

Heavy quarkonium production in the Regge limit of QCD

V. A. Saleev

Samara State University, Samara, Russia



in collaboration with

D. V. Vasin (Samara State University), and B. A. Kniehl (Hamburg University)

1. NRQCD
2. High-energy (or k_T) factorization
3. Reggeized amplitudes
4. B- and D-meson production
5. Charmonium and bottomonium production
6. Polarization effects
7. Conclusions

V. A. Saleev and D. V. Vasin:

- 1) Phys. Rev. D **68**, 114013 (2003); 2) Phys. Atom. Nucl. **68**, 94 (2005) [Yad. Fiz. **68**, 95 (2005)]; 3) In Proc. of First Int. Workshop "HSQCD 2004", 73 (2004); 4) Phys. Lett. B **605**, 311 (2005); 5) Phys. Part. and Nuclei, **38**, 635 (2007)

B. A. Kniehl, V. A. Saleev and D. V. Vasin:

- 6) Phys. Rev. D **73**, 074022 (2006); 7) Phys. Rev. D **74**, 014024 (2006).

NRQCD

The factorization hypothesis of nonrelativistic QCD (NRQCD) [Bodwin, Braaten,Lepage] assumes the separation of the effects of long and short distances in heavy-quarkonium production.

NRQCD is organized as a perturbative expansion in two small parameters, the strong-coupling constant α_s and the relative velocity v of the heavy quarks.

In the framework of the NRQCD factorization approach, the cross section of heavy-quarkonium production in a partonic subprocess $a + b \rightarrow \mathcal{H} + X$ may be presented as a sum of terms in which the effects of long and short distances are factorized as

$$d\hat{\sigma}(a + b \rightarrow \mathcal{H} + X) = \sum_n d\hat{\sigma}(a + b \rightarrow Q\bar{Q}[n] + X) \langle \mathcal{O}^{\mathcal{H}}[n] \rangle, \quad (1)$$

The cross section $d\hat{\sigma}(a + b \rightarrow Q\bar{Q}[n] + X)$ can be calculated in perturbative QCD as an expansion in α_s using the non-relativistic approximation for the relative motion of the heavy quarks in the $Q\bar{Q}$ pair.

The non-perturbative transition of the $Q\bar{Q}$ pair into the physical quarkonium state \mathcal{H} is described by the NMEs $\langle \mathcal{O}^{\mathcal{H}}[n] \rangle$, which can be extracted from experimental data.

To leading order in v , we need to include the $Q\bar{Q}$ Fock states $n = {}^3S_1^{(1)}, {}^3S_1^{(8)}, {}^1S_0^{(8)}, {}^3P_J^{(8)}$ if $\mathcal{H} = \Upsilon(nS), \psi(nS)$, and $n = {}^3P_J^{(1)}, {}^3S_1^{(8)}$ if $\mathcal{H} = \chi_{bJ,cJ}(nP)$, where $J = 0, 1$ or 2 .

High-energy(or k_T) factorization

The hard scale is $\mu \approx M_T = \sqrt{M^2 + |\mathbf{p}_T|^2}$

In the conventional Collinear Parton Model (PM) $S > \mu^2 \gg \Lambda_{QCD}^2$, and $k_T = 0$.

Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equation, with $\ln(\mu/\Lambda_{QCD}) \gg 1$.

In the high-energy Regge limit $S \gg \mu^2 \gg \Lambda_{QCD}^2$ and $x = \mu/\sqrt{S} \ll 1$

Balitsky-Fadin-Kuraev-Lipatov (BFKL) evolution equation, with $\ln(\sqrt{S}/\mu) \gg 1$
 $k_T \neq 0$ for *reggeized t-channel gluons*.

In the high-energy factorization scheme, the hadronic cross section of quarkonium (\mathcal{H}) production in the process

$$p + p \rightarrow \mathcal{H} + X \quad (2)$$

and the partonic cross section for the reggeized-gluon fusion subprocess

$$R + R \rightarrow \mathcal{H} + X \quad (3)$$

are connected as

$$\begin{aligned} d\sigma(p + p \rightarrow \mathcal{H} + X) = & \int \frac{dx_1}{x_1} \int \frac{d^2 \mathbf{k}_{1T}}{\pi} \Phi(x_1, |\mathbf{k}_{1T}|^2, \mu^2) \times \\ & \int \frac{dx_2}{x_2} \int \frac{d^2 \mathbf{k}_{2T}}{\pi} \Phi(x_2, |\mathbf{k}_{2T}|^2, \mu^2) \times d\hat{\sigma}(R + R \rightarrow \mathcal{H} + X), \end{aligned} \quad (4)$$

$$xG(x, \mu^2) = \int \frac{d^2\mathbf{k}_T}{\pi} \Phi(x, |\mathbf{k}_T|^2, \mu^2), \quad (5)$$

The partonic cross section for the two reggeized gluon collision can be presented as follows:

$$d\hat{\sigma}(R + R \rightarrow \mathcal{H} + X) = \frac{\mathcal{N}}{2x_1 x_2 S} \times \overline{|\mathcal{A}(R + R \rightarrow \mathcal{H} + X)|^2} d\Phi, \quad (6)$$

$$\mathcal{N} = \frac{(x_1 x_2 S)^2}{16 |\mathbf{k}_{1T}|^2 |\mathbf{k}_{2T}|^2}. \quad (7)$$

So, that when $\mathbf{k}_{1T} = \mathbf{k}_{2T} = 0$ we obtain the factorization formula of the Collinear Parton Model:

$$d\sigma(p + p \rightarrow \mathcal{H} + X) = \int dx_1 G(x_1, \mu^2) \int dx_2 G(x_2, \mu^2) \times d\hat{\sigma}(g + g \rightarrow \mathcal{H} + X) \quad (8)$$

First new point: unintegrated distribution functions (unDF) $\Phi(x, |\mathbf{k}_T|^2, \mu^2)$. In the stage of the numerical calculations, we have used the JB , JS , and KMR (MRW) unDFs. We have shown that Kimber-Martin-Ryskin-(Watt) unDF describes all data from Tevatron for heavy quark and quarkonium production well.

Second new point: reggeized (off-shell) amplitudes. We are working in the Lipatov-Fadin formalism (QMRK).

Reggeized amplitudes

As the theoretical framework we consider the quasi-multi-Regge kinematics (QMRK) approach.

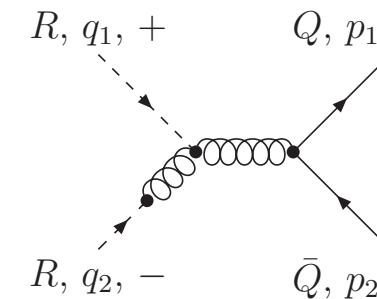
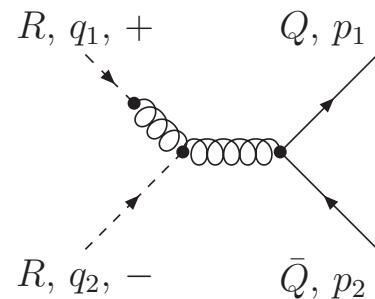
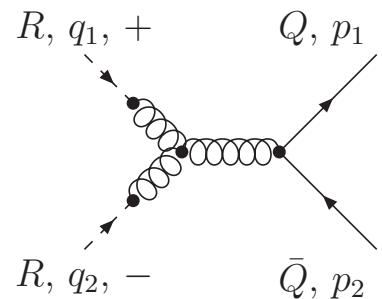
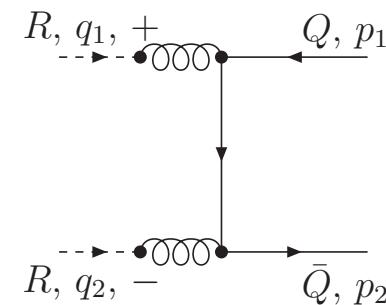
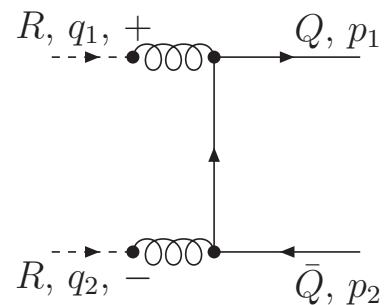
QMRK is based on effective quantum field theory implemented with the non-abelian gauge-invariant action, as was suggested a few years ago [Lipatov, 1995].

