Real-time static potential and quarkonium spectral function in hot QCD perturbation theory

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M. Laine, O. Philipsen, P. Romatschke and M. Tassler, "Real-time static potential in hot QCD," hep-ph/0611300;

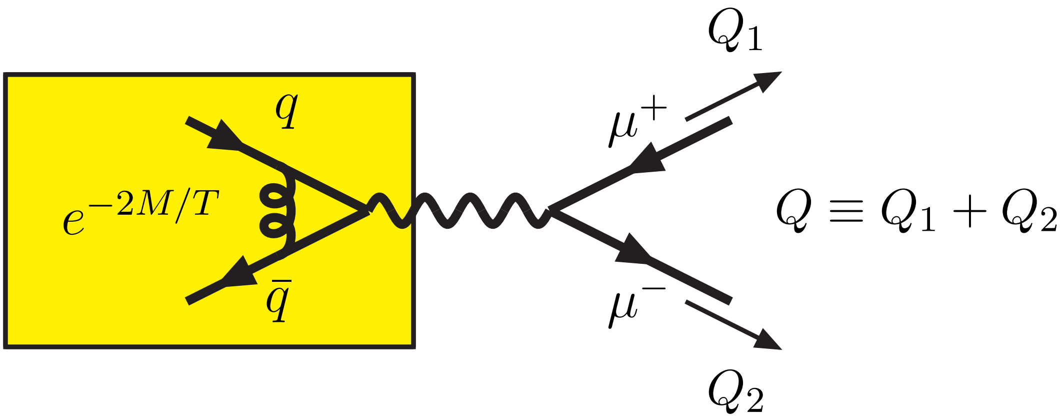
M. Laine, "A resummed perturbative estimate for the quarkonium spectral function in hot QCD," arXiv:0704.1720;

M. Laine, O. Philipsen and M. Tassler, "Thermal imaginary part of a real-time static potential from classical lattice gauge theory simulations," arXiv:0707.2458;

Y. Burnier, M. Laine and M. Vepsäläinen, in preparation.

Physical observable considered:

the heavy quarkonium $(c\bar{c}, b\bar{b})$ contribution to the production rate of lepton–antilepton pairs from a thermal plasma.



Surprisingly, despite asymptotic freedom, the existence of a high temperature does not necessarily make the theoretical determination of the properties of heavy quarkonium any easier than at zero temperature.

In other words, all standard approximation methods develop further systematic errors at $T > 0$.

A. Lattice QCD

$T = 0$: finite volume and lattice spacing;
non-chiral quarks with unphysical masses.

$T > 0$: analytic continuation from numerical data.

B. Potential models

$T = 0$: not theoretically consistent.

$T > 0$: which potential to use?

C. Perturbation theory

At $T > 0$, perturbation theory suffers from serious infrared problems, which require complicated resummations. As a result, the perturbative series is typically of the form ($\alpha_s = g^2/4\pi$; $1/4\pi$'s omitted)

$$\langle O \rangle \sim 1 + \#_1\, g^2 + \#_2\, g^3 + \#_3\, g^4 \ln\frac{1}{g} + \#_4\, g^6 + \ldots\, ,$$

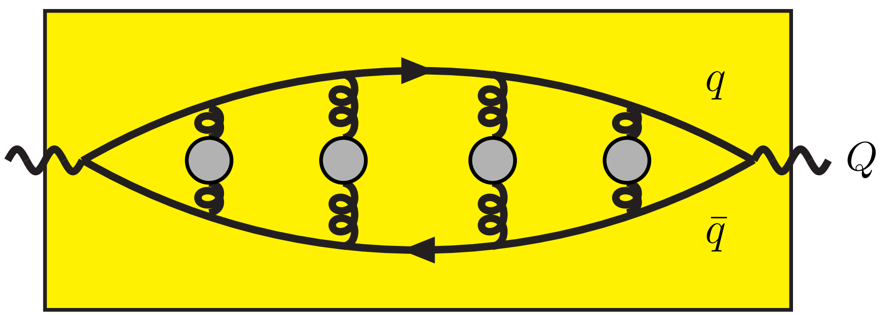
where some coefficients can be non-perturbative.

Moreover, even if the coefficients were known, the convergence of the series can be very slow.

Practical approach

Compute the observable with many different methods, possessing complementary systematic errors, and hope to find a consistent picture!

Here: perturbation theory, i.e. "just" graphs:



Momentum/energy scales

Vacuum: $M, g^2M, g^4M, \ldots$

Finite temperature: $T, gT, g^2T, \ldots$

The procedure now depends on the ratio of M and T.

$T \sim g^2M \Rightarrow$ width $\sim g^6M \ll$ binding energy $\sim g^4M$
$\Rightarrow$ bound state exists.

