = 3/(8\pi^2)\,,\quad b_0 = (11 - \tfrac{2}{3}N_\mathrm{f})/(16\pi^2)\right)$$

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- in HQET

$$\lambda_2^{\mathrm{RGI}} = \tfrac{1}{3}\langle B|\mathcal{O}_{\mathrm{spin}}^{\mathrm{RGI}}|B\rangle \,/\, \langle B|B\rangle$$

$$\mathcal{O}_{\mathrm{spin}}^{\mathrm{RGI}} = \lim_{\mu\to\infty} \left[2b_0\bar{g}^2(\mu) \right]^{-\gamma_0/2b_0} \mathcal{O}_{\mathrm{spin}}^{S}(\mu)\,.$$

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-

$$\mathcal{O}^{S}_{\mathrm{spin}}(\mu) = Z^{S}_{\mathrm{spin}}(\mu, a)\,\overline{\psi}_\mathrm{h}(x)\boldsymbol{\sigma}\cdot\mathbf{B}(x)\psi_\mathrm{h}(x)$$

a = lattice spacing or inverse cutoff
$\mathcal{O}^{S}_{\mathrm{spin}}$: scale- and scheme-dependent (S = scheme)
$\mathcal{O}^{\mathrm{RGI}}_{\mathrm{spin}}$: **not**

Spin splitting

- at finite mass

$$\Delta m^2 \;=\; 2\frac{m_{\mathrm{B}^*}+m_{\mathrm{B}}}{M_{\mathrm{b}}}\underbrace{C_{\mathrm{spin}}(M_{\mathrm{b}}/\Lambda_{\overline{\mathrm{MS}}})}_{\Uparrow}\,\lambda_2^{\mathrm{RGI}}+\underbrace{\mathrm{O}(1/m_{\mathrm{b}})}_{\Uparrow}$$

conversion function   maybe 5-10% for B

M = RGI mass

Conversion function

Solution of RG equation in a general scheme S:

$$\Phi^{S}_{\rm spin}(\mu)/\Phi^{\rm RGI}_{\rm spin} = Z^{S}_{\rm spin}(\mu)/Z^{\rm RGI}_{\rm spin} = U^{S}(\mu)$$

$$U^{S}(\mu) = [2b_0\bar{g}^2_S(\mu)]^{\gamma_0/2b_0}\exp\left\{\int_0^{\bar{g}_S(\mu)} {\rm d}g\left[\frac{\gamma^S(g)}{\beta^S(g)} - \frac{\gamma_0}{b_0 g}\right]\right\}$$

matching of QCD and HQET, conventional form:

$$\mathcal{A}_{\rm qcd} = \frac{1}{m_Q} C_{\rm cm}(m_{\rm Q}) U^{\overline{\rm MS}}(m_Q)\,\mathcal{A}^{\rm RGI}_{\rm hqet}\,, \quad \mathcal{A}^{\rm RGI}_{\rm hqet} = \langle\beta|\mathcal{O}^{\rm RGI}_{\rm spin}|\alpha\rangle$$

m_Q: pole mass

now known to high orders [Grozin et al., 2007]:

$$C_{\rm cm}(m_{\rm Q}) = 1 + 0.6897\alpha_{\overline{\rm MS}}(m_{\rm Q}) + (2.2186 - 0.1938N_{\rm f})\alpha^2_{\overline{\rm MS}}(m_{\rm Q}) + (11.079 - 1.7490N_{\rm f} + 0.0513N_{\rm f}^2)\alpha^3_{\overline{\rm MS}}(m_{\rm Q}) + {\rm O}(\alpha^4_{\overline{\rm MS}})$$

$$\gamma^{\overline{\rm MS}}(\alpha_{\overline{\rm MS}}) = 0.4775\alpha_{\overline{\rm MS}} + (0.4306 - 0.0549N_{\rm f})\alpha^2_{\overline{\rm MS}} + (0.8823 - 0.1472N_{\rm f} - 0.0007N_{\rm f}^2)\alpha^3_{\overline{\rm MS}} + {\rm O}(\alpha^4_{\overline{\rm MS}})$$

