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NRQCD Factorization in Heavy Quarkonium Production

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Based on work with Z.-B. Kang, G.C. Nayak, and G. Sterman

Outline

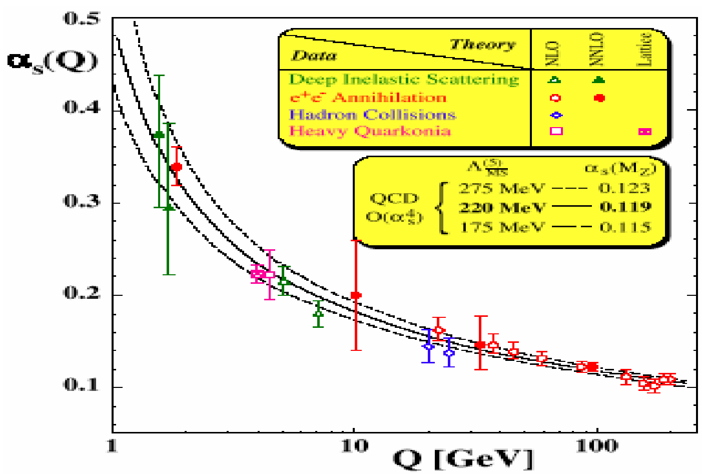
- Perturbative QCD vs NRQCD
- Heavy quarkonium production
- Color transfer in associated production of heavy quarkonium
- Connect pQCD factorization to NRQCD factorization
- Summary and conclusions

Perturbative QCD

- **QCD Lagrangian:**

$$\mathcal{L}_{\mathbf{QCD}} = -\frac{1}{2}\mathrm{tr}\, G_{\mu\nu}G^{\mu\nu} + \sum_{n_F=1}^{6} \bar{q}\, i\not{D} q \qquad D^\mu = \partial^\mu + igA^\mu$$

- **Asymptotic freedom and perturbative QCD:**



$$\sigma_{\mathrm{Pert}}(Q) = \sum_n C_n(Q,\mu)\left[\frac{\alpha_s(\mu)}{2\pi}\right]^n$$

Infrared Safe

- ❖ **Power series of $\alpha_s(\mu)$**
- ❖ **Renormalization scale: μ**

- **PQCD factorization for hadronic observables:**

$$\sigma_{\mathrm{Hadronic}}(Q, 1/\mathrm{fm}) \approx \sum \hat{\sigma}_{\mathrm{Pert}}(Q, \mu_F) \otimes \varphi(\mu_F, 1/\mathrm{fm}) + O\left(\frac{1/\mathrm{fm}}{Q}\right)$$

Infrared safe

Universal – Predictive power

μ_F : **Factorization scale**

Non-Relativistic QCD (NRQCD)

❑ **NRQCD Lagrangian:** $\mathcal{L}_{\rm NRQCD} = \mathcal{L}_{\rm light} + \mathcal{L}_{\rm heavy} + \delta\mathcal{L}$

Caswell, Lepage, Phys. Lett. B, 1986
Bodwin, Braaten, Lepage, PRD, 1995

$$\mathcal{L}_{\rm light} = -\frac{1}{2}{\rm tr}\, G_{\mu\nu}G^{\mu\nu} + \sum_{n_F=1}^{3\text{ or }4} \bar{q}\, i\not{D} q$$

$$\mathcal{L}_{\rm heavy} = \psi^\dagger \left(iD_t + \frac{\mathbf{D}^2}{2M} \right) \psi + \chi^\dagger \left(iD_t - \frac{\mathbf{D}^2}{2M} \right) \chi$$

Pauli spinor for antiquark (χ)

Pauli spinor for heavy quark (ψ)

$$\begin{aligned} \delta\mathcal{L}_{\rm bilinear} = & \frac{c_1}{8M^3}\left(\psi^\dagger(\mathbf{D}^2)^2\psi - \chi^\dagger(\mathbf{D}^2)^2\chi\right) \\ & + \frac{c_2}{8M^2}\left(\psi^\dagger(\mathbf{D}\cdot g\mathbf{E} - g\mathbf{E}\cdot\mathbf{D})\psi + \chi^\dagger(\mathbf{D}\cdot g\mathbf{E} - g\mathbf{E}\cdot\mathbf{D})\chi\right) \\ & + \frac{c_3}{8M^2}\left(\psi^\dagger(i\mathbf{D}\times g\mathbf{E} - g\mathbf{E}\times i\mathbf{D})\cdot\boldsymbol{\sigma}\psi + \chi^\dagger(i\mathbf{D}\times g\mathbf{E} - g\mathbf{E}\times i\mathbf{D})\cdot\boldsymbol{\sigma}\chi\right) \\ & + \frac{c_4}{2M}\left(\psi^\dagger(g\mathbf{B}\cdot\boldsymbol{\sigma})\psi - \chi^\dagger(g\mathbf{B}\cdot\boldsymbol{\sigma})\chi\right), \end{aligned}$$

❑ **Limitation:**

Powerful for a process with available kinetic energy: $Mv^2 \ll Mc^2$

❖ Formalism is ideal for heavy quarkonium decay

Bodwin, Braaten, Lepage, PRD, 1995

❖ Additional complications for production with $s \gg (2M)^2$

Heavy quarkonium production in NRQCD

- NRQCD factorization for heavy quarkonium production:

$$\sigma_{\rm NRQCD}(s,M)=\sum_n \hat{\sigma}_{Q\bar{Q}[n]}(s,M,\alpha_s(M),v)\langle O_n\rangle$$

Bodwin, Braaten, Lepage, PRD, 1995
Brambilla, et al. hep-ph/0412158
Lansberg, hep-ph/0602091, …

Infrared Safe

Universal – Predictive power

- $O_n=\sum_N \chi^\dagger\kappa_n\psi|N,H\rangle\langle N,H|\psi^\dagger\kappa_n'\chi=\chi^\dagger\kappa_n\psi\left(a_H^\dagger a_H\right)\psi^\dagger\kappa_n'\chi$
- $\hat{\sigma}_{Q\bar{Q}[n]}(s,M,\alpha_s(M),v)$: power series of both $\alpha_s(M)$ and v

  with heavy quark velocity: v

- This factorization formalism has not been proved

- Expectations:
  - It works for inclusive heavy quarkonium production when $2M<s<4M$, unless for exclusive production

