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$\Upsilon \to X\gamma$ photon spectrum in the soft-energy region

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Outline

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- The oPs decay spectrum for $\omega \ll m$
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  - QED spectrum in the soft-energy region $(\omega \sim m\alpha)$
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- $\Upsilon \to X\gamma$ spectrum for $\omega \sim m_b \alpha_s$
- Summary

Motivation

- orthopositronium (o-Ps) decay spectrum for $\omega \sim E_0 \sim m\alpha^2$

$$\frac{d\Gamma}{d\omega} = \frac{\omega\alpha^6}{9\pi}\left[|a_m|^2 + \frac{7}{3}|a_e|^2\right]$$

[Manohar, RF '03]

✓ NRQED, includes binding effects

✓ satisfies Low´s theorem for $\omega \to 0$

- valid for the whole region $\omega \ll m$?

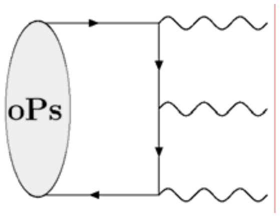
$\longrightarrow$ Voloshin : The dipole approximation for the small-energy photon radiation can be applied for all $\omega \ll m$

$\omega \sim m\alpha$ (soft-energy region) : $a_e = 1 + \#\alpha\sqrt{m/\omega} \sim \sqrt{\alpha}$

$\longrightarrow$ main contribution arises from $r \sim (\sqrt{m\omega})^{-1}$

$e^+e^- \quad : \quad p^0 \sim \mathbf{p}^2/m \sim \omega \sim m\alpha$ ⇨ new NR mode? is the NREFT calculation missing something?

NRQED computation of the oPs spectrum

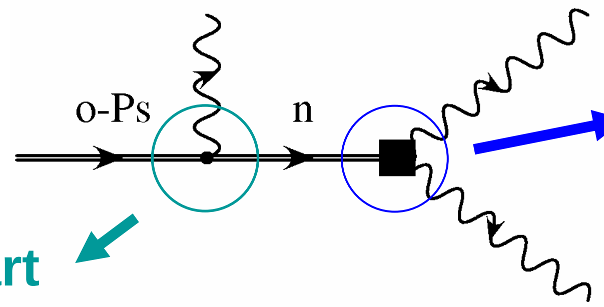


- there is a long-distance part in the decay when one of the photons in the final state is not hard $(\omega \ll m)$

oPs $\left({}^3S_1\right)$

$\omega \ll m$

$C-$odd



short-distance part

hard photons $(E_\gamma \sim m)$

$C-$even

long-distance part

<u>Coulomb Hamiltonian</u>

$$H = H_0 + H_{\rm int}$$

$$H_0 = \frac{\mathbf{p}^2}{m} - \frac{\alpha}{r}$$

$$H_{\rm int} = -\frac{e}{m}\mathbf{p}_1 \cdot \mathbf{A}(\mathbf{x}_1) + \frac{e}{m}\mathbf{p}_2 \cdot \mathbf{A}(\mathbf{x}_2) - \mu\,\boldsymbol{\sigma}_1 \cdot \mathbf{B}(\mathbf{x}_1) - \mu\,\boldsymbol{\sigma}_2 \cdot \mathbf{B}(\mathbf{x}_2)$$

Dipole approximation

$$\mathbf{A}(\mathbf{x}) \simeq \mathbf{A}(\mathbf{0}) \implies e^{-i\mathbf{k}\cdot\mathbf{x}} = \underbrace{1}_{\text{dipole approx.}} - i\mathbf{k}\cdot\mathbf{x} + \dots$$

Atomic transitions in Ps

$\psi_o(r) \propto e^{-r/a}$ constrains $r \sim a = 2/m\alpha$ $\Rightarrow$

Multipole expansion valid for $\omega \ll m\alpha$

o-Ps $\to 3\gamma$ decay with one small-energy photon

$\longrightarrow$ it is the scale $1/m$ and not $1/m\alpha$ which constrains $\omega_{\rm max}$ for multipole expanding the e.m. interaction **[Voloshin '04]**

$$\mathcal{M} \sim \int dr\, \underbrace{G(\mathbf{0}, r; \kappa)}\, \psi_o(r) \quad , \quad \kappa = \sqrt{m\omega - mE_0}$$