Conversion function

include finite renormalization $C_{\rm cm}$ into the definition of the scheme S =mag

$$
\begin{aligned}
\mathcal{A}_{\rm qcd} &= \frac{1}{m_Q}\, C_{\rm cm}(m_{\rm Q}) U^{\overline{\rm MS}}(m_Q)\, \mathcal{A}_{\rm hqet}^{\rm RGI} \\
\downarrow & \qquad\qquad\qquad\qquad\qquad (\ \overline{m}^{\overline{\rm MS}}(\overline{m}_*) = \overline{m}_*\ ) \\
\mathcal{A}_{\rm qcd} &= \frac{1}{m_Q}\, U^{\rm mag}(\overline{m}_*)\, \mathcal{A}_{\rm hqet}^{\rm RGI} = \frac{1}{m_Q}\, C_{\rm mag}(M/\Lambda_{\overline{\rm MS}})\, \mathcal{A}_{\rm hqet}^{\rm RGI}
\end{aligned}
$$

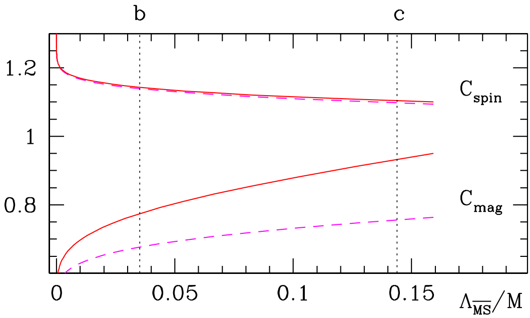
in addition, get rid of m_Q for better behaviour of pert. series

[J. Heitger, A. Juttner, R. S. and J. Wennekers, 2004]

$$
\begin{aligned}
\mathcal{A}_{\rm qcd} &= \frac{1}{M}\, C_{\rm spin}(M/\Lambda_{\overline{\rm MS}})\, \mathcal{A}_{\rm hqet}^{\rm RGI} \\
C_{\rm spin}(M/\Lambda_{\overline{\rm MS}}) &\equiv U^{\rm spin}(\overline{m}_*) = \frac{M}{m_Q}\, C_{\rm mag}(M/\Lambda_{\overline{\rm MS}})
\end{aligned}
$$

defines anomalous dimensions $\gamma^{\rm spin}$, $\gamma^{\rm mag}$

Conversion function

- 

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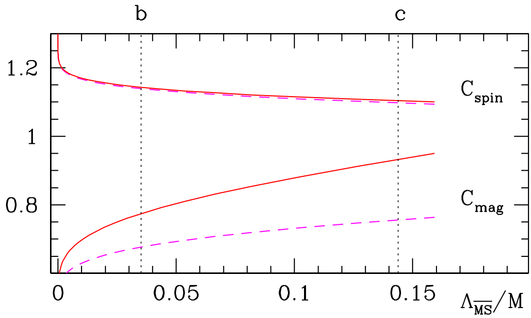
- 

- better "convergence" by changing to

$$C_{\rm spin}(M_{\rm b}/\Lambda_{\overline{\rm MS}}) = \frac{M_{\rm b}}{m_{\rm Q,b}} C_{\rm mag}(M_{\rm b}/\Lambda_{\overline{\rm MS}})$$

$$\Delta m^2 = 2\frac{m_{\rm B^*} + m_{\rm B}}{M_{\rm b}} C_{\rm spin}(M_{\rm b}/\Lambda_{\overline{\rm MS}})\lambda_2^{\rm RGI} + {\rm O}(1/m_{\rm b})$$

(pole mass eliminated; renormalon!) [J. Heitger, A. Juttner, R. S. and J. Wennekers, 2004]

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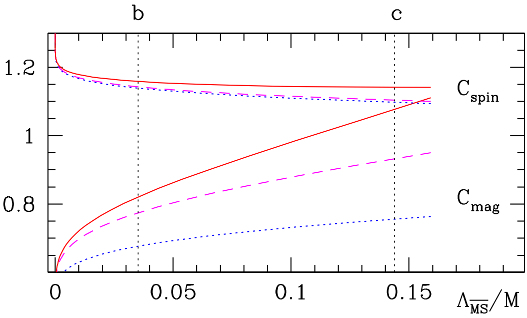
- 