    Nayak, Qiu, Sterman, 2005, 2006

    with a proper insertion of gauge link, the formalism works to two-loop
  - It does not work when additional heavy quark velocity is involved

    such as, the associated production

    Nayak, Qiu, Sterman, PRL 2007

Heavy quarkonium associated production

- Inclusive J/ψ + charm production:

$$\sigma(e^+e^- \longrightarrow J/\psi c\bar{c})$$

Belle: $(0.87^{+0.21}_{-0.19} \pm 0.17)$ pb

NRQCD-LO: ~ 0.07 pb

Kiselev, et al 1994,
Cho, Leibovich, 1996
Yuan, Qiao, Chao, 1997
...
Zhang, Chao, 2007 (NLO)

- Ratio to light flavors:

$$\sigma(e^+e^- \longrightarrow J/\psi c\bar{c})/\sigma(e^+e^- \longrightarrow J/\psi X)$$

Belle: $0.59^{+0.15}_{-0.13} \pm 0.12$

Message:

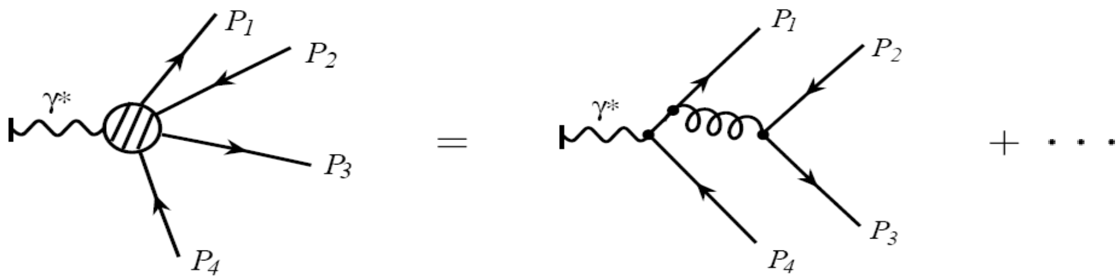
Production rate of $e^+e^- \rightarrow J/\psi c\bar{c}$ is larger than

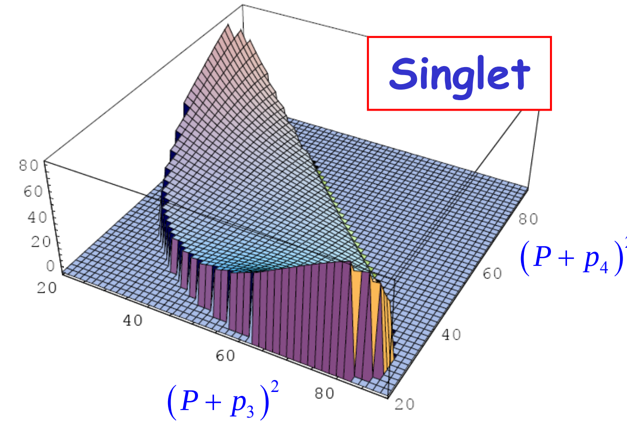
all these channels: $e^+e^- \rightarrow J/\psi gg$, $e^+e^- \rightarrow J/\psi q\bar{q}$, ...

combined ?

Associated production at B-factory

❑ **Kinematically preferred configuration:**

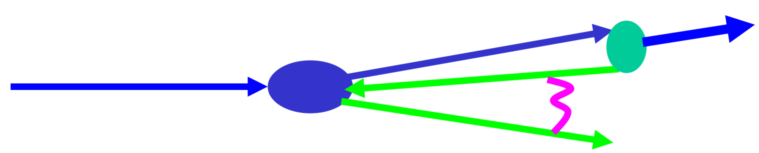




Production rate of a singlet charm quark pair is dominated by the phase space where $s_3=(P_1+P_2+P_3)^2$ or $s_4=(P_1+P_2+P_4)^2$ near its minimum

❑ **NRQCD formalism does not apply when there are more than one heavy quark velocity involved**

❑ **Color transfer enhances associated heavy quarkonium production**

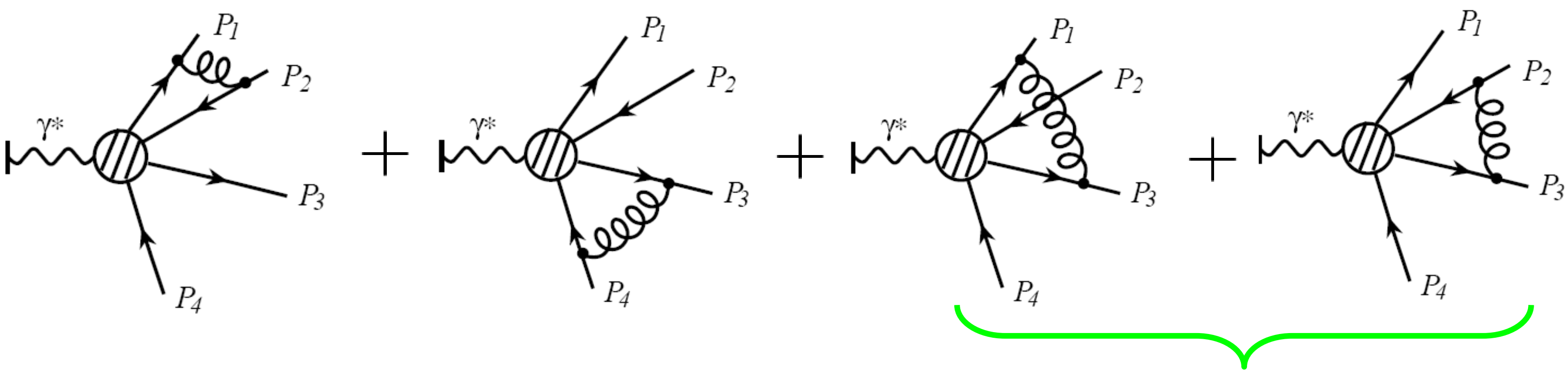


A heavy quark as a color source to enhance the transition rate for an octet pair to become a singlet pair

Nayak, Qiu, Sterman, PRL 2007

Soft gluon enhancement – color transfer

- **Soft gluons between heavy quarks:** **active pair: P_1, P_2; spectators: P_3, P_4**



- **There are three heavy quark velocities:** $\beta_{ij} \equiv \sqrt{1 - 4m^2/(P_i + P_j)^2}$

