New results for excited Charmonia from Lattice QCD

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Introduction

- New experimental results for Charmonium are motivating lattice studies.
- In principle lattice QCD can help to shed light on the nature of the excited spectrum of $c\bar{c}$ states.

- Control the statistical and systematic errors in determining energy levels to resolve a tower of radial and orbital states?
- Discrete group theory to understand particle identification in the observed spectrum.

Determining the $c\bar{c}$ spectrum

- Spin assignment: identification of the observed excitations in lattice data
- Use dynamical (highly) anisotropic lattices
  - $N_f = 2$, 3+1 anisotropy $\rightarrow a_t \ll a_s$, $\xi = a_s/a_t = 6$
  - relativistic charm quarks, $a_t m_c \ll 1$ and $a_s m_c < 1$
- Use all-to-all propagators
  - improved statistical precision
  - facilitates the creation of a large basis of operators to extract excited states
  - allows the inclusion of disconnected diagrams
- Use extended operators, smeared operators and variational analysis
  - allows us to extract multiple excitations

Spin assignment from lattice states (1)

- The lattice explicitly breaks $O(3)$ rotational symmetry to a finite 48-element sub-group O_h, the cubic point group.
- Eigenstates of the lattice hamiltonian will simultaneously be eigenstates of O_h only, so are classified according to their irreducible representation under action of O_h.
- Parity is still a good quantum number (use sub-script g and u for even/odd)
- O_h has 10 irreps:

Irrep	A_{1g}, A_{1u}	A_{2g}, A_{2u}	E_g, E_u	T_{1g}, T_{1u}	T_{2g}, T_{2u}
Dim	1	1	2	3	3

Spin assignment from lattice states (2)

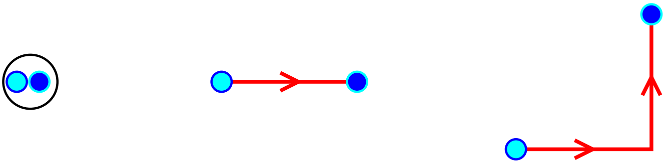
- Continuum spin assignment is made by forming the representations of O_h subduced from $O(3)$. Multiplicities (up to spin 4) are:

J	0	1	2	3	4
A_1	1	0	0	0	1
A_2	0	0	0	1	0
E	0	0	1	0	1
T_1	0	1	0	1	1
T_2	0	0	1	1	1

- Degeneracies appear across lattice irreps as continuum is approached
- Difficulty for charmonium: hyperfine splittings are small so near-degeneracy of many states with different spin assignments.
- A pathological example: it would be hard to distinguish near-degenerate triplet of spin 0,1,2 from spin 4. Both these systems have the same lattice irrep content: $A_1 \oplus T_1 \oplus E \oplus T_2$. Can a radial excitation of P-wave $\bar{c}c$ be distinguished from 4^{++} F-wave?

Operator construction

- Lattice operators are bilinears with path-ordered products between the quark and anti-quark field; different offsets, connecting paths and spin contractions give different projections into lattice irreps.
- This analysis uses a sub-set of the operators given below.



$\mathcal{O}_{\alpha\beta} = \sum_x \bar{\psi}_\alpha(x)\psi_\beta(x)$

$\mathcal{O}^i_{\alpha\beta} = \sum_x \bar{\psi}_\alpha(x)U_i(x)\psi_\beta(x+\hat{\imath})$

$\mathcal{O}^{ij}_{\alpha\beta} = \sum_x \bar{\psi}_\alpha(x)U_i(x)U_j(x+\hat{\imath})\psi_\beta(x+\hat{\imath}+\hat{\jmath})$

- This gives all S and P wave irreps, some D and beyond.
- In progress: more complete basis construction.

Anisotropic lattice QCD, with $N_f = 2$

Use an action designed for high anisotropies, $\xi = a_s/a_t = 6$.

- Fermion action is $\mathcal{O}(a_s^3, a_t, \alpha_s a_s)$ improved
- Gauge action is $\mathcal{O}(a_s^4, a_t^2, \alpha_s a_s^2)$ improved
- Spatial links are stout-smeared, reducing radiative corrections

Introduces 2 new parameters $\xi = a_s/a_t$ in the action which must be simulaneously tuned. [PRD 74 014505 (2006)].

