Matching Coefficients in NRQCD and HQET

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Nucl.Phys.B758:144-160,2006
arXiv:0707.1388; to appear in Nucl. Phys, B

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QWG DESY 2007

Outline

Introduction

- matching coefficients: Connection between full theory (QCD) and an effective theory.

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NRQCD

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- top production at threshold at a future linear collider $\Rightarrow$ determination of top quark mass $\longrightarrow$ talk by K. Schuller
- decay of heavy mesons

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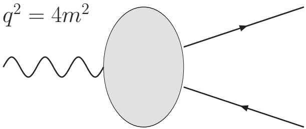
HQET

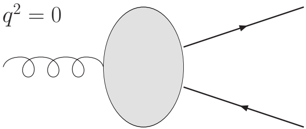
- describes a single heavy quark with small residual momentum
- breaking of the spin symmetry $\Rightarrow B - B^*$ mass splitting cmp talk by R. Sommer

Calculation

NRQCD



HQET



- vector current

$$j_v^k = c_v(\mu)\tilde{j}^k + \frac{d_v(\mu)}{6m_Q^2}\phi^\dagger \sigma^k \vec{D}^2 \chi + \ldots$$

- consider diagrams with at least one closed quark loop

- chromomagnetic operator

$$O_{cm} = \frac{1}{2}\bar{Q}_v G_{\mu\nu}\sigma^{\mu\nu} Q_v$$

- reduces to propagator type diagrams

What's known?

NRQCD

- one-loop MC — Källen et al '55
- two-loop MC

Czarnecki et al '97; Beneke et al '97

Kniehl et al '06

What's known?

NRQCD

- one-loop MC — Källen et al '55
- two-loop MC — Czarnecki et al '97; Beneke et al '97

Kniehl et al '06

HQET

- one-loop MC — Eichten, Hill '90
- one-loop AD — Falk, Grinstein, Luke '91
- two-loop MC — Czarnecki, Grozin '97
- two-loop AD — Amorós et al '97; Czarnecki et al '97
- large-β_0 limit — Grozin, Neubert '97
- m_c effects at two-loops — Davydychev, Grozin '99

General Approach

- diagrams generated with qgraf — Nogueira '91
- topologies identified and mapped with the help of q2e and exp — Harlander, Seidensticker, Steinhauser '96/'97
- calculation done with form — Vermaseren '00
- reduction to master integrals using IBP identities with crusher — PM, Seidel
- master integrals partially known — Melnikov, v. Rittbergen '00; Laporta, Remiddi '96
- master integrals calculated/checked using Mellin-Barnes representations (MB.m) — Czakon '05

Details: NRQCD

Consider renormalized quark-antiquark-photon vertex at threshold

$$Z_2 \Gamma_v = c_v \tilde{Z}_2 \tilde{Z}_v^{-1} \tilde{\Gamma}_v + \dots$$

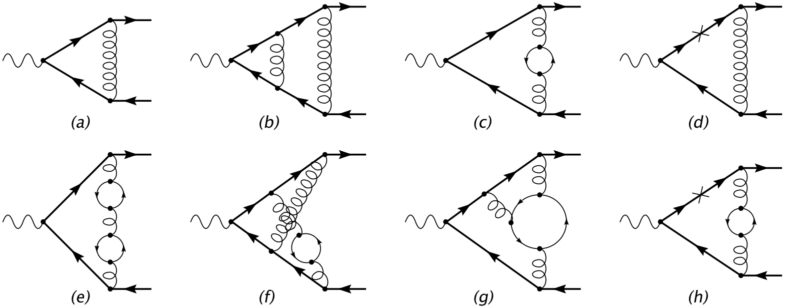
we want to extract c_v and $\tilde{Z}_v$ $\rightarrow$ we need:

- wave function renormalization constant Z_2

  Broadhurst et al '91; Melnikov, v. Rittbergen '00

  $\hookrightarrow$ we recalculated the necessary parts of Z_2
- $\tilde{\Gamma}_v$ only needed at tree level
- $\tilde{Z}_2 = 1$

Details: NRQCD: Graphs



Details: HQET

on QCD side calculate quark-antiquark-gluon vertex with onshell quarks and zero momentum transfer

- decompose into form factors

$$\Gamma^\mu = \gamma^\mu F_1(q^2) - \frac{i}{2m_Q}\sigma^{\mu\nu} q_\nu F_2(q^2)$$