In 2005 the Feynman rules for the effective theory based on the non-abelian gauge-invariant action were derived for the induced and the some important effective vertices [Antonov, Kuraev, Lipatov, Cherednikov].

The QMRK approach gives the following:

1. The set of Feynman's rules for the gauge invariant amplitudes with reggeized gluons (and reggeized quarks [Fadin, Bogdan, 2006]).
2. The opportunity to perform calculations in the NLO approximation in α_s .

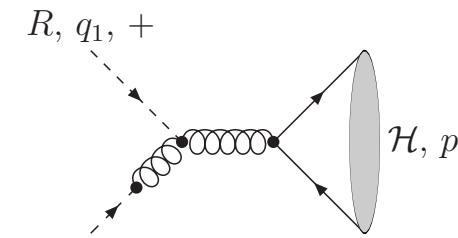
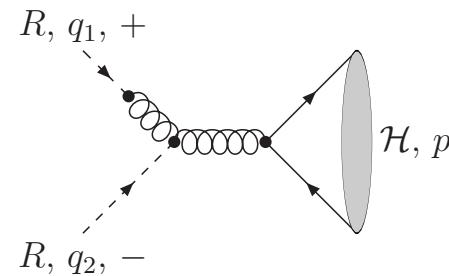
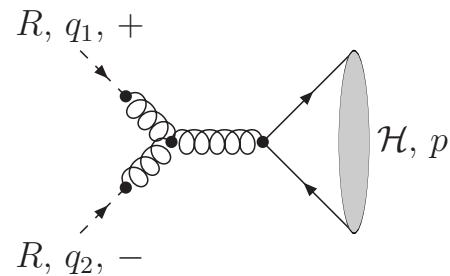
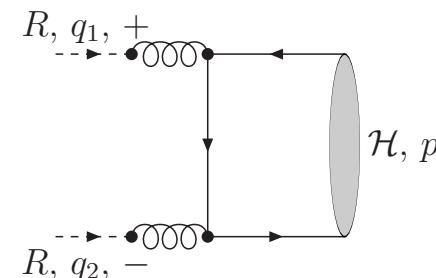
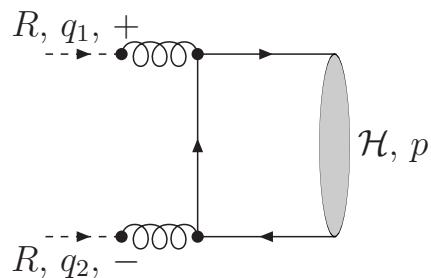
Heavy quark production by reggeized gluons

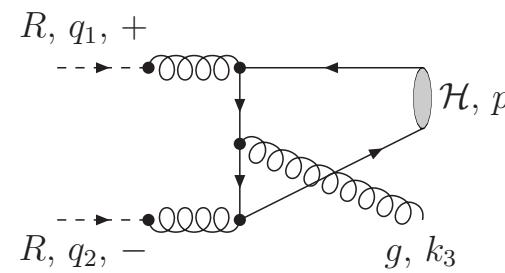
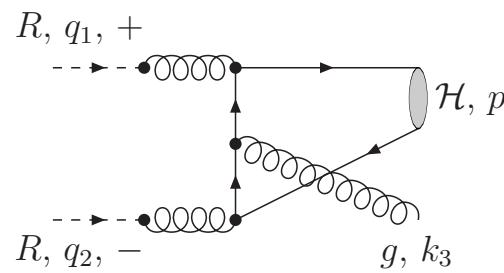
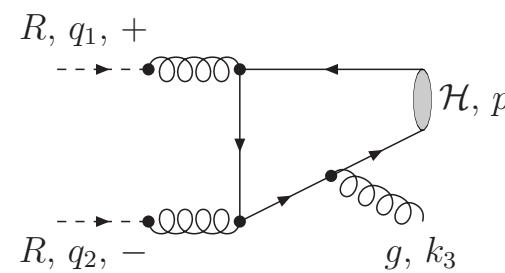
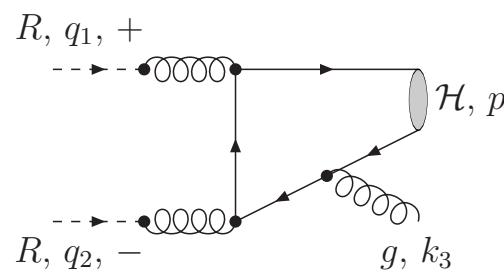
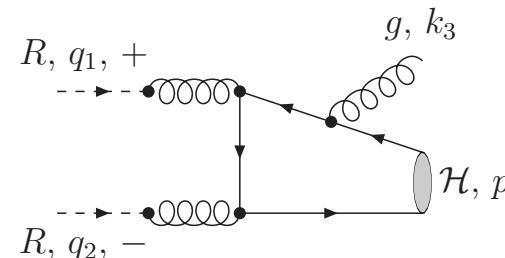
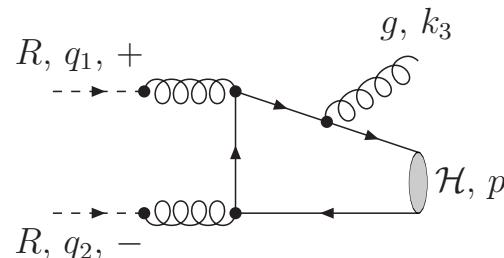


Heavy quarkonium production by reggeized gluons

$$R + R \rightarrow \mathcal{H}[{}^3P_J^{(1)}, {}^3S_1^{(8)}, {}^1S_0^{(8)}, {}^3P_J^{(8)}],$$

$$R + R \rightarrow \mathcal{H}[{}^3S_1^{(1)}] + g,$$





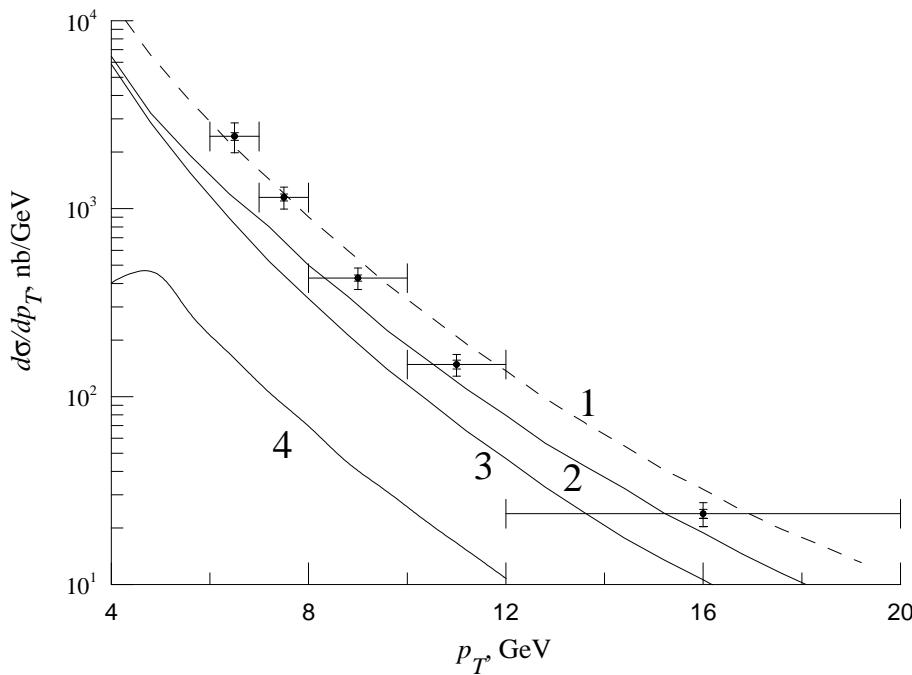
B- and D- meson production

We use universal FFs by Kniehl, Kramer, Schienbein, Spiesberger and work in the, so-called, mass-less scheme ($p_T \gg m_c$).