Simulation Parameters

Light quarks	$m_\pi/m_\rho = 0.54$
Lattice spacing	$a_t = 0.025\text{fm}$, $a_s = 0.17\text{fm}$
Lattice volume	$12^3 \times 80$
$a_t m_c$	0.117
$a^{-1}_{(1\mathrm{P}-1\mathrm{S})}$	7.23GeV

Estimating all elements of the charm quark propagator (1)

- "Traditional" lattice calculations exploit translation invariance and γ_5-hermiticity to re-write correlation function in terms of a "point-to-all" propagator.
- Point-to-all propagator, computed by applying sparse matrix inverter to vector with single non-zero entry.
- This wastes information. Can we get access to all elements of the propagator? Yes, provided we are satisfied with an unbiased estimator.

Estimating all elements of the charm quark propagator (2)

- Fill a vector with random elements of $Z_4 = \{+1, i, -1, -i\}$ so that $E[\eta_i \eta_j^*] = \delta_{ij}$, then apply sparse matrix inverter to form $\psi = M^{-1}\eta$. Now
$$E[\psi_i \eta_j^*] = [M^{-1}]_{ij}$$

- Variance reduction is critical, since the estimator is noisy.
- Use "dilution": break the vector space of quark fields on the lattice into sub-components using a complete set of projection operators, $\{\mathcal{P}_1, \mathcal{P}_2, \ldots \mathcal{P}_N\}$ so $\sum_i P_i = I$ then variance is reduced by inverting on "diluted" noise vectors.
$$\eta^{(d)} = P_d \eta, \ \psi^{(d)} = M^{-1}\eta^{(d)} \text{ and } \sum_d E[\psi_i^{(d)} \eta_j^{(d)*}] = [M^{-1}]_{ij}$$

but
$$\text{var}\,(\sum_d \psi_i^{(d)} \eta_j^{(d)*}) < \text{var}\,(\psi_i \eta_j *)$$

All-to-all propagators

Foley et al, Comput.Phys.Commun. 172 (2005) 145-162

Advantages:

- Can dilute in any combination of time, space, colour, spin.
- All-to-all means any correlation function can be computed easily and precisely.
- Disconnected diagrams accessible (but still intrinsically noisy)
- Code is organised so extra dilution and/or smearing can be added *post hoc* without restarting from scratch
- Construction of molecular operators more intuitive

Determination of energies using the variational method

Use the variational analysis of Michael, NPB 259, 58 (1985) and Lüscher and Wolf, NPB339, 222, (1990).

- Create a basis of interpolating operators for the state of interest, $\mathcal{O}_\alpha$, $\alpha = 1, 2, \ldots, n$ using point and extended operators as well as different smearings.
- construct the matrix

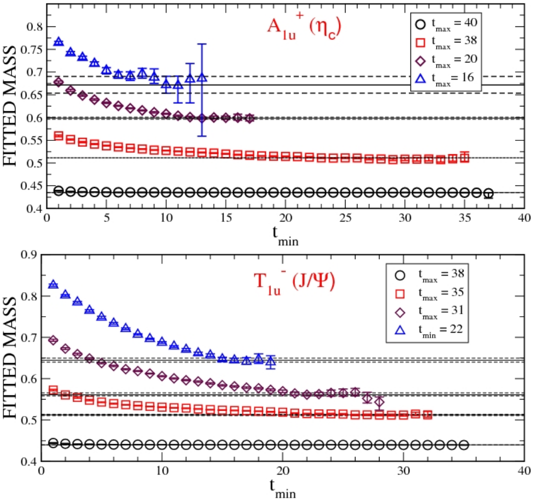
$$C_{\alpha\beta} = \langle 0|\mathcal{O}_\alpha(t)\mathcal{O}^\dagger_\beta(0)|0\rangle$$

- solve for the eigenvalues λ_α of the matrix $C(t_0)^{-1/2}C(t)C(t_0)^{-1/2}$ where t_0 is some small reference time, $(\lambda_1 \geq \lambda_2 \geq \lambda_3 \ldots)$
- Then

