

# QCD corrections to $J/\psi$ and $\Upsilon$ production at hadron colliders

Fabio Maltoni

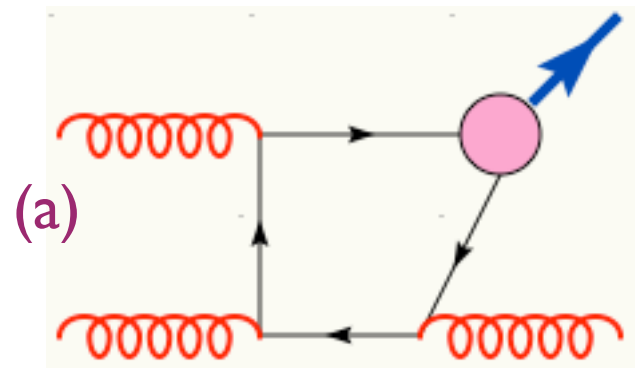
Center for Particle Physics and Phenomenology  
Université Catholique de Louvain

J. Campbell, F. M., F. Tramontano, PRL98:252002,2007  
P. Artoisenet, J. Campbell, J.-P. Lansberg, F. M., F. Tramontano, in progress

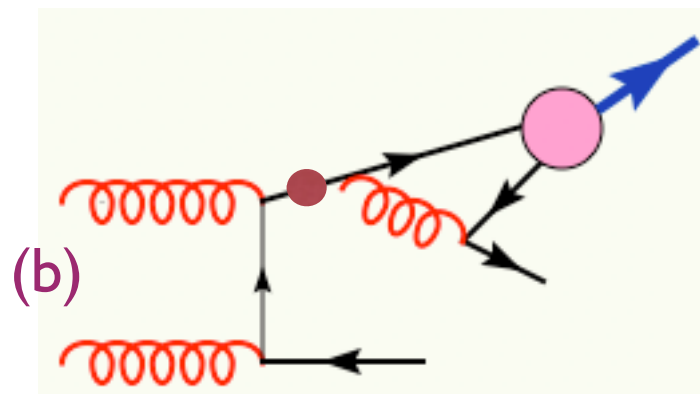
# Outline

- Motivations
- Calculation
- Phenomenology
- Conclusions & Outlook

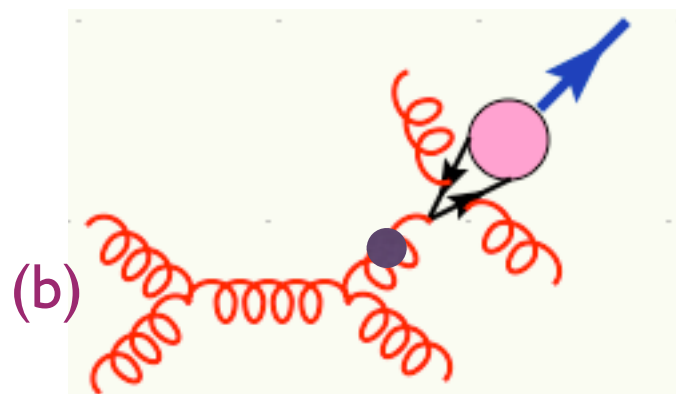
# J/ψ at the Tevatron



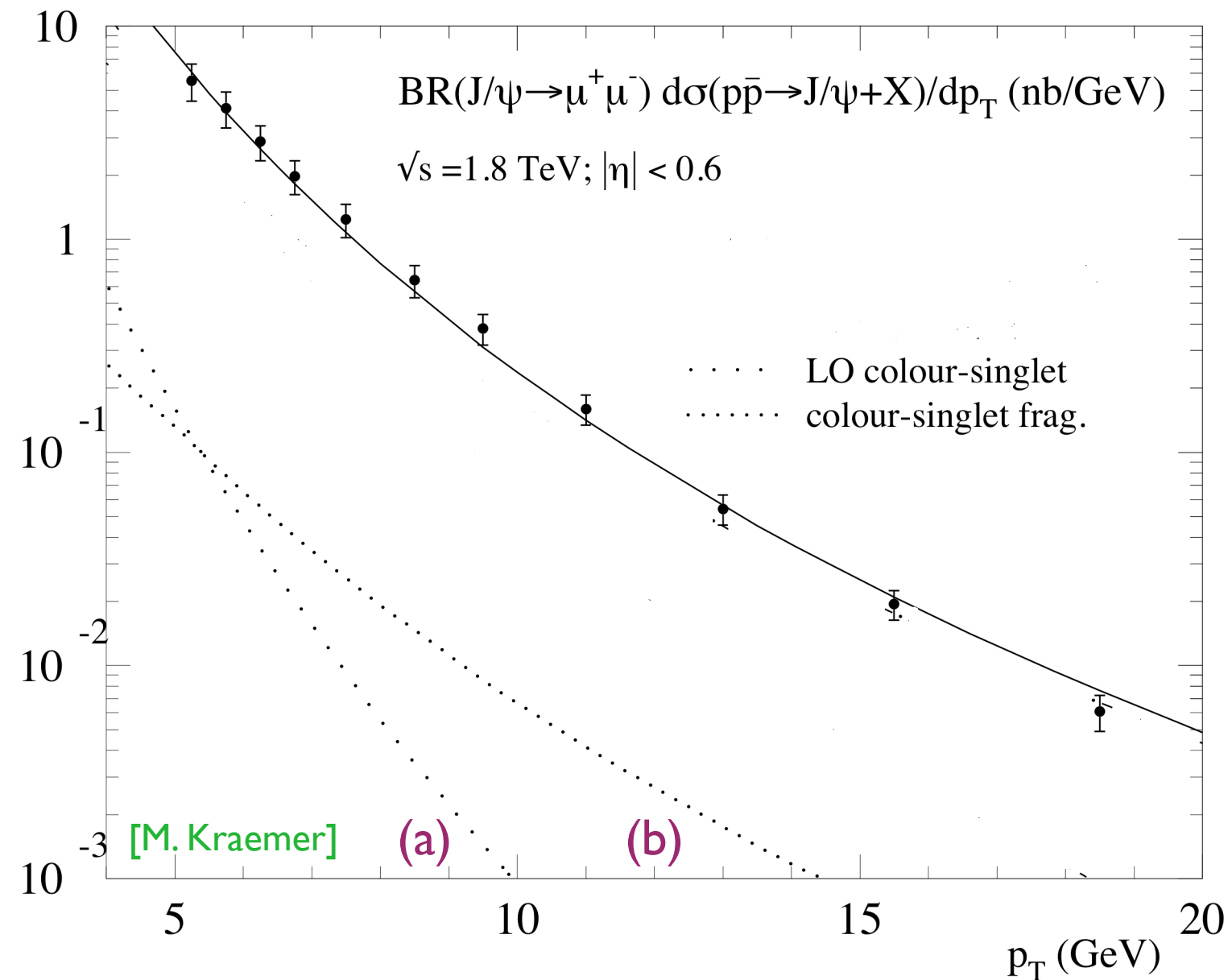
$$\alpha_S^3 \frac{(2m_c)^4}{p_T^8}$$



$$\alpha_S^4 \frac{1}{p_T^4}$$



$$\alpha_S^5 \frac{1}{p_T^4}$$



The overall shape (a)+(b) is ok, but the normalization is off by a large factor!  
Some higher order contributions included through the fragmentation approximation.

# The NRQCD revolution

In 1995, Braaten, Bodwin and Lepage proposed NRQCD, where a quarkonium state can be written as an expansion in powers of  $v$  in a Fock space.

For a  $J/\psi$

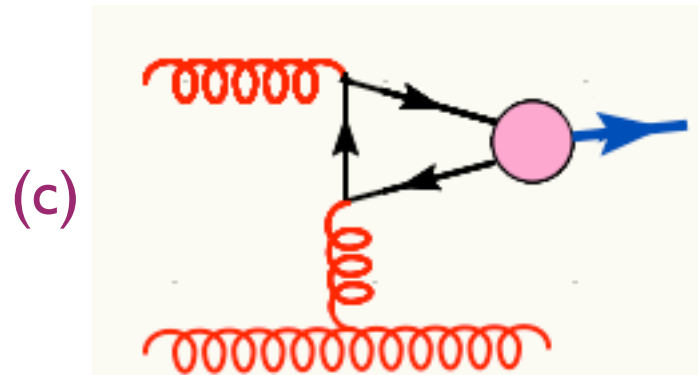
$$\begin{aligned}
 |J/\psi\rangle &= O(1)|Q\bar{Q}(^3S_1^{[1]})\rangle \\
 &+ O(v)|Q\bar{Q}(^3P_J^{[8]})g\rangle \\
 &+ O(v^2)|Q\bar{Q}(^1S_0^{[8]})g\rangle \\
 &+ O(v^2)|Q\bar{Q}(^3S_1^{[8]})gg\rangle + O(v^4)
 \end{aligned}$$

while for  $\chi_J$

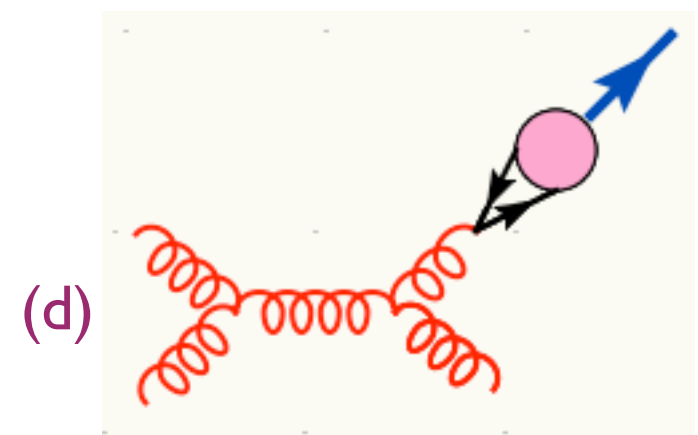
$$|\chi_J\rangle = O(1)|Q\bar{Q}(^3P_J^{(1)})\rangle + O(v)|Q\bar{Q}(^3S_1^{(8)})g\rangle + O(v^2)$$

The final scaling of a process is obtained through a power counting in  $v$  (long distance) and  $\alpha_s$  (short distance).