$$d\sigma(ij \rightarrow HX) = \sum_k \int d\sigma(ij \rightarrow kX) D_{k \rightarrow H}(z, \mu^2) dz \quad (9)$$

$$\mu^2 \frac{dD_k(z, \mu^2)}{d\mu^2} = \sum_i \int_z^1 \frac{dx}{x} P_{ik}\left(\frac{z}{x}\right) D_i(x, \mu^2), \quad (10)$$

$$\text{where } k = q, g, c \quad (11)$$



p_T spectrum of D^{*+} -mesons. Curve 1 is the sum of all contributions, curve 2 is the c -quark contribution, curve 3 is the gluon contribution, curve 4 is the b -quark contribution. The CDF data are from Tevatron at $\sqrt{S} = 1.96$ TeV and $|y| < 1$.

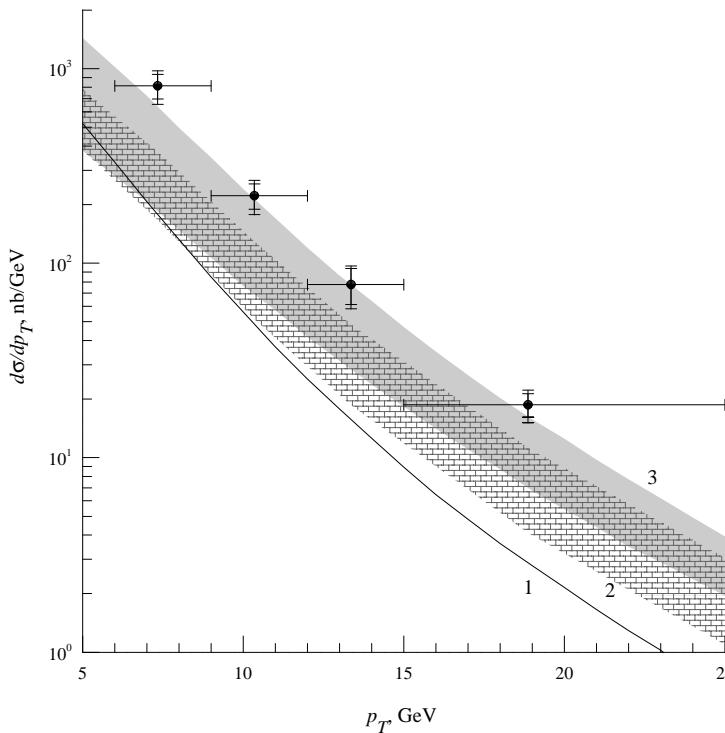


Figure 1: Theoretical results of B^+ -meson hadroproduction at $\sqrt{S} = 1.8$ GeV and $|y| < 1$ compared with CDF data from Tevatron . Curve 1 — collinear parton model prediction at LO, 2 — collinear parton model prediction at NLO , 3 — LO predictions in QMRK.

Charmonium and bottomonium production

Nowadays Tevatron CDF data incorporate p_T -spectra for prompt $\Upsilon(1S, 2S, 3S)$ at the $\sqrt{S} = 1.8$ TeV and for prompt $\Upsilon(1S)$ in the different intervals of rapidity at the $\sqrt{S} = 1.96$ TeV; for direct J/ψ , for J/ψ from ψ' decays, for J/ψ from χ_{cJ} decays at the $\sqrt{S} = 1.8$ TeV; for prompt J/ψ at the $\sqrt{S} = 1.96$ TeV.

$$\begin{aligned}\sigma^{prompt}(J/\psi) = & \sigma^{direct}(J/\psi) + \sigma(\psi' \rightarrow J/\psi) + \\ & + \sigma(\chi_{cJ} \rightarrow J/\psi) + \sigma(\psi' \rightarrow \chi_{cJ} \rightarrow J/\psi)\end{aligned}$$

In contrast to previous analysis in the collinear parton model we perform a joint fit to the run-I and run-II CDF data for all p_T , including the region of small p_T , to obtain the color-octet NMEs for $\psi(nS)$, $\Upsilon(nS)$ and $\chi_{cJ}(1P), \chi_{bJ}(nP)$.

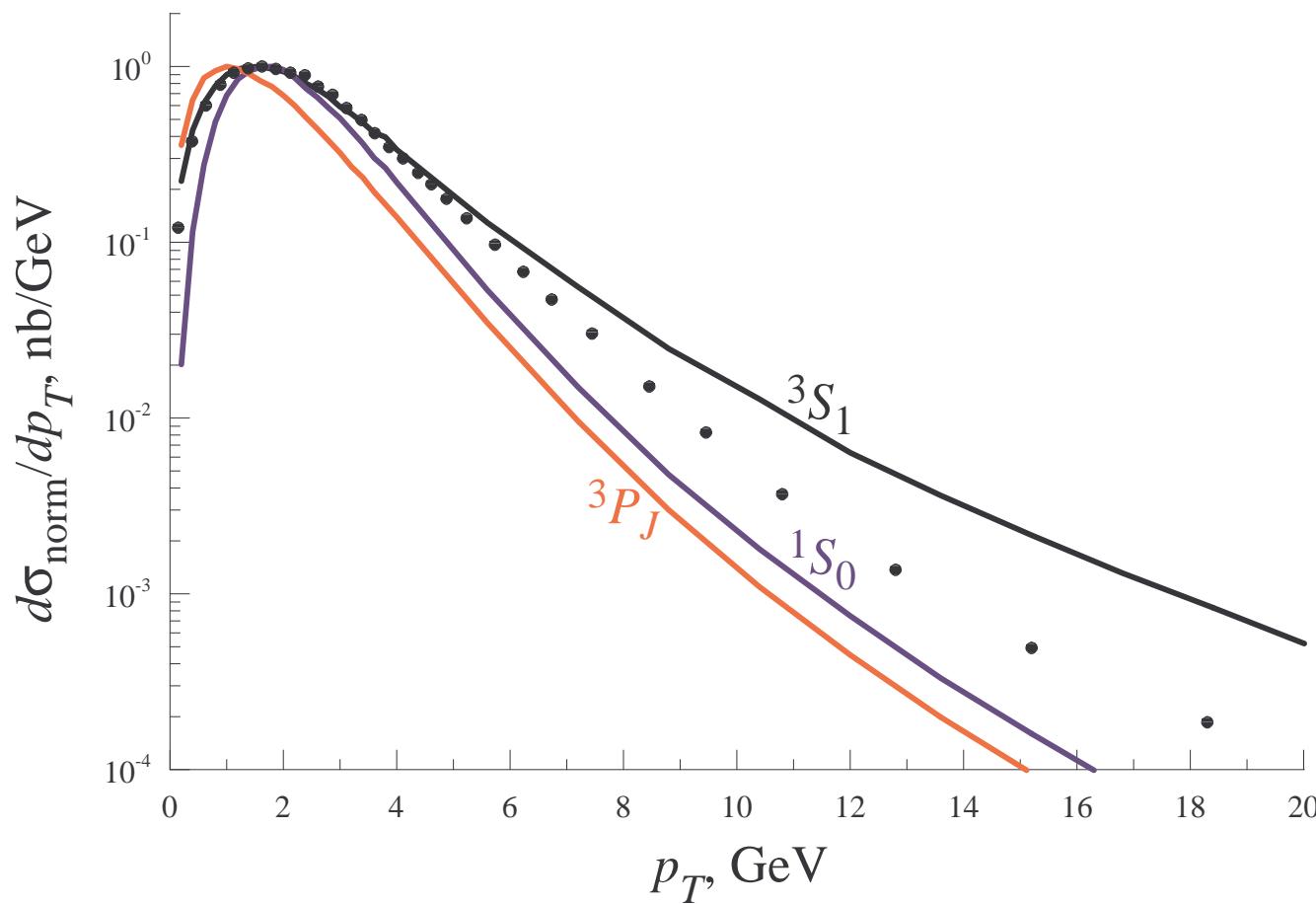
Our calculations are based on exact analytical expressions for the relevant squared amplitudes, obtained in the QMRK approach.

