Relativistic corrections to the interquark potential from Lattice QCD

Miho Koma
(Universität Mainz)

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A sequel to the talk "Relativistic corrections to the static potential"
by Y.Koma (QWG4,'06)

[Y.Koma, M.Koma & H.Wittig, Phys.Rev.Lett.97('06)122003,
Y.Koma, M.Koma, Nucl.Phys.B769('07)79,
Y.Koma, M.Koma & H.Wittig, PoS(LATTICE 2007)111]

INTRODUCTION

- ▷ **Effective field theory for heavy quarkonium $\Longrightarrow$ potential NRQCD (pNRQCD)**
  [Brambilla,Pineda,Soto&Vairo('99-)]

- ▷ **Effective Hamiltonian for quarkonium up to $O(1/m^2)$** [Pineda&Vairo('01)]

$$H = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} + V^{(0)}(r) + \frac{1}{m_1}V^{(1,0)}(r) + \frac{1}{m_2}V^{(0,1)}(r) + \frac{1}{m_1^2}V^{(2,0)}(r) + \frac{1}{m_2^2}V^{(0,2)}(r) + \frac{1}{m_1 m_2}V^{(1,1)}(r) + O(1/m^3)$$

- ▷ **Interquark potential $V(r) =$ static potential $V^{(0)}$ + relativistic corrections**

- ▷ **Once $V(r)$ is obtained, one can compute full spectrum and wavefunctions**

- ▷ **These potentials need to be determined nonperturbatively**
  - **Static potential contains a linearly rising term (confinement)**
  - **Relativistic corrections are related to the static potential through Poincaré invariance (Gromes relation, BBMP relations)**

OUR PROJECT

Nonperturbative determination of the interquark potential including relativistic corrections from lattice QCD simulations

We have developed a new method to determine these corrections

▷ $O(1/m)$:

- **first lattice result** [Koma,Koma,Wittig('06)] [QWG4]
- **update [THIS TALK]**

▷ $O(1/m^2)$:

- **spin-dependent potential** [Koma,Koma('07)] [QWG4]
- **spin-independent (velocity-dependent) potential [THIS TALK]**

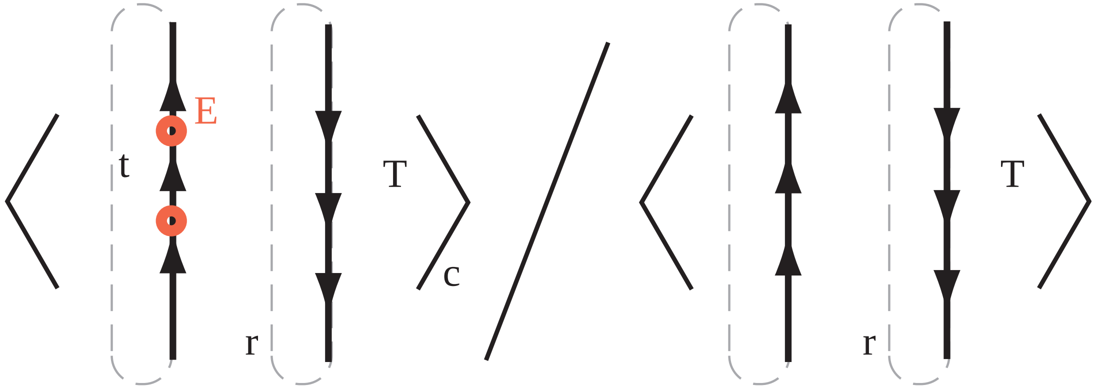
$O(1/m)$ POTENTIAL — DEFINITIONS

- ▷ **Nonperturbative expression** [Brambilla,Pineda,Soto&Vairo('01)]

$$V^{(1)}(r) = -\frac{1}{2}\lim_{\tau'\to\infty}\int_0^{\tau'} dt\; t\,\langle\langle g^2\vec{E}(0,0)\cdot\vec{E}(0,t)\rangle\rangle_c$$

- ▷ **Field strength correlator on the L^3T lattice**

$$\langle\langle g^2 E^i(0,0)\,E^i(0,t)\rangle\rangle_c =$$



We measure this quantity accurately by utilizing the multilevel algorithm and fit it to its spectral representation. The integral is performed analytically.