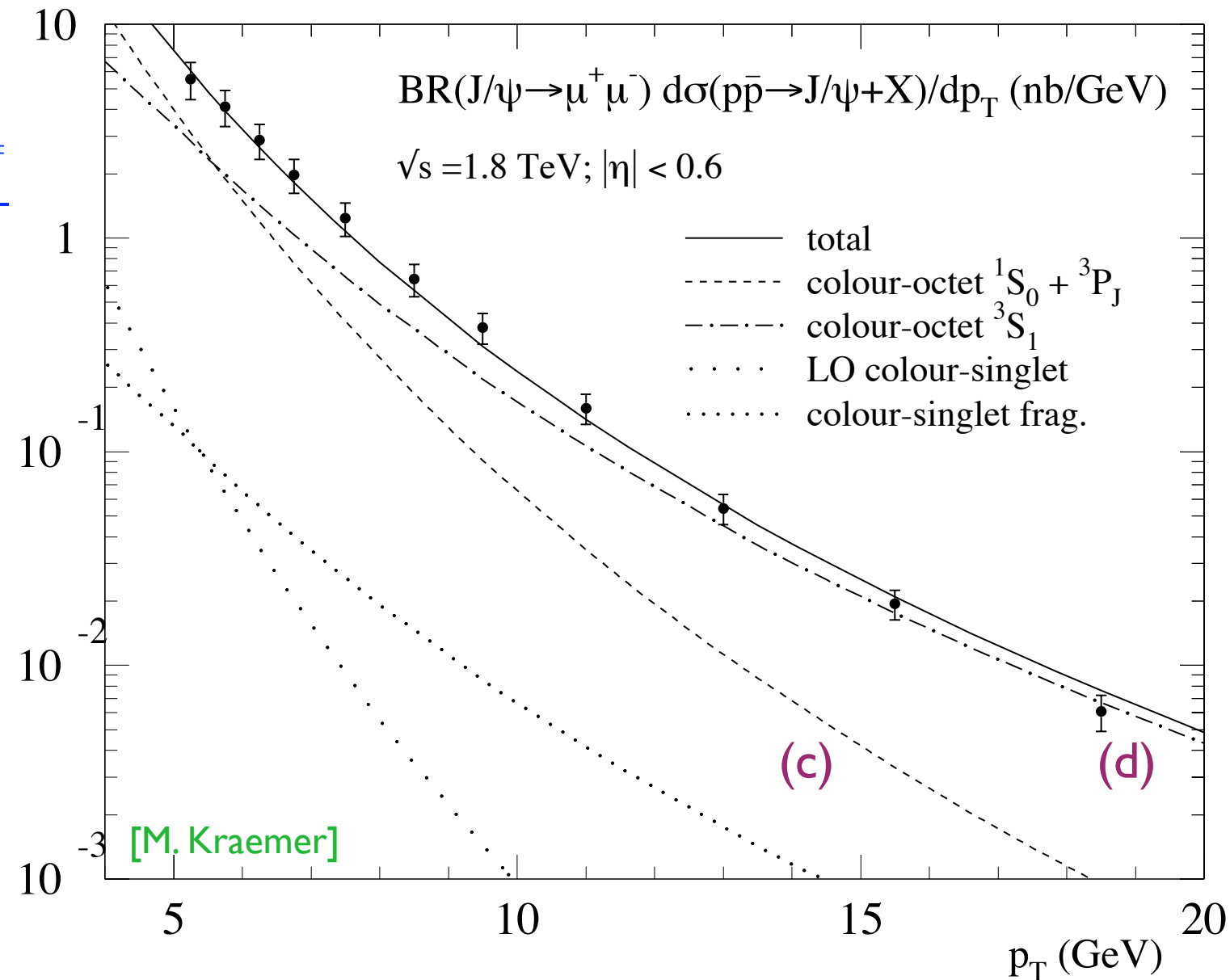
# J/ψ at the Tevatron



$$\alpha_S^3 \frac{(2m_c)^2 v^4}{p_T^6}$$



$$\alpha_S^3 \frac{(2m_c)^2 v^4}{p_T^4}$$



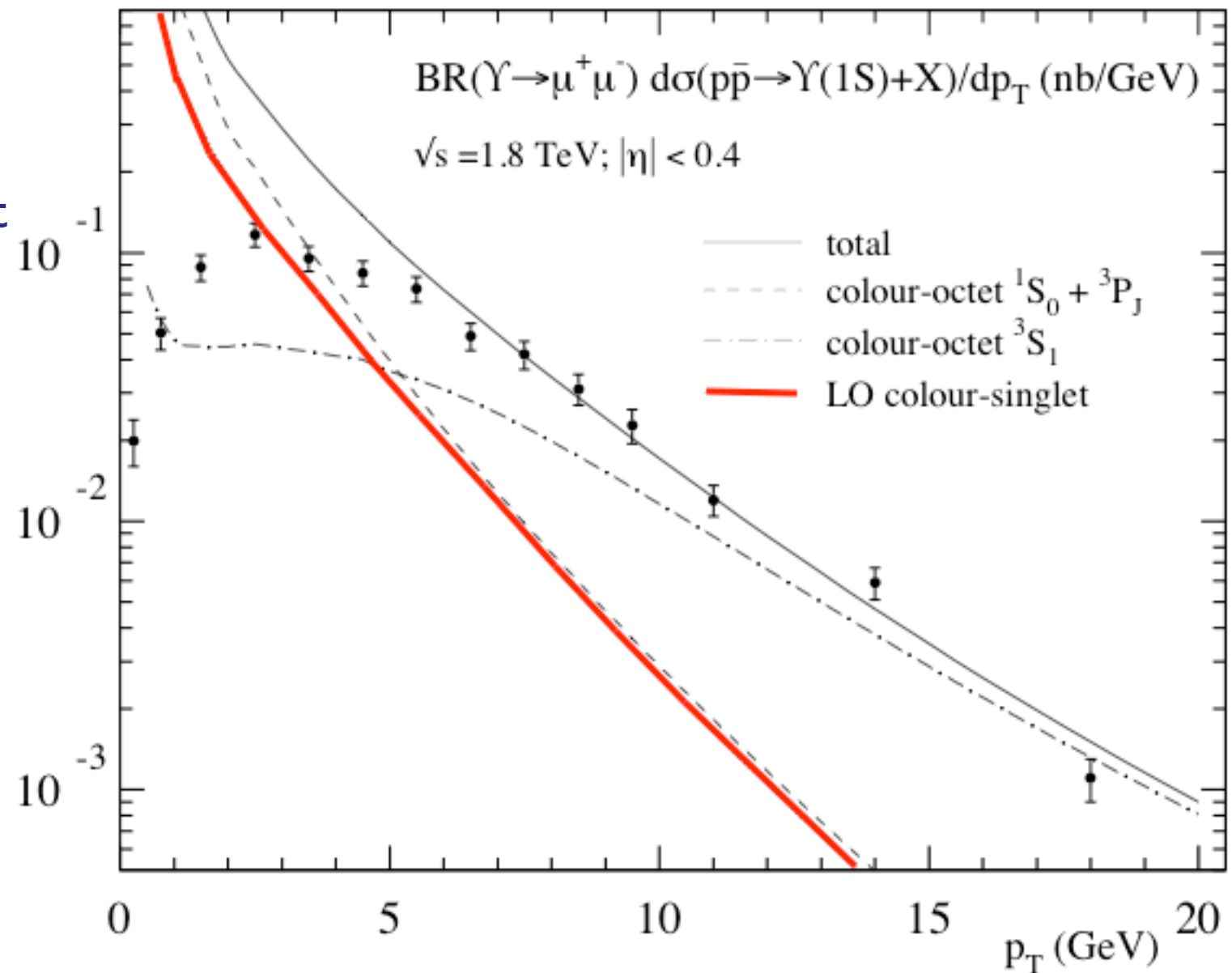
(c) & (d) are fitted to the data, but the resulting NP MEs are of the order predicted by the scaling rules of NRQCD.

# $\Upsilon$ at the Tevatron

Situation is similar to the charmonium, but much less dramatic.

This is expected from the fact that  $v^2 \approx 0.1$  and therefore color-octet contributions are more suppressed.

Actually, the total rate here is not too far from the LO singlet.

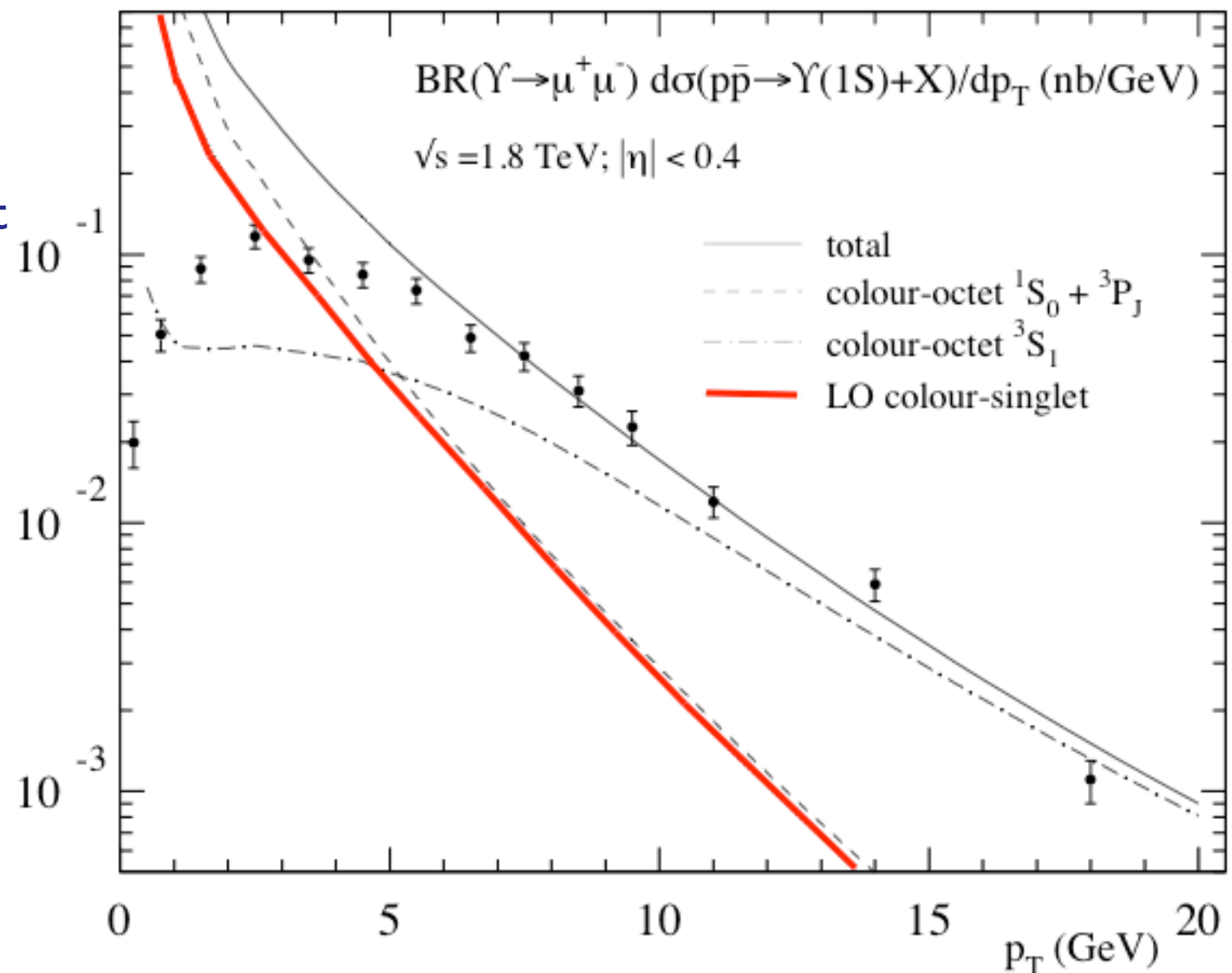


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## Are we done?



## Open questions

- Universality of the NP matrix elements is not well established.
- Octets are important in pp at high-pt , probably in  $\gamma\gamma$ , but not in  $e^+e^-$ ,  $\gamma p$  or fixed target experiments.
- The J/psi is unpolarized in pp at high-pt, contrary to expectations.



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- The J/psi is unpolarized in pp at high-pt, contrary to expectations.

A critical (re)-analysis  
of our TH predictions is needed!