$T \sim gM \Rightarrow$ width $\sim g^3M \gg$ binding energy $\sim g^4M$
$\Rightarrow$ bound state has melted.

In the following assume, formally, $g^2M < T < gM$.
Then the computation proceeds in the following steps:

1. Relation of production rate to Green's function

$$\frac{\mathrm{d}\Gamma_{\mu^+\mu^-}}{\mathrm{d}^4Q} = -\frac{e^2}{3(2\pi)^5 Q^2}\Big(1+\frac{2m_\mu^2}{Q^2}\Big)\Big(1-\frac{4m_\mu^2}{Q^2}\Big)^{\frac{1}{2}} e^{-\frac{q^0}{T}}\tilde{C}_>(Q) \; ,$$

$$\tilde{C}_>(Q) \equiv \int_{-\infty}^{\infty}\mathrm{d}t \int \mathrm{d}^3\mathbf{x}\, e^{iQ\cdot x}\langle \hat{\mathcal{J}}^\mu(x)\hat{\mathcal{J}}_\mu(0)\rangle \; ,$$

$$\hat{\mathcal{J}}^\mu(x) = \ldots + \frac{2}{3}e\,\hat{\bar{c}}\,(x)\gamma^\mu\hat{c}(x) - \frac{1}{3}e\,\hat{\bar{b}}\,(x)\gamma^\mu\hat{b}(x) \; ,$$

$$\langle\ldots\rangle \equiv \mathcal{Z}^{-1}\,\mathrm{Tr}[\exp(-\hat{H}/T)(\ldots)] \; .$$

[Rather than $\tilde{C}_>$ one often considers the spectral function:

$$\rho(Q) = \frac{1}{2}(1 - e^{-\frac{q^0}{T}})\tilde{C}_>(Q) \; .$$
]

2. Properties of the Green's function at large M

Restrict to $\mathbf{q} = \mathbf{0}$ and introduce point-splitting:

$$C_{>}(t,r) \equiv \int \mathrm{d}^3\mathbf{x} \left\langle \hat{\bar{\psi}} \left(t, \mathbf{x} + \frac{\mathbf{r}}{2}\right) \gamma^{\mu} \, W \, \hat{\psi}\left(t, \mathbf{x} - \frac{\mathbf{r}}{2}\right) \; \hat{\bar{\psi}}\,(0,\mathbf{0}) \gamma_{\mu} \hat{\psi}(0,\mathbf{0}) \right\rangle \, .$$

The r-dependence is not physical ...

$$\tilde{C}_{>}(Q) = \int_{-\infty}^{\infty} \mathrm{d}t \, e^{iq^0 t} C_{>}(t,0) \ ,$$

... but it facilitates perturbative solution:

$$\left\{ i\partial_t - \left[2M - \frac{\nabla_{\mathbf{r}}^2}{M} + \mathcal{O}\left(\frac{1}{M^3}\right) \right] \right\} C_{>}(t,r) = 0 \ ,$$

$$C_{>}(0,r) = -6 N_{\mathrm{c}} \, \delta^{(3)}(\mathbf{r}) + \mathcal{O}\left(\frac{1}{M^2}\right) \ .$$

3. Definition of a real-time static potential

The static potential $V_>(t,r)$ is defined to be the term independent of M in the “exact” Schrödinger equation:

$$\left\{i\partial_t - \left[2M + V_>(t,r) - \frac{\nabla_{\mathbf{r}}^2}{M} + \mathcal{O}\left(\frac{1}{M^2}\right)\right]\right\}C_>(t,r) = 0 \; .$$

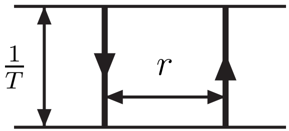
It can thus be obtained in the limit $M \to \infty$, whereby the heavy quarks can be replaced by Wilson lines.

Noting that $C_>(t,r) = C_E(it,r)$, where $C_E(\tau,r)$ is the Euclidean Wilson loop, we are lead to

$$i\partial_t C_E(it,r) \equiv V_>(t,r) C_E(it,r) \; .$$

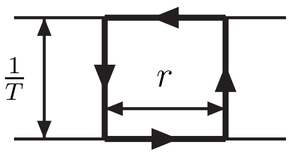
Remark: A few different static potentials

From Polyakov loops:



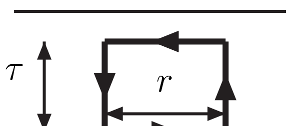
$$\langle \mathrm{Tr}[P]\,\mathrm{Tr}[P^\dagger]\rangle \equiv e^{-\frac{V_{\mathsf{a}}(r)}{T}}\ .$$

From a Wilson loop:



$$\langle \mathrm{Tr}[W_E(\frac{1}{T}, r)]\rangle \equiv e^{-\frac{V_{\mathsf{b}}(r)}{T}}\ .$$