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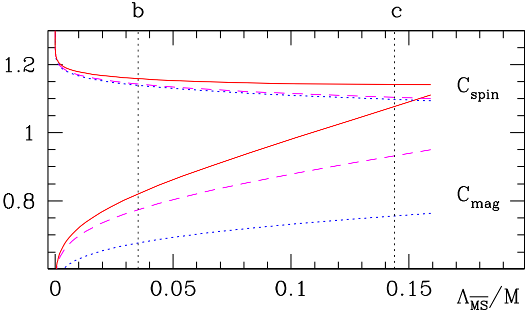
- 

[A. G. Grozin, P. Marquard, J. H. Piclum and M. Steinhauser, 2007]

normal perturbative series for $C_{\rm spin}$

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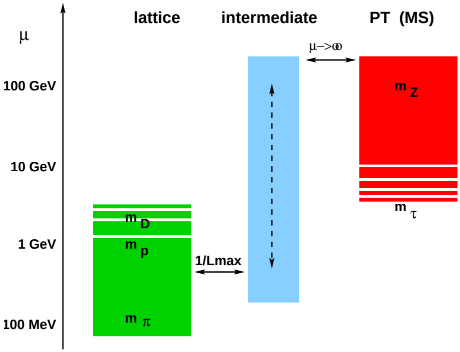

$$\Phi^{\rm RGI}_{\rm spin} = \Phi^{\rm SF}_{\rm spin}(\mu_{\rm had}) \underbrace{\frac{\Phi^{\rm SF}_{\rm spin}(\mu_{\rm pert})}{\Phi^{\rm SF}_{\rm spin}(\mu_{\rm had})}}_{\rm NP} \underbrace{\frac{\Phi^{\rm RGI}_{\rm spin}}{\Phi^{\rm SF}_{\rm spin}(\mu_{\rm pert})}}_{\rm pert.\ theory}$$

$$Z^{\rm RGI}_{\rm spin}(g_0) = Z^{\rm SF}_{\rm spin}(\mu_{\rm had}, g_0) \frac{\Phi^{\rm SF}_{\rm spin}(\mu_{\rm pert})}{\Phi^{\rm SF}_{\rm spin}(\mu_{\rm had})} U^{\rm SF}(\mu_{\rm pert})$$

$$(g_0 \Leftrightarrow a)$$

NP running from finite volume schemes

need $\Lambda_{\rm QCD} \ll \mu_{\rm pert} \ll a^{-1} = \Lambda_{\rm cut}$
trick [Lüscher, Weisz & Wolff] $\mu = 1/L$: finite volume scheme

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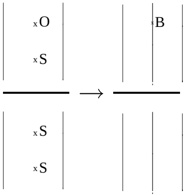

NP definition of $Z_{\rm spin}^{\rm SF}$: ($\mathcal{O}_{\rm spin}, S_k$ at a distance $L/2$, S_k Noether charge of spin trafo)

$$Z_{\rm spin}^{\rm SF} \frac{\langle\, \mathcal{O}_{\rm spin}\, S_1 \rangle}{\langle\, S_1\, S_1 \rangle} = \frac{\langle\, \mathcal{O}_{\rm spin}\, S_1 \rangle}{\langle\, S_1\, S_1 \rangle}\Big|_{\text{tree level}}$$

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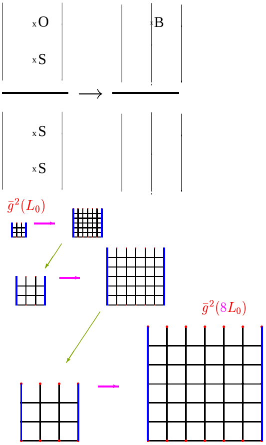

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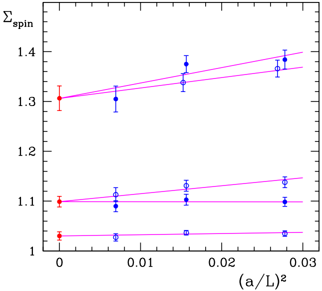
recursively: $L_0 \to L_1 = 2L_0 \to \ldots 2^n L_0$

- coupling: $\bar{g}^2(2L) = \sigma(\bar{g}^2(L))$,
  $\sigma(\bar{g}^2(L)) = \lim_{a/L\to 0} \Sigma(\bar{g}^2(L), a/L)$
-