- use projectors to obtain form factors
- colour charge

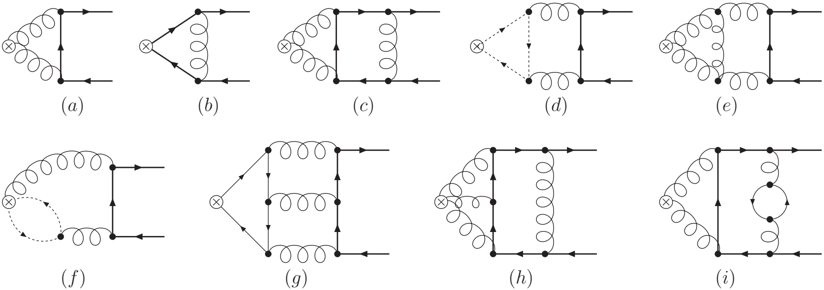
$$Z_2^{OS} F_1(0) = 1 \quad \Rightarrow \quad F_1(0)^{-1} = Z_2^{OS}$$

Melnikov, v. Rittbergen '00; PM, Mihaila, Piclum, Steinhauser '07

- chromomagnetic moment

$$\gamma_{cm} = Z_2^{OS} F_2(0) \quad \Rightarrow \quad C_{cm}(\mu) = Z_{cm}(\alpha_s(\mu))[1+\mu_{cm}(\alpha_s(\mu))]$$

Details: HQET: Graphs



Results for NRQCD

- bottom system
- top system

Bottom System

Decay rate of $\Upsilon(1S)$ into leptons

$$\Gamma(\Upsilon(1S) \to l^+ l^-) = \Gamma_{\mathrm{LO}} \rho_1 \left[c_v^2(m_b) + \frac{C_F^2 \alpha_s^2(\mu_s)}{12} c_v(m_b)(d_V(m_b) + 3) \right]$$

with $\alpha_s(M_Z) = 0.118$, $m_b = 5.3$ GeV and $\mu_s = 2.0967$ GeV

$$\Gamma_1 \approx \Gamma_1^{LO}(1 - 0.446_{NLO} + 1.75_{NNLO} - 1.67_{NNNLO})$$

changes to

$$\Gamma_1 \approx \Gamma_1^{LO}(1 - 0.446_{NLO} + 1.75_{NNLO} - 1.20_{NNNLO})$$

convergence improved

Top System

normalized $t\bar{t}$ peak cross section at a linear collider

$$R(e^+e^- \to t\bar{t}) = \frac{\sigma(e^+e^- \to t\bar{t})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

$$= R_{\mathrm{LO}}\rho_1 \left[c_v^2(m_b) + \frac{C_F^2 \alpha_s^2(\mu_s)}{12} c_v(m_b)(d_V(m_b) + 3) \right]$$

with $\alpha_s(M_Z) = 0.118$, $m_t = 175$ GeV and $\mu_s = 32.625$ GeV

$$R_1 \approx \Gamma_1^{LO}(1 - 0.243_{NLO} + 0.435_{NNLO} - 0.268_{NNNLO})$$

changes to

$$R_1 \approx \Gamma_1^{LO}(1 - 0.243_{NLO} + 0.435_{NNLO} - 0.195_{NNNLO})$$

Sum of all corrections ~ 0.003

Results for HQET

- mass splitting
- magnetic moments

Results

matching coefficient

$$
\begin{aligned}
C_{cm}(m_c) &= 1 + 0.2309 + 0.1835 + 0.2362 = 1.6506 \\
C_{cm}(m_b) &= 1 + 0.1492 + 0.0676 + 0.0497 = 1.2664 \\
C_{cm}(m_t) &= 1 + 0.0741 + 0.0144 + 0.0045 = 1.0930
\end{aligned}
$$

anomalous dimension

$$
\begin{aligned}
\gamma_{cm}(m_c) &= 0.1599 + 0.0298 + 0.0163 = 0.2060 \\
\gamma_{cm}(m_b) &= 0.1033 + 0.0099 + 0.0029 = 0.1160 \\
\gamma_{cm}(m_t) &= 0.0513 + 0.0018 + 0.0002 = 0.0533
\end{aligned}
$$

Heavy Quark Mass Splittings: Lattice

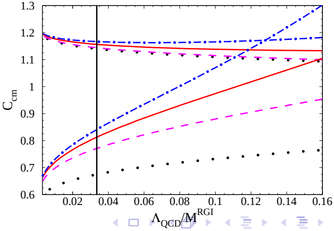
mass splitting of heavy mesons

$$m_{B^*}^2 - m_B^2 = \frac{4}{3} C_{cm}^{(4)}(\mu)\mu_{G(4)}^2(\mu) + \mathcal{O}\left(\frac{\Lambda_{QCD}}{m_b}\right)$$