In previous fit of CDF data were used for the region of large $|\mathbf{p}_T| > 8(4)$ GeV only, and the linear combination

$$M_r^{\mathcal{H}} = \langle \mathcal{O}^{\mathcal{H}}[{}^1S_0^{(8)}] \rangle + \frac{r}{m_Q^2} \langle \mathcal{O}^{\mathcal{H}}[{}^3P_0^{(8)}] \rangle \quad (12)$$

was fixed because it was unfeasible to separate the contributions proportional to $\langle \mathcal{O}^{\mathcal{H}}[{}^1S_0^{(8)}] \rangle$ and $\langle \mathcal{O}^{\mathcal{H}}[{}^3P_0^{(8)}] \rangle$.

By contrast, QMRK fit allow us to determine $\langle \mathcal{O}^{\mathcal{H}}[{}^1S_0^{(8)}] \rangle$ and $\langle \mathcal{O}^{\mathcal{H}}[{}^3P_0^{(8)}] \rangle$ separately, which is due to the different $|\mathbf{p}_T|$ dependence of the respective contributions for $|\mathbf{p}_T| < 8(4)$ GeV.

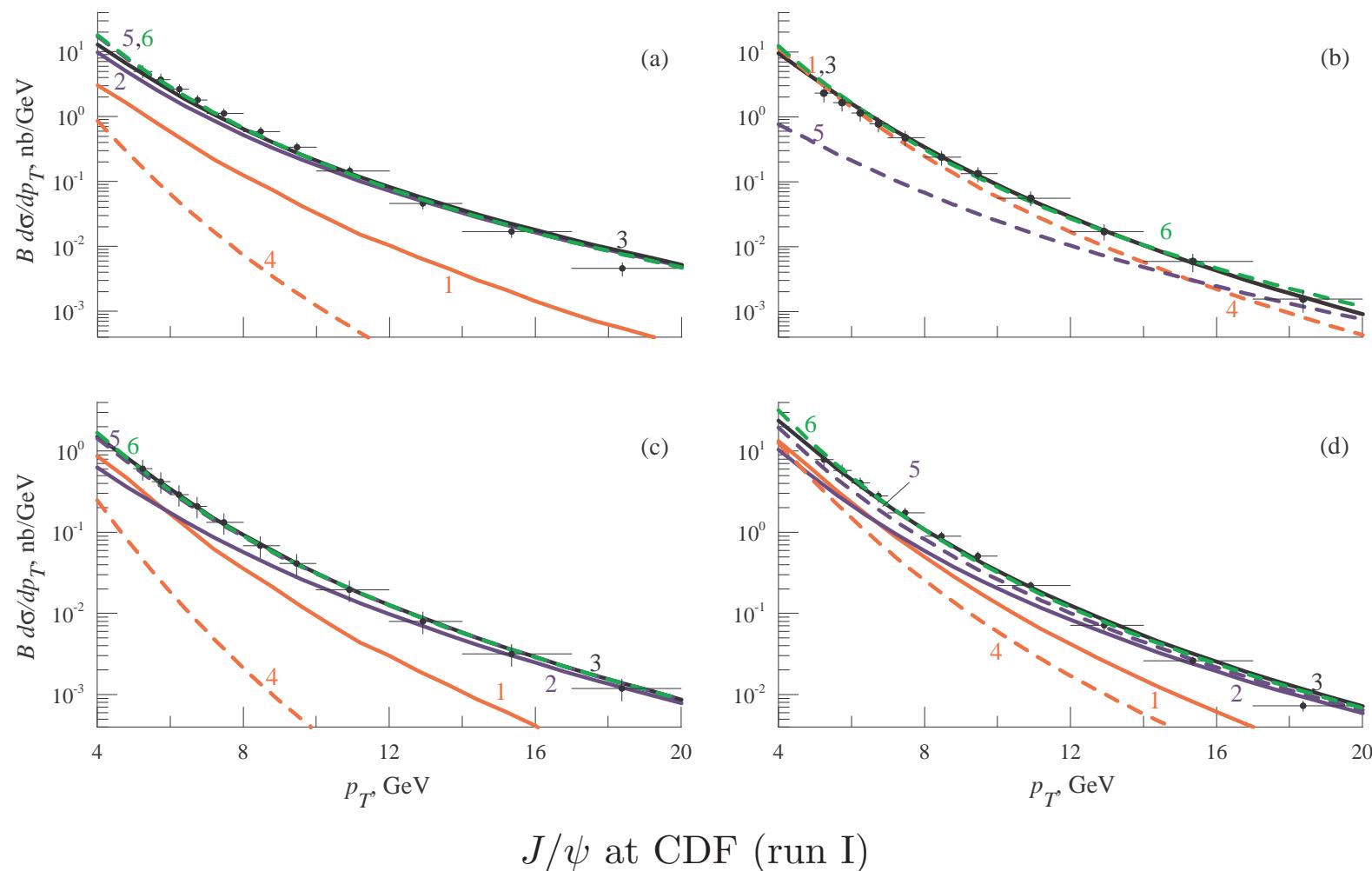


Contributions to the p_T distribution of direct $\Upsilon(1S)$ hadroproduction in $p\bar{p}$ scattering with $\sqrt{S} = 1.8$ TeV and $|y| < 0.4$ from the relevant color-octet states. All distributions are normalized on unit in their peak values.