NRQCD approach is not well defined in this region

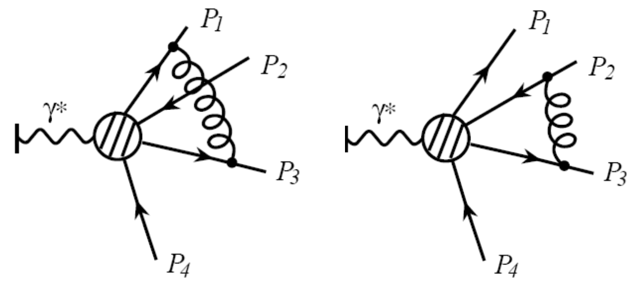
- **Soft gluon between a heavy quark pair:**

$$-i\, g^2 \int \frac{d^D k}{(2\pi)^D} \frac{4 P_i \cdot P_j}{[2P_i \cdot k + k^2 + i\epsilon][-2P_j \cdot k + k^2 + i\epsilon][k^2 + i\epsilon]}$$

$$= \frac{\alpha_s}{2\pi} \left[-\frac{1}{2\varepsilon} \left(\frac{1}{\beta_{ij}} + \beta_{ij} \right) (2\beta_{ij} - i\pi) \; + \ldots \right] \Longrightarrow i\,\frac{1}{\varepsilon}\,\frac{\alpha_s}{\beta_{ij}}$$

Associated production is enhanced

❑ NLO correction to the amplitude:



$$\mathrm{Im}\left[\mathcal{A}_{13}+\mathcal{A}_{23}\right] = \frac{\alpha_s}{4\varepsilon}\mathcal{A}^{(0)}(P_i)\left[\frac{1+\beta_{13}^2}{\beta_{13}} - \frac{1+\beta_{23}^2}{\beta_{23}}\right]$$

Does not contribute to NLO production rate in NRQCD

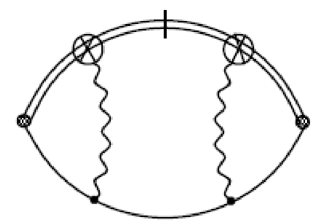
Zhang, Chao, PRL 2007

❑ Estimate the enhancement factor from NNLO in NRQCD approach:

❖ Velocity expansion:

$$\frac{1}{\beta_{13}} - \frac{1}{\beta_{23}} \sim -\frac{4}{\beta_S^3}\frac{q_S\cdot q}{m^2} \sim \frac{4}{\beta_S^2}\, v\, \cos\phi_S$$

$$\beta_S = \sqrt{\frac{-q_S^2}{m^2-q_S^2}}$$



$$P_3^\mu = \frac{P_0^\mu}{2}\sqrt{1-\frac{q_S^2}{m^2}} + q_S^\mu$$

❖ Velocity-ordered region:

$$\beta_s < 1, \quad \frac{v}{\beta_s} < 1$$

$$P_0^\mu = (2m, 0) \text{ and } q_S\cdot P_0 = 0$$

❖ the enhancement factor:

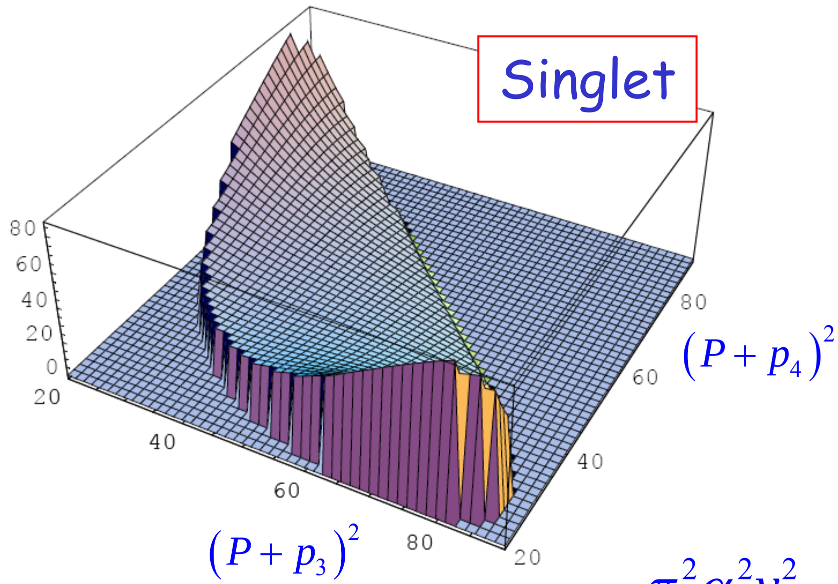
$$\left|A_{Singlet}^{NNLO}\right|^2 \sim \left(C_{8\to 1}\frac{\alpha_s^2 v^2}{\varepsilon^2}\right)\left(\frac{\pi^2}{\beta_s^4}\right)\left|A_{Octet}^{LO}\right|^2$$

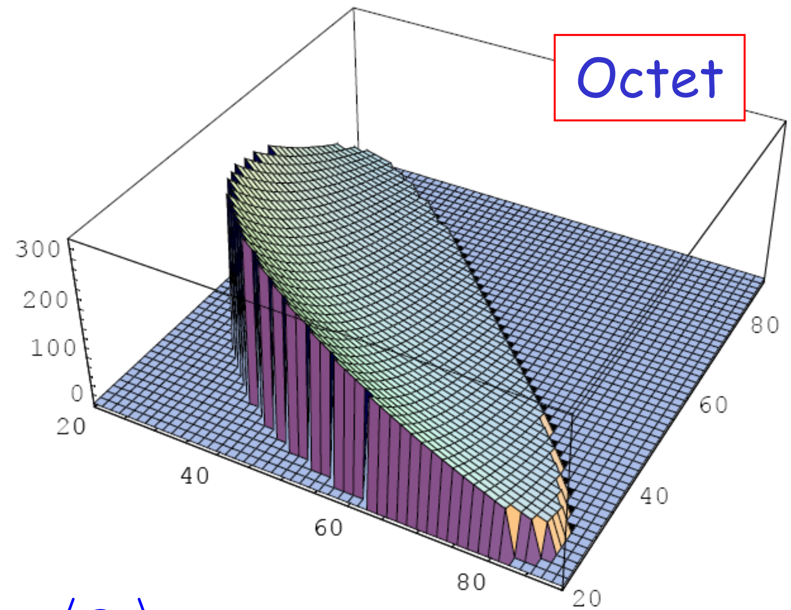
All other two-loop diagrams give a single pole !