- ▷ **for all technical details, see** [Koma, Koma, NPB769('07)79]

SIMULATION DETAILS

▷ **Setting of the simulation**

- **Wilson gauge action**
- **Multilevel algorithm, 6 sublattices, 50000 internal updates**

$\beta = 6/g^2$	a	Volume	$N_{\rm conf}$
5.85	0.123 fm	$18^3 24$	100
6.00	0.093 fm	$24^3 32$	45

- **NEC SX8@RCNP Osaka University**

▷ **Electric field strength operator:** $ga^2E^i \equiv (U_{4i} - U^\dagger_{4i})/(2i)$
(traceless, with two-leaf-type modification)

▷ **Huntley-Michael (HM) factor** [Huntley & Michael '87]**:**

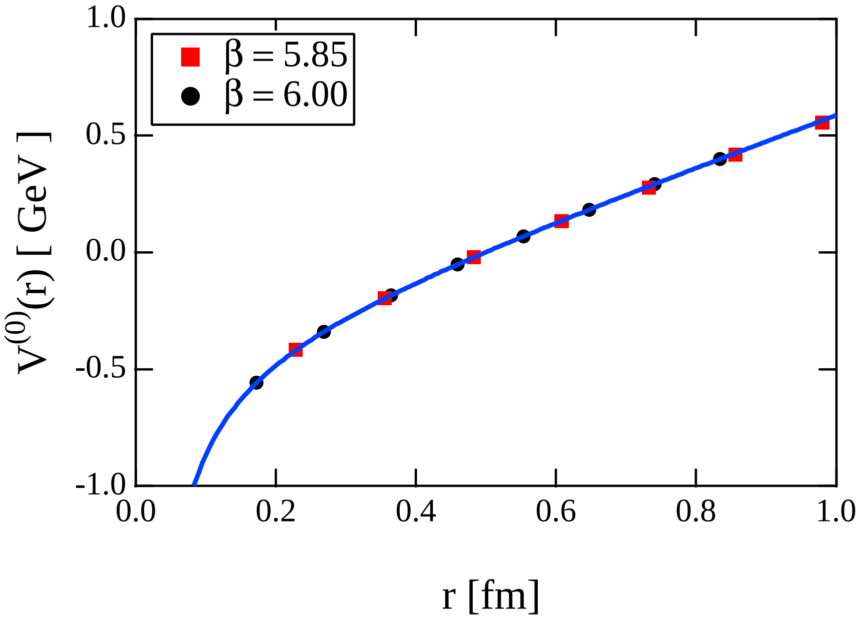
$$Z_{F_{\mu\nu}} = \langle PP^* \rangle / \langle {\rm Re}\, U_{\mu\nu} \rangle_{PP^*}$$

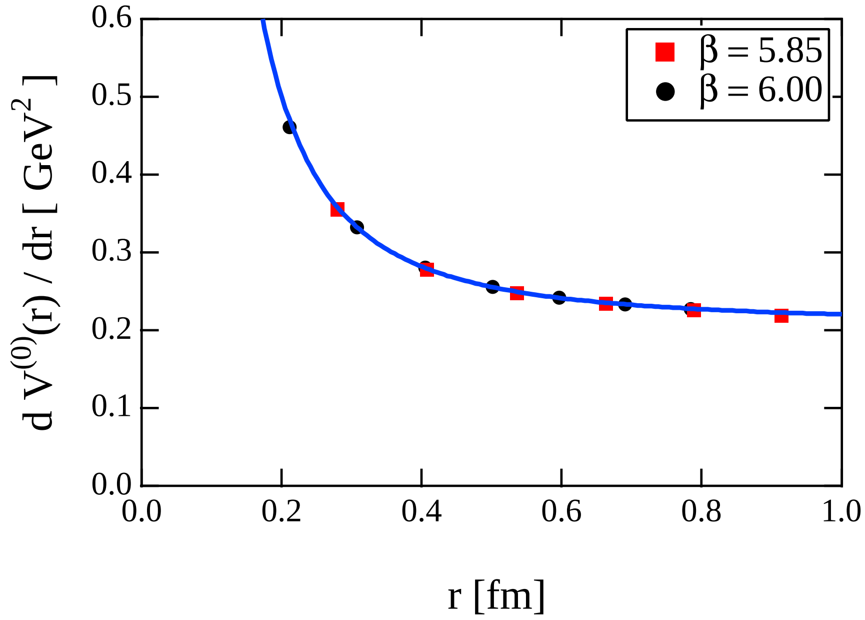
(cancel self energies in field strength correlators at $O(g^2)$)