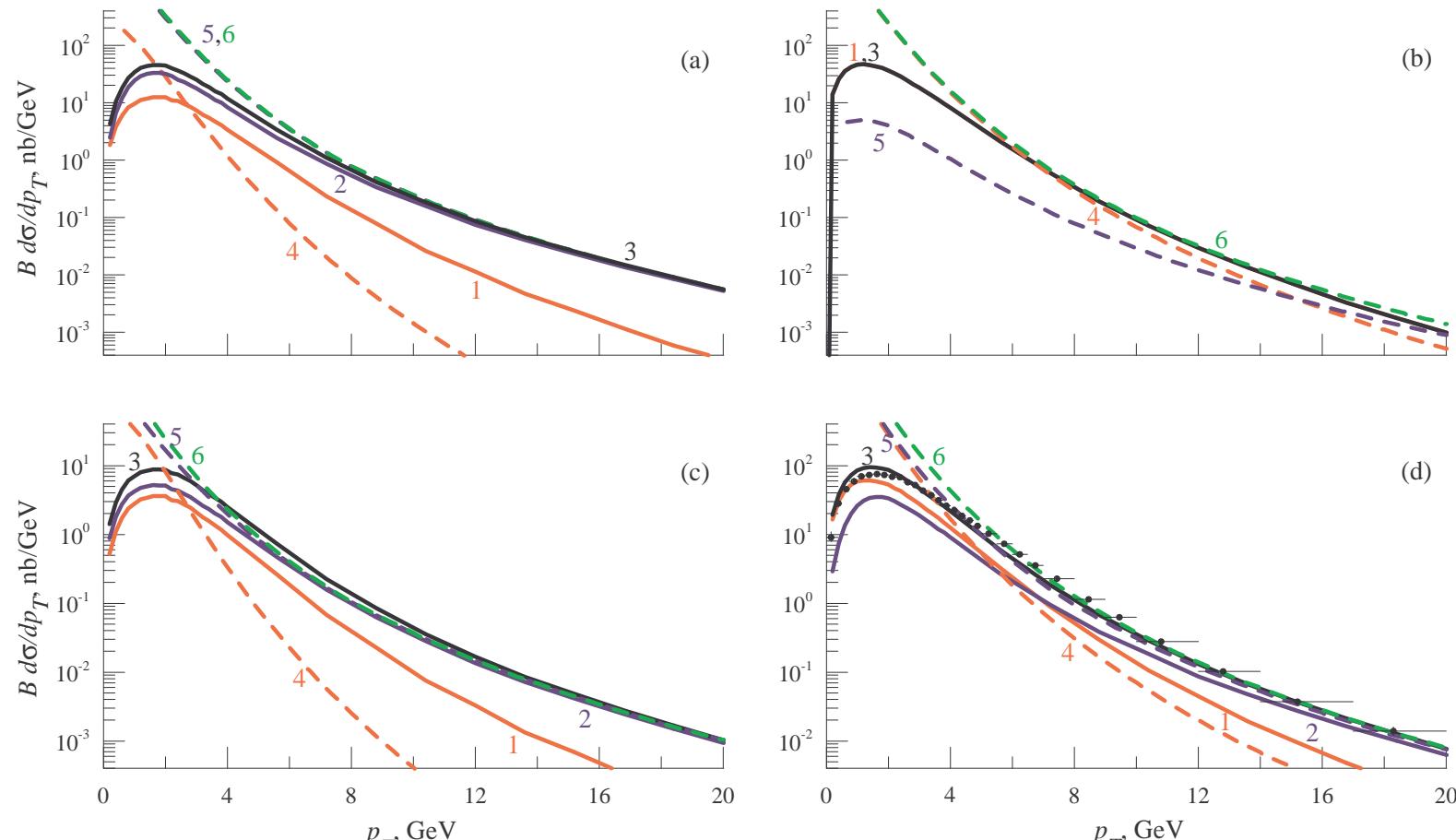
Table: NMEs for J/ψ , ψ' and χ_{cJ}

NME	PM	Fit JB	Fit JS	Fit KMR
$\langle \mathcal{O}^{J/\psi}[{}^3S_1^{(1)}] \rangle / \text{GeV}^3$	1.3	1.3	1.3	1.3
$\langle \mathcal{O}^{J/\psi}[{}^3S_1^{(8)}] \rangle / \text{GeV}^3$	$4.4 \cdot 10^{-3}$	$1.5 \cdot 10^{-3}$	$6.1 \cdot 10^{-3}$	$2.7 \cdot 10^{-3}$
$\langle \mathcal{O}^{J/\psi}[{}^1S_0^{(8)}] \rangle / \text{GeV}^3$	$4.3 \cdot 10^{-2}$	$6.6 \cdot 10^{-3}$	$9.0 \cdot 10^{-3}$	$1.4 \cdot 10^{-2}$
$\langle \mathcal{O}^{J/\psi}[{}^3P_0^{(8)}] \rangle / \text{GeV}^5$	$2.8 \cdot 10^{-2}$	0	0	0
$\langle \mathcal{O}^{\psi'}[{}^3S_1^{(1)}] \rangle / \text{GeV}^3$	$6.5 \cdot 10^{-1}$	$6.5 \cdot 10^{-1}$	$6.5 \cdot 10^{-1}$	$6.5 \cdot 10^{-1}$
$\langle \mathcal{O}^{\psi'}[{}^3S_1^{(8)}] \rangle / \text{GeV}^3$	$4.2 \cdot 10^{-3}$	$3.0 \cdot 10^{-4}$	$1.5 \cdot 10^{-3}$	$8.3 \cdot 10^{-4}$
$\langle \mathcal{O}^{\psi'}[{}^1S_0^{(8)}] \rangle / \text{GeV}^3$	$6.9 \cdot 10^{-3}$	0	0	0
$\langle \mathcal{O}^{\psi'}[{}^3P_0^{(8)}] \rangle / \text{GeV}^5$	$3.9 \cdot 10^{-3}$	0	0	0
$\langle \mathcal{O}^{\chi_{c0}}[{}^3P_0^{(1)}] \rangle / \text{GeV}^5$	$8.9 \cdot 10^{-2}$	$8.9 \cdot 10^{-2}$	$8.9 \cdot 10^{-2}$	$8.9 \cdot 10^{-2}$
$\langle \mathcal{O}^{\chi_{c0}}[{}^3S_1^{(8)}] \rangle / \text{GeV}^3$	$4.4 \cdot 10^{-3}$	0	$2.2 \cdot 10^{-4}$	$4.7 \cdot 10^{-5}$
$\chi^2/\text{d.o.f}$	–	2.2 (*)	4.1	3.0

$$\Delta L \approx \Delta S \approx 0$$



(a) direct, (b) χ_{cJ} -decays, (c) ψ' -decays, (d) prompt; solid lines - QMRK, dashed lines - PM; 1,4 - color-singlet, 2,5 - color-octet, 3,6 - total.

 J/ψ at CDF (run II)

(a) direct, (b) χ_{cJ} -decays, (c) ψ' -decays, (d) prompt; solid lines - QMRK, dashed lines - PM; 1,4 - color-singlet, 2,5 - color-octet, 3,6 - total.

Table: Inclusive branchings fractions for transitions between spin-triplet bottomonium states.

