Semileptonic $bc$ to $cc$ Baryon Decay and Heavy Quark Spin Symmetry

J. Flynn, E. Hernández, J.M. Verde-Velasco and JN


**Motivation**: Separate heavy quark spin symmetries make it possible to describe the semileptonic decays

\[ \Sigma_{bc}^{(*)} \rightarrow \Sigma_{cc}^{(*)} l \bar{\nu}_l, \quad \Omega_{bc}^{(*)} \rightarrow \Omega_{cc}^{(*)} l \bar{\nu}_l \]

in the limit $m_{b,c} \gg \Lambda_{QCD}$ and close to the zero recoil point.
\[
q^2 = m_{bc}^2 + m_{cc}^2 - 2m_{bc}m_{cc}\omega, \quad \frac{1}{2\pi} \leq \omega \leq \frac{m_{bc}^2 + m_{cc}^2 - m_l^2}{2m_{bc}m_{cc}}
\]

\[
HQS \text{ constraints on SL FF’s and } \Gamma \text{’s of doubly heavy baryons.}
\]

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<th>$J^P$</th>
<th>$I$</th>
<th>$S^\pi_{hh}$</th>
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For instance, let us study \( \Xi^{(*)}_{bc} \to \Xi^{(*)}_{cc} \) SL decays,

\[
p_\mu = m_{\Xi^{(*)}_{bc}} v_\mu, \quad p'_\mu = m_{\Xi^{(*)}_{cc}} v'_\mu = m_{\Xi^{(*)}_{cc}} v_\mu + k_\mu
\]

Near the zero-recoil point \( \omega = 1 \) (\( \omega = v \cdot v' \)) \( k \) small residual momentum \( \Rightarrow k \cdot v = O(1/m_{\Xi^{(*)}_{cc}}) \).

To represent the lowest-lying \( S \)-wave \( bcq \) baryons we use wavefunctions comprising tensor products of Dirac matrices and spinors, namely:

\[
B'_{bc} = -\left[ \frac{(1 + \gamma^5)}{2} \right]_{\alpha\beta} \gamma_5 u_\gamma(v, r)
\]

\[
B_{bc} = \left[ \frac{(1 + \gamma^5)}{2} \gamma_\mu \right]_{\alpha\beta} \left[ \frac{1}{\sqrt{3}} (v^\mu + \gamma^\mu) \gamma_5 u(v, r) \right]_\gamma
\]

\[
B^*_{bc} = \Xi^*_{bc} = \left[ \frac{(1 + \gamma^5)}{2} \gamma_\mu \right]_{\alpha\beta} u^\mu_\gamma(v, r)
\]

\( \alpha, \beta, \gamma \) Dirac indices and \( r \) baryon helicity label. These wavefunctions can be considered as matrix elements of the form \( \langle 0 | c_\alpha \bar{q}^c \beta b_\gamma | B_{bc}^{(i*)} \rangle \) where \( \bar{q}^c = q^T C \) with \( C \) the charge-conjugation matrix.
Under a Lorentz ($\Lambda$), and $b$ and $c$ quark spin ($S_b$ and $S_c$) transformations, a wavefunction $\Gamma_{\alpha\beta} u_\gamma$ transforms as:

$$\Gamma u \rightarrow S(\Lambda)\Gamma S^{-1}(\Lambda) S(\Lambda) u$$

$$\Gamma u \rightarrow S_c \Gamma S_b u$$

**States** normalised using $\bar{u}u \text{Tr}(\Gamma \Gamma)$: mutually orthogonal and have a common normalisation ($\bar{\Gamma} = \gamma^0 \Gamma^\dagger \gamma^0$). States where the $b$ and $c$ quarks are coupled to definite spin,

$$|S_{bc} = 0; J = \frac{1}{2}\rangle = -\frac{1}{2} |S_{cq} = 0; J = \frac{1}{2}\rangle + \frac{\sqrt{3}}{2} |S_{cq} = 1; J = \frac{1}{2}\rangle$$

$$|S_{bc} = 1; J = \frac{1}{2}\rangle = \frac{\sqrt{3}}{2} |S_{cq} = 0; J = \frac{1}{2}\rangle + \frac{1}{2} |S_{cq} = 1; J = \frac{1}{2}\rangle$$

$$|S_{bc} = 1; J = \frac{3}{2}\rangle = |S_{cq} = 1; J = \frac{3}{2}\rangle$$
Remarks:

- **We have not used definite spin combinations directly for the $b$ and $c$ quarks.** The reason is to make both the spin transformations on the heavy quarks and the Lorentz transformation of the states convenient, making it straightforward to build spin-invariant and Lorentz covariant quantities.

- **We could have combined the $b$ quark with the light quark to a definite spin.** This would clearly interchange the spin transformations and alter the appearance of spin-invariant and Lorentz covariant quantities. Physical results should of course be unchanged.
For the $cc$ baryons,

$$B'_{cc} = -\sqrt{\frac{2}{3}} \left[ \frac{1 + \psi'}{2} \gamma_5 \right]_{\alpha\beta} u_\gamma(v, r)$$

$$B_{cc} = \sqrt{2} \left[ \frac{1 + \psi}{2} \gamma_\mu \right]_{\alpha\beta} \left[ \frac{1}{\sqrt{3}} (v^\mu + \gamma^\mu) \gamma_5 u(v, r) \right]_\gamma$$

$$B^*_c = \Xi^*_c = \sqrt{\frac{1}{2}} \left[ \frac{1 + \psi}{2} \gamma_\mu \right]_{\alpha\beta} u_\mu(v, r)$$

- the two charm quarks can only be in a symmetric spin-1 state: $B'_{cc}$ and $B_{cc}$ correspond to the same baryon state $\Xi_{cc}$.

- normalisation: there are two ways to contract the charm quark indices, leading to $\bar{u}u \text{Tr} (\Gamma \Gamma) + \bar{u} \Gamma \Gamma u$. To have the same normalisation as for the $bc$ case, we have to include extra numerical factors.
... construct spin-invariant and Lorentz covariant amplitudes for the weak transition matrix elements, 
SL $\Xi_{bc}^{(*)} \rightarrow \Xi_{cc}^{(*)}$ decays $\leftrightarrow$ ME weak current $J_\mu = \bar{c}\gamma_\mu(1-\gamma_5)b$

We first build transition amplitudes between the $B_{bc}^{(*)}$ and $\Xi_{cc}^{(*)}$ states and subsequently take linear combinations to obtain transitions from $\Xi_{bc}^{(*)}$ states. The most general form for the ME respecting the HQSS is ($j_\mu = \gamma_\mu(1-\gamma_5)$):

$$
\langle \Xi_{cc}^{(*)}, v, k, M' | J_\mu(0) | B_{bc}^{(*)}, v, M \rangle = \bar{u}_{cc}(v, k, M')j_\mu u_{bc}(v, M) \text{Tr}[\Gamma_{bc}\Omega\Gamma_{cc}]
+ \bar{u}_{cc}(v, k, M')\Gamma_{bc}\Omega\Gamma_{cc}j_\mu u_{bc}(v, M)
$$

$$
\begin{align*}
\Gamma_{bc} & \rightarrow S_c\Gamma_{bc}, & u_{bc} & \rightarrow S_b u_{bc} \\
\Gamma_{cc} & \rightarrow \Gamma_{cc}S_c^\dagger, & \bar{u}_{cc} & \rightarrow \bar{u}_{cc}S_c^\dagger \\
\bar{c}j_\mu b : j_\mu & \rightarrow S_c j_\mu S_b^\dagger
\end{align*}
$$