STATIC POTENTIAL & FORCE

▷ $V^{(0)}(r_I) = -\frac{1}{T}\ln\langle P(0)P(r)^*\rangle + O(e^{-(\Delta E_{10})T})$

▷ $V^{(0)\prime}(\bar{r}) = \frac{1}{a}\{V^{(0)}(r) - V^{(0)}(r-a)\}$

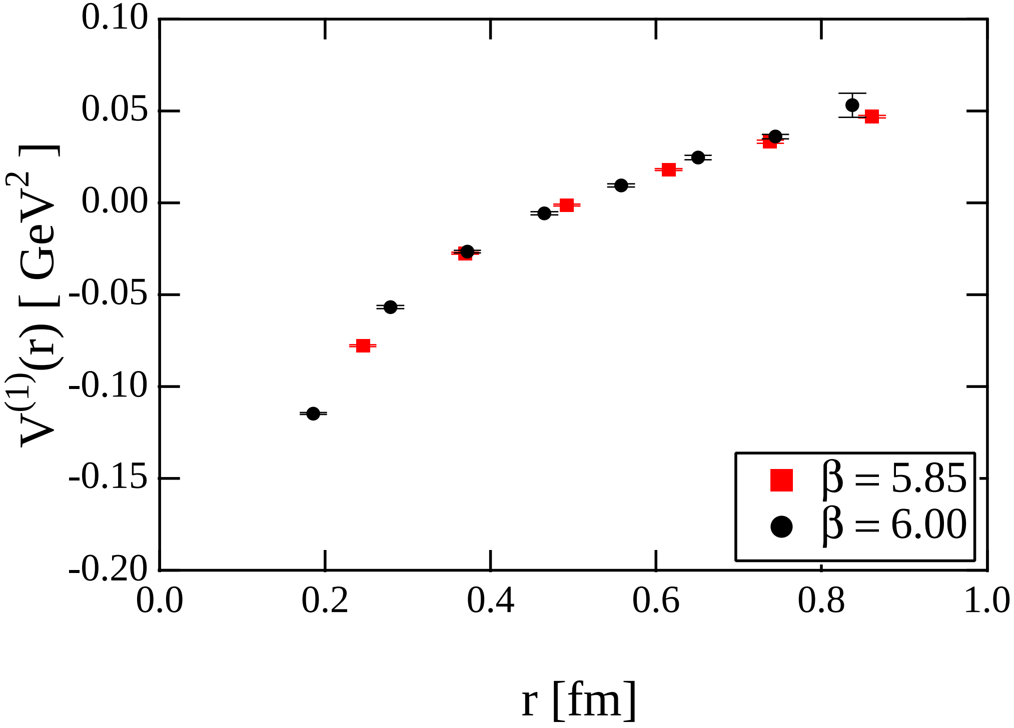


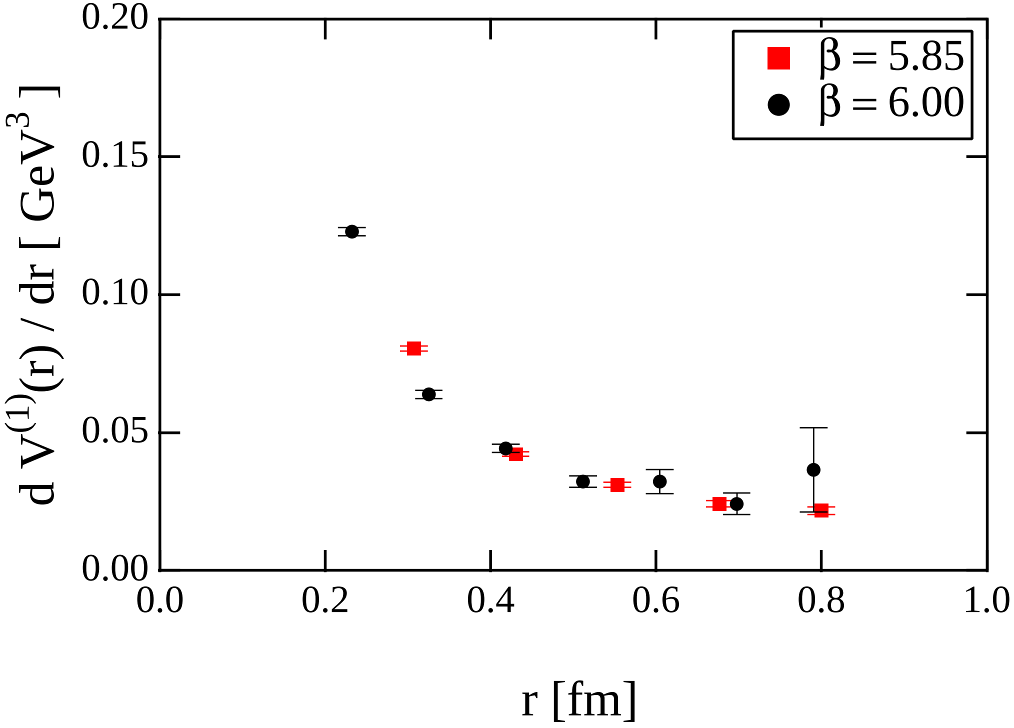


- **Fit to** $V^{(0)\prime}(r) = \sigma + \frac{c}{r^2}$ **(β=6.00)** $\Rightarrow$ $\sigma a^2 = 0.0465(1), c = 0.303(1)$