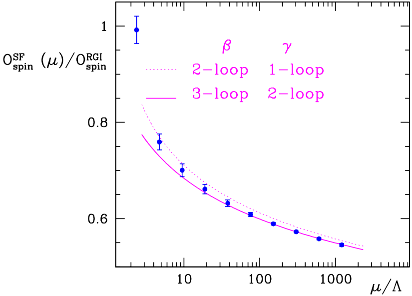
$$\mathcal{O}_{\rm spin}(1/2L) = \sigma_{\rm spin}(\bar{g}^2(L))\, \mathcal{O}_{\rm spin}(1/L)$$

$$\sigma_{\rm spin}(\bar{g}^2(L)) = \lim_{a/L\to 0} \frac{Z_{\rm spin}(\frac{1}{2L}, g_0)}{Z_{\rm spin}(\frac{1}{L}, g_0)}$$

The running of $\mathcal{O}_{\mathrm{spin}}$: $N_{\mathrm{f}} = 0$



Continuum limit



reconstructed running

The total renormalization factor

- $$\begin{aligned} Z_{\rm spin}^{\rm RGI} &= Z_{\rm spin}^{\rm SF}(L/a, g_0) \times \frac{\Phi_{\rm RGI}}{\Phi_{\rm SF}(1/L)} \quad \text{at} \quad L = 2L_{\rm max} \\ &= 2.62(2) \quad \text{at} \quad \beta = 6.0 \quad \text{EH action} \\ &= 2.63(2) \quad \text{at} \quad \beta = 6.0 \quad \text{HYP2 action} \end{aligned}$$

The spin splitting using bare matrix elements from the literature

- $\beta = 6.0$

$$\text{Ref. [Bochicchio et al., 93]:} \qquad a^2\lambda_2^{\rm bare} = 0.0100(19)\,,$$
$$\text{Ref. [JLQCD, 03]:} \qquad a^2\lambda_2^{\rm bare} = 0.0138(15)\,.$$

They then quote

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low compared to experiment

$$\Delta m^2 = 0.485{\rm GeV}^2$$

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- with $C_{\rm spin}(M_{\rm b}/\Lambda_{\overline{\rm MS}}) = 1.15$, $M_{\rm b} = 6.76(9)\,{\rm GeV}$ and $Z_{\rm spin}^{\rm RGI} = 2.6$, $a = 1/(2\,{\rm GeV})$ we find

$$\text{Ref. [Bochicchio et al., 93] and NP } Z_{\rm spin}^{\rm RGI}: \qquad \Delta m^2 = 0.38(7)(?)\,{\rm GeV}^2\,,$$
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- seems closer to experiment

(?): no cont. limit, quenched!

remark:
spin-dependent potentials renormalize the same way

- [Eichten & Feinberg; Gromes]: relativistic corrections to static potential

$$V = \ldots + \frac{\mathbf{s}_1 \cdot \mathbf{s}_2}{3 m_1 m_2} V_4(r) + \frac{1}{m_1 m_2} \left[\frac{\mathbf{x} \cdot \mathbf{s}_1\, \mathbf{x} \cdot \mathbf{s}_2}{r^2} - \frac{\mathbf{s}_1 \cdot \mathbf{s}_2}{3} \right] V_3(r) \,.$$

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- periodic boundary conditions in all directions, $r^2 = x_1^2 + x_2^2 + x_3^2$

$$\frac{x_1 x_2}{r^2} V_3^{\mathrm{RGI}}(r) =$$

$$[Z_{\mathrm{spin}}^{\mathrm{RGI}}]^2 \lim_{L_0 \to \infty} a \sum_{x_0} \frac{\langle\, \mathrm{Tr}(\mathcal{P}_0(0) B_1(0)) \mathrm{Tr}(\mathcal{P}_0(x)^\dagger B_2(x)) \rangle}{\langle\, \mathrm{Tr}\mathcal{P}_0(0) \mathrm{Tr}\mathcal{P}_0(x)^\dagger\, \rangle}$$

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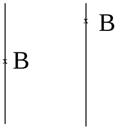

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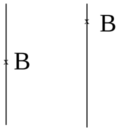

- this should replace the Huntley Michael factor