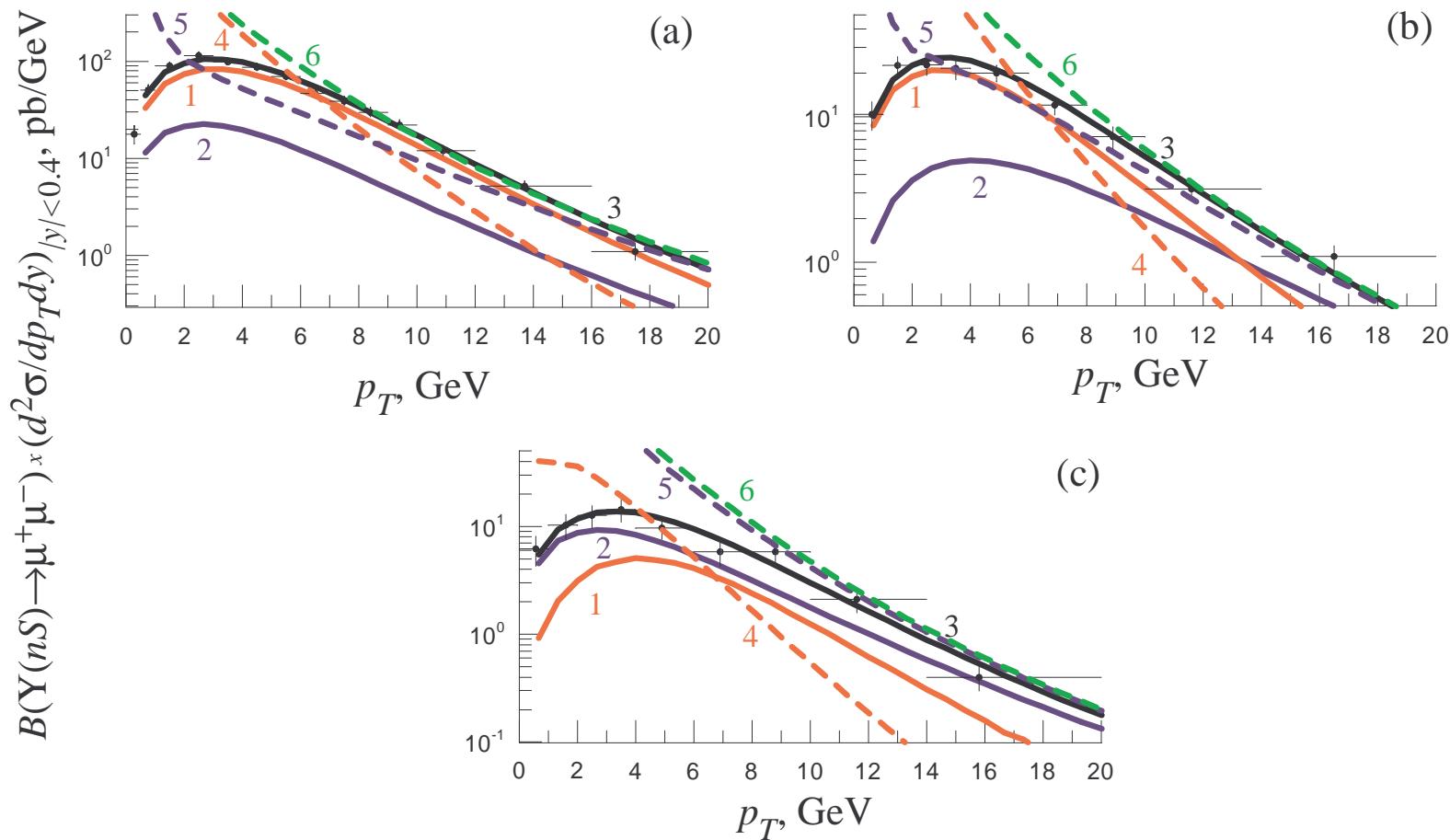
In\Out	$\Upsilon(3S)$	$\chi_{b2}(2P)$	$\chi_{b1}(2P)$	$\chi_{b0}(2P)$	$\Upsilon(2S)$	$\chi_{b2}(1P)$	$\chi_{b1}(1P)$	$\chi_{b0}(1P)$	$\Upsilon(1S)$
$\Upsilon(3S)$	1	0.114	0.113	0.054	0.106	0.007208	0.00742	0.004028	0.102171
$\chi_{b2}(2P)$	—	1	—	—	0.162	0.011016	0.01134	0.006156	0.129565
$\chi_{b1}(2P)$	—	—	1	—	0.21	0.01428	0.0147	0.00798	0.160917
$\chi_{b0}(2P)$	—	—	—	1	0.046	0.003128	0.00322	0.001748	0.0167195
$\Upsilon(2S)$	—	—	—	—	1	0.068	0.07	0.038	0.319771
$\chi_{b2}(1P)$	—	—	—	—	—	1	—	—	0.22
$\chi_{b1}(1P)$	—	—	—	—	—	—	1	—	0.35
$\chi_{b0}(1P)$	—	—	—	—	—	—	—	1	0.06
$\Upsilon(1S)$	—	—	—	—	—	—	—	—	1

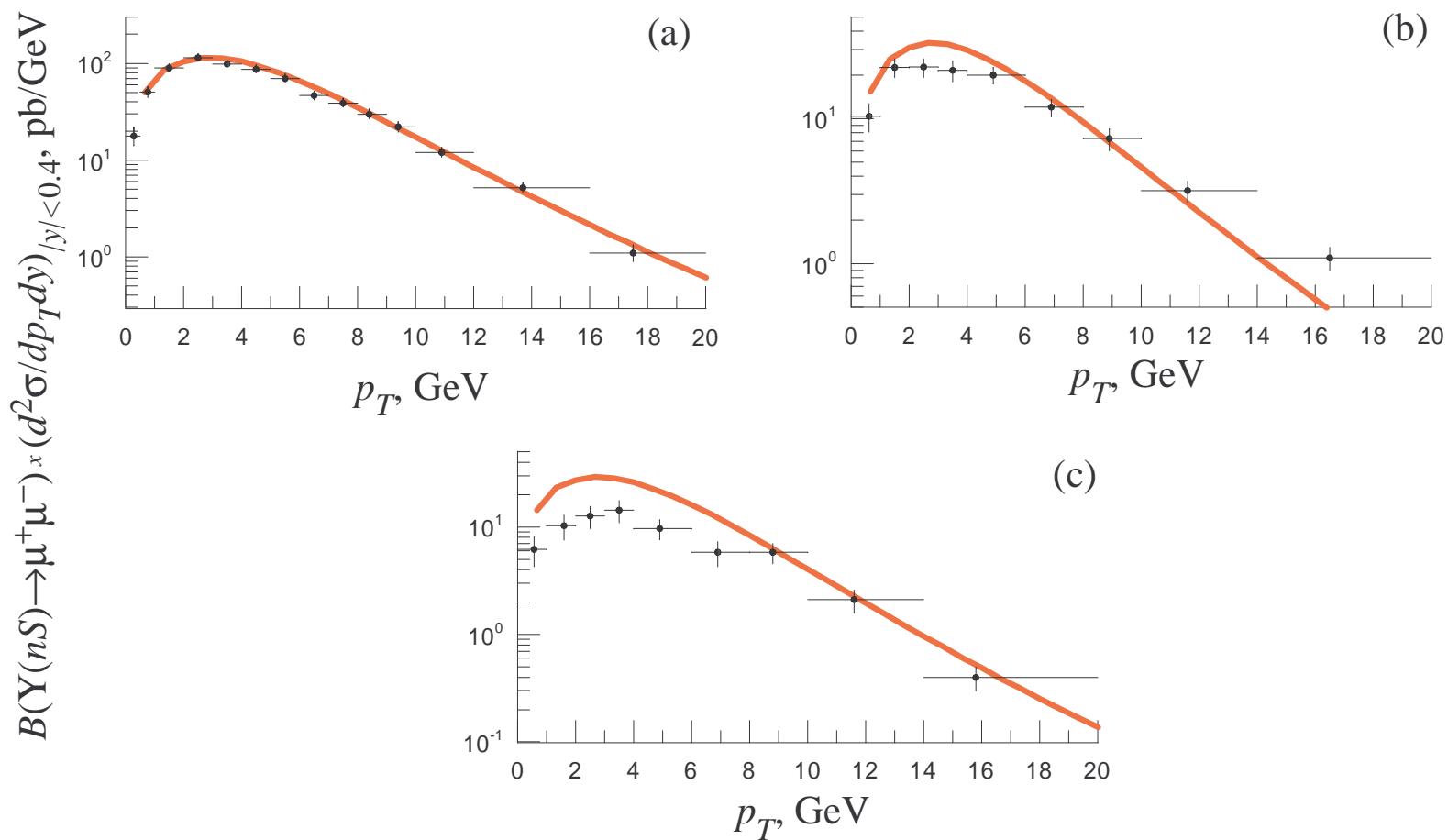
Table: NMEs for $\Upsilon(1S, 2S, 3S)$, and χ_{bJ}