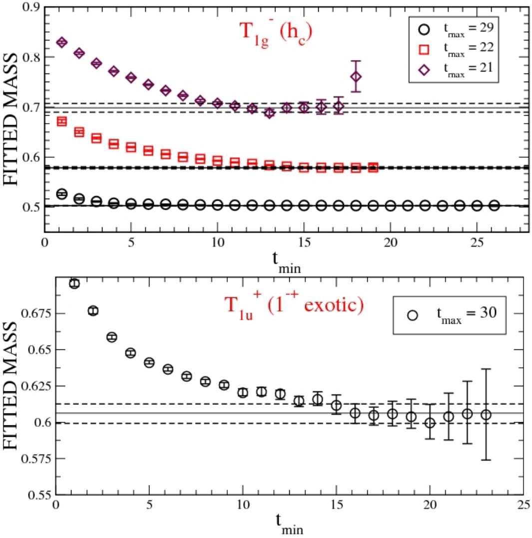
$$\lim_{t\to\infty} \lambda_\alpha(t, t_0) = e^{-(t-t_0)E_\alpha}[1 + O(e^{(-t\Delta E_\alpha)}]$$

Perform single-exponential fits to the diagonal elements to extract energies.

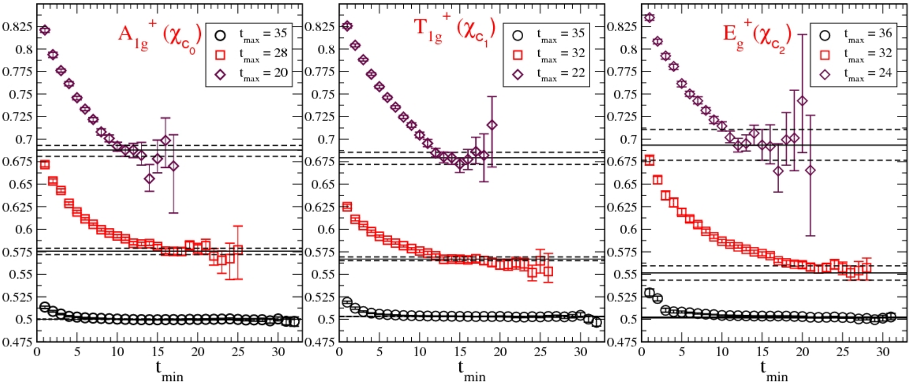
S waves: $\eta_c(0^{-+})$ and $J/\Psi(1^{--})$