  $\Rightarrow$ σ**= 1.06[GeV/fm]**

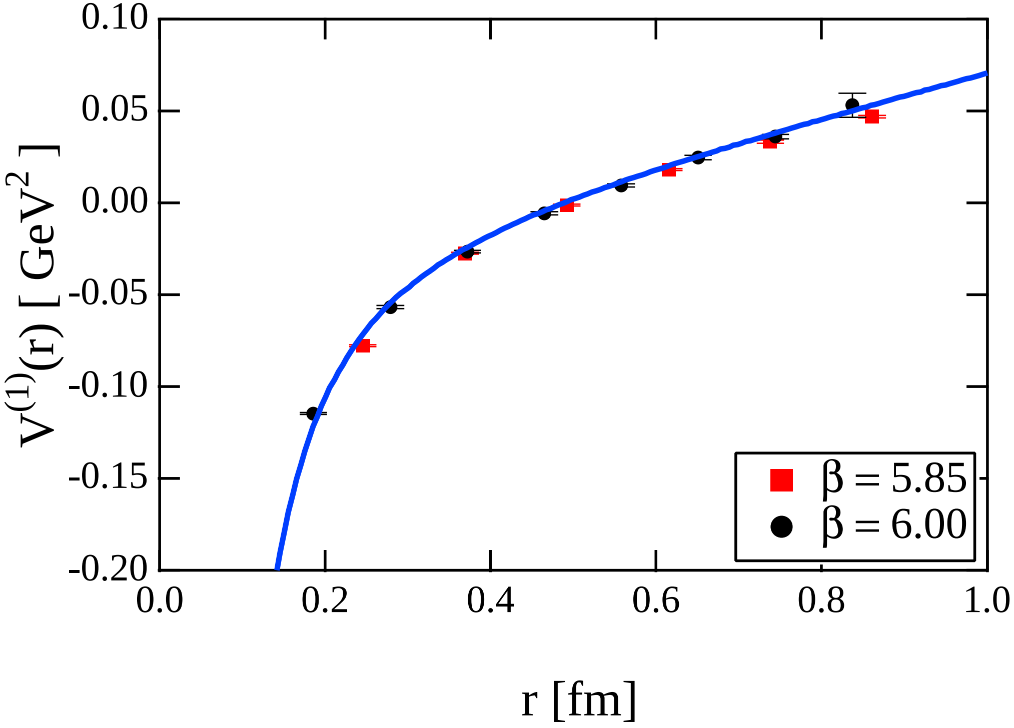
$O(1/m)$ POTENTIAL — RESULTS





- **Potential is normalized at $r = 0.5$ fm**
- **Force:** $V^{(1)\prime}(\bar{r}) = \frac{1}{a}\{V^{(1)}(r) - V^{(1)}(r-a)\}$
- **Good scaling behavior**
- **Linear behavior at long distance (force: non-zero, constant)**