# Directions for TH improvements in quarkonium production

## 1. Improve predictions systematically:

- Calculate higher order corrections in  $v$  and  $\alpha_s$ .
- Build better MC tools

### Example Talks:

FM, J.-P. Lansberg,  
A. Leibovich, Y-J Zhang  
G. Bodwin

P.Artoisenet, A. Kraan

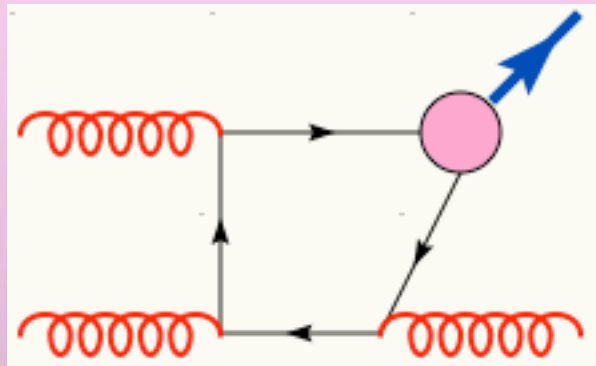
## 2. Propose and test alternative approaches

S. Baranov, V. Saleev,  
V. Braguta

## 3. Propose new and/or more exclusive measurements.

TH/EXP Round Tables

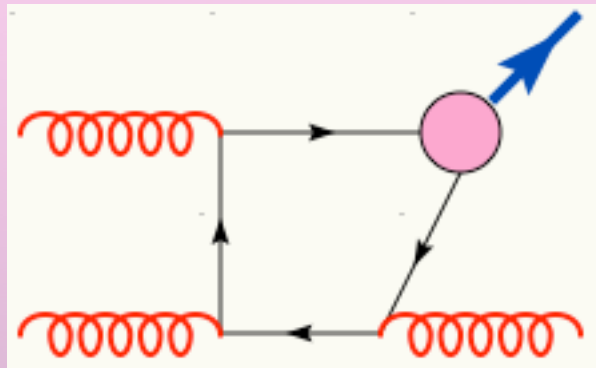
# Improving on $pp \rightarrow {}^3S_1[1] + X$



$$\alpha_S^3 \frac{(2m_c)^4}{p_T^8}$$

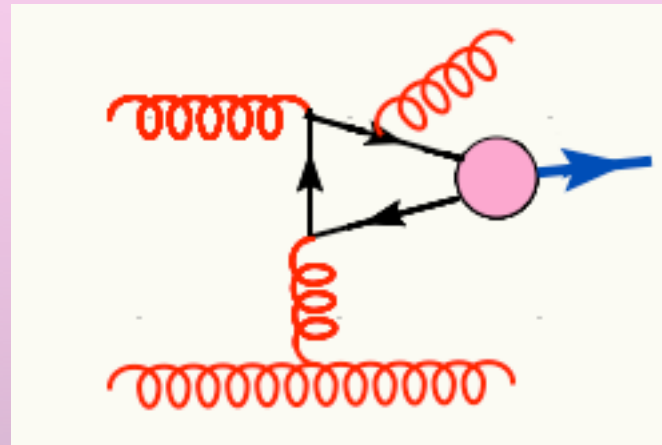
LO

# Improving on $pp \rightarrow {}^3S_1[1] + X$

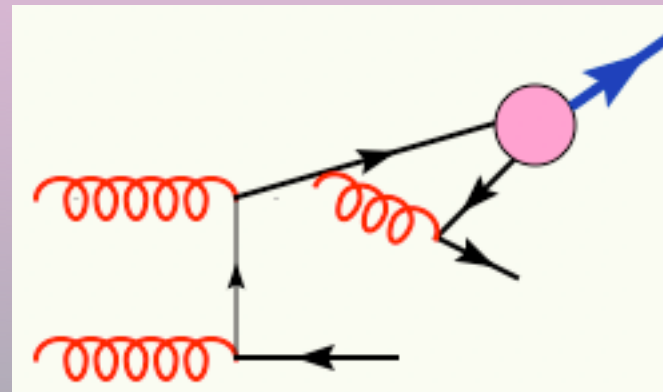


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LO



$$\alpha_S^4 \frac{(2m_c)^2}{p_T^6}$$

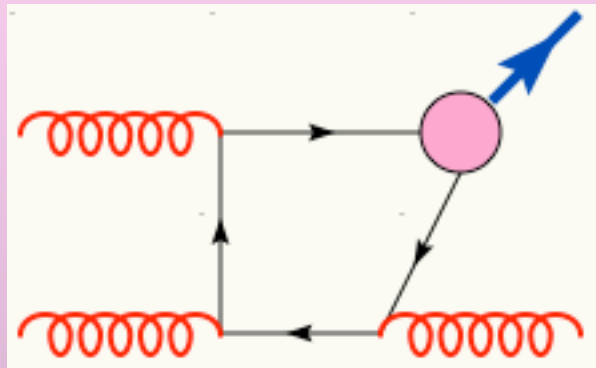


$$\alpha_S^4 \frac{1}{p_T^4}$$

NLO

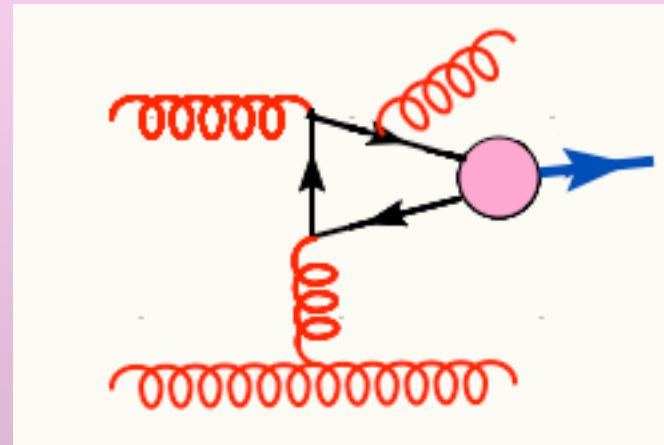
- \* At NLO both  $p_T^{-6}$  and  $p_T^{-4}$  scalings appear, which compete with the octet contributions:  $\alpha_s$  vs  $v^4$
- \* Fragmentation approach might just be a bad approximation.
- \* Need for exact matrix element calculations.

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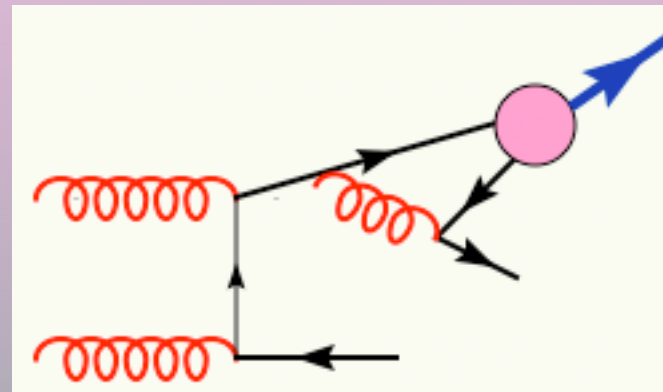


$$\alpha_S^3 \frac{(2m_c)^4}{p_T^8}$$

LO



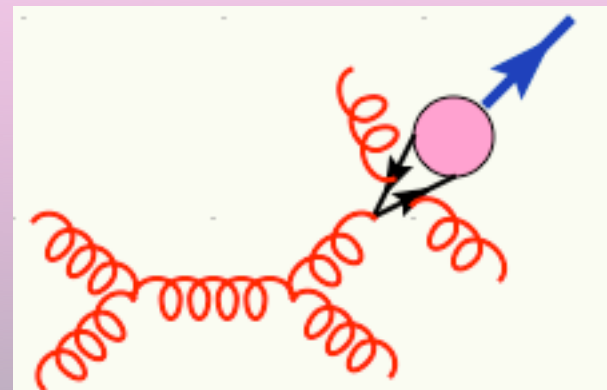
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NLO

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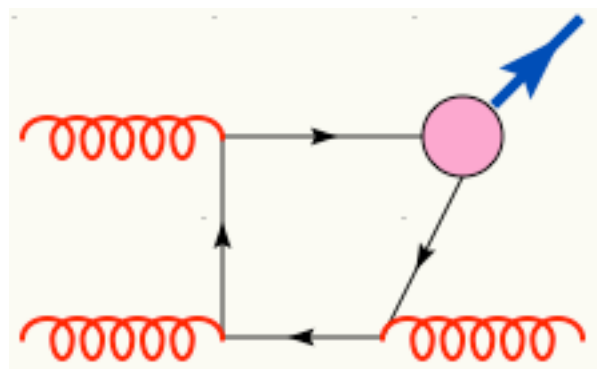


$$\alpha_S^5 \frac{1}{p_T^4}$$

NNLO

# Leading order

$$\sigma(pp \rightarrow \mathcal{Q} + X) = \sum_{i,j,n} \int dx_1 dx_2 f_{i/p} f_{j/p} \hat{\sigma}[ij \rightarrow (Q\bar{Q})_n + x] \langle 0 | \mathcal{O}_n^{\mathcal{Q}} | 0 \rangle$$



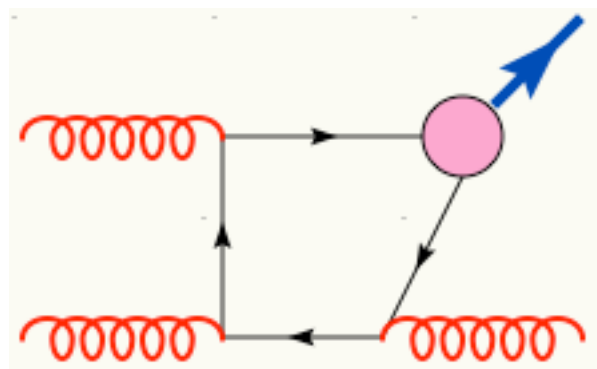
$$ggg \rightarrow {}^3S_1^{[1]} \rightarrow \ell\bar{\ell}$$

Calculation is involved  
 $\Rightarrow$  make use of modern NLO techniques:

1. Helicity amplitudes in 4 dimensions
2. Color decomposition (calculate dual amplitudes)
3. Decompose the  ${}^3S_1^{[1]}$  momentum in two light-like momenta = include the decay.

# Leading order

$$\sigma(pp \rightarrow Q + X) = \sum_{i,j,n} \int dx_1 dx_2 f_{i/p} f_{j/p} \hat{\sigma}[ij \rightarrow (Q\bar{Q})_n + X] \langle 0 | \mathcal{O}_n^Q | 0 \rangle$$



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3. Decompose the  $^3S_1^{[1]}$  momentum in two light-like momenta = include the decay.

$$ggg \rightarrow ^3S_1^{[1]} \rightarrow \ell\bar{\ell}$$

$$A(1^+, 2^+, 3^+; 5_\ell^+, 6_{\bar{\ell}}^-) = 0$$

$$A(1^+, 2^+, 3^-; 5_\ell^+, 6_{\bar{\ell}}^-) = 8\sqrt{2}g_s^3 e^2 m_Q \frac{\langle 35 \rangle^2 [12]^2 [56]}{(s_{12} + s_{13})(s_{13} + s_{23})(s_{12} + s_{23})}$$

$$\mathcal{A}(1, 2, 3) = \sum_{\sigma_{23}} \text{Tr}(t^{a_1} t^{a_{\sigma_2}} t^{a_{\sigma_3}}) A(1, \sigma_2, \sigma_3)$$

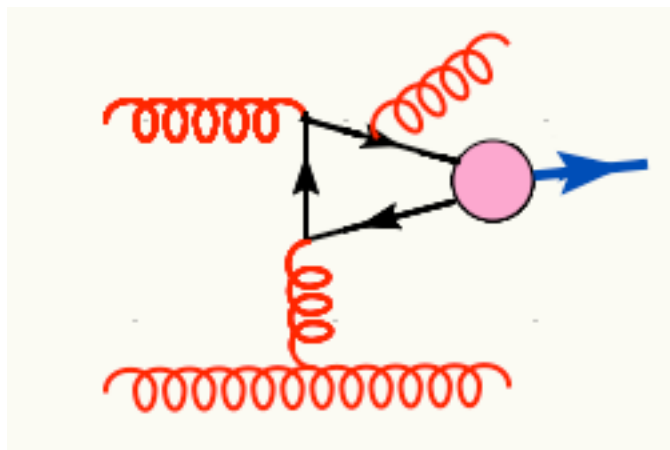
$$\mathcal{A}^{LO} = g_s^3 \frac{d^{abc}}{\sqrt{N_c}} T(1, 2, 3)$$





# Next-to-leading order: real contributions

$$\sigma^{NLO} = \int |\mathcal{A}^{\text{Real}}|^2 d\Phi_3 + \int 2\text{Re}(\mathcal{A}^{\text{Virt}} \mathcal{A}^{\text{LO}*}) d\Phi_2$$



$$gggg, gggq\bar{q} \rightarrow {}^3S_1^{[1]} \rightarrow \ell\bar{\ell}$$

$$\mathcal{A}(1, 2, 3, 4) = \sum_{\sigma_{234}} \text{Tr}(t^{a_1} t^{a_{\sigma_2}} t^{a_{\sigma_3}} t^{a_{\sigma_4}}) A(1, \sigma_2, \sigma_3, \sigma_4)$$