n / n	PM	Fit JB	Fit JS	Fit KMR
$\langle \mathcal{O}^{\Upsilon(1S)} [{}^1 S_0^{(8)}] \rangle, \text{GeV}^3$	$1.4 \cdot 10^{-1}$	0.0	0.0	0.0
$\langle \mathcal{O}^{\Upsilon(1S)} [{}^3 S_1^{(1)}] \rangle, \text{GeV}^3$	$1.1 \cdot 10^1$	$1.1 \cdot 10^1$	$1.1 \cdot 10^1$	$1.1 \cdot 10^1$
$\langle \mathcal{O}^{\Upsilon(1S)} [{}^3 S_1^{(8)}] \rangle, \text{GeV}^3$	$2.0 \cdot 10^{-2}$	$5.3 \cdot 10^{-3}$	0.0	0.0
$\langle \mathcal{O}^{\Upsilon(1S)} [{}^3 P_0^{(8)}] \rangle, \text{GeV}^5$	0.0	0.0	0.0	$9.5 \cdot 10^{-2}$
$\langle \mathcal{O}^{\chi_{b0}(1P)} [{}^3 S_1^{(8)}] \rangle, \text{GeV}^3$	$1.5 \cdot 10^{-2}$	0.0	0.0	0.0
$\langle \mathcal{O}^{\chi_{b0}(1P)} [{}^3 P_0^{(1)}] \rangle, \text{GeV}^5$	2.4	2.4	2.4	2.4
$\langle \mathcal{O}^{\Upsilon(2S)} [{}^1 S_0^{(8)}] \rangle, \text{GeV}^3$	0.0	0.0	0.0	0.0
$\langle \mathcal{O}^{\Upsilon(2S)} [{}^3 S_1^{(1)}] \rangle, \text{GeV}^3$	4.5	4.5	4.5	4.5
$\langle \mathcal{O}^{\Upsilon(2S)} [{}^3 S_1^{(8)}] \rangle, \text{GeV}^3$	$1.6 \cdot 10^{-1}$	0.0	0.0	$3.3 \cdot 10^{-2}$
$\langle \mathcal{O}^{\Upsilon(2S)} [{}^3 P_0^{(8)}] \rangle, \text{GeV}^5$	0.0	0.0	0.0	0.0
$\langle \mathcal{O}^{\chi_{b0}(2P)} [{}^3 S_1^{(8)}] \rangle, \text{GeV}^3$	$8.0 \cdot 10^{-3}$	$1.1 \cdot 10^{-2}$	0.0	0.0
$\langle \mathcal{O}^{\chi_{b0}(2P)} [{}^3 P_0^{(1)}] \rangle, \text{GeV}^5$	2.6	2.6	2.6	2.6
$\langle \mathcal{O}^{\Upsilon(3S)} [{}^1 S_0^{(8)}] \rangle, \text{GeV}^3$	$5.4 \cdot 10^{-2}$	0.0	0.0	0.0
$\langle \mathcal{O}^{\Upsilon(3S)} [{}^3 S_1^{(1)}] \rangle, \text{GeV}^3$	4.3	4.3	4.3	4.3
$\langle \mathcal{O}^{\Upsilon(3S)} [{}^3 S_1^{(8)}] \rangle, \text{GeV}^3$	$3.6 \cdot 10^{-2}$	$1.4 \cdot 10^{-2}$	$5.9 \cdot 10^{-3}$	$1.1 \cdot 10^{-2}$
$\langle \mathcal{O}^{\Upsilon(3S)} [{}^3 P_0^{(8)}] \rangle, \text{GeV}^5$	0.0	$2.4 \cdot 10^{-2}$	$3.4 \cdot 10^{-3}$	$5.2 \cdot 10^{-2}$
$\chi^2/\text{d.o.f}$	—	2.9	$2.7 \cdot 10^1$	$4.9 \cdot 10^{-1}$

$$\frac{\text{Color Octet Contribution}}{\text{Color Singlet Contribution}} \ll 1$$

$$v_{c\bar{c}}^2 \simeq 0.3, \quad v_{b\bar{b}}^2 \simeq 0.1$$

Prompt $\Upsilon(nS)$ p_T -spectra. $\Upsilon(1S)$ — a, $\Upsilon(2S)$ — b, $\Upsilon(3S)$ — c, KMR distribution function



Prompt $\Upsilon(nS)$ p_T -spectra. $\Upsilon(1S)$ — a, $\Upsilon(2S)$ — b, $\Upsilon(3S)$ — c, KMR distribution function, Color Singlet Model with $\chi_{bJ}(3P)$ contribution.

Polarization effects

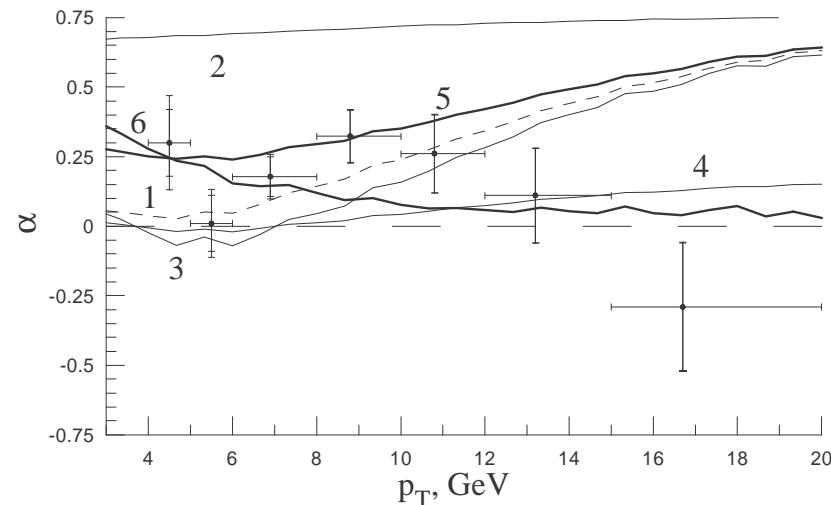
For the polarized J/ψ or ψ' production via direct channel we can write [Cho, Leibovich]:

$$\begin{aligned}\sigma_L^{J/\psi, \psi'} = & \sigma_1^{J/\psi, \psi'}(^3S_1^{(1)}) + \sigma_0^{J/\psi, \psi'}(^3S_1^{(8)}) + \frac{1}{3}\sigma_1^{J/\psi, \psi'}(^1S_0^{(8)}) + \frac{1}{3}\sigma_1^{J/\psi, \psi'}(^3P_0^{(8)}) + \\ & \frac{1}{2}\sigma_1^{J/\psi, \psi'}(^3P_1^{(8)}) + \frac{2}{3}\sigma_0^{J/\psi, \psi'}(^3P_2^{(8)}) + \frac{1}{2}\sigma_1^{J/\psi, \psi'}(^3P_2^{(8)})\end{aligned}$$