J. Nieves, U. Granada
... construct spin-invariant and Lorentz covariant amplitudes for the weak transition matrix elements,

\[ \text{SL } \Xi_{bc}^{(*)} \rightarrow \Xi_{cc}^{(*)} \text{ decays } \leftrightarrow \text{ME weak current } J^\mu = \bar{c}\gamma^\mu(1-\gamma_5)b \]

We first build transition amplitudes between the \( B_{bc}^{(*)} \) and \( \Xi_{cc}^{(*)} \) states and subsequently take linear combinations to obtain transitions from \( \Xi_{bc}^{(*)} \) states. The most general form for the ME respecting the HQSS is \( (j^\mu = \gamma^\mu (1-\gamma_5)) \):

\[
\langle \Xi_{cc}^{(*)}, v, k, M' | J^\mu (0) | B_{bc}^{(*)}, v, M \rangle = \bar{u}_{cc}(v, k, M') S_c^\dagger S_c j^\mu S_b^\dagger S_b u_{bc}(v, M) \text{Tr}[S_c \Gamma_{bc} \Omega \bar{\Gamma}_{cc} S_c^\dagger] + \bar{u}_{cc}(v, k, M') S_c^\dagger S_c \Gamma_{bc} \Omega \bar{\Gamma}_{cc} S_c^\dagger S_c j^\mu S_b^\dagger S_b u_{bc}(v, M)
\]

\[
\begin{align*}
\Gamma_{bc} & \rightarrow S_c \Gamma_{bc}, & u_{bc} & \rightarrow S_b u_{bc} \\
\bar{\Gamma}_{cc} & \rightarrow \bar{\Gamma}_{cc} S_c^\dagger, & \bar{u}_{cc} & \rightarrow \bar{u}_{cc} S_c^\dagger \\
\bar{c}j^\mu b : j^\mu & \rightarrow S_c j^\mu S_b^\dagger
\end{align*}
\]
where $M$ and $M'$ are the helicities of the initial and final states

$$\Omega = -\frac{1}{\sqrt{2}} \eta(v \cdot v')$$

is the most general Dirac matrix that can be written in terms of the vectors $k$ and $v$.

- terms with a factor of $\psi$ can be omitted because of the equations of motion ($\psi u = u$, $\psi \Gamma = \Gamma$, $\gamma_\mu u^\mu = 0$, $v_\mu u^\mu = 0$),

- terms with $k$ will always lead to contributions proportional to $v \cdot k = \mathcal{O}(1/m_{\Xi_{cc}^*})$. 
\[
\begin{align*}
\Xi_{bc} \rightarrow \Xi_{cc} & \quad \eta \bar{u}_{cc} \left(2\gamma^{\mu} - \frac{4}{3}\gamma^{\mu}\gamma_{5}\right)u_{bc} \\
\Xi'_{bc} \rightarrow \Xi_{cc} & \quad -\frac{2}{\sqrt{3}}\eta \bar{u}_{cc} (-\gamma^{\mu}\gamma_{5})u_{bc} \\
\Xi_{bc} \rightarrow \Xi^*_{cc} & \quad -\frac{2}{\sqrt{3}}\eta \bar{u}_{cc}^{\mu}u_{bc} \\
\Xi'_{bc} \rightarrow \Xi^*_{cc} & \quad -2\eta \bar{u}_{cc}^{\mu}u_{bc} \\
\Xi^*_{bc} \rightarrow \Xi_{cc} & \quad -\frac{2}{\sqrt{3}}\eta \bar{u}_{cc}^{\mu}u_{bc} \\
\Xi^*_{bc} \rightarrow \Xi^*_{cc} & \quad -2\eta \bar{u}_{cc}^{\lambda} (\gamma^{\mu} - \gamma^{\mu}\gamma_{5})u_{bc}^{\lambda}
\end{align*}
\]
Remarks:

- If the $b$ and $c$ quarks become degenerate, then vector current conservation ensures that $\eta(1) = 1$.