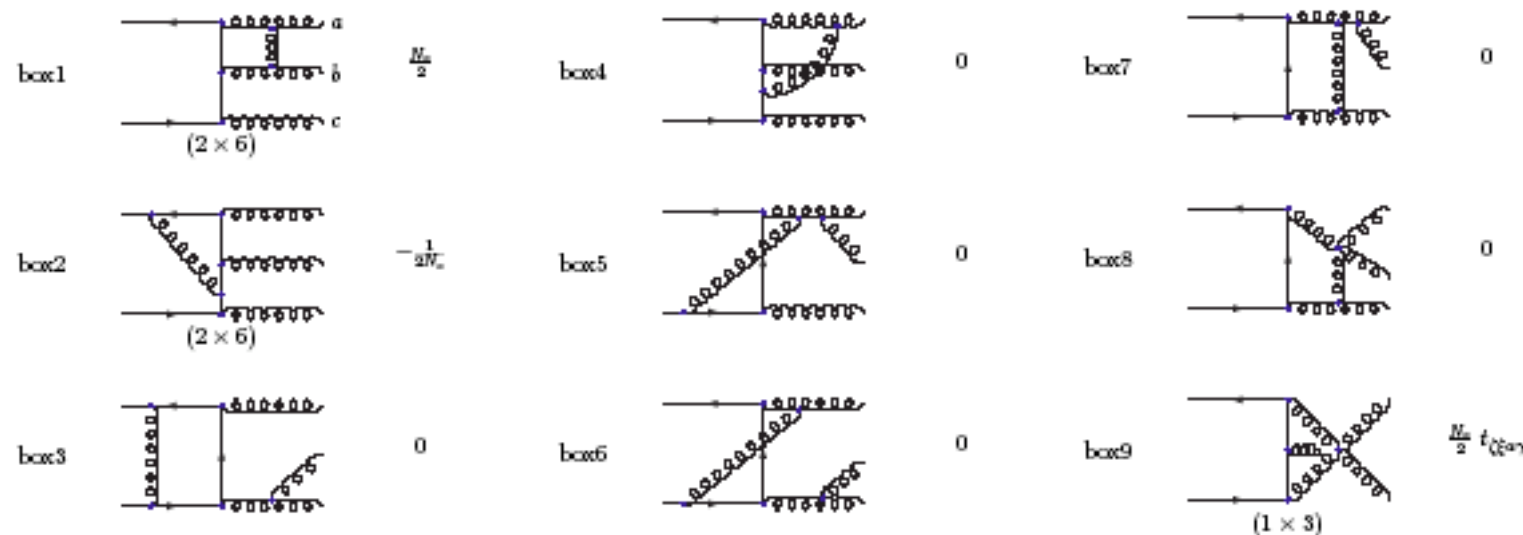
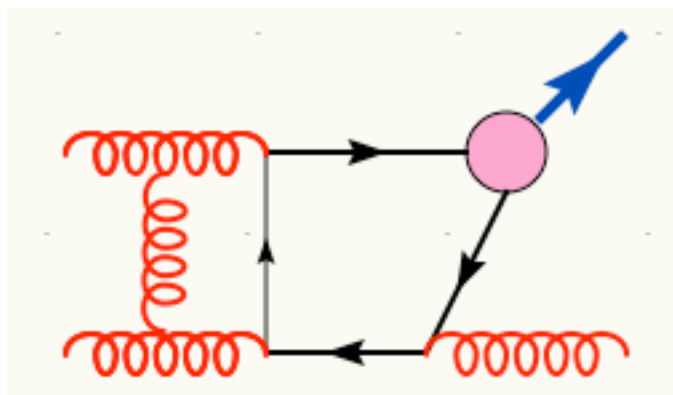
$$|\mathcal{A}|^2 = C_2(F) N_c \sum_{\sigma_{234}} |A_{\sigma}|^2 \quad C_2(F) = (d^{abc}/\sqrt{N_c})^2$$

C	A	A	A	A	B
A	C	A	B	A	A
A	A	C	A	B	A
A	B	A	C	A	A
A	A	B	A	C	A
B	A	A	A	A	C

Impressive “color magic” simplification!

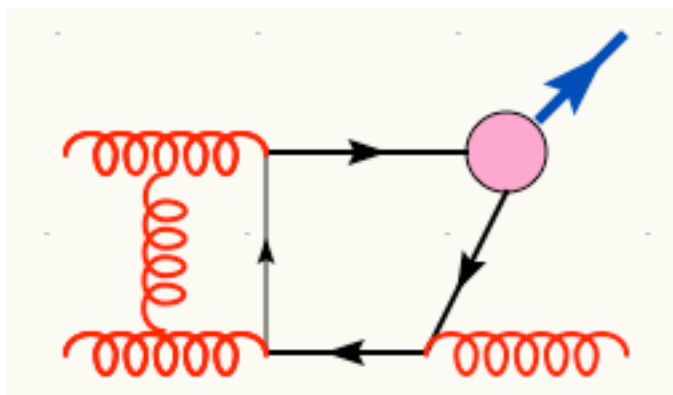
# Next-to-leading order: virtual contributions

$$\sigma^{NLO} = \int |\mathcal{A}^{\text{Real}}|^2 d\Phi_3 + \int 2\text{Re}(\mathcal{A}^{\text{Virt}} \mathcal{A}^{\text{LO}*}) d\Phi_2$$



# Next-to-leading order: virtual contributions

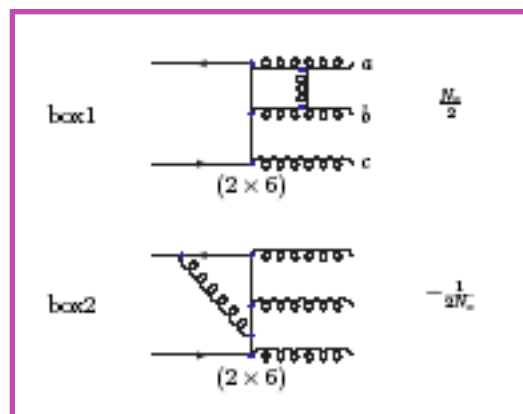
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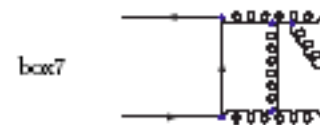
vtx 10 → 5

box 9 → 3

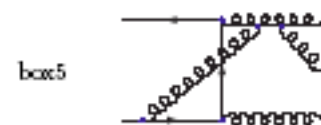
penta 4 → 3



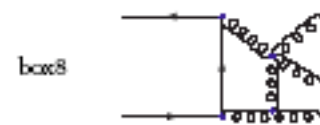
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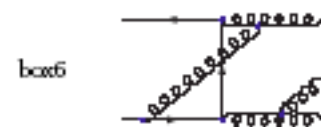
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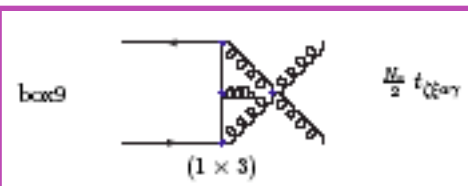
0



0



0



\* color plays entails again a huge simplification.

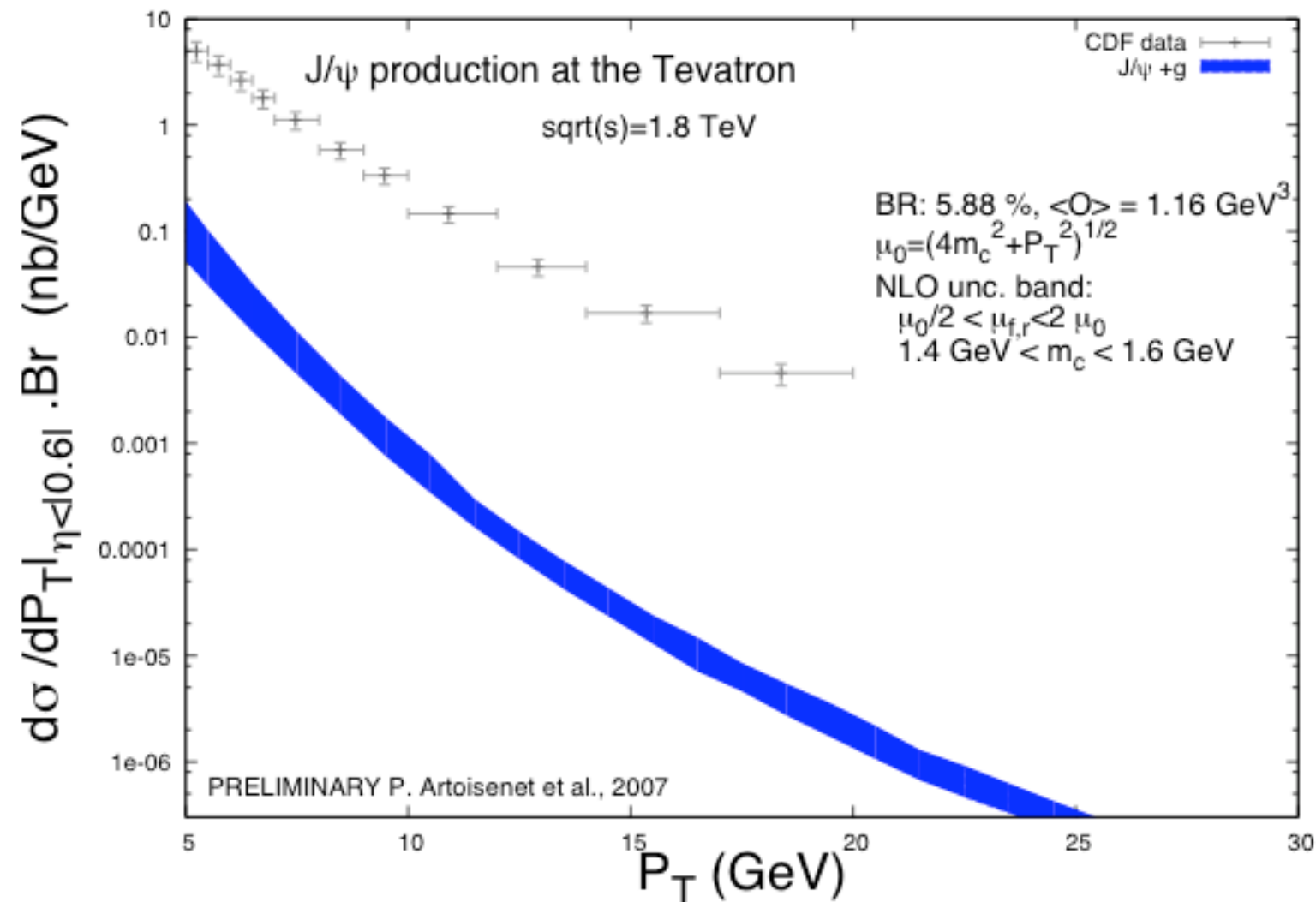
\* only the same diagrams as  $\Upsilon_{gg} \rightarrow {}^3S_1^{[1]}$  at 1 loop survive. (but color prefactors are different).

$$\mathcal{A}^{\text{Virt}} = g_s^5 \frac{d^{abc}}{\sqrt{N_c}} T^{\text{Virt}}(1, 2, 3)$$

