Inclusive Electromagnetic decays of the Bottomonium ground state

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Outline

Introduction

Are we ready to describe heavy quarkonium experiments? In particular inclusive electromagnetic decay data?

→ We have an effective field theory, Potential Non-Relativistic QCD, which describes the heavy quarkonium dynamics in the weak and strong coupling situation. $m \gg mv \gg mv^2$

$$\left.\begin{array}{l}\left(i\partial_0 - \frac{\mathbf{p}^2}{2m} - V_0(r)\right)\Phi(\mathbf{r}) = 0\\ +\text{corrections to the potential}\\ +\text{interaction with other low}\\ \quad\text{energy degrees of freedom}\end{array}\right\}\text{potential NRQCD} \qquad E \sim mv^2$$

In the weak coupling regime the starting point is $V_0 = -C_f \frac{\alpha_s}{r}$.
In the strong coupling regime case

$$V_0(\mathbf{r}) = \lim_{T\to\infty} \frac{i}{T} \log \langle W_\Box \rangle \quad \text{Wilson, Susskind}$$

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In particular searches for η_b (talks by Fabio Maltoni, Yu Jia, Pietro Santorelli, Cong-Feng Qiao).

Decays eventual important for the determination of α_s (talk by Xavier Garcia).

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Which states belong to the weak/strong coupling regime?
In particular, does the bottomoniun ground state belong to the weak coupling regime?

Analysis of the spectrum supports this hypothesis, but what about decays?

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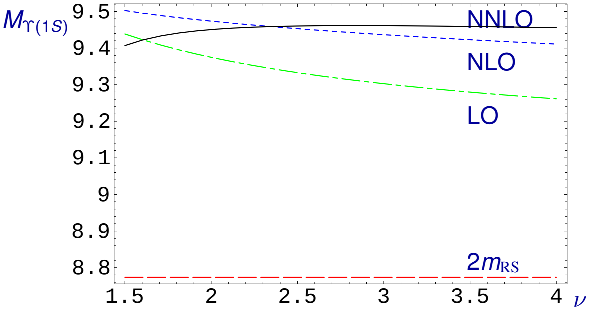
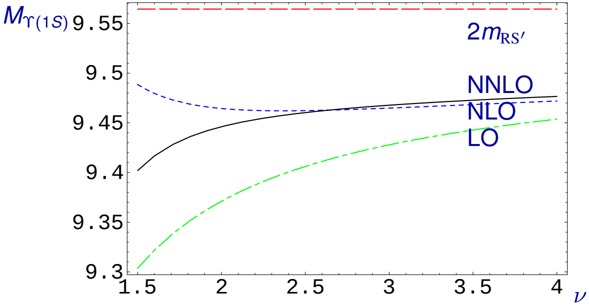

$M_{\Upsilon(1S)}$
9.5
9.4
9.3
9.2
9.1
9
8.9
8.8
NNLO
NLO
LO
$2m_{\rm RS}$
1.5
2
2.5
3
3.5
4
ν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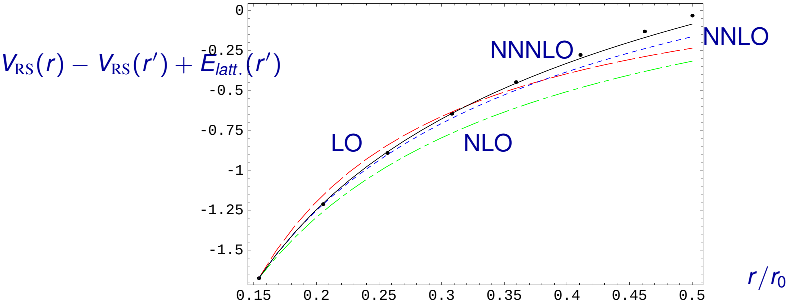
$V_{\rm RS}(r) - V_{\rm RS}(r') + E_{latt.}(r')$
NNNLO
NNLO
LO
NLO
r/r_0
0
-0.25
-0.5
-0.75
-1
-1.25
-1.5
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0.25
0.3
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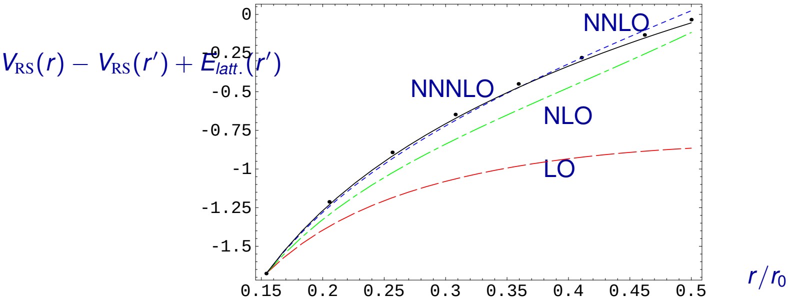
0.5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Inclusive electromagnetic decays: bottomonium