- $\omega \sim m\alpha \quad : \quad \sim e^{-\kappa r} \quad \longrightarrow \quad r \sim \kappa^{-1} \ll a$

$$\mathbf{k}\cdot\mathbf{x} \sim \omega/\kappa \sim \sqrt{\alpha} \ll 1 \longrightarrow \text{multipole expansion still valid}$$

- oPs calculation without multipole expanding the e.m. field

$$\mathcal{M}_e = -e\,\psi_0(0)\,\langle\chi^\dagger\,\mathbf{W}_1\cdot\epsilon\,\phi\rangle_{\varepsilon_0}\Big(1+\mathcal{D}(\omega)\Big)$$

$$\mathcal{D}(\omega) = -\frac{8\pi}{ma}\int dr\,r^3\,G_1(0,r;\kappa)\,e^{-r/a}\left[\frac{24}{(\omega r)^3}\sin\left(\frac{\omega r}{2}\right)-\frac{12}{(\omega r)^2}\cos\left(\frac{\omega r}{2}\right)\right]$$

- $\omega \sim m\alpha^2 \quad : \quad \kappa = \sqrt{m\omega - mE_0} \sim m\alpha \sim 1/a$
- $\omega \sim m\alpha \quad : \quad \kappa = \sqrt{m\omega} \sim m\sqrt{\alpha}$

$\longrightarrow$ we can always expand in $\omega\,r \sim \omega/\kappa$ for $\omega \ll m$

$$\mathcal{D}(\omega) = \underbrace{d_e(\omega)}_{\text{dipole approx.}}\Big(1+\underbrace{\mathcal{O}\left(\omega^2/\kappa^2\right)}\Big)$$

dipole approx.

$\mathcal{O}(\alpha^2)$ for $\omega \sim m\alpha^2$

$\mathcal{O}(\alpha)$ for $\omega \sim m\alpha$

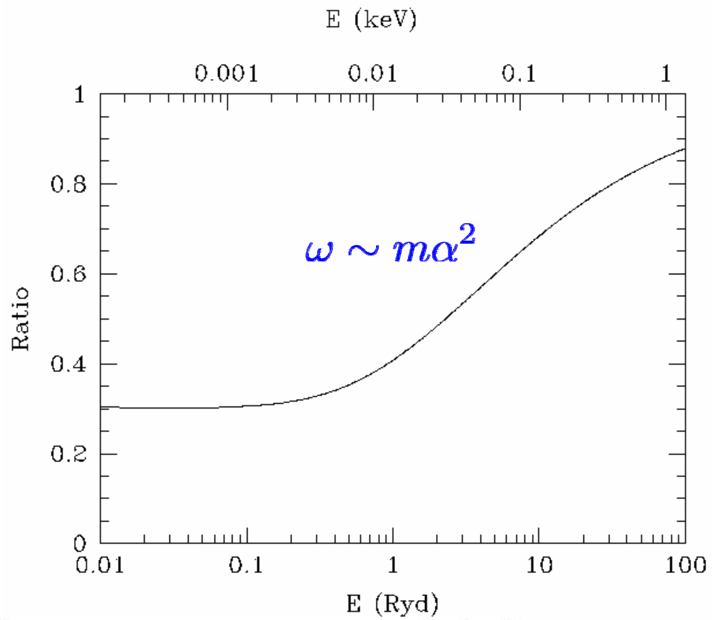
EFT oPs spectrum: results

$$\omega \ll m \; : \quad a_e = 1 + d_e \quad , \quad a_m = \frac{-\omega}{\Delta E_{\rm hfs} - \omega + i(\Gamma_o + \Gamma_p)/2}$$

$$\frac{d\Gamma}{dx} = \frac{m\alpha^6}{9\pi} x \left[|a_m|^2 + \frac{7}{3} |a_e|^2 \right]$$

$$\mathrm{Ratio} = \frac{\frac{d\Gamma^{\rm EFT}}{dx}}{\frac{d\Gamma^{\rm QED}}{dx}}$$

$$1\,\mathrm{Ryd} = m\alpha^2/2$$



- $\omega \sim m\alpha \; : \quad a_e = 1 - \frac{2}{3}\frac{\alpha}{\sqrt{x}} + \mathcal{O}(\alpha) \quad , \quad a_m = 1 + \mathcal{O}(\alpha)$

