Pion Interactions in the X(3872)

S.Fleming, M. Kusunoki, T.M., U. van Kolck, PRD76:034006 (2007)

T. Mehen, Duke U.

QWG 2007, DESY, Hamburg, 10/16/2007

X(3872)

- New charmonium state discovered in B decays by Belle
- Seen in several decay channels

$$X(3872) \to J/\psi\pi^+\pi^-$$
$$\to J/\psi\pi^+\pi^-\pi^0$$
$$\to D^0\bar{D}^0\pi^0$$
$$\to J/\psi\gamma$$

not conventional charmonium

$$\frac{\Gamma(X(3872) \to \gamma\chi_{c1})}{\Gamma(X(3872) \to \pi^+\pi^- J/\psi)} < 0.89$$

isospin violating

$$\frac{\mathrm{Br}[X \to J/\psi\,\pi^+\pi^-\pi^0]}{\mathrm{Br}[X \to J/\psi\,\pi^+\pi^-]} = 1.0 \pm 0.4 \pm 0.3$$

$$J^{PC} = 1^{++}$$

$$M_X - (M_{D^{*0}} + M_{D^0}) < -0.6 \pm 0.6\,\mathrm{MeV}$$

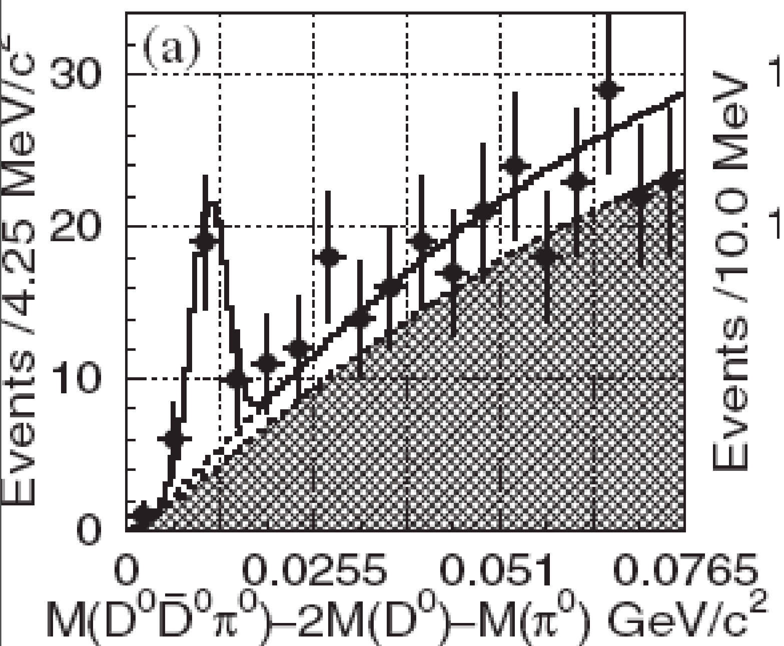
- Extremely shallow S-wave hadronic bound state

$$X = \frac{1}{\sqrt{2}}\left(D^{*0}\bar{D}^0 + D^0\bar{D}^{*0}\right)$$

Charm Meson analog of the Deuteron

- Near Threshold $D^0 \bar{D}^0 \pi^0$ Enhancement in $B \to D^0 \bar{D}^0 \pi^0 K$

(G. Gokhroo,et. al., PRL97:162002 (2006))



$$\frac{\Gamma[X \to D^0 \bar{D}^0 \pi^0]}{\Gamma[X \to J/\psi\, \pi^+ \pi^-]} = 8.8^{+3.1}_{-3.6}$$

large b.r. consistent w/ molecule

$$M_X = 3875.2 \pm 0.7^{+0.3}_{-1.6} \pm 0.8\,\mathrm{MeV}$$

4 MeV (~2σ) > world avg. (3871.2 MeV)

- New Particle? Above threshold DD* resonance?

(Bugg, Hanhart et. al.)

- Isospin violation in $\frac{\Gamma[X \to J/\psi\, \pi^+ \pi^0 \pi^-]}{\Gamma[X \to J/\psi\, \pi^+ \pi^-]}$ consistent with conventional $Q\bar{Q}$ interpretation

(Colangelo, de Fazio, Nicotri)

- Tetraquarks, ...

- Effective Range Theory (ERT) of X(3872)

(Braaten, et. al., Voloshin)

- ERT - suitable for long wavelength probes of systems bound by short-range interactions:

$$p \ll 1/R \qquad R \sim \text{range}$$

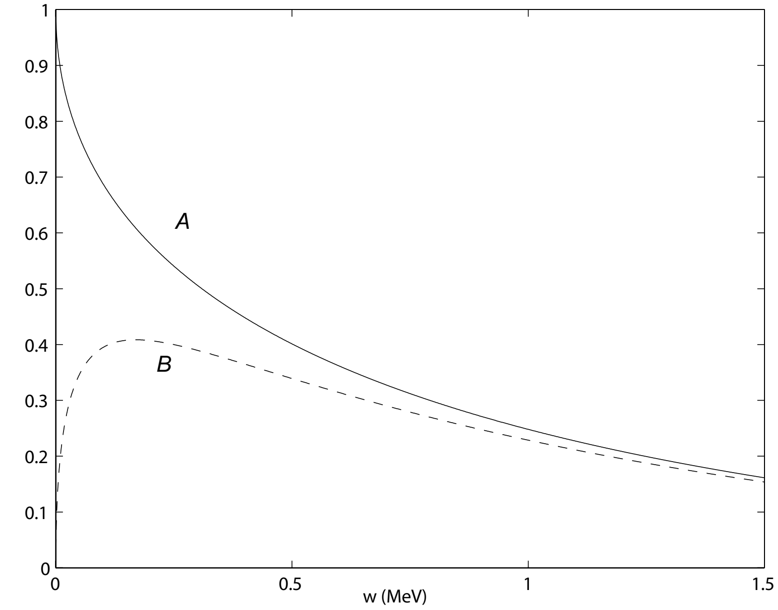
- $\psi_{DD^*}(r) \propto \frac{e^{-r/a}}{r} \qquad a \geq 6.3\,\text{fm} \qquad B.E. = \frac{1}{2\mu_{DD^*} a^2}$

- Long distance physics of X(3872) calculable in terms of scattering length, known properties of D mesons

- Three-body decays : $X(3872) \to D^0 \bar{D}^0 \pi^0$

(M.B.Voloshin, PLB579: 316 (2004))

$$\Gamma[X \to D^0 \bar{D}^0 \pi^0] =$$
$$\Gamma[D^{*0} \to D^0 \pi^0] \times [A(\omega) + B(\omega)]$$

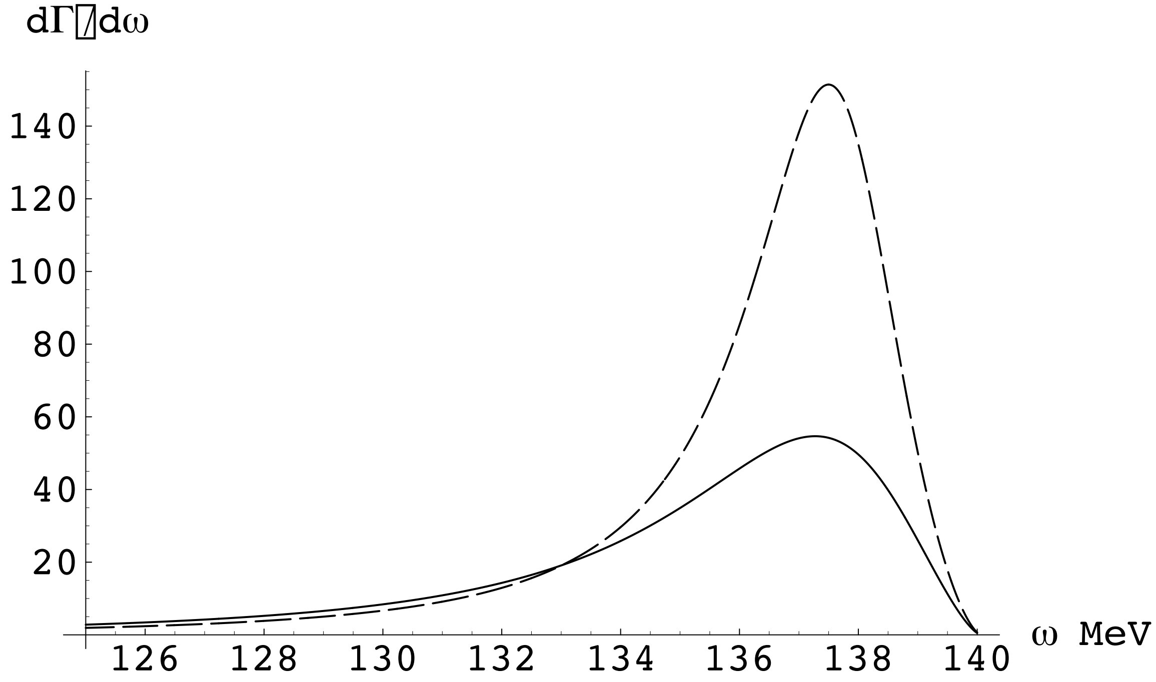


w - binding energy

B(w) - interference term

Three-body decays : $X(3872) \to D^0 \bar{D}^0 \gamma$

(M.B.Voloshin, Int. J. Mod. Phys. A21: 1239 (2006))