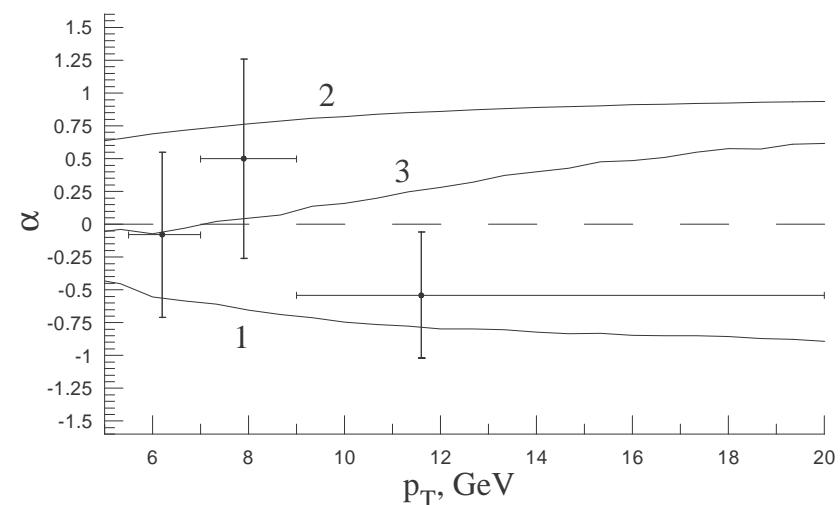
As it can be shown [Kniehl, Lee] for the polarized J/ψ production via prompt mechanism one has:

$$\begin{aligned}\sigma_L^{prompt} = & \sigma_L^{J/\psi} + \sigma_L^{\chi_c \rightarrow J/\psi} + \sigma_L^{\psi' \rightarrow J/\psi} + \sigma_L^{\psi' \rightarrow \chi_c \rightarrow J/\psi} \\ \sigma_L^{\chi_c \rightarrow J/\psi} = & [\frac{1}{3}\sigma_0^{\chi_{c0}}(^3P_0^{(1)}) + \frac{1}{3}\sigma_0^{\chi_{c0}}(^3S_1^{(8)})]Br(\chi_{c0} \rightarrow J/\psi + \gamma) + \\ + & \frac{1}{3}[\frac{1}{2}\sigma_1^{\chi_{c1}}(^3P_1^{(1)}) + \frac{1}{2}\sigma_0^{\chi_{c1}}(^3S_1^{(8)}) + \frac{1}{4}\sigma_1^{\chi_{c1}}(^3S_1^{(8)})]Br(\chi_{c1} \rightarrow J/\psi + \gamma) + \\ + & \frac{1}{5}[\frac{2}{3}\sigma_0^{\chi_{c2}}(^3P_2^{(1)}) + \frac{1}{2}\sigma_1^{\chi_{c2}}(^3P_2^{(1)}) + \frac{17}{30}\sigma_0^{\chi_{c2}}(^3S_1^{(8)}) + \frac{13}{60}\sigma_1^{\chi_{c2}}(^3S_1^{(8)})]Br(\chi_{c2} \rightarrow J/\psi + \gamma) \\ \sigma_L^{\psi' \rightarrow J/\psi} = & \sigma_L^{\psi'} Br(\psi' \rightarrow J/\psi + X)\end{aligned}$$

$$\begin{aligned}\sigma_L^{\psi' \rightarrow \chi_c \rightarrow J/\psi} &= \frac{1}{3} \sigma_L^{\psi'} Br(\psi' \rightarrow \chi_{c0}) Br(\chi_{c0} \rightarrow J/\psi + \gamma) + \\ &+ (\frac{1}{2} \sigma_L^{\psi'} + \frac{1}{4} \sigma_T^{\psi'}) Br(\psi' \rightarrow \chi_{c1}) Br(\chi_{c1} \rightarrow J/\psi + \gamma) + \\ &+ (\frac{17}{30} \sigma_L^{\psi'} + \frac{13}{60} \sigma_T^{\psi'}) Br(\psi' \rightarrow \chi_{c2}) Br(\chi_{c2} \rightarrow J/\psi + \gamma)\end{aligned}$$



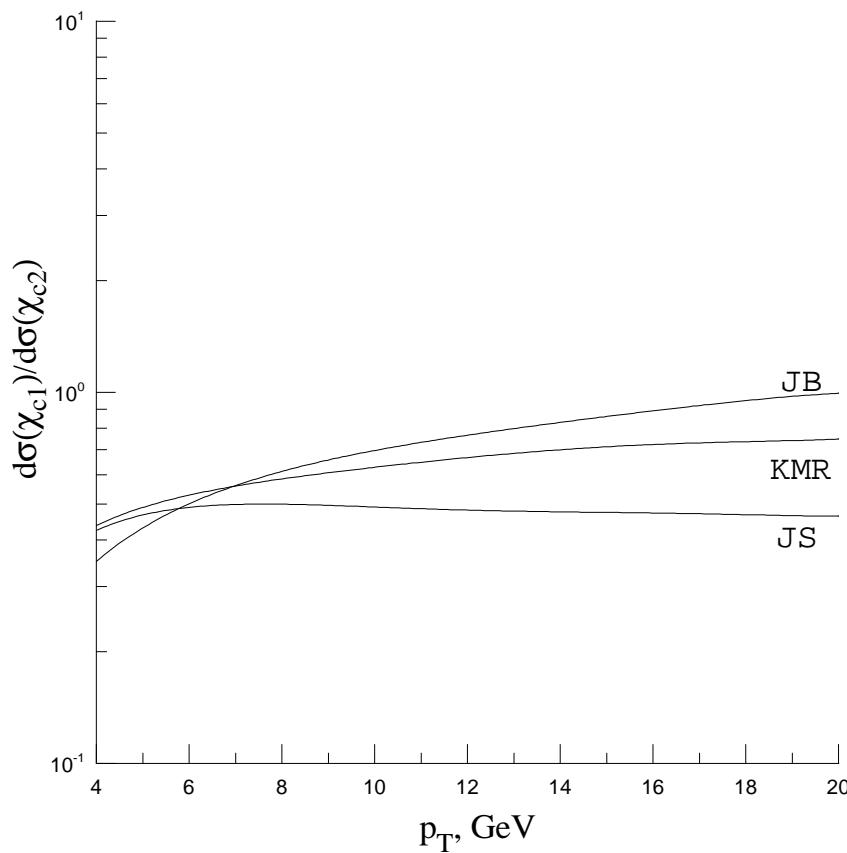
(a)



(b)

- a) Polarization parameter $\alpha(p_T)$ for prompt J/ψ meson production. Curve 1 — direct, 2 — $\chi_c \rightarrow J/\Psi$, 3 — $\psi' \rightarrow J/\psi$, 4 — $\psi' \rightarrow \chi_c \rightarrow J/\psi$, 5 — total, 6 — CSM only.
- b) Polarization parameter $\alpha(p_T)$ for direct ψ' meson production. Curve 1 — singlet, 2 — octet, 3 — direct.

Ratio χ_{c1}/χ_{c2} as function of p_T



Conclusions

1. Working at LO in the QMRK plus NRQCD approach, we analytically evaluated the squared amplitudes of heavy quarks and direct heavy quarkonium production in two reggeized gluon collisions.
2. We extracted the relevant color-octet NMEs, $\langle \mathcal{O}^{\mathcal{H}}[{}^3S_1^{(8)}] \rangle$, $\langle \mathcal{O}^{\mathcal{H}}[{}^1S_0^{(8)}] \rangle$, and $\langle \mathcal{O}^{\mathcal{H}}[{}^3P_0^{(8)}] \rangle$ for $\mathcal{H} = \Upsilon(1S, 2S, 3S)$, J/ψ , ψ' , $\chi_{cJ}(1P)$ and $\chi_{bJ}(1P, 2P)$, through fits to p_T distributions measured by the CDF Collaboration in $p\bar{p}$ collisions at the Tevatron with $\sqrt{S} = 1.8$ TeV and 1.96 TeV. Our fit to the Tevatron CDF data turned out to be satisfactory with the KMR unintegrated gluon distribution function in the proton.
3. $\Delta S \simeq \Delta L \simeq 0$.
4. $\langle \mathcal{O}^{(b\bar{b})}[{}^{2S+1}L_J^{(8)}] \rangle \ll \langle \mathcal{O}^{(c\bar{c})}[{}^{2S+1}L_J^{(8)}] \rangle$