$x = \omega/m$

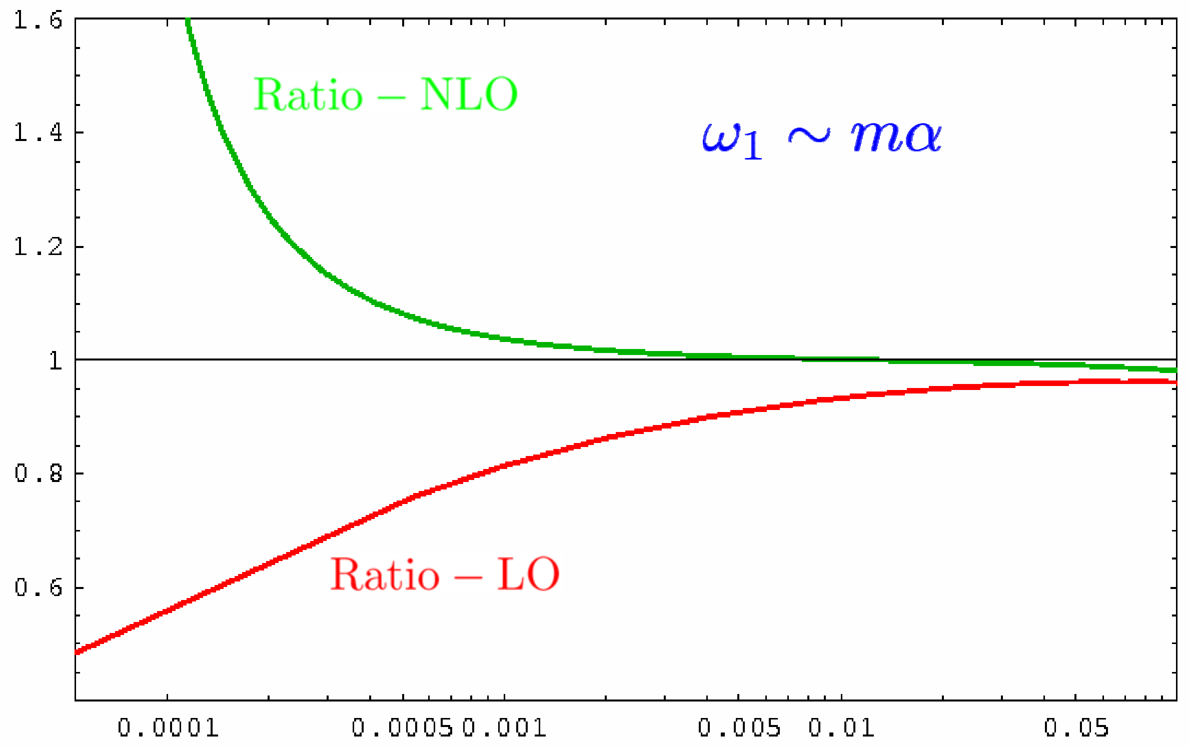
$$\frac{d\Gamma}{dx} = \frac{2m\alpha^6}{27\pi} \left[5x - \frac{14}{3} \alpha \sqrt{x} + \mathcal{O}(\alpha\, x) \right]$$

tree-level QED

one-loop QED?

EFT oPs spectrum: $\omega_1 \sim m\alpha$

$$\text{Ratio} = \frac{\frac{d\Gamma^{\text{EFT}}}{dx}}{\frac{d\Gamma^{\text{QED}}}{dx}}$$



$x = \omega_1/m$

QED computation

- from the NRQED computation in the $\omega \sim m\alpha$ region

$$G = G_0 + G_0\, V_c\, G_0 + \ldots$$

$$\text{since } r \sim \kappa^{-1} \sim 1/\sqrt{m\omega} \;\longrightarrow\; \int d^3x\, V_c\, G_0 \sim \frac{m\alpha}{\kappa} \sim \alpha\sqrt{\frac{m}{\omega}} \ll 1$$

$\longrightarrow$ Coulomb int. can be treated as a perturbation in the e^+e^- system after soft-photon radiation

- exact phase-space distribution at one-loop is known **[Adkins '05]**

$$\frac{d\Gamma_1}{dx_1} = \frac{m\alpha^7}{36\pi^2}\int_{-1}^{1} dx_2 \frac{1}{x_1 x_2 x_3}\left[\underbrace{F(x_1,x_2)} + \text{perm.}\right]$$

$F(x_1,x_2)$: can be expanded in $x_1 \to 0$ before integration

$$x_1 \to 0 \quad : \quad \frac{d\Gamma_1}{dx_1} = \frac{2m\alpha^6}{27\pi}\left[-\frac{14}{3}\alpha\sqrt{x_1} + \left(\frac{3\pi^2}{4} - \frac{35}{2}\right)\alpha\, x_1 + \mathcal{O}(\alpha\, x_1^{3/2})\right]$$

$\left(-\frac{14}{3}\alpha\sqrt{x_1}\right)$ $\rightarrow$ agrees with NLO term in EFT calculation!