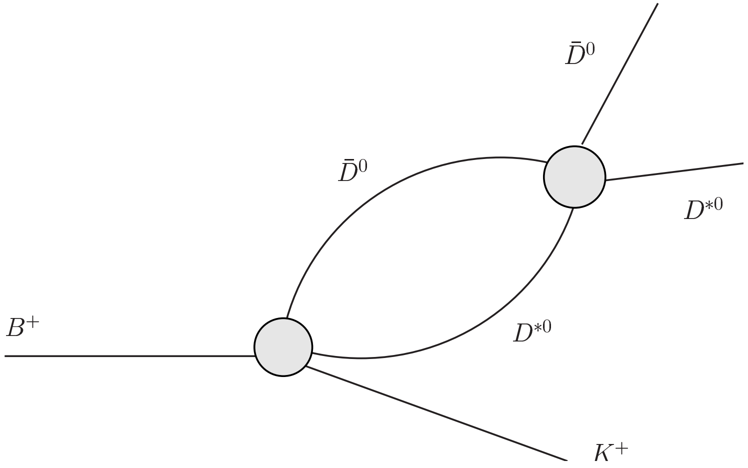
dashed: w = 0.3 MeV

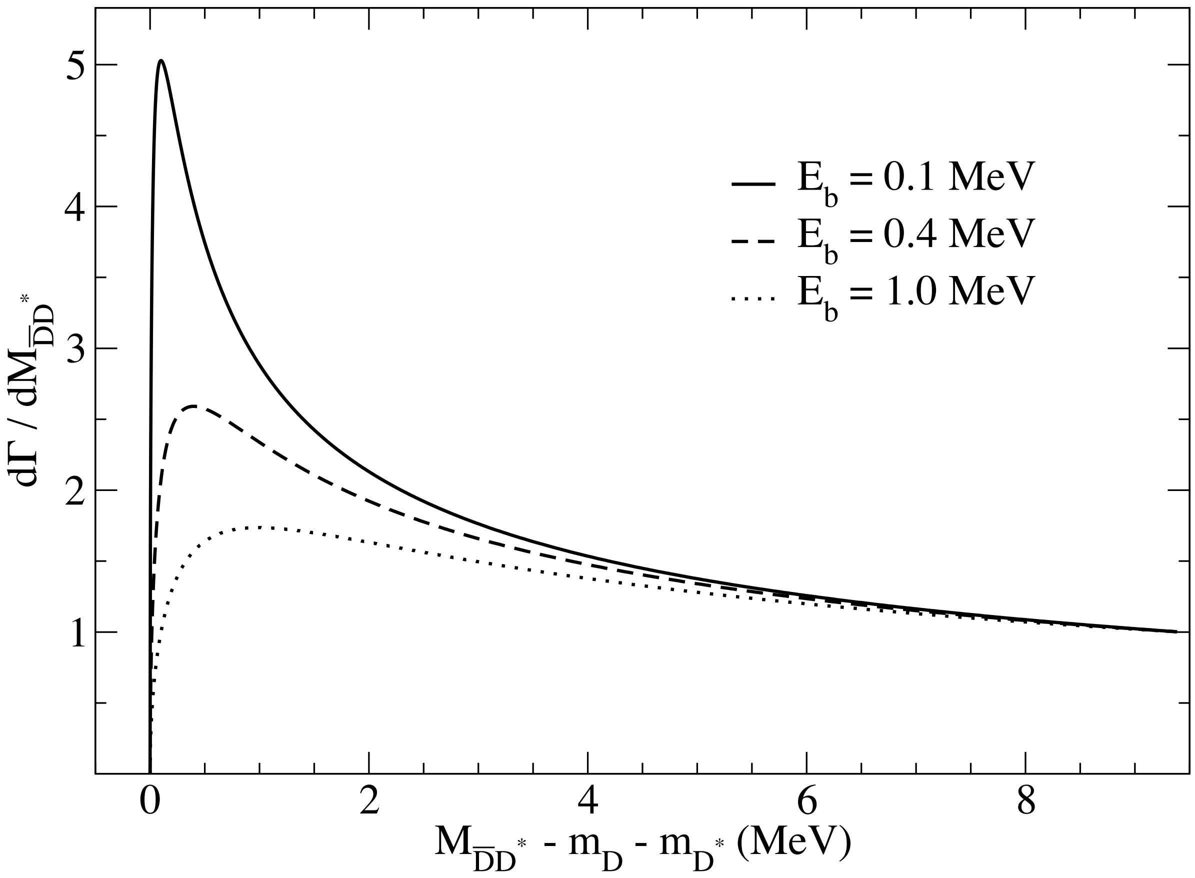
solid: w = 1.0 MeV

- Inputs: binding energy, $\Gamma[D^{*0} \to D^0\gamma]$

Invariant Mass Distribution in $B \to \bar{D}^0 D^{*0} K$

E. Braaten, M. Kusunoki, & S. Nussinov, PRL 93: 162001 (2006)

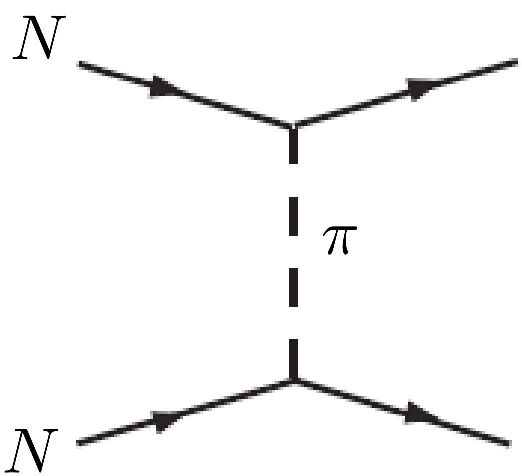




- ERT in Nuclear Physics

works for NN phase shifts, deuteron physics if $p \ll m_\pi$

- For $p \geq m_\pi$ must include explicit pions



$$\frac{g_A^2}{2f^2}\frac{\vec{q}\cdot\sigma_1\,\vec{q}\cdot\sigma_2}{\vec{q}^2+m_\pi^2}$$