From an analytic continuation:



$$\langle \mathrm{Tr}[W_E(\tau, r)]\rangle \equiv C_E(\tau, r)\ .$$

4. Result for our real-time static potential

In Hard Thermal Loop perturbation theory, to g^2:

$$
\begin{aligned}
\mathrm{Re}\, V_>^{(2)}(\infty, r) &= -\frac{g^2 C_F}{4\pi}\left[m_\mathrm{D} + \frac{\exp(-m_\mathrm{D} r)}{r}\right] \;, \\
\mathrm{Im}\, V_>^{(2)}(\infty, r) &= -\frac{g^2 T C_F}{4\pi}\,\phi(m_\mathrm{D} r) \;,
\end{aligned}
$$

where $m_\mathrm{D} \sim gT$ is the Debye mass, $C_F \equiv 4/3$, and

$$
\phi(x) = 2\int_0^\infty \frac{\mathrm{d}z\, z}{(z^2+1)^2}\left[1 - \frac{\sin(zx)}{zx}\right] \;,
$$

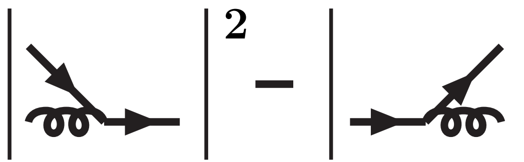
is finite and strictly increasing, with the limiting values $\phi(0) = 0$, $\phi(\infty) = 1$.

Physics of real part:

$2 \times$ thermal mass correction for a heavy quark $+$ r-dependent Debye-screened potential.

Physics of imaginary part:

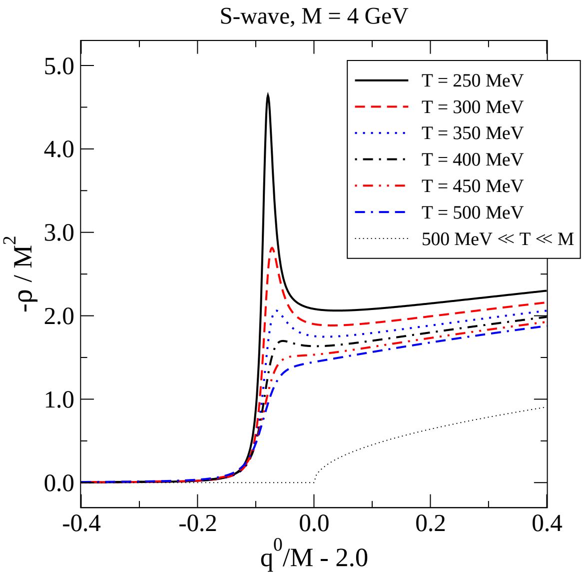
almost static (off-shell) gluons may disappear due to interactions with hard particles in the plasma.

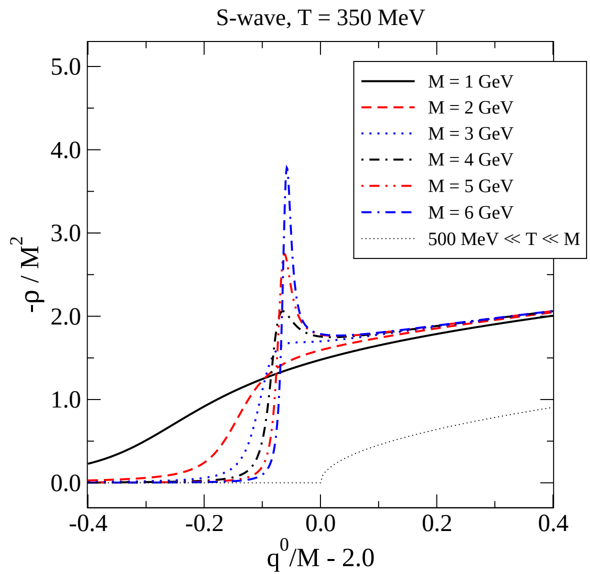
$n_{\mathbf{F}} n_{\mathbf{B}} (1 - n_{\mathbf{F}})$  $n_{\mathbf{F}} (1 + n_{\mathbf{B}}) (1 - n_{\mathbf{F}})$

This is the phenomenon of Landau-damping.

Consequently, there is no stationary wave function: the bound state is a short-lived transient!

5. Result for spectral function





Basic qualitative structure as suggested by Matsui and Satz (1986) from phenomenological arguments.

Conclusions

Useful definition of a finite-temperature real-time static potential is non-trivial. $V_>$ originates from a physical observable; it has both a real and an imaginary part.

Using this potential, the existence and disappearance of the quarkonium peak in the dilepton production rate is qualitatively a **weak-coupling** phenomenon.

The conceptual point: at high temperatures, there is no stationary wave function. The bound state is a short-lived transient!

In the end, for quantitative understanding, need to compare with other methods.