h_c and the hybrid, 1^{-+}

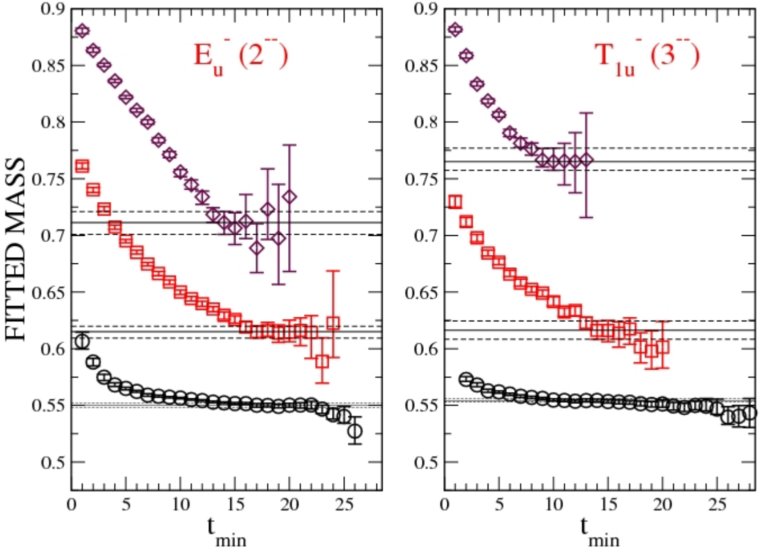


P waves



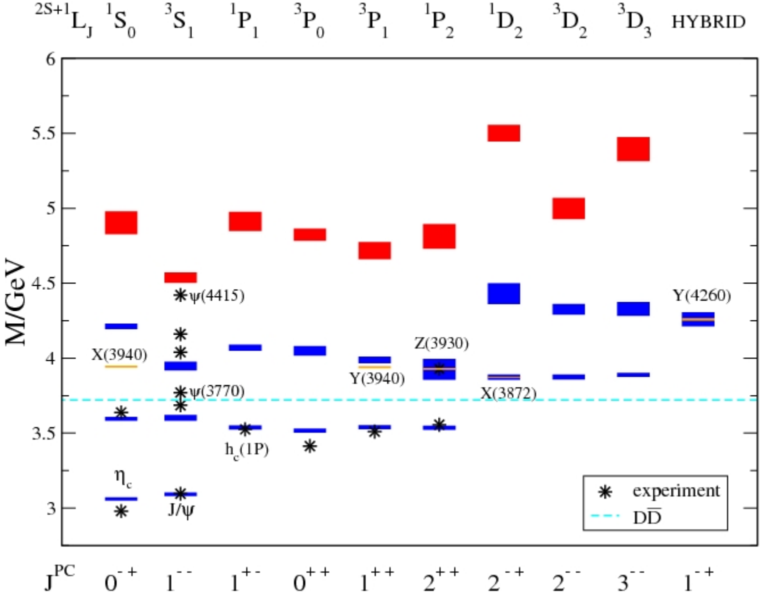
$A_{1g}^{+}(\chi_{c_0})$
$t_{max} = 35$
$t_{max} = 28$
$t_{max} = 20$
$T_{1g}^{+}(\chi_{c_1})$
$t_{max} = 35$
$t_{max} = 32$
$t_{max} = 22$
$E_{g}^{+}(\chi_{c_2})$
$t_{max} = 36$
$t_{max} = 32$
$t_{max} = 24$
t_{min}
0.825
0.8
0.775
0.75
0.725
0.7
0.675
0.65
0.625
0.6
0.575
0.55
0.525
0.5
0.475
0
5
10
15
20
25
30

D waves



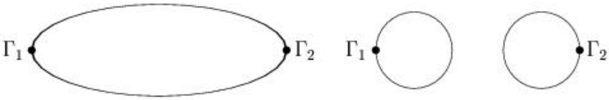
FITTED MASS
E_u^- (2^{--})
T_{1u}^- (3^{--})
0.9
0.85
0.8
0.75
0.7
0.65
0.6
0.55
0.5
0
5
10
15
20
25
t_{min}

The spectrum: preliminary



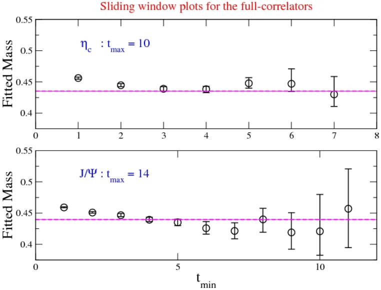
Including disconnected diagrams (1)

$c\bar{c}$ interpolating operators are singlet ($\bar{c}\Gamma c$) $\longrightarrow$ bubble diagrams in Wick contractions.



OZI suppressed $\Rightarrow$ small. Unless other nonperturbative effects play a role.
Usually non-singlet (connected) correlators are calculated. The disconnected diagrams require all-to-all propagators.

Including disconnected diagrams (2)



- Full correlator seems to lie (marginally) above the connected.

- From fits $m_{\eta_c(C-D)} \approx m_{\eta_c(C)} + 10 MeV$
- Mixing of the non-singlet with a nearby glueball (as in Morningstar & Peardon PRD60, 034509) $\longrightarrow$ states repel. Predicts the singlet contribution raises m_{η_c} and lowers $m_{J/\Psi}$.
- Disagrees with earlier indications from McNeile & Michael.
- Preliminary analysis only.

Conclusions

- The combination of anisotropic lattices, all-to-all propagators and variational fitting techniques mean the orbital and radial excitations of Charmonia can be resolved.
- Additional operators are required for a definitive spin identification of calculated excited states. In progress.
- A similar analysis is in progress for the heavy-light ($D_{(s)}$) system
- Simulations at finer lattice spacings and at lighter sea quark masses are underway.