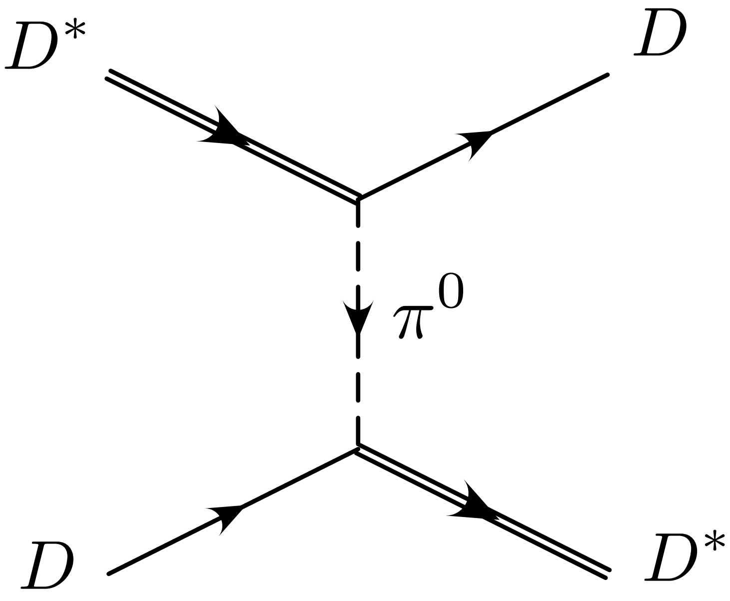
$$\xrightarrow[\text{F.T.}]{} \frac{g_A^2}{8\pi f^2}\left(\sigma_1\cdot\sigma_2-3\,\sigma_1\cdot\hat{r}\,\sigma_2\cdot\hat{r}\right)\frac{e^{-m_\pi r}}{r^3}+\ldots$$

nonperturbative spin-tensor force

What is impact of pion exchange in X(3872)?

- Curious features of π^0 exchange in the X(3872)

$$\Delta \equiv m_{D^*} - m_D \approx 142\,\text{MeV} \qquad m_{\pi^0} \approx 135\,\text{MeV}$$



$$\frac{g^2}{2f^2}\frac{\vec{q}\cdot\epsilon\,\vec{q}\cdot\epsilon^*}{\vec{q}^{\,2}-\Delta^2+m_\pi^2} = \frac{g^2}{2f^2}\frac{\vec{q}\cdot\epsilon\,\vec{q}\cdot\epsilon^*}{\vec{q}^{\,2}-\mu^2}$$

- $\mu^2 \equiv \Delta^2 - m_\pi^2 \approx (44\,\text{MeV})^2$ - new long-distance scale

- wrong sign Yukawa mass!

$$\frac{\vec{q}\cdot\epsilon\,\vec{q}\cdot\epsilon^*}{\vec{q}^{\,2}-\mu^2} \xrightarrow[\text{F.T.}]{} \frac{1}{4\pi}\left(\epsilon\cdot\epsilon^* - 3\,\epsilon\cdot\hat{r}\,\epsilon^*\cdot\hat{r}\right)\left(\frac{\cos\mu r}{r^3} + \frac{\mu\,\sin\mu r}{r^3}\right) + \ldots$$

How this has been dealt with in the past?

- acknowledge cancellation, but fudge sign of μ^2!

  predicts shallow bound state "deuson" (Tornqvist, 1993)

$$\mathrm{B.E.} \sim \frac{\mu^2}{2\mu_{DD^*}} \approx 1\,\mathrm{MeV}$$

- ignore long range force, argue that δ-function can't bind

  (Suzuki, PRD 72, 114013 (2005))

  1st to point out non-Yukawa nature of $D\,D^*$ potential

- use wrong sign AND magnitude μ^2 (~130 MeV)!

  (Swanson, PLB 588, 189 (2004))

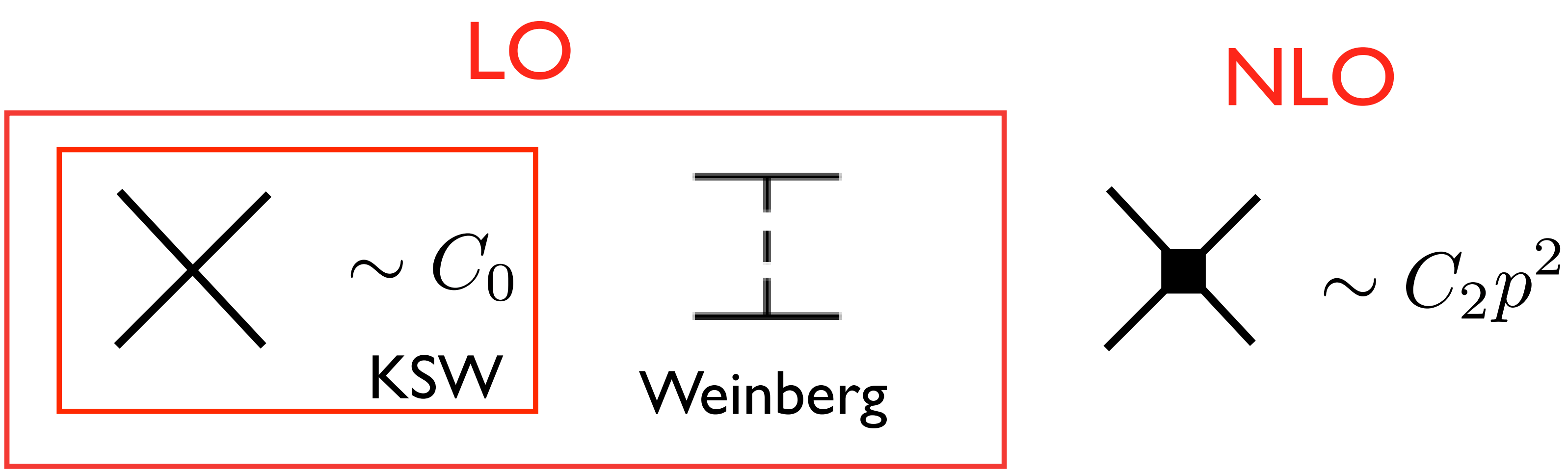
  successful post-diction of X(3872) anyways!