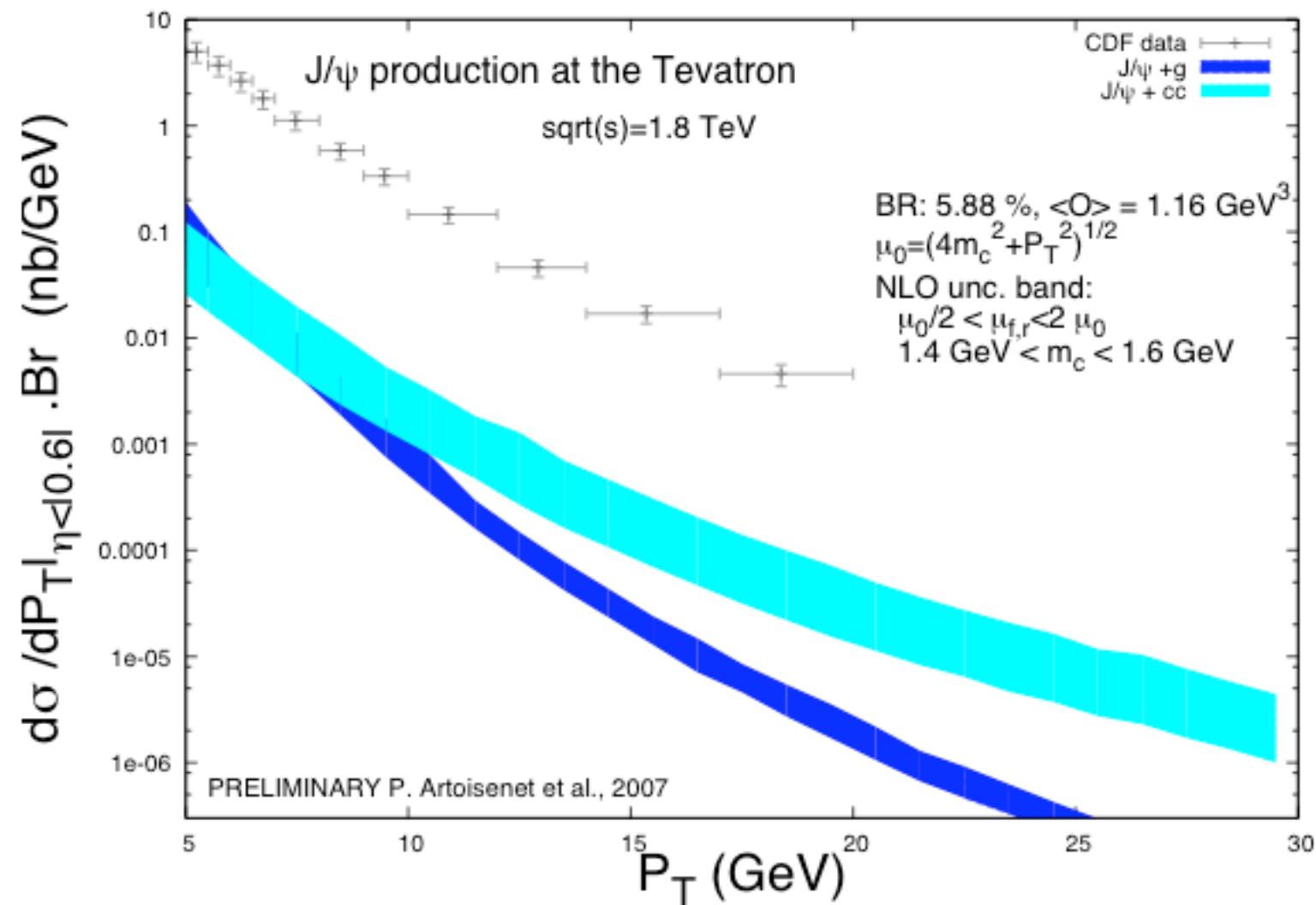
# Checks

- Gauge invariance (reference vectors for the polarization)
- Real and virtual pole structure via the dipole subtraction scheme
- Positronium decay at NLO in em
- $\Upsilon$  decay into hadrons and into  $\gamma$ +hadrons

# J/psi production at the Tevatron Run I

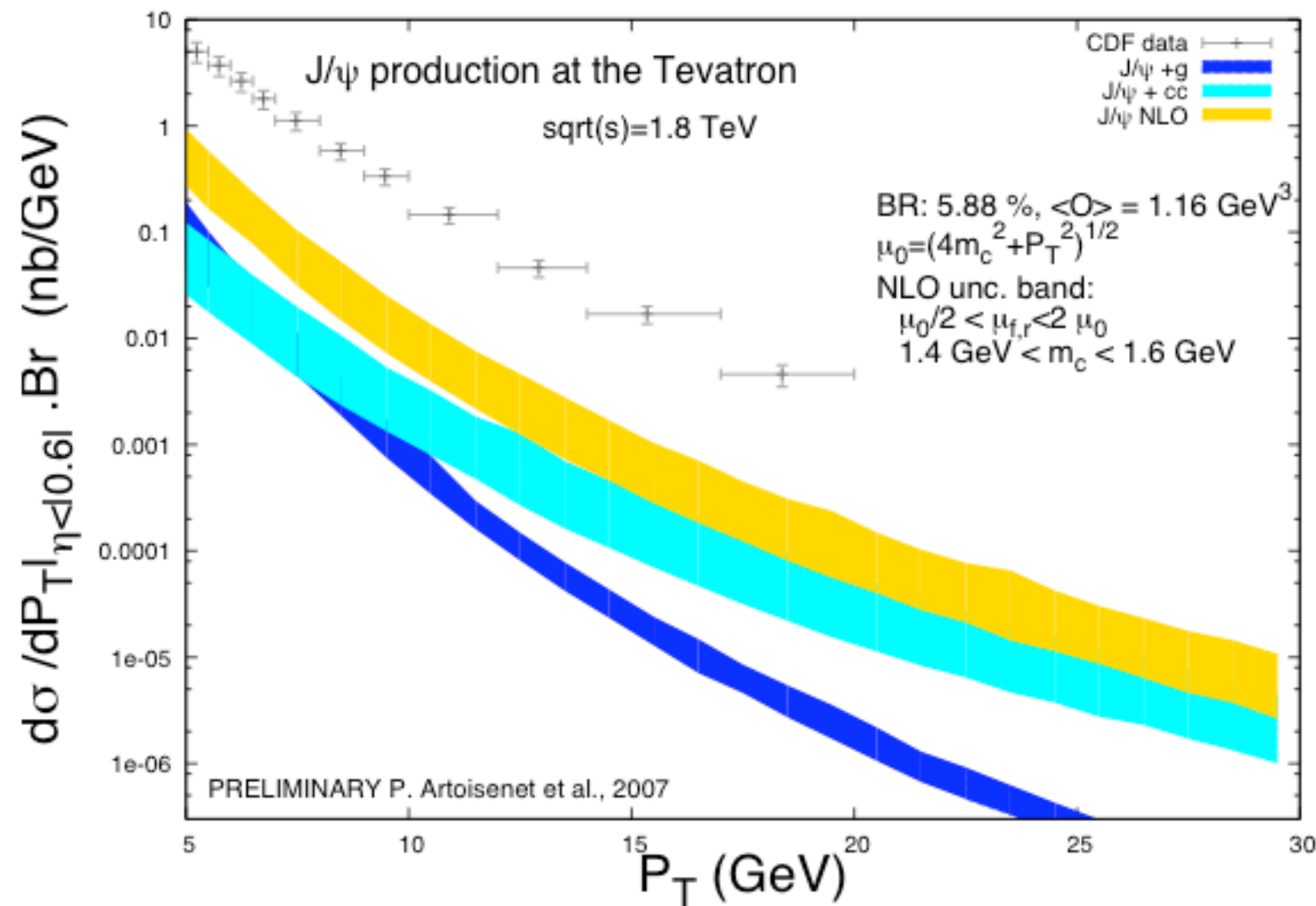


# J/psi production at the Tevatron Run I



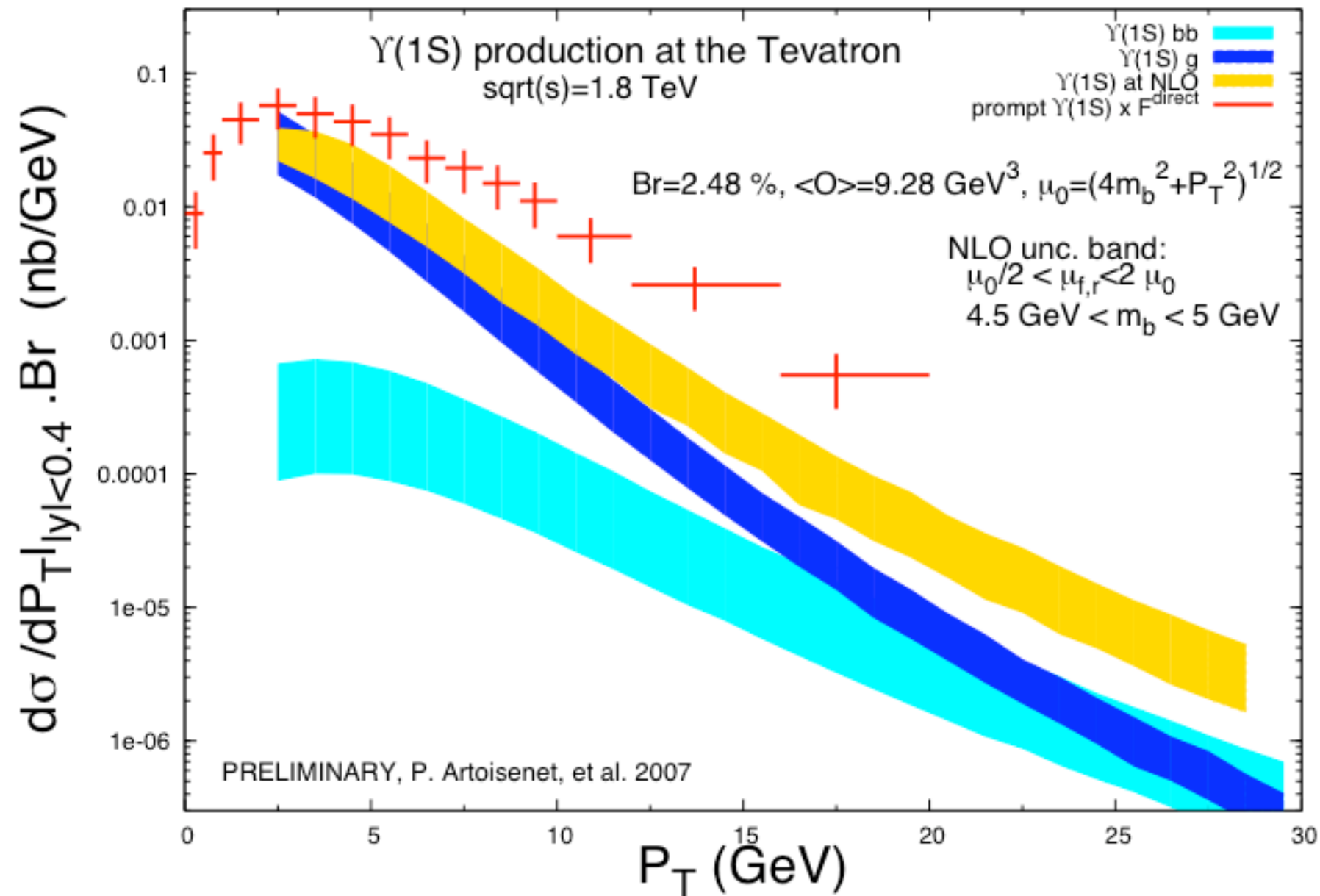


# J/psi production at the Tevatron Run I



Dramatic improvement in shape and normalization  
but certainly not close enough to data.