Pineda-Signer

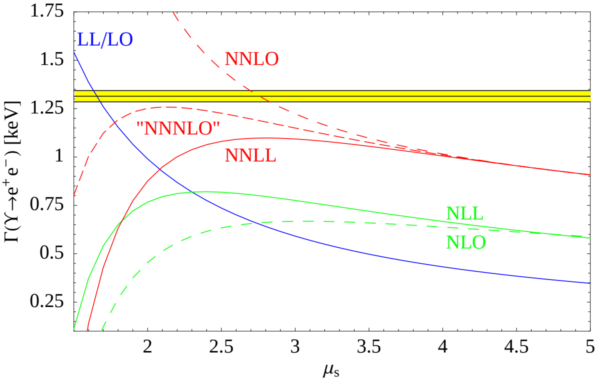


Figure: Prediction for the $\Upsilon(1S)$ decay rate to e^+e^-. We work in the RS' scheme.

The effect of the resummation of logarithms is important if compared with just keeping the single logarithm.

$$\Gamma(\Upsilon(nS) \to e^+e^-) = 16\pi \frac{C_A}{3} \left[\frac{\alpha_{EM}\, e_Q}{M_{\Upsilon(nS)}}\right]^2 \left|\phi_n^{(s=1)}(\mathbf{0})\right|^2 \left\{c_1 - d_1 \frac{M_{\Upsilon(nS)} - 2m_Q}{6m_Q}\right\}^2 ;$$

$$\Gamma(\eta_b(nS) \to \gamma\gamma) = 16\pi\, C_A \left[\frac{\alpha_{EM}\, e_Q^2}{M_{\eta_b(nS)}}\right]^2 \left|\phi_n^{(s=0)}(\mathbf{0})\right|^2 \left\{c_0 - d_0 \frac{M_{\eta_b(nS)} - 2m_Q}{6m_Q}\right\}^2 .$$

The corrections to the wave function at the origin are obtained by taking the residue of the Green function at the position of the poles

$$\left|\phi_n^{(s)}(\mathbf{0})\right|^2 = \left|\phi_n^{(0)}(\mathbf{0})\right|^2 \left(1 + \delta\phi_n^{(s)}\right) = \underset{E=E_n}{\mathrm{Res}} G_s(\mathbf{0}, \mathbf{0}; E) ,$$

where the LO wave function is given by

$$\left|\phi_n^{(0)}(\mathbf{0})\right|^2 = \frac{1}{\pi} \left(\frac{m_Q C_F \alpha_s}{2n}\right)^3 .$$

Note that $\left|\phi_n^{(s)}(\mathbf{0})\right|^2$ is SCHEME and SCALE dependent.

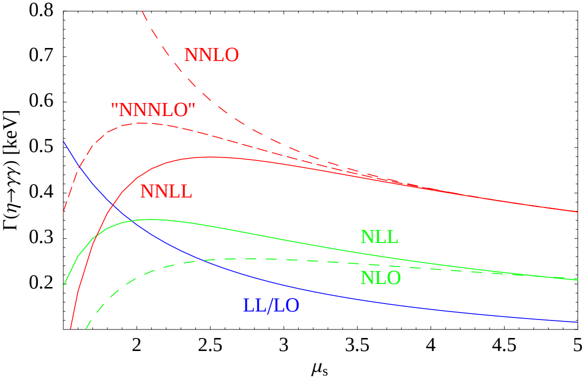


Figure: Prediction for the $\eta_b(1S)$ decay rate to two photons. We work in the RS' scheme.

Decay Ratio at NNLL

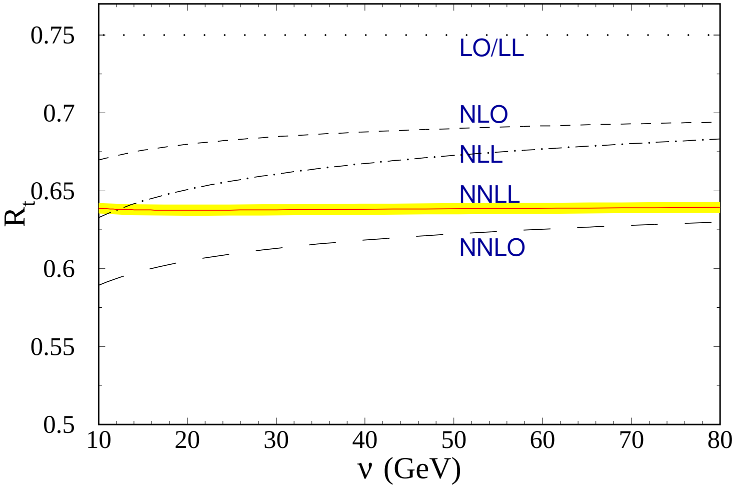
Penin, Smirnov, Steinhauser, Pineda

$$\frac{\Gamma(V_Q(nS)\to e^+e^-)}{\Gamma(P_Q(nS)\to\gamma\gamma)} \sim 1+\alpha\ln\alpha+\alpha^2\ln^2\alpha+\cdots$$
$$+\alpha+\alpha^2\ln\alpha+\alpha^3\ln^2\alpha+\cdots$$
$$+\alpha^2+\alpha^3\ln\alpha+\alpha^4\ln^2\alpha+\cdots$$