Modern EFT Approach to Nuclear Bound States

- Potentials Singular   require regularization/renormalization

- Shallow bound state result of fine-tuned cancellation between long range part of potential (π exchange) and short-distance part (vector-meson exchange, quark substructure, ...)

- $p \gtrsim m_\pi$ Short-distance physics: contact interactions ("pseudo-potentials")
  Long-distance physics: explicit pions

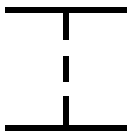
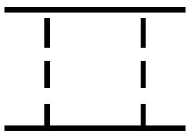


$C_{0,2}$ - tuned to reproduce scattering length, effective range

- $p \ll m_\pi$ Integrate out pions, recover ERT

Perturbative Pions and the X(3872)

Nuclear Physics: NN scattering

 $= \dfrac{g_A^2}{2f^2} A\left(\dfrac{p}{m_\pi}\right),$  $= \left(\dfrac{g_A^2}{2f^2}\right)^2 \dfrac{M m_\pi}{4\pi} B\left(\dfrac{p}{m_\pi}\right)$

Expansion parameter: $\dfrac{g_A^2 M_N m_\pi}{8\pi f^2} \sim \dfrac{1}{2}$

NLO ~30% accuracy, fails at NNLO

S. Fleming, T.M., I. Stewart, NPA 677, 313 (2000)

X(3872): $g_A = 1.25 \rightarrow g \sim 0.5 - 0.7$ $\qquad m_\pi \rightarrow \mu$

$$\frac{g^2 M_D \mu}{8\pi f^2} \sim \frac{1}{20} - \frac{1}{10}$$

Low energy theory of non-relativistic D^0, D^{*0}, and π^0

(Fleming, Kusunoki, T.M., van Kolck, PRD76:034006 (2007))

- ## Non-Relativistic Propagators

D $\qquad D^*$ $\qquad \pi$ $\qquad \sim \dfrac{1}{Q^2}$

- ## Contact interactions, Pion Exchange

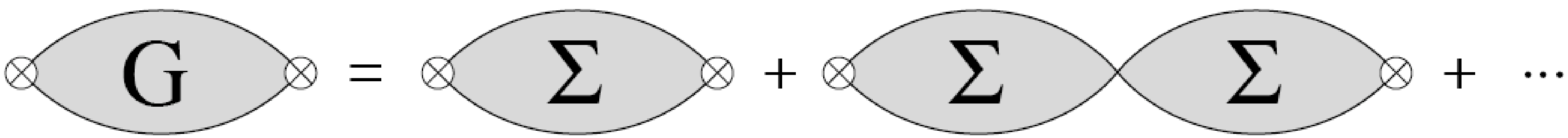
$C_0 \sim Q^{-1}$ $\qquad C_2\, p^2 \sim Q^0$ $\qquad B_1\, \epsilon \cdot p_\pi \sim Q^{-1}$ $\qquad \sim Q^0$

- ## Power Counting

$$p_D \sim p_\pi \sim \mu \sim \gamma \sim Q \qquad \gamma \equiv \sqrt{-2\mu_{DD^*}\mathrm{B.E.}}$$

- Calculation of Pion Corrections to $X(3872) \to D^0 \bar{D}^0 \pi^0$

$$G(E)\delta^{ij} = \int d^4x\, e^{-iEt} \langle 0|T[X^i(x)X^j(0)]|0\rangle = i\,\delta^{ij} \frac{Z(-E_X)}{E+E_X+i\Gamma/2} + \ldots$$



$$G(E) = \frac{-i\,\Sigma(E)}{1+C_0\,\Sigma(E)} = \frac{-i\,\mathrm{Re}\,\Sigma(E) + \mathrm{Im}\,\Sigma(E)}{1+C_0\mathrm{Re}\,\Sigma(E) + i\,C_0\,\mathrm{Im}\,\Sigma(E)}$$

- Expanding about $E = -E_X$

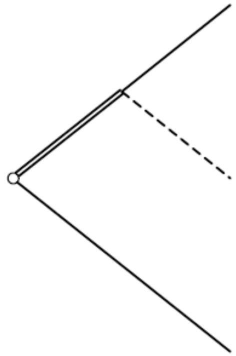
$$Z(E) = \frac{1}{C_0^2\,\mathrm{Re}\,\Sigma'(-E_X)}, \qquad \Gamma = \frac{2\,\mathrm{Im}\,\Sigma(-E_X)}{\mathrm{Re}\,\Sigma'(-E_X)}$$