Nayak, Qiu, Sterman, 2007

Numerical enhancement from NNLO

- LO hard parts with color factor:





- Matrix elements: $\dfrac{\pi^2\alpha_s^2 v^2}{\varepsilon^2} \Rightarrow \langle O_8 \rangle$

$$d\sigma^{\rm tot}_{e^+e^-\to H+X}(p_H) \sim d\hat{\sigma}_{e^+e^-\to Q\bar{Q}[S_1]+Q'(\beta_S)}(p_H)\,\langle {}^3\mathcal{S}_1^H\rangle$$
$$+\, d\hat{\sigma}_{e^+e^-\to Q\bar{Q}[S_8]+Q'(\beta_S)}(p_H)\,\frac{\langle {}^3\mathcal{S}_8^H\rangle}{\beta_S^4}$$

Two terms are equally important if $\beta_s \sim 0.3$

Nayak, Qiu, Sterman, 2007

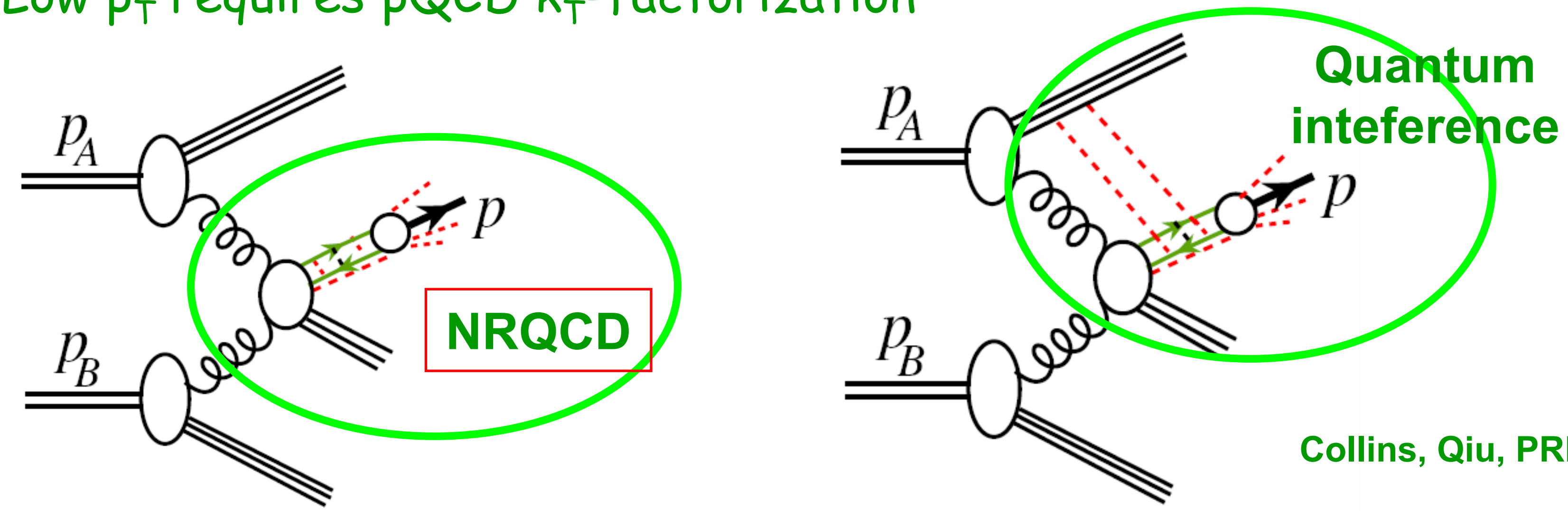
Same feature for heavy quark fragmentation

Kang, et al. 2007

Hadronic heavy quarkonium production

- **NRQCD factorization has not been proved theoretically**
- **NRQCD Factorization fails for low p_T:**

  Low p_T requires pQCD k_T-factorization



Collins, Qiu, PRD 2007

- **NRQCD Factorization might work for large p_T**

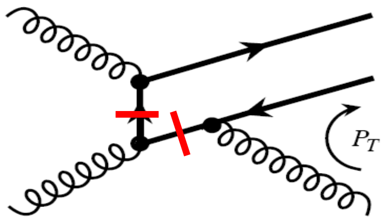
  Spectator interactions are suppressed by $(1/p_T)^n$

Factorization is necessary for the predictive power

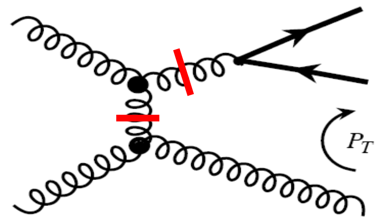
Combination of pQCD and NRQCD

Also see Lansberg
In the workshop

- **Heavy quarkonium production when $P_T \gg 2M$**



$$\propto \left(\frac{1}{P_T^2}\right)^2$$

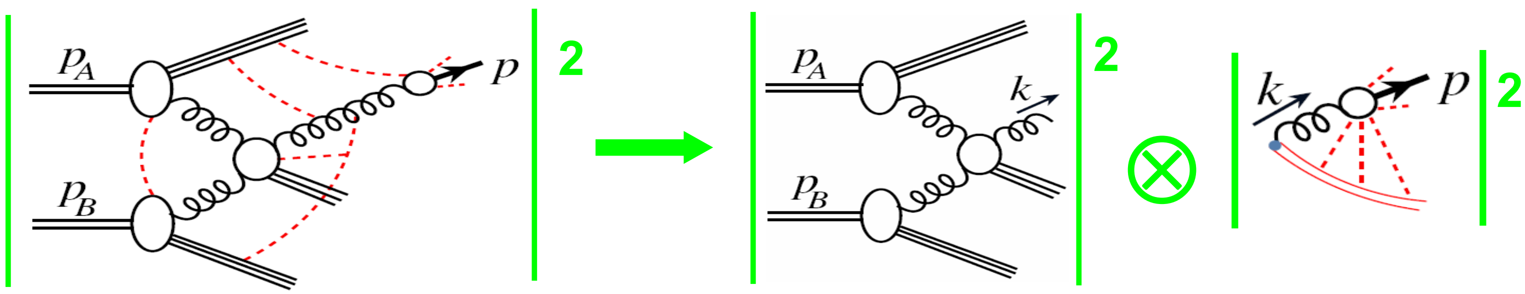


$$\propto \frac{1}{P_T^2} \times \frac{1}{(2M)^2}$$

When $P_T^2 \gg (2M)^2$, fragmentation contribution dominates the production

- **Combination of pQCD and NRQCD factorization (not yet proved):**

  - pQCD factorization to isolate heavy quarkonium physics into the universal fragmentation function