- Savage and White (PLB 271 (1991) 410) found similar results: approach where the two heavy quarks bind into a colour antitriplet which appears as a pointlike colour source to the light degrees of freedom + “superflavor” formalism of Georgi and Wise. We find two differences to their results (one of these was already pointed out by Sanchis-Lozano PLB 321 (1994) 407).

- Our approach, where we consider the spin transformations of each heavy quark explicitly, is straightforward and similar to that used to describe $B_c$ meson decays: Jenk-

- **Spin symmetry** for both the $b$ and $c$ quarks **enormously simplifies** the description of all $\Xi_{bc}^{(*)} \rightarrow \Xi_{cc}^{(*)} \nu \bar{\nu}$ decays in the heavy quark limit and near the zero recoil point. **All the weak transition matrix elements are given in terms of a single universal function.** Lorentz covariance alone allows a large number of form factors (six form factors to describe $\Xi_{bc} \rightarrow \Xi_{cc}$, another six for $\Xi'_{bc} \rightarrow \Xi_{cc}$, eight each for $\Xi_{bc} \rightarrow \Xi^*_{cc}, \Xi'_{bc} \rightarrow \Xi^*_{cc}$ and $\Xi^*_{bc} \rightarrow \Xi_{cc}$, and even more for $\Xi^*_{bc} \rightarrow \Xi^*_{cc}$).
Test: QM [EPJA 32 (2007) 183]

\[
\eta(v \cdot v') = \int d^3r_1 d^3r_2 \exp[-i\vec{k} \cdot \vec{r}_{12}/2][\Psi_{cc}^\Xi(r_1, r_2, r_{12})]^* \Psi_{bc}^\Xi(r_1, r_2, r_{12})
\]

\[
\begin{align*}
\text{Re}lative \text{ Motion of } q \text{ and a Pointlike } Q_c \text{ Diquark} \\
r_{12} \ll r_1, r_2 \rightarrow \Psi_{Qc}^\Xi(r_1, r_2, r_{12}) \approx \Phi_{Qc}(r_{12}) \phi(r_{Qc}) \varphi_{Qc}(\vec{r}_{12} \cdot \vec{r}_{Qc}) \\
\end{align*}
\]

⇐ Jacobi's coordinates, \(Q, Q' = c, b\).
\[
\begin{align*}
&\left\langle \Xi_{cc}, r' \bar{p}' \left| \bar{c} \gamma^\mu (1 - \gamma_5) b(0) \right| \Xi_{bc}'^{(i)}, r \bar{p} \right\rangle = \bar{u}_{r' c \bar{c}}(\bar{p}') \left\{ \gamma^\mu \left( F_1(w) - \gamma_5 G_1(w) \right) \\
&\quad + u^\mu \left( F_2(w) - \gamma_5 G_2(w) \right) + v'^\mu \left( F_3(w) - \gamma_5 G_3(w) \right) \right\} u_{r \bar{b} c}(\bar{p})
\end{align*}
\]
... $1/2 \rightarrow 3/2$ spin transitions

$$
\langle \Xi_{cc}, r' \bar{p}' | \bar{c} \gamma^\mu (1 - \gamma_5) b(0) | \Xi^{(i)}_{bc}, r \bar{p} \rangle = \bar{u}_{\lambda r'}(\bar{p}') \Gamma^\lambda \mu u_{\lambda, r}(\bar{p})
$$

$$
\Gamma^\lambda \mu = \left( \frac{C^V_3(\omega)}{m_{\Xi^{(i)}_{bc}}} (g^{\lambda \mu} \bar{q} - q^\lambda \gamma^\mu) + \frac{C^V_4(\omega)}{m_{\Xi^{(i)}_{bc}}^2} (g^{\lambda \mu} q p' - q^\lambda p'^\mu) 
\right.
$$

$$
+ \frac{C^V_5(\omega)}{m_{\Xi^{(i)}_{bc}}^2} (g^{\lambda \mu} q p - q^\lambda p^\mu) + C^V_6(\omega) g^{\lambda \mu}) \gamma_5
$$