$2\,\mathrm{Im}\Sigma(-E_X)$ diagrams for $X(3872) \to D^0 \bar{D}^0 \pi^0$

$2\,\mathrm{Re}\Sigma'(-E_X)$ wavefunction renormalization

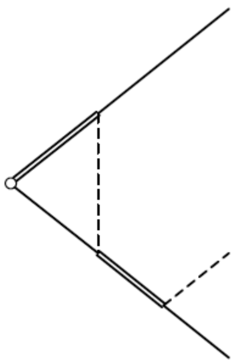
- Line Shape: (see Braaten, Lu, Lee, PRD 76:054010 (2007), Braaten, Lu, hep-ph/0709.2697)

LO - reproduce ERT of X(3872)



$$\frac{d\Gamma_{\rm LO}}{dp_D^2 dp_{\bar{D}}^2} = \frac{g^2}{32\pi^3 f_\pi^2} 2\pi\gamma(\vec{p}_\pi \cdot \vec{\epsilon}_X)^2 \left[\frac{1}{p_D^2+\gamma^2} + \frac{1}{p_{\bar{D}}^2+\gamma^2}\right]^2$$

NLO - range corrections, non-analytic corr. from π^0 exchange



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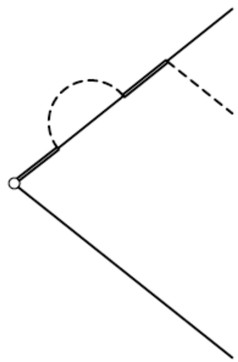
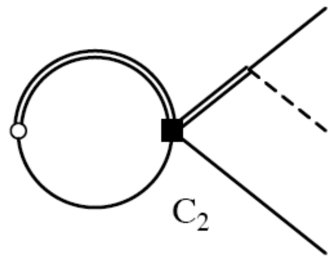


b)



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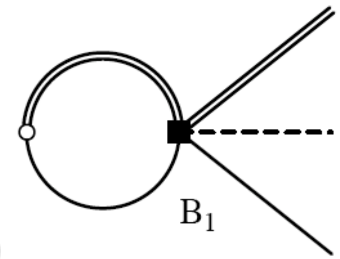
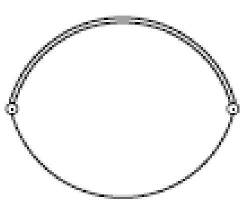
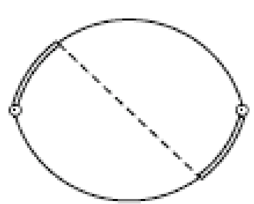


b)

Wavefunction Renormalization





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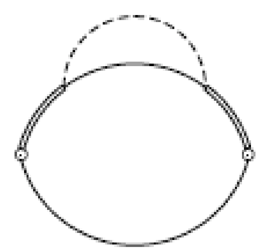

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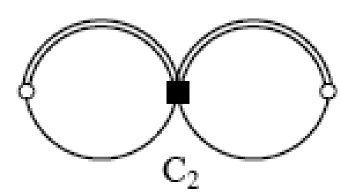


c)

LO

NLO

NLO Corrections

$$\frac{d\Gamma_{\rm NLO}}{dp_D^2 dp_{\bar D}^2} = \frac{d\Gamma_{\rm LO}}{dp_D^2 dp_{\bar D}^2}\left(1+\frac{g^2 M_{DD^*}\gamma}{6\pi f_\pi^2}\left(\frac{4\gamma^2-\mu^2}{4\gamma^2+\mu^2}\right)+C_2(\Lambda_{\rm PDS})\frac{M_{DD^*}\gamma(\gamma-\Lambda_{\rm PDS})^2}{\pi}\right) \qquad (23)$$

$$-\frac{g\gamma}{16\pi^3 f_\pi}\left(\frac{gM_{DD^*}}{f_\pi}C_2(\Lambda_{\rm PDS})+B_1(\Lambda_{\rm PDS})\right)(\Lambda_{\rm PDS}-\gamma)\,(\vec p_\pi\cdot\vec\epsilon_X)^2\left[\frac{1}{p_D^2+\gamma^2}+\frac{1}{p_{\bar D}^2+\gamma^2}\right]$$

$$-\frac{g^4 M_{DD^*}\gamma}{64\pi^3 f_\pi^4}(\vec p_\pi\cdot\vec\epsilon_X)^2\left[\frac{\mathrm{Re}\,h_1(p_D)}{p_D^2+\gamma^2}+\frac{\mathrm{Re}\,h_1(p_{\bar D})}{p_{\bar D}^2+\gamma^2}\right]\left[\frac{1}{p_D^2+\gamma^2}+\frac{1}{p_{\bar D}^2+\gamma^2}\right]$$