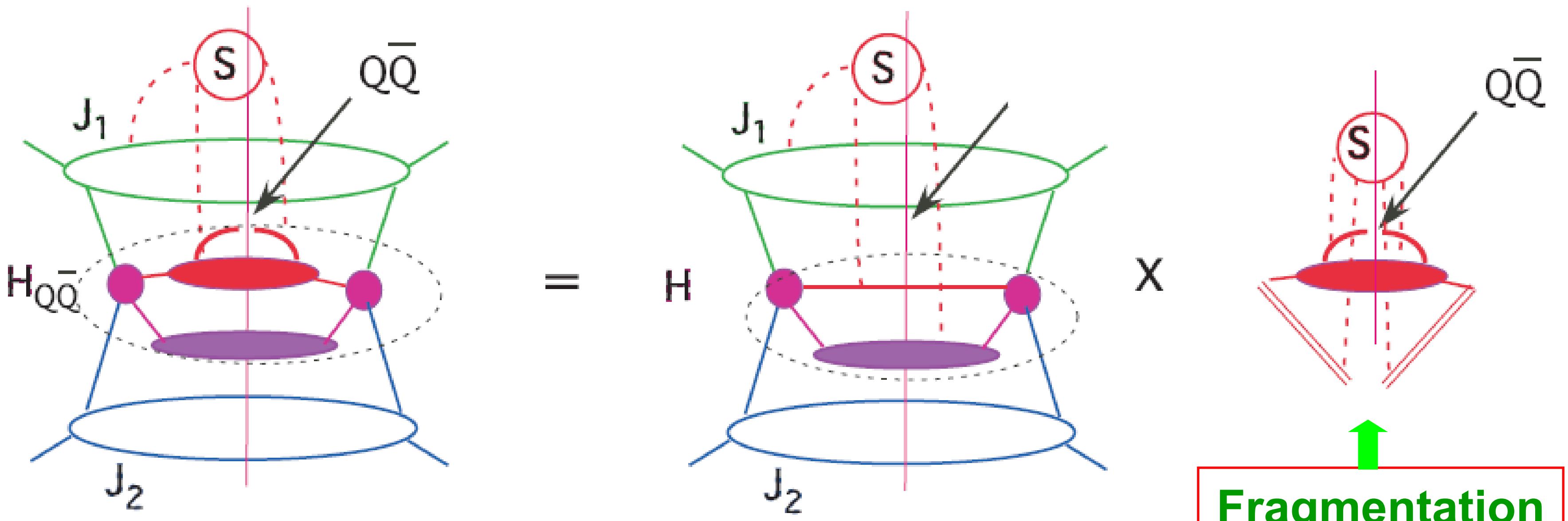
$$+ \frac{M^2}{P_T^2}$$

  - NRQCD factorization to isolate non-perturbative physics of the fragmentation function into NRQCD matrix elements

PQCD collinear factorization – 2 steps

Nayak, Qiu, Stermen, 2005

- Step 1: Fragmentation factorizes from the rest



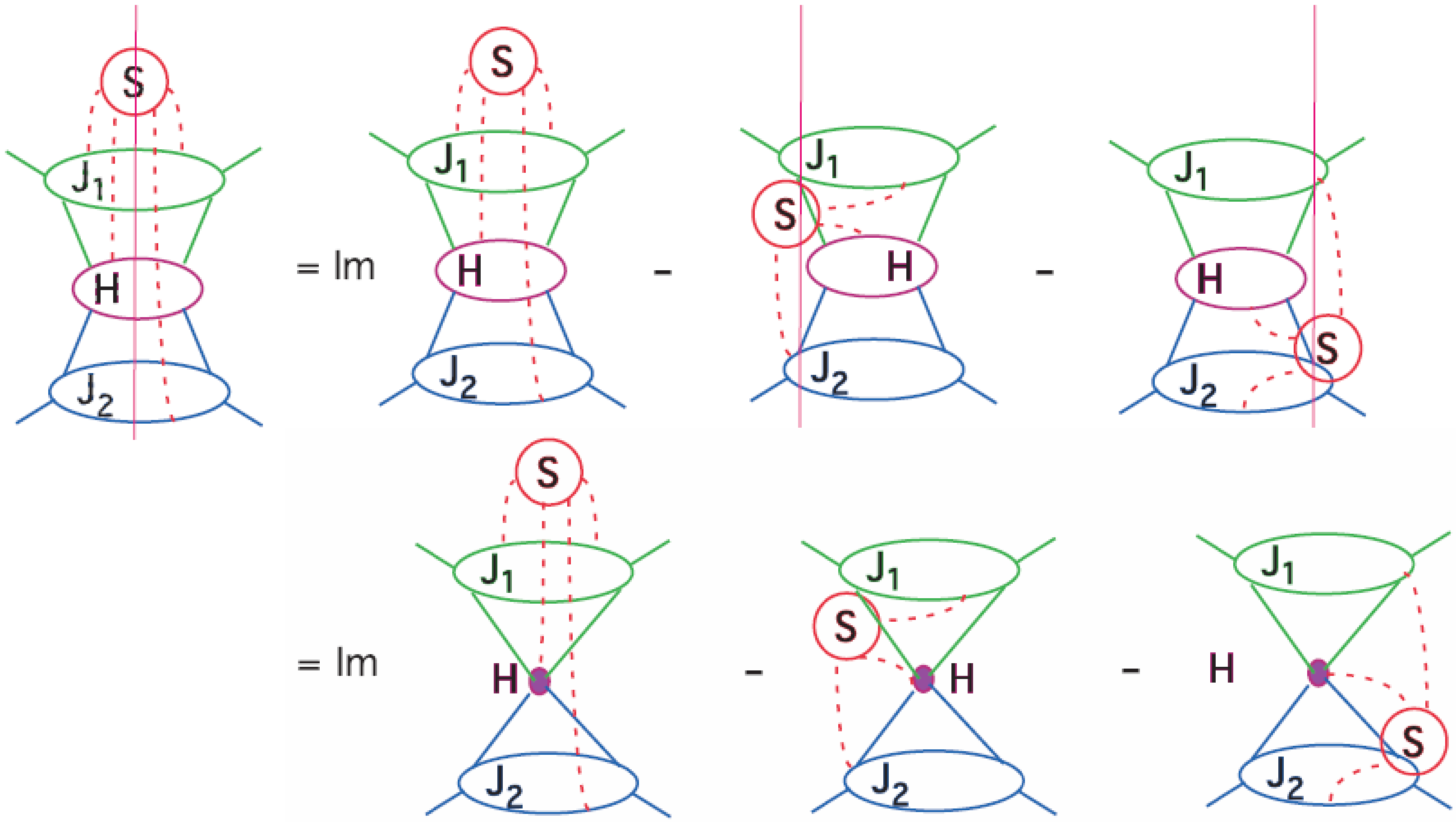
Fragmentation function

Reduced diagram

Still has long-distance physics due to incoming hadrons

❑ Step 2: Cancellation of remaining IR final state:

Note: Uncut loops are short distance



Remaining soft-interaction absorbed into the Wilson lines of PDFs

H is IR safe!

Fragmentation function

- **PQCD Factorization at large p_T:**

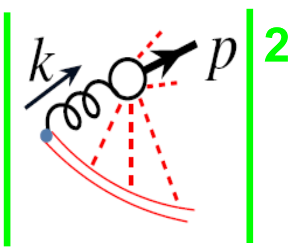
$$d\sigma_{A+B\to H+X}(p_T) = \sum_i d\tilde{\sigma}_{A+B\to i+X}(p_T/z,\mu) \otimes D_{H/i}(z,m_c,\mu) + \mathcal{O}(m_H^2/p_T^2)$$

- **Universal fragmentation function – single parton to a hadron H:**

e.g. a gluon to a hadron H (such as J/ψ):

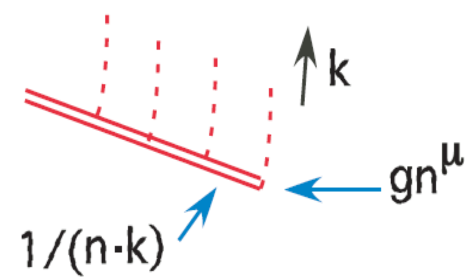
$$D_{H/g}(z,m_c,\mu) \propto \frac{1}{P+}\mathrm{Tr}_{color}\int dx^- \mathrm{e}^{-ik^+x^-}$$
$$\times\langle F^{+\lambda}(0)\,[\Phi_-^{(g)}(0)]^\dagger\, a_H(P^+)\, a_H^\dagger(P^+)\,\Phi_-^{(g)}(x^-)\, F_\lambda^+(y^-)\rangle$$

- **For the fragmentation function, all that is left is the gluon source:**



Gauge link:

$$\Phi_-^{(g)}(x^-) = P\exp\left[-ig\int_0^\infty n\cdot A^{(adj)}\left((x^-+\lambda)n\right)\right]$$



- **A necessary condition for the universality:**

The fragmentation function is independent of the n^μ

Connection to NRQCD factorization

- **Proposed NRQCD factorization:**

$$d\sigma_{A+B\to H+X}(p_T) = \sum_n d\hat{\sigma}_{A+B\to c\bar{c}[n]+X}(p_T)\,\langle \mathcal{O}_n^H \rangle$$

- **Proved pQCD factorization for single hadron production:**

$$d\sigma_{A+B\to H+X}(p_T) = \sum_i d\tilde{\sigma}_{A+B\to i+X}(p_T/z,\mu) \otimes D_{H/i}(z,m_c,\mu) + \mathcal{O}(m_H^2/p_T^2)$$

- **Prove NRQCD Factorization**

↔ **To prove: at** $\mu_0 \sim 2m_c$

$$D_{H/i}(z,m_c,\mu) = \sum_n d_{i\to c\bar{c}[n]}(z,\mu,m_c)\,\langle \mathcal{O}_n^H \rangle$$

with
- $d_{g\to c\bar{c}[n]}(z,\mu,m_c)$ IR safe
- $\langle \mathcal{O}_n^H \rangle$ gauge invariant and universal
- independent of the direction of the Wilson lines

Need to modify NRQCD matrix elements

- Conventional operator definition $(\mathrm{Q\bar{Q}}\ \text{rest frame})$

$$\mathcal{O}_n^H(0) = \chi^\dagger \mathcal{K}_n \psi(0) \left(a_H^\dagger a_H \right) \psi^\dagger \mathcal{K}_n' \chi(0)$$

- ψ, χ are heavy quark, antiquark fields

- $\mathcal{K}_n,\ \mathcal{K}_n'$: Products of color and spin matrices,
  covariant derivatives

- Fields at $x = 0$ *but* $\mathcal{O}_n^H$ not truly local

- Operator-valued gauge transformations
  (as to $A^+ = 0$ gauge) do not commute with $a_H^\dagger a_H$

- Only color-singlet $\mathcal{K}$'s give gauge invariant $\mathcal{O}$'s,
  the color-octet operators are not gauge invariant

Nayak, Qiu, Stermen, 2005

- **Resolution: supplement fields by Wilson lines – gauge links:**

$$\Phi_l[x, A] = \exp\left[-ig\int_0^\infty d\lambda\, l\cdot A(x+\lambda l)\right]$$

- **Our new, gauge invariant operators:**

$$\mathcal{O}_n^H(0) \to \chi^\dagger \mathcal{K}_{n,c}\psi(0)\, \Phi_l^\dagger[0,A]_{cb}\left(a_H^\dagger a_H\right)\, \Phi_l[0,A]_{ba}\, \chi^\dagger \mathcal{K}'_{n,a}\psi(0)$$

- **Two remaining questions for NRQCD factorization:**

$$D_{H/i}(z, m_c, \mu) = \sum_n d_{i\to c\bar{c}[n]}(z,\mu,m_c)\,\langle \mathcal{O}_n^H\rangle$$

- Are the "coefficient" functions $d_{g\to c\bar{c}[n]}(z,\mu,m_c)$ IR safe?

  Our NNLO answer is *no* ⟹ The gauge links are necessary

- Do the gauge links absorb all IR divergences?