# Y(1S) production at the Tevatron Run I



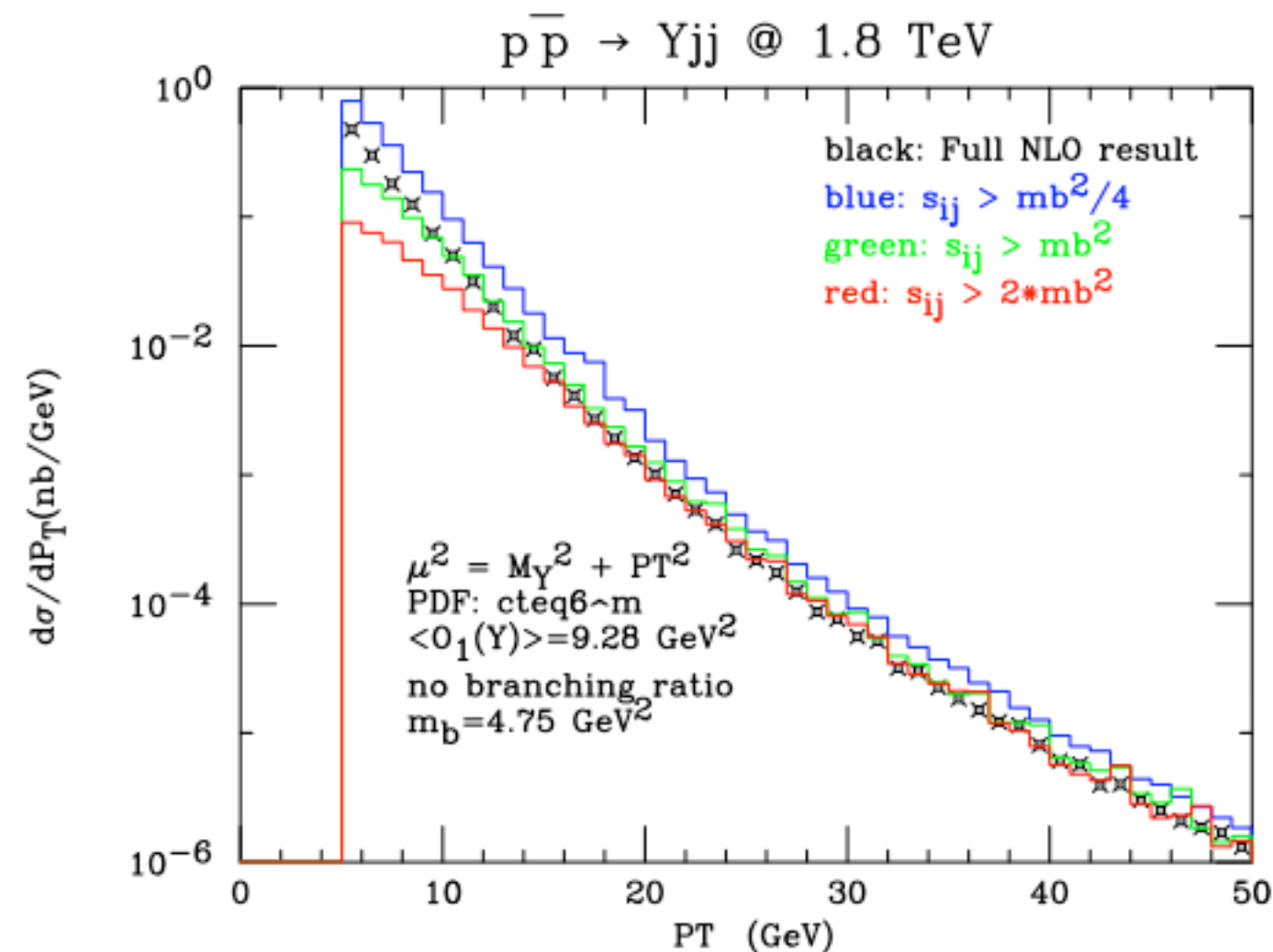
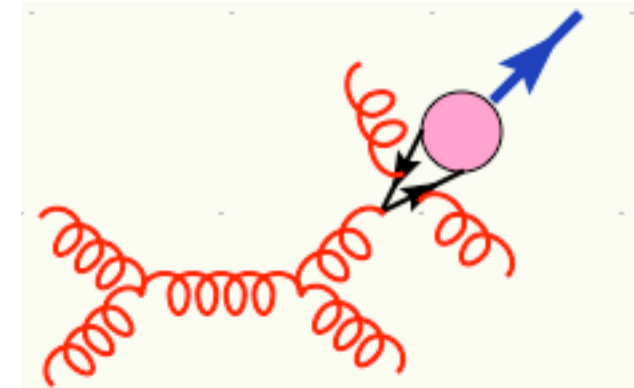
Improvement in shape and normalization.  $p_T$  is harder,  
but not too far...

# Estimating the impact of NNLO contributions

Can we improve our description and include the  $\alpha_s^5 / p_T^4$  terms,  
E.g.,  $gg \rightarrow {}^3S_1^{[1]} ggg$ , not using the fragmentation approximation?

[see Lansberg's talk]

Technically challenging ( $\sim$ deca subprocesses,  $\sim$ kilo diagrams) but  
now feasible! [see Artoisenet's talk]



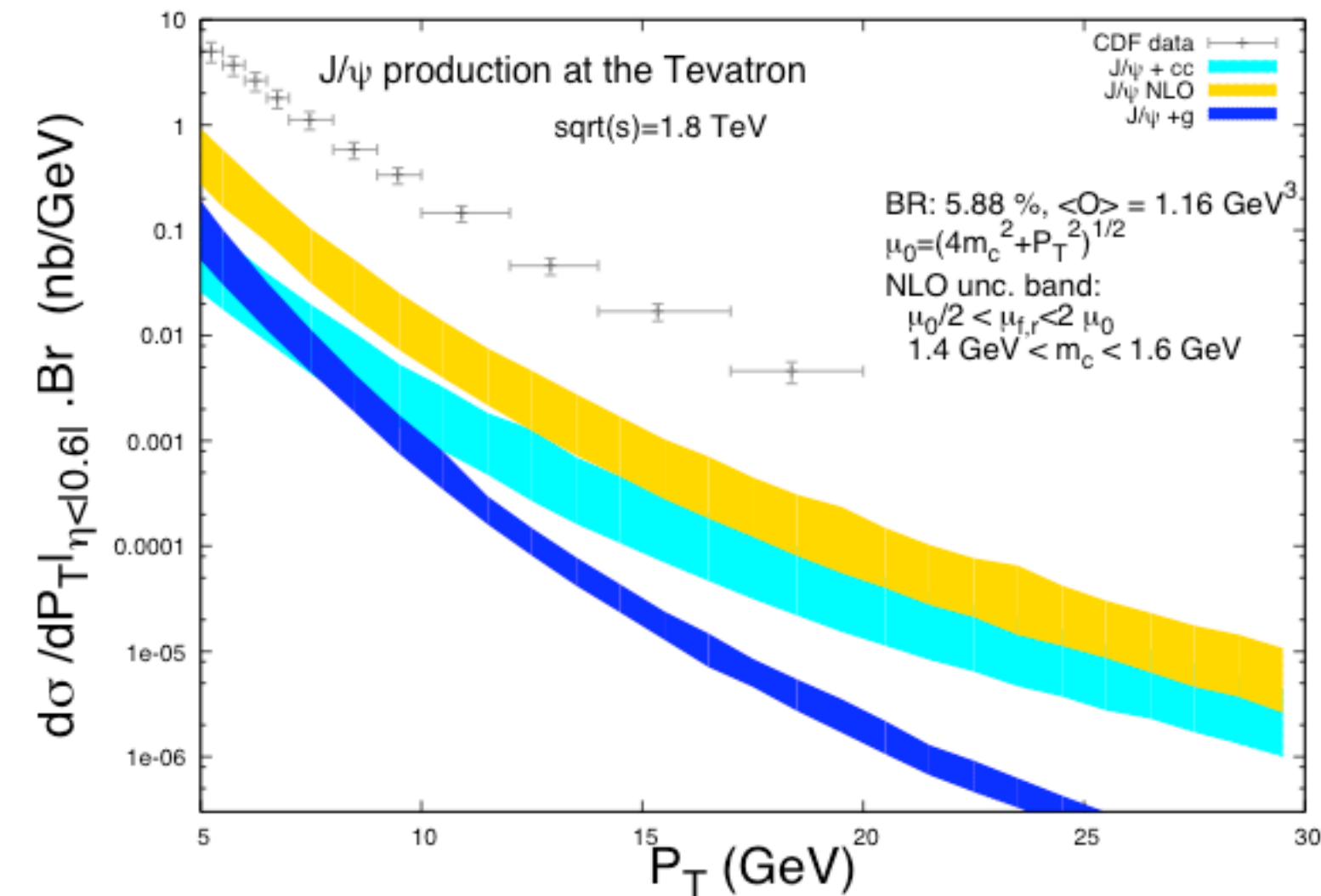
Strategy:

The real corrections with a democratic cut on  $mb^2/4 < s_{ij} < 2 mb^2$  approximate the NLO result very well.

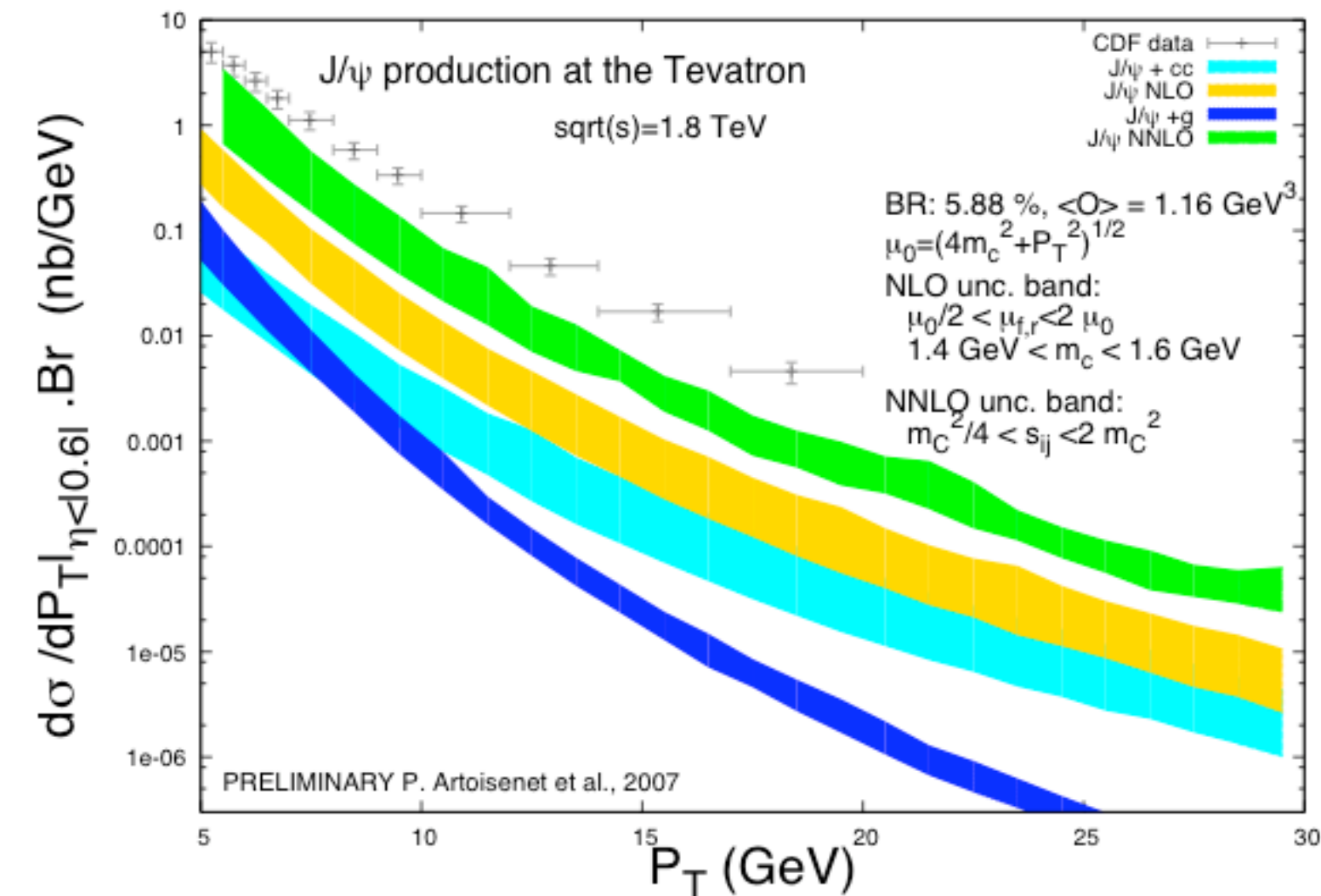
As a first approximation we use the NLO result to find a reasonable interval of  $s_{ij}$  and give a rough estimate the NNLO contributions.

A more solid approach is based on the matching prescription a la CKKW...

# Estimating the impact of NNLO contributions



# Estimating the impact of NNLO contributions



\*NNLO\* contribution is a very large effect.