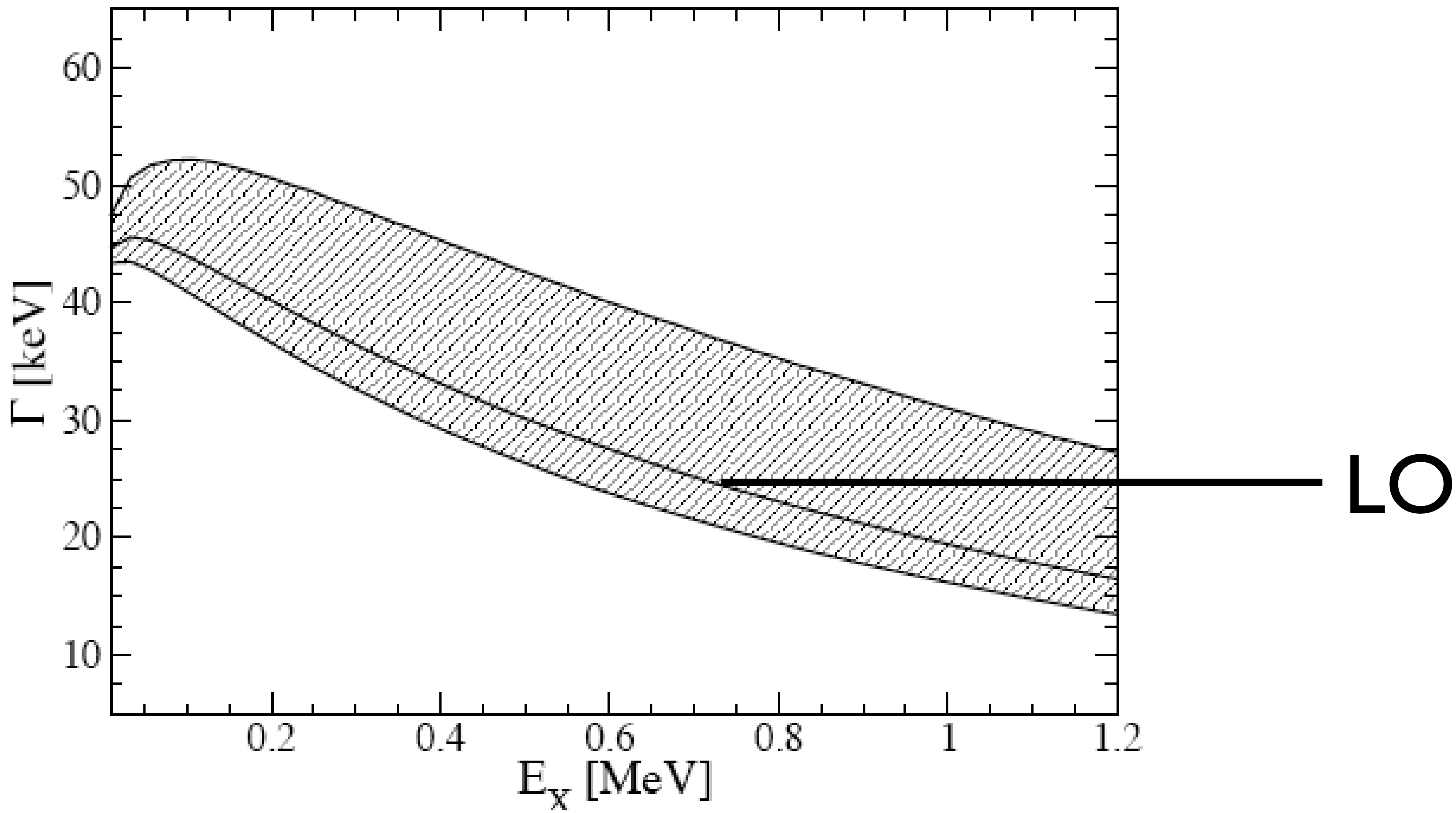
$$+\frac{g^4 M_{DD^*}\gamma}{64\pi^3 f_\pi^4}\left[\frac{\mathrm{Re}\,h_2(p_D)}{p_D^2+\gamma^2}\vec p_\pi\cdot\vec\epsilon_X\,\vec p_D\cdot\vec\epsilon_X\,\vec p_\pi\cdot\vec p_D+(p_D\to p_{\bar D})\right]\left[\frac{1}{p_D^2+\gamma^2}+\frac{1}{p_{\bar D}^2+\gamma^2}\right].$$

$$h_1(p)=\int_0^1 dx\sqrt{-p^2x^2+(p^2+\gamma^2+\mu^2)x-\mu^2-i\epsilon} \qquad h_2(p)=\int_0^1 dx\frac{x^2}{\sqrt{-p^2x^2+(p^2+\gamma^2+\mu^2)x-\mu^2-i\epsilon}}$$

Effective Range Corrections

$$C_2(\Lambda_{\rm PDS})=\frac{2\pi}{M_{DD^*}}\frac{r_0}{2}\frac{1}{(\Lambda_{\rm PDS}-\gamma)^2} \qquad \psi^{\rm ER}(r)=\sqrt{\frac{\gamma}{4\pi(1-\gamma r_0)}}\frac{e^{-\gamma r}}{r}$$

- $X(3872) \to D^0 \bar{D}^0 \pi^0$ at NNLO



$g = 0.6 \qquad 0 \leq r_0 \leq (100\,\text{MeV})^{-1} \qquad -1 \leq \eta \leq 1$

$$\left(\frac{g M_{DD^*}}{f_\pi} C_2(\Lambda_{\text{PDS}}) + B_1(\Lambda_{\text{PDS}})\right)(\Lambda_{\text{PDS}} - \gamma) = \frac{\eta}{(100\,\text{MeV})^3}$$

- Corrections Dominated by Counterterms

Summary

- One Pion exchange introduces new scale

$$\mu^2 = \Delta^2 - m_\pi^2$$

- Novel, non-Yukawa-like force between D and D*

- Can use KSW-like theory to analyze X(3872)

  resum contact interaction at LO, recover ERT

  pion exchange, higher dimension operators treated in p.t.