$$
+ \left( \frac{C^A_3(\omega)}{m_{\Xi^{(i)}_{bc}}} (g^{\lambda \mu} \bar{q} - q^\lambda \gamma^\mu) + \frac{C^A_4(\omega)}{m_{\Xi^{(i)}_{bc}}^2} (g^{\lambda \mu} q p' - q^\lambda p'^\mu) + C^A_5(\omega) g^{\lambda \mu} + \frac{C^A_6(\omega)}{m_{\Xi^{(i)}_{bc}}^2} q^\lambda q^\mu \right)
$$

and $3/2 \rightarrow 1/2$ transitions...

$$
\langle \Xi_{cc}, r' \bar{p}' | \bar{c} \gamma^\mu (1 - \gamma_5) b(0) | \Xi^{*}_{bc}, r \bar{p} \rangle = \bar{u}_{\lambda r'}(\bar{p}') \hat{\Gamma}^\lambda \mu u_{\lambda, r}(\bar{p})
$$
\[ (F_1 + F_2 + F_3)/2 \]

\[ 3/4 G_1 \]

\[ F_1 + F_2 + F_3 \]

\[ -\sqrt{3} G_1/2 \]

\[ \Xi_{bc} \rightarrow \Xi_{cc} \]

\[ \Xi'_{bc} \rightarrow \Xi_{cc} \]

\[ \Xi_{bb} \rightarrow \Xi_{bc} \]

\[ \Xi'_{bb} \rightarrow \Xi'_{bc} \]

\[ \Xi^*_{bc} \rightarrow \Xi_{cc} \]

\[ \Xi^*_{bb} \rightarrow \Xi^*_{bc} \]

\[ \Xi^*_{bb} \rightarrow \Xi'_{bc} \]
and \( \frac{3}{2} \to \frac{3}{2} \) transitions, \( \Xi_{bc}^{*} \to \Xi_{cc}^{*} \sim 50 \) FF’s

\[
(F_1 + F_2 + F_3)/2 \\
3/4 G_1 \\
F_1 + F_2 + F_3 \\
-\sqrt{3} G_1/2 \\
\Xi_{bc} \to \Xi_{cc}^{*} \\
\Xi_{bc}^{*} \to \Xi_{cc}^{*} \\\n\Xi_{bc}^{*} \to \Xi_{cc}^{*} \\
\Xi_{bb} \to \Xi_{bc} \\
\Xi_{bb} \to \Xi_{bc}^{*} \\\n\Xi_{bb}^{*} \to \Xi_{bc}^{*} \\
vect \rightarrow r'=3/2 \\
vector, r=1/2 \rightarrow r'=1/2 \\
axial, r=1/2 \rightarrow r'=-1/2 \\
axial, r=1/2 \rightarrow r'=3/2
\]
HQSS constraints on semileptonic decay widths

\[ \Gamma = \frac{G_F^2 |V_{cb}|^2}{32\pi^4} \frac{m_{\Xi_{cc}}}{m_{\Xi_{bc}^{(*)}}} \int_{1}^{\omega_{\text{max}}} d\omega \frac{\sqrt{\omega^2 - 1}}{\mathcal{L}^{\mu\nu} \mathcal{H}_{\mu\nu}} \]

\[ \mathcal{L}^{\mu\nu} = \int \frac{d^3k_1}{2E_1} \frac{d^3k_2}{2E_2} \delta^{(4)}(q - k_1 - k_2) \left( k_1^\mu k_2^\nu + k_1^\nu k_2^\mu - g^{\mu\nu} k_1 \cdot k_2 + i \epsilon^{\mu\nu\alpha\beta} k_1^\alpha k_2^\beta \right) \]

\[ = A(q^2) g^{\mu\nu} + B(q^2) \frac{q^\mu q^\nu}{q^2} \]