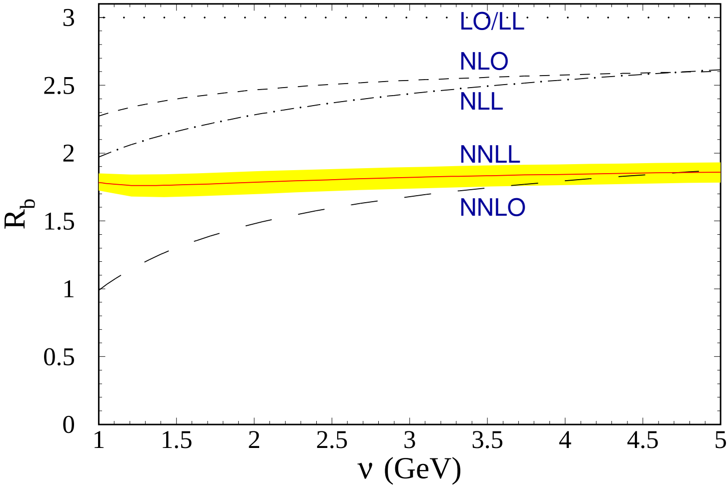
$$R_t=\frac{\Gamma(T(1S)\to e^+e^-)}{\Gamma(\eta_t(1S)\to\gamma\gamma)}=\frac{1}{3Q_t^2}\left(1-0.13198-0.0179492\right)\,.$$

$$R_b=\frac{\Gamma(\Upsilon(1S)\to e^+e^-)}{\Gamma(\eta_b(1S)\to\gamma\gamma)}=\frac{1}{3Q_b^2}\left(1-0.302-0.111\right)\,.$$

$$\Gamma(\eta_b(1S)\to\gamma\gamma)=0.659\pm0.089(\mathrm{th.})^{+0.019}_{-0.018}(\delta\alpha_s)\pm0.015(\mathrm{exp.})\ \mathrm{KeV}\,,$$



The spin ratio as the function of the renormalization scale ν for the (would be) toponium ground state. The yellow band reflects the errors due to $\alpha_s(M_Z)$.



The spin ratio as the function of the renormalization scale ν for the bottomonium ground state. The yellow band reflects the errors due to $\alpha_s(M_Z)$.

NNNLO ?

Coulomb corrections. Penin, Smirnov, Steinhauser; Beneke, Kiyo, Schuller

$$\frac{\delta_3|\phi_1(0)|^2_C}{|\phi_1^{(0)}(0)|^2} \simeq -0.47 \sim \alpha_s^3(\mu) \qquad \text{for } \mu = \mu_B \sim 2\text{GeV}$$

Relativistic corrections. Beneke, Kiyo, Schuller

$$\frac{\delta_3|\phi_1(0)|^2_R}{|\phi_1^{(0)}(0)|^2} \simeq -1.4 \sim \alpha_s^3(\mu) \qquad \text{for } \mu = \mu_B \sim 2\text{GeV}$$

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$$\frac{\delta_3|\phi_1(0)|^2_{US}}{|\phi_1^{(0)}(0)|^2} \simeq 1.93 \sim \alpha_s^3(\mu) \qquad \text{for } \mu = \mu_B \sim 2\text{GeV}$$

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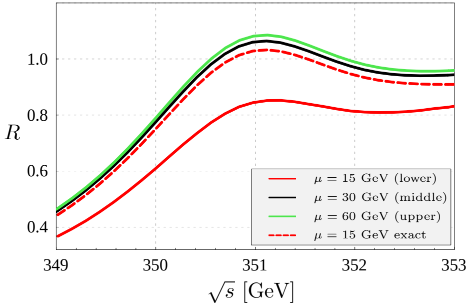
There is an strong scale dependence for scales below two GeV (20 for the case of toponium, they seem to have a similar origin) even after the resummation of logarithms.

Possible origin. The scale dependence of the Coulomb corrections.

Possible solution. Solving the Schroedinger equation with the Coulomb equation exactly (numerically). This significantly reduces the scale dependence.