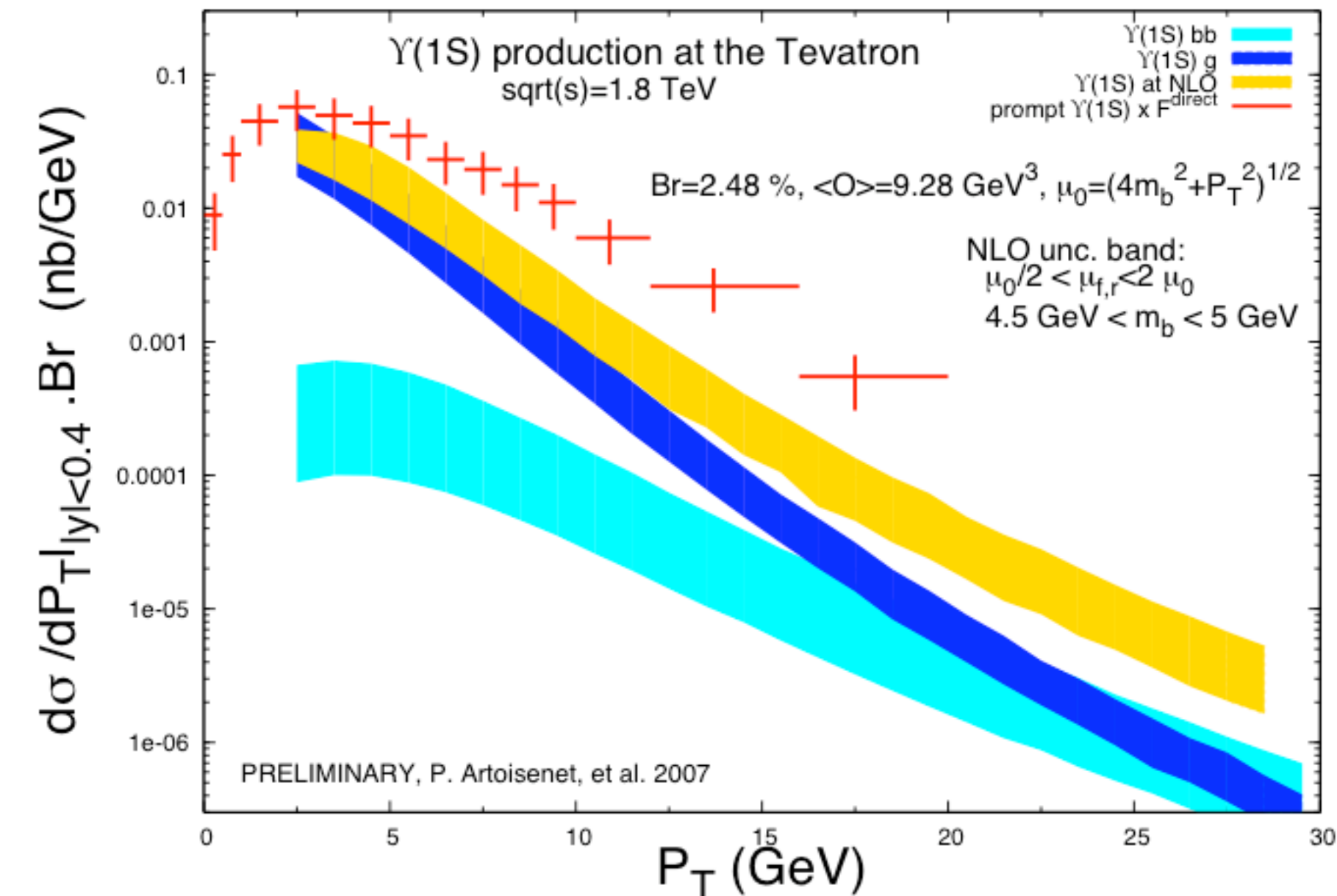
\*Predictions much closer to the data in shape and normalization.

\*Singlet alone not able to describe the data alone but situation much less dramatic!

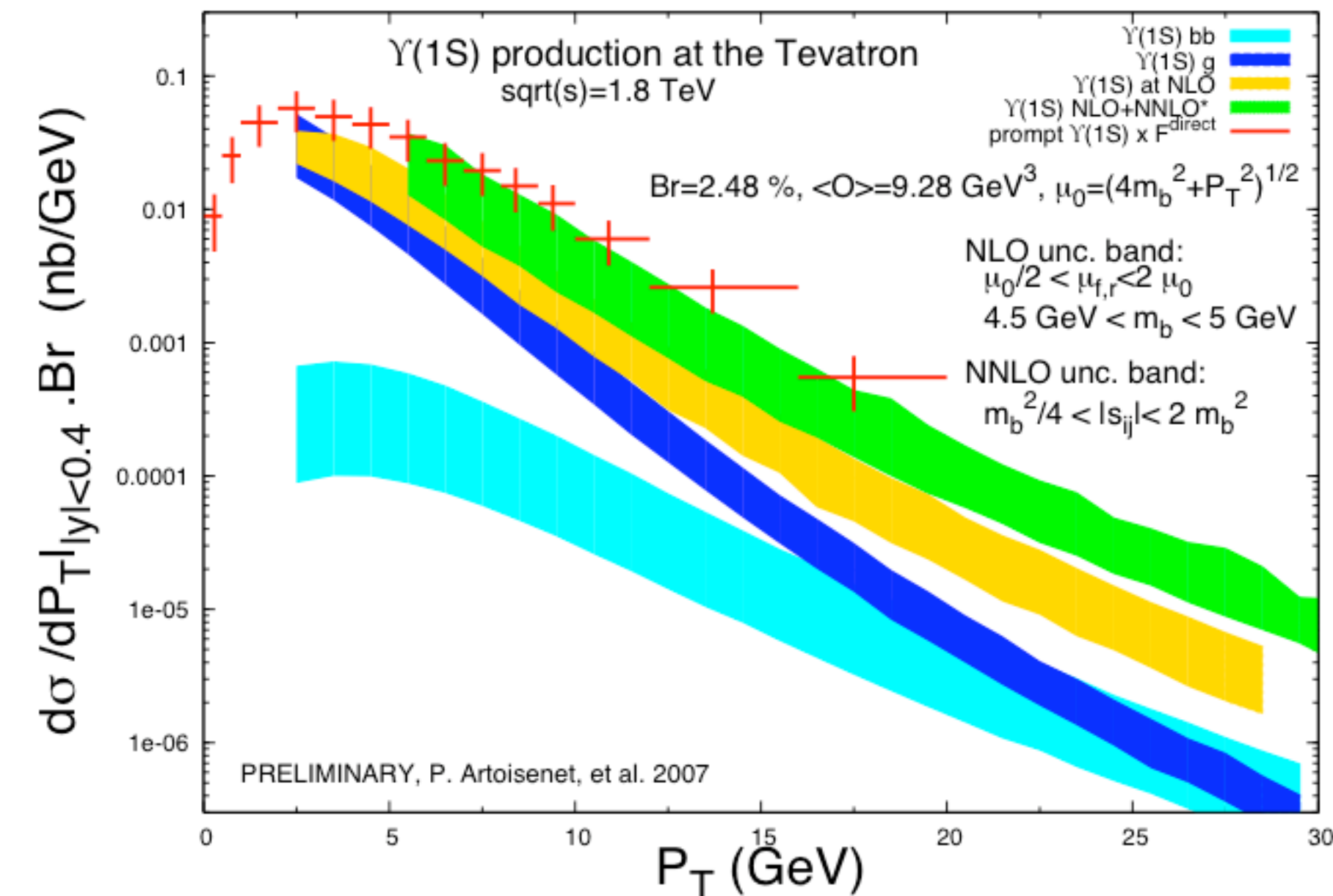
\*More study on the TH systematics is needed (comparison with the fragmentation approach, smaller band at very high  $p_T$ ,...)



# Estimating the impact of NNLO contributions



# Estimating the impact of NNLO contributions



Mild effect, but exactly what is needed in terms of normalization and shape.

Inclusion of NNLO real contribution, i.e. of  $p_T^4$  terms, allows a pretty good description of the data in terms of color singlet only.

More work on the systematics needed but looks promising!

Possible improvement:  
do this with a better matching!



# Outlook

On-going and future applications:

- \* Predictions for the polarization at Tevatron.
- \* Complete fixed target data analysis at NLO in NRQCD.
- \* RHIC
- \*  $\gamma p \rightarrow {}^3S_1^{[1]} + X$  with polarization info and single resolved contributions [comparison with M. Kraemer's calculation (1995)]
- \*  $\gamma\gamma \rightarrow {}^3S_1^{[1]} + X$  with polarization info + single and double resolved contributions + direct color singlet contribution
- \*  $\gamma\gamma \rightarrow {}^3S_1^{[8]} + X$  with polarization info and single resolved contributions [comparison with Klasen et al. calculation (2005)]

# Conclusions

- Quarkonium production phenomenology is very challenging.
- NLO strong corrections are now available for the color singlet production of  $J/\psi$  and  $Y$  at hadron colliders.
- Preliminary comparison with Tevatron data shows that the color singlet at NLO :
  - is well below the  $J/\psi$  data.
  - is just below the  $Y$  data.
- Evidence that NNLO corrections play an important role
- Further TH and EXP work needed!

# Backup slides

# Quarkonium inclusive NLO cross sections

	$^1S_0^{[1]}$	$^3S_1^{[1]}$	$^3P_J^{[1]}$	$^1P_1^{[1]}$	$^1S_0^{[8]} \quad ^3P_J^{[8]}$ $^1P_1^{[8]}$	$^3S_1^{[8]}$
$e^+e^-$						
$\Upsilon\Upsilon$						
$\Upsilon p$						
$pp$						

[1] Kuhn and Mirkes, 1993;