Threshold expansion of QED 1-loop diagrams

Beneke, Smirnov '98

- asymptotic expansion of loop integrals involving heavy massive particles close to threshold

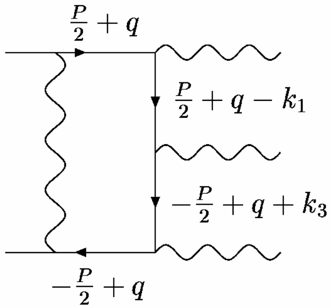
relevant regions in loops involving non-relativistic fermion pairs

hard	:	$q^0 \sim \mathbf{q} \sim m$
soft	:	$q^0 \sim \mathbf{q} \sim mv$
potential	:	$q^0 \sim \mathbf{q}^2/m \sim mv^2$
ultrasoft	:	$q^0 \sim \mathbf{q} \sim mv^2$

- separation of regions achieved by expanding loop momenta according to the v-scaling above (each region contributes to a single power of v)

$\longrightarrow$ apply to the 1-loop oPs$\rightarrow 3\gamma$ for $\omega \sim m\alpha$ (where QED calculation is valid)

Ladder graph



$\longrightarrow$ gives the leading term in the ω_1/m expansion

5-point scalar integral

$$I_0 = \int \frac{[d^D q]}{q^2\,[(q+P/2)^2-m^2]\,[(q-P/2)^2-m^2]\,[(q+P/2-k_1)^2-m^2]\,[(q-P/2+k_3)^2-m^2]}$$

$q \ll m$:
$$I_0^{(\text{small})} = -\frac{1}{2m\omega_3}\int \frac{[d^D q]}{(q_0^2-\mathbf{q}^2)\,(2mq_0-\mathbf{q}^2)\,(-2mq_0-\mathbf{q}^2)\,(2mq_0-2m\omega_1-\mathbf{q}^2)}$$

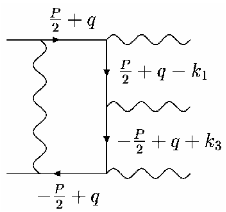
$\longrightarrow$ pick residue from the pole of the massive propagator at $q_0 = -\mathbf{q}^2/m + i\epsilon$

$$I_0^{(\text{S-r})} = -\frac{i}{16m^2\omega_3}\int [d^n\mathbf{q}]\,\frac{1}{(\mathbf{q}^2)^2\,(\mathbf{q}^2+m\omega_1)} = \frac{1}{64\pi m^2\omega_3}(m\omega_1)^{-\frac{3}{2}}$$

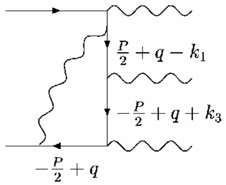
dominated by loop-momentum $\mathbf{q} \sim \sqrt{m\omega_1}$ $\Longrightarrow$

$q^0 \sim \mathbf{q}^2/m \sim \omega_1$
'soft-radiation' region

1-loop QED result for $\omega_1 \sim m\alpha$



$$= e\,\langle\chi^\dagger\,\mathbf{W}_1\cdot\epsilon\,\phi\rangle\,\frac{\alpha}{3}\sqrt{\frac{m}{\omega_1}} + ie\,\frac{W_0}{m}\,\delta_1\cdot\langle\chi^\dagger\sigma\,\phi\rangle\,\frac{\alpha}{2}\sqrt{\frac{m}{\omega_1}} + \cdots$$