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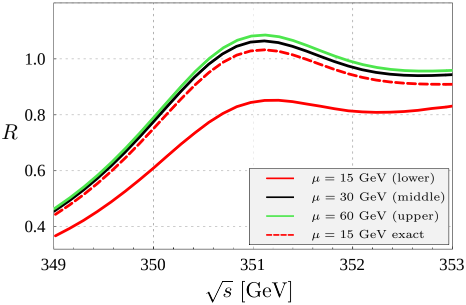

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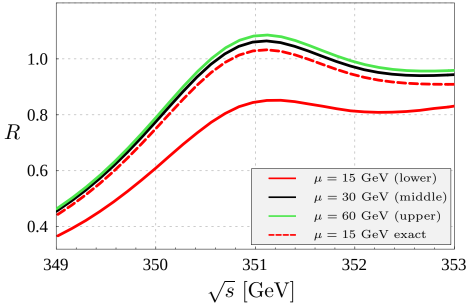

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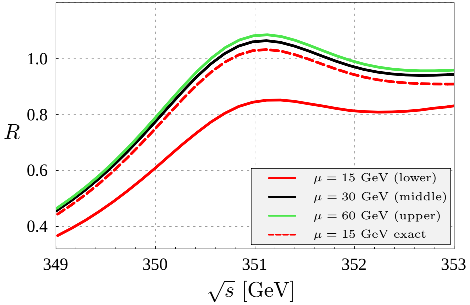

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Large corrections at each order in perturbation theory.

Besides the mass (for which we know how to handle the renormalon), the matching coefficients have renormalons themselves. In particular c_1.

So far they have been mainly studied at the theoretical level in the large β_0 approximation by Braaten and Yu-Qi Chen and by Bodwin and Yu-Qi Chen. Consistency shown. Renormalon contribution from the hard matching coefficient cancels with the renormalon contribution from the relativistic corrections.

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Well, I am optimistic

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BACK UP SLIDES

Vacuum polarization in the non-relativistic limit

$$J^{\mu} = \bar{Q}\gamma^{\mu}Q = c_1\psi^{\dagger}\boldsymbol{\sigma}\chi + \cdots\,, \qquad c_1 = 1 + a_1\alpha_s + a_2\alpha_s^2 + \cdots$$

B_1 at NNLO: Hoang(QED); Beneke, Signer, Smirnov; Czarnecki, Melnikov
B_1, B_0 at NLL: Pineda; Hoang, Stewart
B_1/B_0 at NNLL: Penin, Pineda, Smirnov, Steinhauser
B_1, B_0 at NNLL (partial): Pineda, Signer

$$(q_\mu q_\nu - g_{\mu\nu})\Pi(q^2) = i\int d^4x e^{iqx}\langle \mathrm{vac}|J_\mu(x)J_\nu(0)|\mathrm{vac}\rangle$$

$$\Pi(q^2) \sim c_1^2\langle \mathbf{r}=\mathbf{0}|\frac{1}{E-H}|\mathbf{r}=\mathbf{0}\rangle$$

$$G(0,0,E) = \sum_{m=0}^{\infty}\frac{|\phi_{0m}(0)|^2}{E_{0m}-E+i\epsilon-i\Gamma_t} + \frac{1}{\pi}\int_0^{\infty} dE'\,\frac{|\phi_{0E'}(0)|^2}{E_{0E'}-E+i\epsilon-i\Gamma_t}$$

$M(V_Q(nS))$ is also needed in order to obtain expressions for the t-$\bar{t}$ production near threshold with NNLL accuracy:
$M(V_Q(nS))$ at NNLL: Pineda; Hoang, Stewart
$M(V_Q(nS)) - M(P_Q(nS))$ at NNNLL: Kniehl, Penin, Pineda, Smirnov, Steinhauser

Relation of the vacuum polarization with $\sigma_{t\bar{t}}$, non-relativistic sum rules and $\Gamma(V_Q(nS) \rightarrow e^+e^-)$

Determination of m_b, m_t, α_s, Higgs-top yukawa coupling, ...

$$\Gamma(V \rightarrow e^+e^-) \sim \frac{1}{m^2} c_1^2 |\phi(\mathbf{0})|^2$$

$$\sigma_{t-\bar{t}} \sim c_1(\nu)^2 \mathrm{Im} G(0,0,\sqrt{s}) + \cdots$$

$$M_n \equiv \frac{12\pi^2 e_b^2}{n!} \left(\frac{d}{dq^2}\right)^n \Pi(q^2)\Big|_{q^2=0} = \int_0^\infty \frac{ds}{s^{n+1}} R_{b\bar{b}}(s),$$

$$M_n = 48\pi e_b^2 N_c \int_{-\infty}^{\infty} \frac{dE}{(E+2m_b)^{2n+3}} \left(c_1^2 - c_1 d_1 \frac{E}{3m_b}\right) \mathrm{Im}\, G(0,0,E)$$