For the actual doubly heavy baryon masses \( \omega_{\text{max}} \approx 1.22 (1.08) \) for \( bc \rightarrow cc (bb \rightarrow bc) \) transitions. The different differential decay widths \( d\Gamma/d\omega \) show a maximum at around \( \omega \approx 1.05 (1.01) \implies \)

\[ \eta(\omega) \rightarrow \mathcal{H}_{\mu\nu} \]

and approximating

\[ m_{\Xi_{bb}} \approx m_{\Xi_{bb}^*}; \quad m_{\Xi_{bc}} \approx m_{\Xi_{bc}'^*} \approx m_{\Xi_{bc}^*}; \quad m_{\Xi_{cc}} \approx m_{\Xi_{cc}^*} \]
predict that some ratios between different decay widths should be approximately 1...

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<td>$\frac{\Gamma(B_{bc}^{\prime}\rightarrow B_{cc}^{<em>}l\bar{\nu}<em>l)}{3\Gamma(B</em>{bc}\rightarrow B_{cc}^{</em>}l\bar{\nu}_l)}$</td>
<td>1.04$^{+0.03}_{-0.01}$</td>
<td>1.04$^{-0.03}_{-0.01}$</td>
<td>0.79</td>
<td>0.82</td>
<td>0.68</td>
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<td>0.82$^{+0.06}_{-0.01}$</td>
<td>0.84$^{+0.13}_{-0.01}$</td>
<td>1.22</td>
<td>1.17</td>
<td>2.72</td>
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<td>1.14$^{+0.08}_{-0.06}$</td>
<td>1.16$^{+0.04}_{-0.01}$</td>
<td>1.05</td>
<td>1.08</td>
<td>3.90</td>
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<td>$\frac{\Gamma(B_{bc}^{<em>}\rightarrow B_{cc}^{</em>}l\bar{\nu}<em>l)}{\Gamma(B</em>{bc}\rightarrow B_{cc}l\bar{\nu}<em>l)+\frac{1}{2}\Gamma(B</em>{bc}\rightarrow B_{cc}^{*}l\bar{\nu}_l)}$</td>
<td>0.89$^{+0.11}_{-0.06}$</td>
<td>0.94$^{+0.13}_{-0.01}$</td>
<td>1.01</td>
<td>1.01</td>
<td>1.08</td>
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### $bb \to bc$

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<td>$\Omega$</td>
<td>$\Xi$</td>
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<tr>
<td>$3 \Gamma(B^*<em>{bb} \to B</em>{bc} l\bar{\nu}_l)$</td>
<td>$1.00^{+0.01}_{-0.04}$</td>
<td>$1.00^{+0.03}_{-0.01}$</td>
<td>$0.99$</td>
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<tr>
<td>$\frac{2}{3} \Gamma(B_{bb} \to B_{bc}^* l\bar{\nu}_l)$</td>
<td>$0.86^{+0.08}_{-0.06}$</td>
<td>$0.86^{+0.05}_{-0.0}$</td>
<td>$0.96$</td>
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<tr>
<td>$\frac{1}{2} \Gamma(B_{bb} \to B_{bc}^* l\bar{\nu}_l)$</td>
<td>$1.14^{+0.04}_{-0.05}$</td>
<td>$1.13^{+0.01}_{-0.17}$</td>
<td>$1.05$</td>
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<tr>
<td>$\Gamma(B^<em><em>{bb} \to B</em>{bc}^</em> l\bar{\nu}_l)$</td>
<td>$0.94^{+0.07}_{-0.06}$</td>
<td>$0.93^{+0.11}_{-0.10}$</td>
<td>$1.01$</td>
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<tr>
<td>$\Gamma(B_{bb} \to B_{bc} l\bar{\nu}<em>l) + \frac{1}{2} \Gamma(B</em>{bb} \to B_{bc}^* l\bar{\nu}_l)$</td>
<td>$1.05$</td>
<td>$1.04$</td>
<td>$0.31$</td>
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J. Nieves, U. Granada
**Diquark Picture and Link to $B_c$ Meson Decays**