- Non analytic corrections due to pion exchange in

  $\Gamma[X \to D^0 \bar{D}^0 \pi^0]$ negligible

Future Work

- Other decays to analyze:

$$\frac{d\Gamma[X \to D^0 \bar{D}^0 \gamma]}{dE_\gamma}$$

(Voloshin; Colangelo, de Fazio, Nicotri)

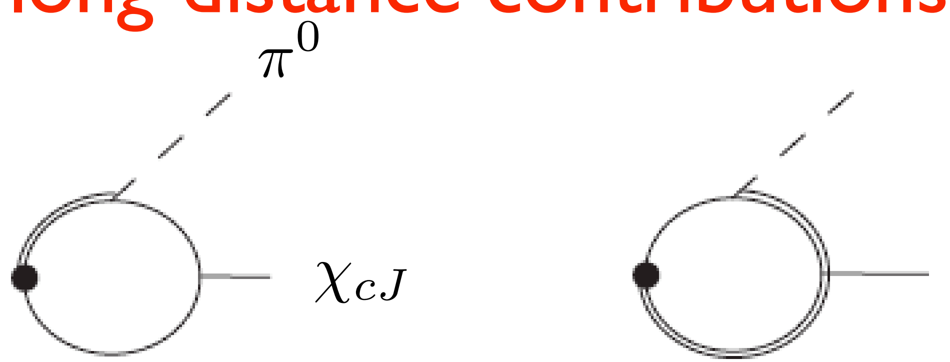
$$\Gamma[X \to \chi_{cJ}\,\pi], \Gamma[X \to \chi_{cJ}\,\pi\pi]$$

(Dubynskiy & Voloshin; hep-ph/0709.4474)

relative rates should distinguish conventional, molecule interpretations

Can calculate long-distance contributions in EFT!

X(3872)



need relativistic treatment of pions, couplings of D, D* to chi_cJ

- Does the scale μ play a role in the binding of the X(3872)?

$$\Delta^2 \propto \frac{1}{m_c^2} \qquad\qquad m_\pi^2 \propto m_q$$

Could lattice simulation of X(3872) reveal dependence of B.E. on these parameters? This could shed light on nature of X(3872)!

Extra Slides

- Low energy theory of non-relativistic D, D^*, and π^0

(Fleming, Kusunoki, T.M., van Kolck, PRD76:034006 (2007))

obtain from $\mathrm{HH\chi PT}$ integrating out m_π

$$
\begin{aligned}
\mathcal{L} \;=\; & \boldsymbol{D}^\dagger\left(i\partial_0+\frac{\vec\nabla^2}{2m_{D^*}}\right)\boldsymbol{D}+D^\dagger\left(i\partial_0+\frac{\vec\nabla^2}{2m_D}\right)D \\
& +\bar{\boldsymbol{D}}^\dagger\left(i\partial_0+\frac{\vec\nabla^2}{2m_{D^*}}\right)\bar{\boldsymbol{D}}+\bar{D}^\dagger\left(i\partial_0+\frac{\vec\nabla^2}{2m_D}\right)\bar{D}+\pi^\dagger\left(i\partial_0+\frac{\vec\nabla^2}{2m_\pi}+\delta\right)\pi \\
& +\left(\frac{g}{\sqrt{2}f_\pi}\right)\frac{1}{\sqrt{2m_\pi}}\left(D\boldsymbol{D}^\dagger\cdot\vec\nabla\pi+\bar{D}^\dagger\bar{\boldsymbol{D}}\cdot\vec\nabla\pi^\dagger\right)+\mathrm{h.c.} \\
& -\frac{C_0}{2}\left(\bar{\boldsymbol{D}}D+\boldsymbol{D}\bar{D}\right)^\dagger\cdot\left(\bar{\boldsymbol{D}}D+\boldsymbol{D}\bar{D}\right) \\
& -\frac{C_2}{16}\left(\bar{\boldsymbol{D}}D+\boldsymbol{D}\bar{D}\right)^\dagger\cdot\left(\bar{\boldsymbol{D}}(\overleftrightarrow{\nabla})^2D+\boldsymbol{D}(\overleftrightarrow{\nabla})^2\bar{D}\right)+h.c. \\
& +\frac{B_1}{2}\left(\bar{\boldsymbol{D}}D+\boldsymbol{D}\bar{D}\right)^\dagger\cdot D\bar{D}\vec\nabla\pi+h.c. \\
& +\cdots,
\end{aligned}
$$

non-relativistic D^0, D^{*0}, π^0; π^0 exchange, contact interaction

Power Counting $\quad p_D \sim p_\pi \sim \mu \sim \gamma \sim Q$

$$\gamma \equiv \sqrt{-M_D\, B.E.}$$