[2] Kraemer, 1995

[3] Petrelli et al., 1998

[4] FM, Mangano Petrelli et al., 1999

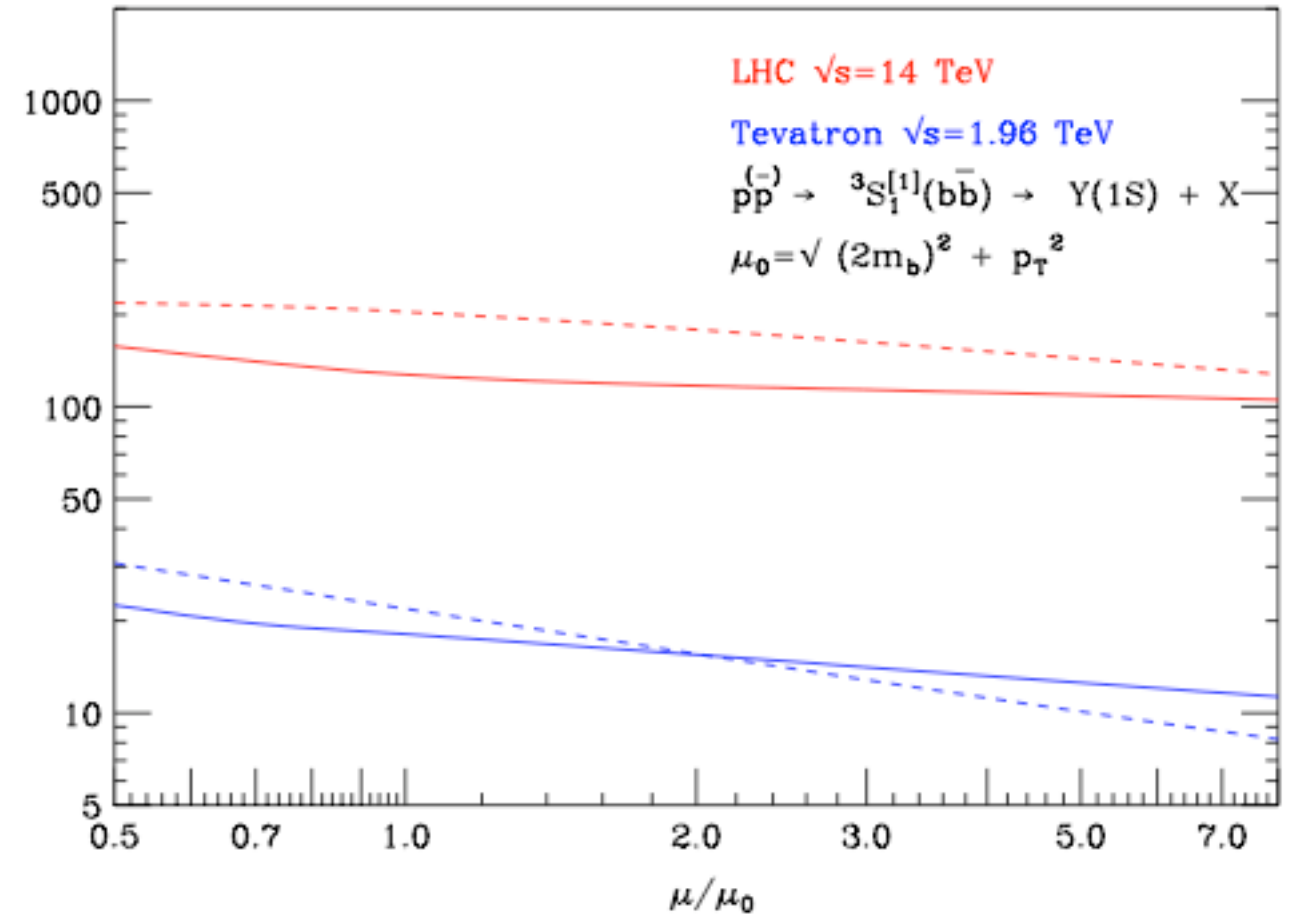
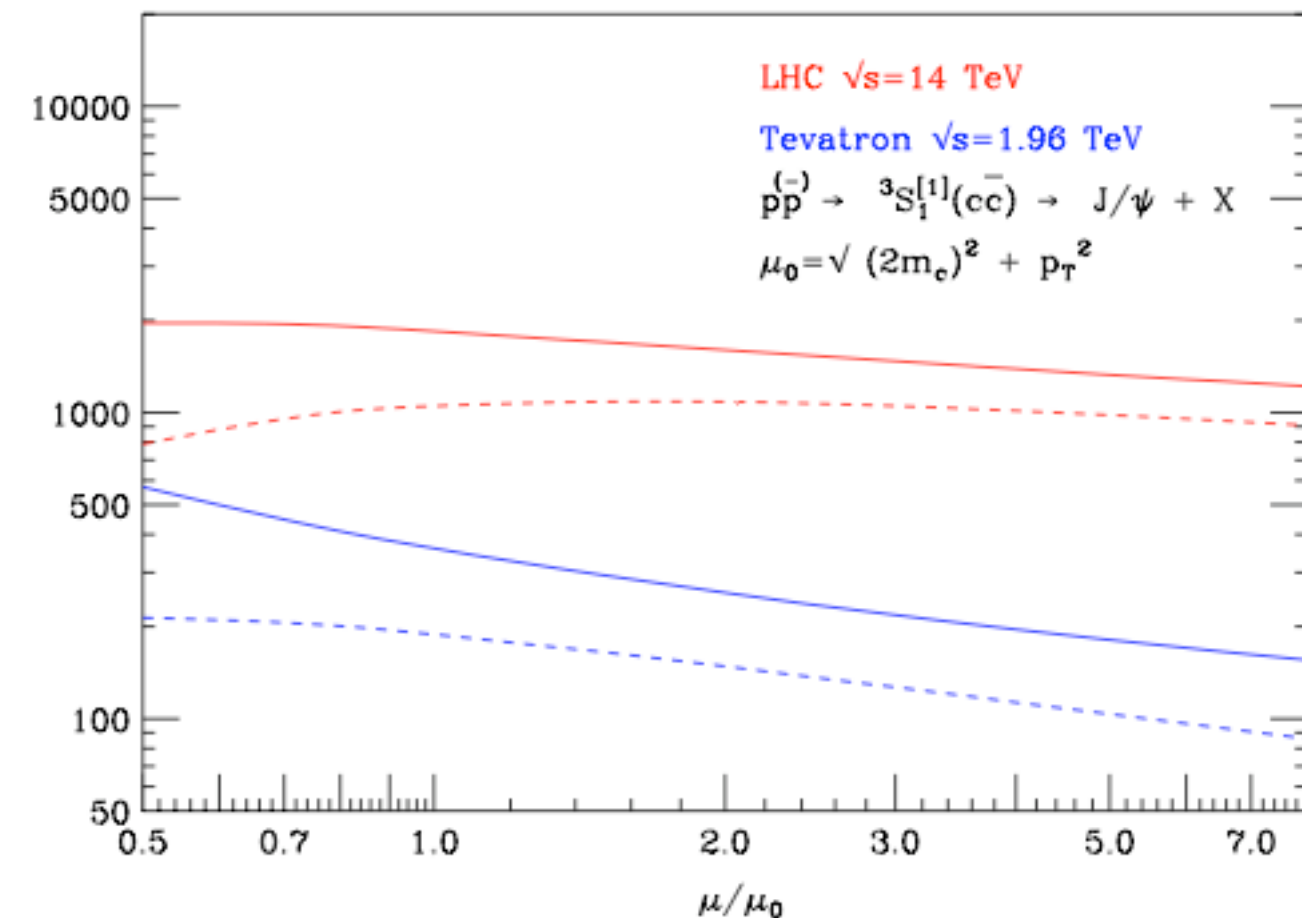
[5] Klasen et al., 2005

[6] Y.-J. Zhang and K.-T. Chao, 2007

[7] Campbell, FM, Tramontano, 2007

= desiderata

# Results

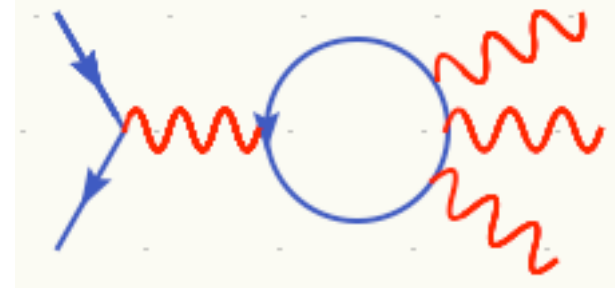


- \*  $p_T > 3$  GeV,  $|y| < 3$
- \* “Total” K-factors are mild.
- \* Scale dependence slightly improved.

# Checks

Orthopositronium decay:

\* abelian subset of diagrams + new diagram



$$\Gamma^{\text{NLO}}(\text{Orthopositronium} \rightarrow 3\gamma) = \Gamma^{\text{LO}} \left[ 1 - 10.28665 \frac{\alpha_{EM}}{\pi} \right], \text{ [Adkins, et al., 2000]}$$

$\Upsilon$  inclusive decay:

$$\Gamma^{\text{NLO}}(\Upsilon \rightarrow LH) = \Gamma^{\text{LO}} \left[ 1 + \frac{\alpha_S(\mu)}{\pi} \left( -9.471 C_F + 4.106 C_A - 1.150 n_f + \frac{3}{2} \beta_0 \log \frac{\mu}{m_b} \right) \right],$$

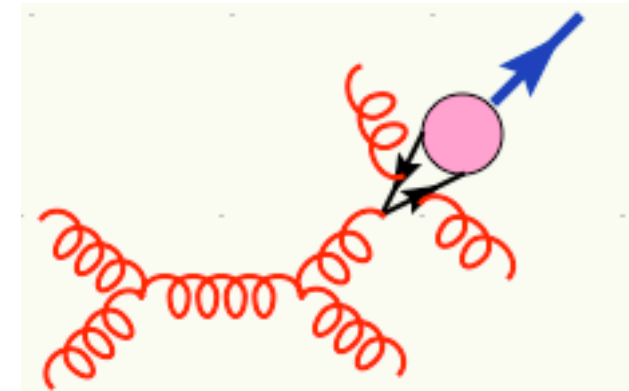
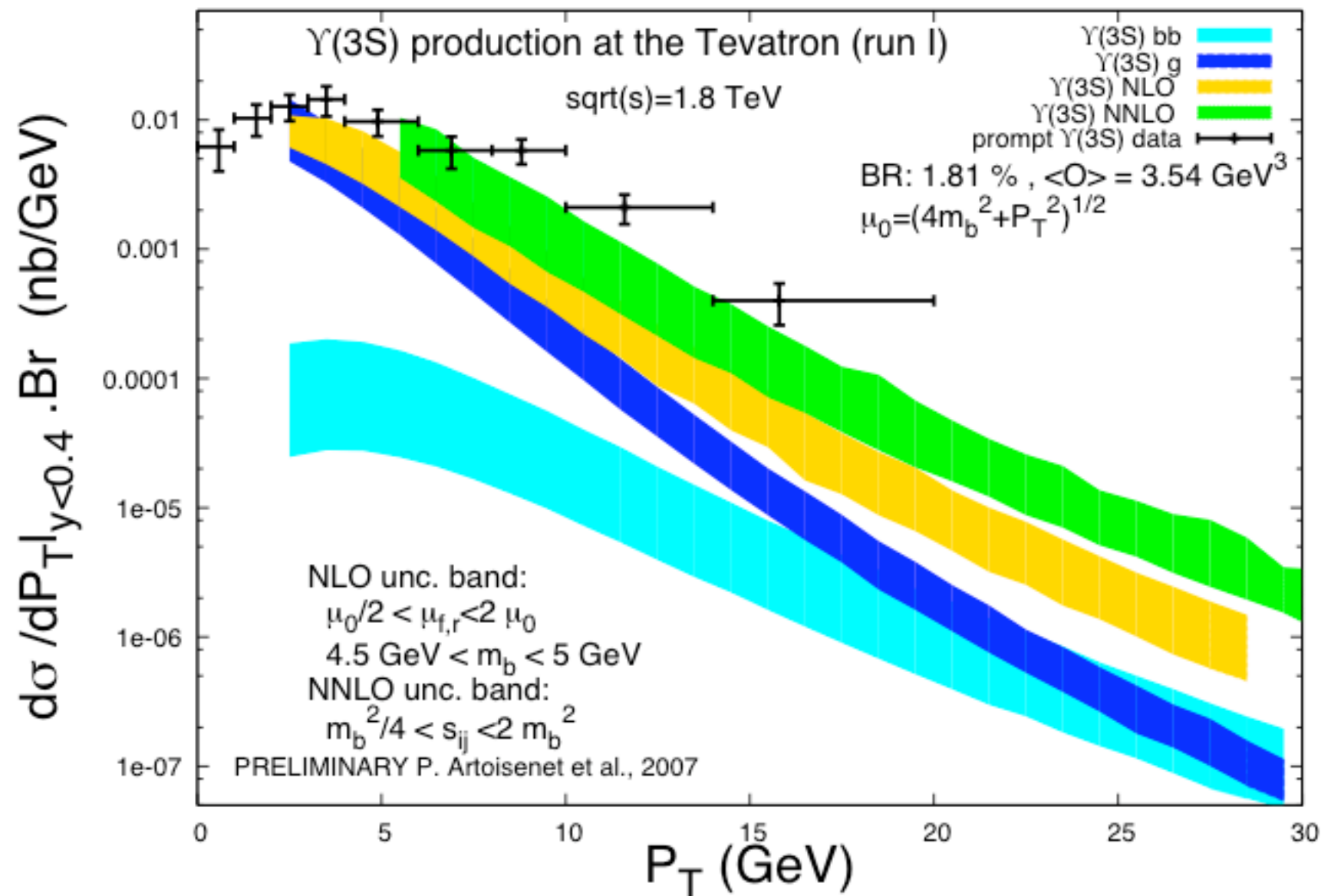
$$\text{[Mackenzie \& Lepage, 1981]} \left[ 1 + (-9.46(2)C_F + 4.13(17)C_A - 1.161(2)n_f) \frac{\alpha_s}{\pi} \right]$$

$\Upsilon$  photon inclusive decay:

$$\Gamma^{\text{NLO}}(\Upsilon \rightarrow \gamma + LH) = \Gamma^{\text{LO}} \left[ 1 + \frac{\alpha_S(\mu)}{\pi} \left( -9.471 C_F + \frac{2}{3} (4.106 C_A - 1.150 n_f + \frac{3}{2} \beta_0 \log \frac{\mu}{m_b}) \right) \right],$$

$$\text{[Mackenzie \& Lepage, 1981; M. Kraemer 1998]} \left[ 1 + (-9.46(2)C_F + 2.75(11)C_A - 0.774(1)n_f) \frac{\alpha_s}{\pi} \right]$$

# Estimating the impact of NNLO contributions



A similar study can be also performed on the Y(3S) data which don't have feeddown contributions but are only direct.