$O(1/m)$ POTENTIAL — RESULTS



Preliminary fit result (β=6.00)

$$V^{(1)}(r) = -\frac{A}{r^2} + Br + C$$

A = 0.090(5), Ba^3=0.024(1), Ca^2=0.389

cf. perturbation theory $\propto 1/r^2$

[Melnikov etal('98), Hoang('99), Brambilla etal('01)]

Correction to the string tension

- **For charmonium (m_c=1.3 GeV)**
  $\delta\sigma = \frac{2B}{m_c} = 0.18(1)$[GeV/fm] = 0.17(1)$\sigma$
- **For bottomonium (m_b=4.7 GeV)**
  $\delta\sigma = \frac{2B}{m_b} = 0.049(2)$[GeV/fm] = 0.046(2)$\sigma$

- **Potential is normalized at $r = 0.5$ fm**
- **Force:** $V^{(1)\prime}(\bar{r}) = \frac{1}{a}\{V^{(1)}(r) - V^{(1)}(r-a)\}$
- **Good scaling behavior**
- **Linear behavior at long distance (force: non-zero, constant)**

$O(1/m^2)$ POTENTIAL — DEFINITIONS

- **Spin-dependent potential** [Koma, Koma, NPB769('07)79] [QWG4]

- **Spin-independent potential** [Pineda&Vairo('01)]

$$V_{\rm SI} = \frac{1}{m_1^2}\left(\frac{1}{2}\left\{p_1^2, V_{p^2}^{(2,0)}(r)\right\} + \frac{V_{l^2}^{(2,0)}(r)}{r^2} l_1^2 + V_r^{(2,0)}(r)\right) + (1 \to 2)$$

$$+\frac{1}{m_1 m_2}\left(-\frac{1}{2}\left\{p_1 \cdot p_2, V_{p^2}^{(1,1)}(r)\right\} + \frac{V_{l^2}^{(1,1)}(r)}{2r^2}(l_1 \cdot l_2 + l_2 \cdot l_1) + V_r^{(1,1)}(r)\right)$$

- **Velocity-dependent potentials** V_b, V_c, V_d, V_e

$$V_{p^2}^{(2,0)} = V_d - \frac{2}{3}V_e, \quad V_{l^2}^{(2,0)} = V_e, \quad V_{p^2}^{(1,1)} = -V_b + \frac{2}{3}V_c, \quad V_{l^2}^{(1,1)} = -V_c$$

VELOCITY DEPENDENT POTENTIALS — DEFINITIONS

▷ **Nonperturbative expression**

$$V_b(r) = -\frac{1}{3}\int_0^\infty dt\, t^2 \,\langle\langle g^2 \vec{E}(\vec{0},0)\cdot\vec{E}(\vec{r},0)\rangle\rangle_c$$

$$V_d(r) = \frac{1}{6}\int_0^\infty dt\, t^2 \,\langle\langle g^2 \vec{E}(\vec{0},0)\cdot\vec{E}(\vec{0},0)\rangle\rangle_c$$