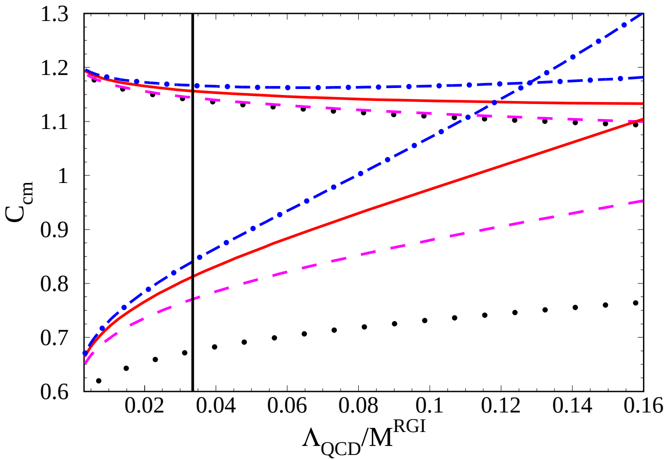
- transform matching coefficient to RGI scheme and rewrite it in terms of Λ_{QCD}/M^{RGI}

$$M^{RGI} = \bar{m}_* \left(2\beta_0 \frac{\alpha_s(\bar{m}_*)}{\pi}\right)^{-\frac{\gamma_{m,0}}{\beta_0}} \exp\left[-\int_0^{\alpha_s(\bar{m}_*)} \left(\frac{\gamma_m}{\beta} - \frac{\gamma_{m,0}}{\beta_0}\right)\frac{d\alpha_s}{\alpha_s}\right]$$

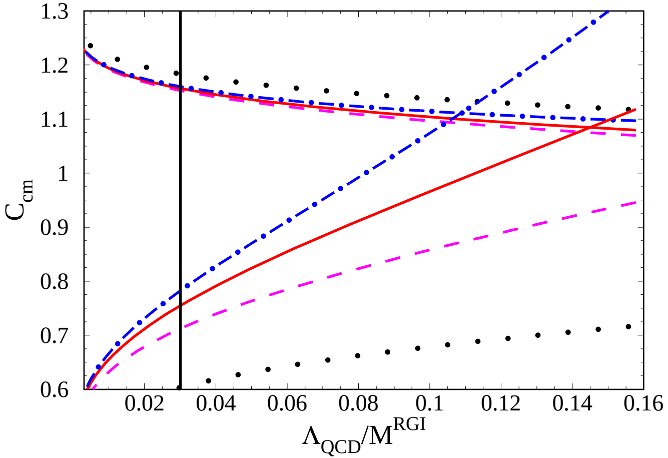
- improved convergence of the matching coefficient
  $\hookrightarrow$ error ≈ 1 % Grozin, Guazzini, PM, Meyer, Piclum, Sommer, Steinhauser '07



Heavy Quark Mass Splittings: Lattice: N_f=0



Heavy Quark Mass Splittings: Lattice: N_f=4



Heavy Quark Mass Splittings: Perturbative

- matrix element perturbatively not accessible
  $\hookrightarrow$ consider ratio
- matrix elements related by renormalization and decoupling relations

Grozin '00

$$R = \frac{m_{B^*}^2 - m_B^2}{m_{D^*}^2 - m_D^2} = 0.88 \quad (\text{exp.})$$

resummed logs

$$R = 0.8517 - 0.0696 - 0.0908 + [-0.1285] = 0.6914 + [-0.1285]$$

fixed order

$$R = 1 - 0.1113 - 0.0780 - 0.0755 + \ldots = 0.7352 + \ldots$$

Magnetic moments

replace external gluon by photon

$$\frac{a_Q}{Q_Q} = 0.2122\,\alpha_s^{(n_f)}(m_Q) + (0.8417 - 0.0469\,n_l)\left[\alpha_s^{(n_f)}(m_Q)\right]^2 + \left(4.5763 - 0.5856\,n_l + 0.0145\,n_l^2\right)\left[\alpha_s^{(n_f)}(m_Q)\right]^3 + \mathcal{O}(\alpha_s^4)$$

$$\begin{aligned}
a_c(m_c) &= 0.0478 + 0.0533 + 0.0758 = 0.1770 \\
a_b(m_b) &= -0.0153 - 0.0103 - 0.0084 = -0.0340 \\
a_t(m_t) &= 0.0152 + 0.0047 + 0.0017 = 0.0215
\end{aligned}$$

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$$a_b(m_Z) = -0.0083 - 0.0066 - 0.0056 = -0.0206$$

exp: $a_b > -0.135$ (68% CL) Escribano, Massó '94

Magnetic moments

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- no measurement of the magnetic moment of the top quark
- sensitive to new physics

Conclusion

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  - generally improves convergence
  - still incomplete, full calculation in progress

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- NRQCD
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- HQET
  - chromomagnetic moment calculated
  - important input for lattice calculations
  - convergence improved in RGI scheme
  - pert. results for mass splittings not in good agreement with experimental value
  - byproduct: magnetic moments