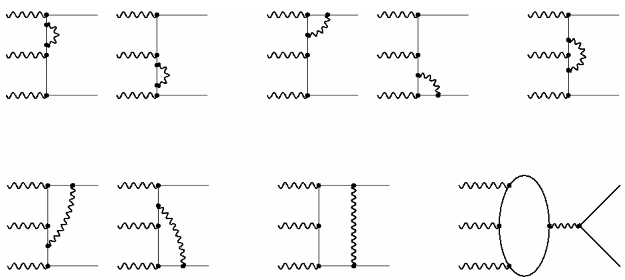


$$= -ie\,\frac{W_0}{m}\,\delta_1\cdot\langle\chi^\dagger\sigma\,\phi\rangle\,\frac{\alpha}{2}\sqrt{\frac{m}{\omega_1}} + \cdots$$

$$\mathcal{M}^{1-\mathrm{loop}} = e\,\langle\chi^\dagger\,\mathbf{W}_1\cdot\epsilon\,\phi\rangle\,\frac{2\alpha}{3}\sqrt{\frac{m}{\omega_1}} + \cdots$$

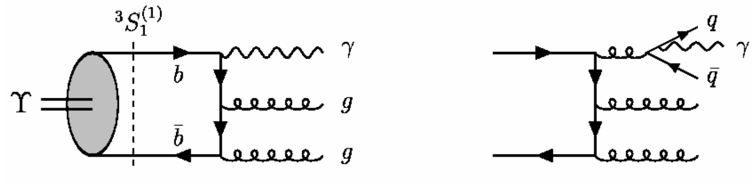
✓ agrees with NLO term of NRQED calculation

- the rest of 1-loop diagrams in oPs$\to 3\gamma$ give subleading contributions in ω_1/m



Relevance for Heavy Quarkonium

$\Upsilon \to X\gamma$



- photon spectrum can be written as a sum of a direct contrib. and a fragmentation contrib.

$$\frac{d\Gamma}{dx} = \frac{d\Gamma^{\rm dir}}{dx} + \frac{d\Gamma^{\rm frag}}{dx}$$

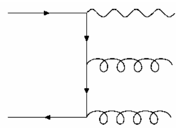
- $\frac{d\Gamma^{\rm dir}}{dx}$ can be calculated within the NRQCD factorization

$$\frac{\mathrm{d}\Gamma^{\rm dir}}{\mathrm{d}x} = \sum_{Q\bar{Q}[n]} C_n(x)\langle\Upsilon|\mathcal{O}_n|\Upsilon\rangle$$

$C_n(x)$: short-distance Wilson coefficients

$\mathcal{O}_n$: NRQCD operators

⟶ leading order: $\langle\Upsilon|\mathcal{O}_1(^3S_1)|\Upsilon\rangle = 2N_c\,|\psi(0)|^2$

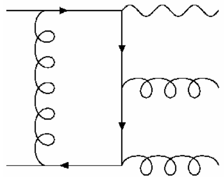


$$C_1(x) = \frac{32}{27}\frac{\alpha\alpha_s^2 Q_b^2}{m_b^2}\left[\frac{2-x}{x} + \frac{(1-x)\,x}{(2-x)^2} - \frac{2\,(1-x)^2\,\log(1-x)}{(2-x)^3} + \frac{2\,(1-x)\,\log(1-x)}{x^2}\right]$$

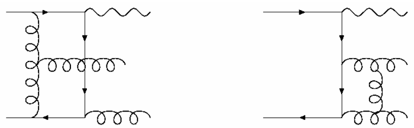
$\Upsilon \to X\gamma$: 1-loop QCD correction for $x \sim \alpha_s$

$\longrightarrow$ analyze the 1-loop diagrams for the direct Υ spectrum for $\omega_1 \sim m_b \alpha_s$ with the threshold expansion

- leading term comes from ladder diagram



- non-abelian diagrams are sub-leading in ω_1/m_b



$$x \sim \alpha_s \quad : \quad \frac{1}{\Gamma_\Upsilon}\frac{d\Gamma_\Upsilon^{\rm NLO}}{dx} = \frac{1}{\Gamma_{\rm oPs}}\frac{d\Gamma_{\rm oPs}^{\rm LO}}{dx} + \frac{16}{27}\frac{1}{\Gamma_{\rm oPs}}\frac{d\Gamma_{\rm oPs}^{\rm NLO}}{dx}$$