\[ \eta(\mathbf{v} \cdot \mathbf{v'}) = \int d^3r_1 d^3r_2 \exp[-i\mathbf{k} \cdot \mathbf{r}_{12}/2] [\Psi_{cc}^{\Xi}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_{12})]^* \Psi_{bc}^{\Xi}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_{12}) \]

\[ \xi_{bb}, \xi_{bc}, \xi_{cc} \text{ fm} \]

\[ P_{h_{1/2}} [\text{fm}^{-1}] \]

\[ \Omega_{bb}, \Omega_{bc}, \Omega_{cc} \text{ fm} \]

\[ r_{12} \ll r_1, r_2 \rightarrow \Psi_{Qc}^{\Xi}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_{12}) \approx \Phi_{Qc}(\mathbf{r}_{12}) \phi(r_{Qc}) \varphi_{Qc}(\mathbf{r}_{12} \cdot \mathbf{r}_{Qc}) \]

\[ \text{DIQUARK} \quad \text{VARIATIONAL}\approx1 \]

\[ \text{RELATIVE MOTION OF } q \text{ AND A POINTLIKE } Qc \text{ DIQUARK} \]
\[ \eta(v \cdot v') = \int d^3 r_{12} \exp[-i\vec{k} \cdot \vec{r}_{12}/2][\Phi_{cc}(r_{12})]^*\Phi_{bc}(r_{12}) \int d^3 r \phi^*(r)\phi(r) \]

where \( \vec{r} = \vec{r}_{ccq} \) and in the \( d^3 r \) integral we have replaced \( \phi(r_{bcq}) \) by \( \phi(r) \) since \( \vec{r}_{bcq} = \vec{r}_{ccq} + O(\vec{r}_{12}) \). This approximation leads to uncertainties of \( O(r_{12}^2) \) after integration,

\[ \eta(v \cdot v') = \int d^3 r_{12} \exp[-i\vec{k} \cdot r_{12}/2][\Phi_{cc}(r_{12})]^*\Phi_{bc}(r_{12}) \]

which has an identical form to the expression of the form factor \( \Delta \), unique form factor which describes the \( B_c \) to \( \eta_c \) and \( J/\psi \) semileptonic decays, in terms of wavefunctions of the \( \bar{b}c \) and \( \bar{c}c \) bound states (Jenkins et al., NPB 390 (1993) 463).
This does not mean that $\eta$ and $\Delta$ are identical because the $QQ$ and $Q\bar{Q}$ potentials used to compute the diquark and meson wavefunctions are not the same. For example a $\lambda_i\lambda_j$ colour dependence ($\lambda_i$ are the usual Gell-Mann matrices) would lead to $V^{QQ} = V^{Q\bar{Q}}/2$. [approx wfnt overlaps (solid lines) vs IW funcs (dahsed lines)]
The $\omega^2$ slope of the $\Delta$ form factor is indeed smaller than that of $\eta$, but the ratio is around 1 to 3 rather than 1 to 6, so there are significant corrections to the Coulomb wavefunction description.

**Conclusions**

- Separate HQSS make it possible to describe all $SL_{bc} \rightarrow SL_{cc} l \bar{\nu}_l$, $\Omega_{bc}^{(*)} \rightarrow \Omega_{cc}^{(*)} l \bar{\nu}_l$

  decays using a single form factor. Similarly for $bb \rightarrow bc$ decays.

- We have discussed the resemblance of the $bc$ baryon decays to those of $B_c$ mesons to $\eta_c$ and $J/\psi$ mesons.
• **Lattice QCD** simulations work best near the zero-recoil point and thus are well-suited to **check the validity of the results**.

• **QM calculations consistent with HQSS?**
  
  – Results by **Hernández et al.** (FF’s and decay width ratios), and **Ebert et al.** (decay width ratios), **compare well**, within expectations **with HQSS**
  
  – **We detect problems** either in the model or in the calculation performed by Guo et al.