  Can't tell yet for sure. OK at NNLO

An all-order proof of NRQCD factorization at high P_T is still lacking, and urgently needed

Summary and conclusions

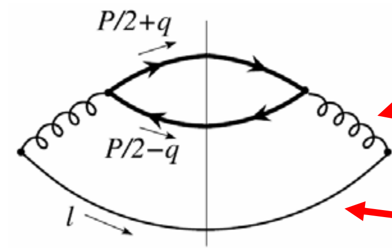
- NRQCD formalism is ideal for heavy quarkonium decay
- NRQCD formalism does not apply for associated production due to multiple heavy quark "velocities"
- Associated production of heavy quarkonium could be strongly enhanced by the soft color transfer
- Combination of PQCD and NRQCD factorization might work for hadronic heavy quark production at high P_T
- NRQCD factorization for the fragmentation function is valid to the NNLO order in α_s at a finite v
- An all-order proof of NRQCD factorization at high P_T is still lacking, and urgently needed

Thank you!

Backup slices

Factorization works to NLO at v^2

- **LO:**

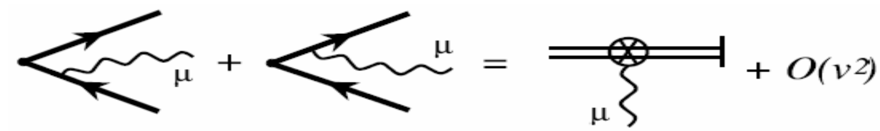


Short-distance

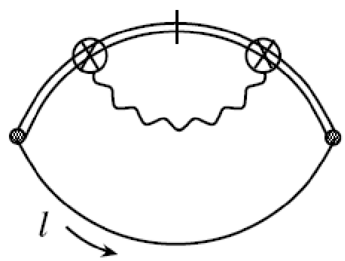
Eikonal line

$n=[Q\bar{Q}]_{singlet}$

Velocity expansion:



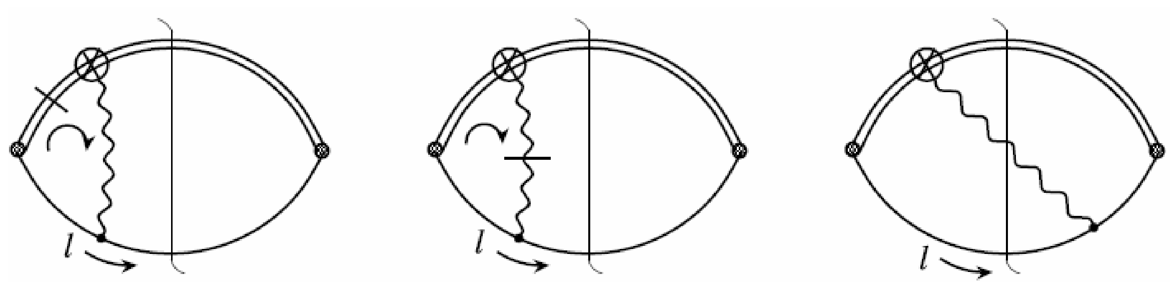
- **NLO:**



$$= \frac{16}{3}\frac{\alpha_s}{\pi}\frac{\vec{q}^{\,2}}{P^2}\frac{1}{-\varepsilon} + \ldots$$

→ **Topologially-factorized the matrix element** $\langle \mathcal{O}_n^H \rangle$

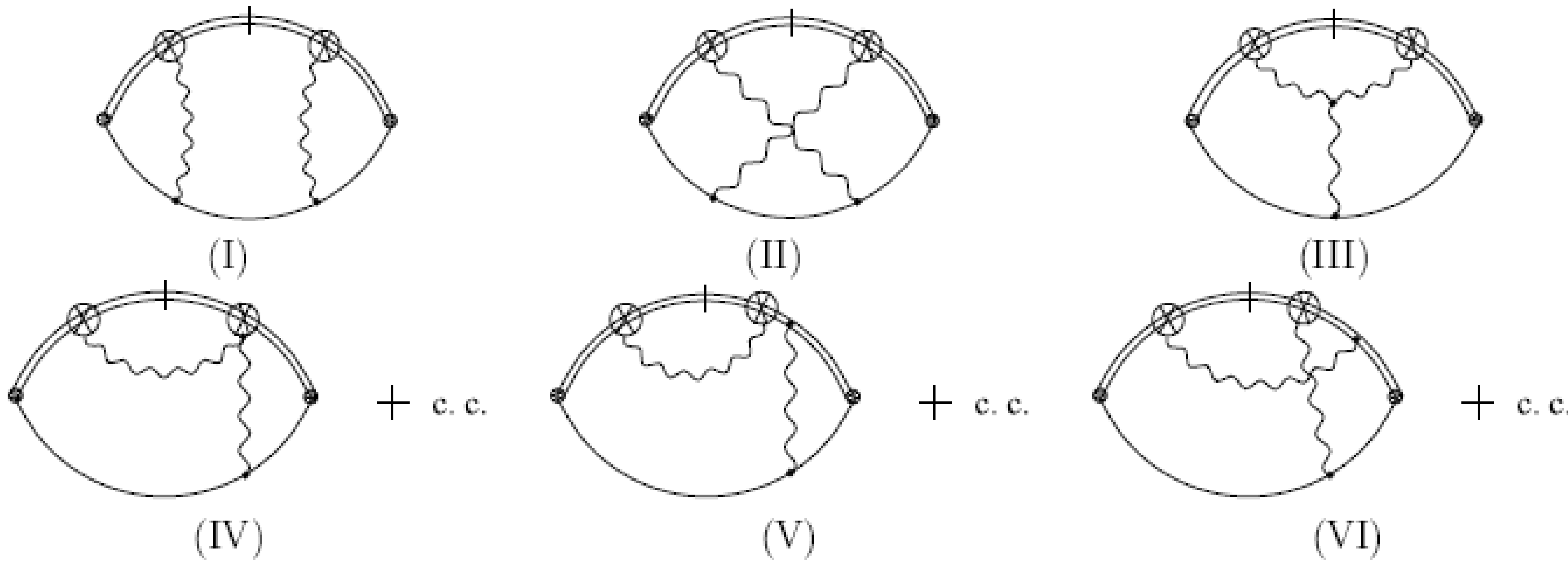
Color neutralization is IR divergent – nonperturbative!