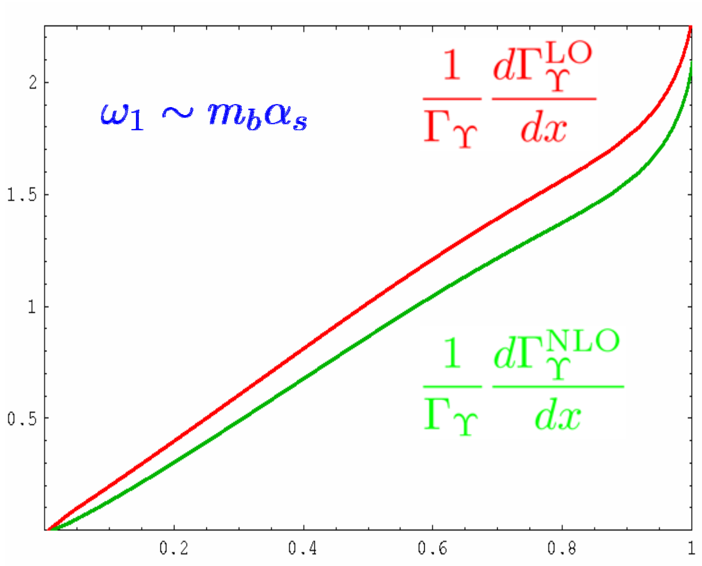
$$\frac{1}{\Gamma_{\rm oPs}}\frac{d\Gamma_{\rm oPs}^{\rm LO}}{dx} = \frac{2}{\pi^2-9}\left[\frac{2-x}{x} + \cdots\right] = \frac{2}{\pi^2-9}\left[\frac{5}{6}x + \mathcal{O}(x^2)\right]$$

$$\frac{1}{\Gamma_{\rm oPs}}\frac{d\Gamma_{\rm oPs}^{\rm NLO}}{dx} = \frac{-14}{9(\pi^2-9)}\alpha_s\sqrt{x} + \mathcal{O}(\alpha_s\, x)$$

Results for the direct Upsilon decay spectrum

$(\alpha_s = 0.2)$

threshold expansion



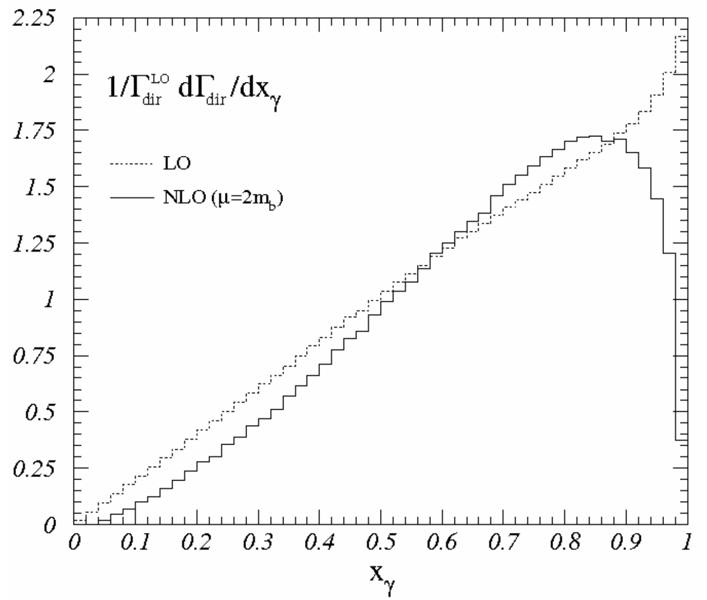
$x = \omega_1/m$

Recall: frag. contrib. to the γ spectrum become important in the low-x region

Catani, Hautmann '95
Maltoni, Petrelli '99

num. $\mathcal{O}(\alpha_s)$ calculation

Krämer '99



Summary

- NRQED calculation of the $\mathrm{oPs} \to 3\gamma$ photon spectrum valid for all $\omega \ll m$

$\longrightarrow$ dipole approximation ok

$\longrightarrow$ $\omega \sim m\alpha$: NLO EFT term and 1-loop QED calculation agree

- Threshold expansion of 1-loop diagrams

$\longrightarrow$ LO $\mathcal{O}(\alpha)$ contribution arises from loop-momentum region

$$p^0 \sim \mathbf{p}^2/m \sim \omega \sim m\alpha$$

- Application to the soft-energy direct photon spectrum in Upsilon decays is straightforward

$$x \sim \alpha_s : \quad \frac{1}{\Gamma_\Upsilon}\frac{d\Gamma_\Upsilon^{\mathrm{NLO}}}{dx} = -\frac{224}{243}\frac{1}{(\pi^2-9)}\alpha_s\sqrt{x} + \mathcal{O}(\alpha_s\, x)$$