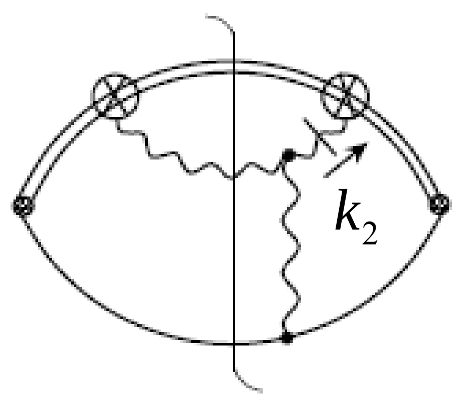
IR divergences cancel between real and virtual diagrams

Factorization fails at NNLO

❑ **Diagrams:**



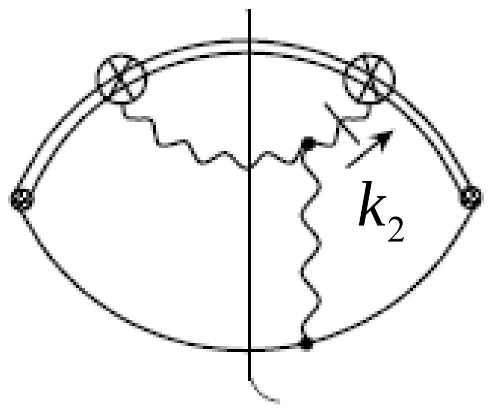
❑ **All IR divergences cancel between real and virtual diagrams, except**



Explicit calculation at NNLO at v^2 – I

❑ **The infrared divergent expression to order $q^2 \sim v^2$:**

$$\Sigma^{(2c)}(P,q,l) \;=\; -16i\, g^4 \mu^{4\varepsilon} \int \frac{d^D k_1}{(2\pi)^D} \frac{d^D k_2}{(2\pi)^D}, 2\pi\delta(k_1^2) l^\lambda V_{\nu\mu\lambda}[k_1,k_2]$$
$$\times\; [q^\mu (P\cdot k_1) - (q\cdot k_1)P^\mu]\;\; [q^\nu (P\cdot k_1) - (q\cdot k_2)P^\nu]$$
$$\times \frac{1}{[P\cdot k_1 + i\epsilon]^2\; [P\cdot k_2 - i\epsilon]^2}$$
$$\times \frac{1}{[k_2^2 - i\epsilon]\;[(k_2-k_1)^2 - i\epsilon]\;[l\cdot(k_1-k_2) - i\epsilon]},$$



❑ **The result is:**

$$\Sigma^{(2)}(P,q,l) \;=\; \alpha_s^2\, \frac{4}{3\varepsilon}\, [\frac{(P\cdot q)^2}{P^4} - \frac{q^2}{P^2}]$$

Explicit calculation at NNLO at v^2 - II

- In heavy quark pair's rest frame:

$$\Sigma(P,q,l) = \alpha_s^2 \frac{4}{3\varepsilon}\frac{\vec{q}^{\,2}}{4m_c^2} = \alpha_s^2 \frac{1}{3\varepsilon}\frac{\vec{v}^{\,2}}{4}$$

- IR poles would appear in coefficient function at NNLO unless we have eikonal interactions to absorb them

- This non-topological IR divergence Cannot be absorbed into the conventional NRQCD matrix elements

- Can be absorbed into the modified matrix elements

Factorization at a finite v?

- **Velocity expansion is not efficient for charmonium**
  - Large phase space available for gluon radiation:

$$Q^2 - 4M_C^2 \Rightarrow 4M_D^2 - 4M_C^2 \approx 6\ \text{GeV}^2$$

  - Large possible velocity in production:

$$v_{\text{prod}} \sim \frac{|k_c|}{M_c} \sim \sqrt{\frac{4M_D^2 - 4M_c^2}{4M_c^2}} \sim 0.88$$

  - Very different from decay:

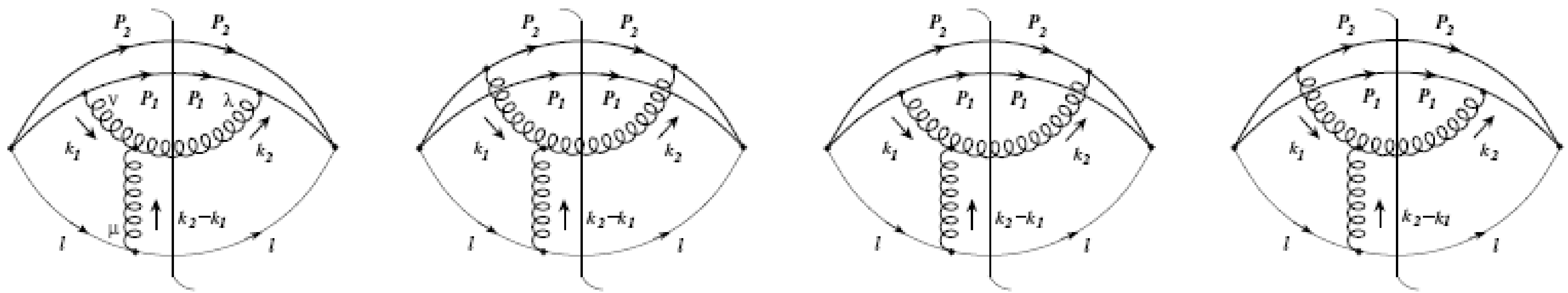
$$v_{\text{decay}} \sim \sqrt{\frac{4M_{J/\psi}^2 - 4M_c^2}{4M_c^2}} \sim 0.48$$

- **High order terms are very important**

Still no solution for the polarization data
even if NRQCD factorization is valid

Factorization at NNLO and all order in v^2

- Calculation with a finite v



$$\mathcal{I}^{(8\to 1)} = \frac{\alpha_s^2}{4\varepsilon}\left[1-\frac{1}{2f(|\vec{v}|)}\ln\left[\frac{1+f(|\vec{v}|)}{1-f(|\vec{v}|)}\right]\right]$$

with

$$f(v)=\frac{2v}{1+v^2} \qquad \vec{v}=\vec{q}/E^*$$

$2E^*$ is the total energy of the heavy quark pair

($Q\bar{Q}$ rest frame)

- Reproduce the v^2 result when expanded

Significance?

- The IR poles at all orders of v-expansion at NNLO are independent of the direction of the Wilson line and universal
  – consistent with factorization

- Although limited to NNLO, our result suggests that the decoupling of light parton dynamics from heavy quark pair production is robust in perturbation theory at the level of infrared divergence
  – high orders?

- Although the eikonal approximation do not cover many terms in general NRQCD velocity expansion, in particular, those dealing with spin, it should cover all perturbative infrared divergences