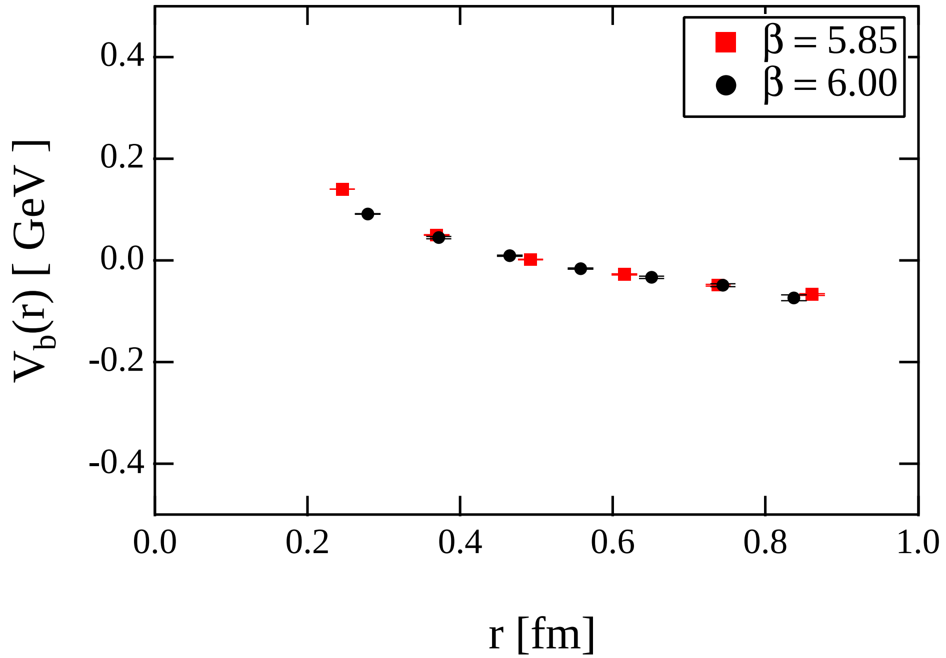
$$\left(\frac{r_i r_j}{r^2} - \frac{\delta_{ij}}{3}\right) V_c(r) = \int_0^\infty dt\, t^2 \left[\langle\langle g^2 E^i(\vec{0},0)\, E^j(\vec{r},0)\rangle\rangle_c - \frac{\delta_{ij}}{3}\langle\langle g^2 \vec{E}(\vec{0},0)\cdot\vec{E}(\vec{r},0)\rangle\rangle_c\right]$$

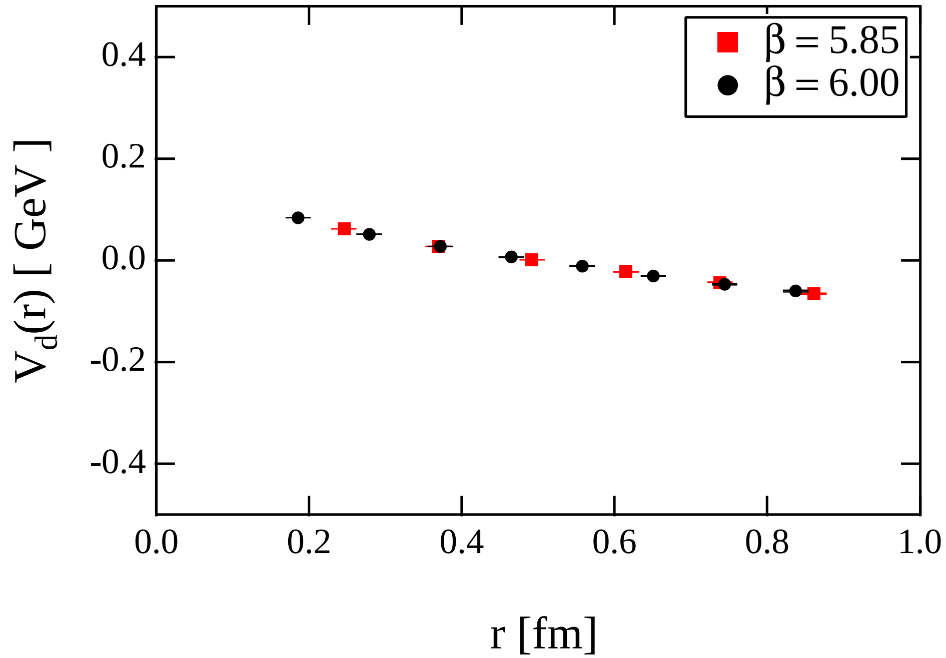
$$\left(\frac{r_i r_j}{r^2} - \frac{\delta_{ij}}{3}\right) V_e(r) = -\frac{1}{2}\int_0^\infty dt\, t^2 \left[\langle\langle g^2 E^i(\vec{0},0)\, E^j(\vec{0},0)\rangle\rangle_c - \frac{\delta_{ij}}{3}\langle\langle g^2 \vec{E}(\vec{0},0)\cdot\vec{E}(\vec{0},0)\rangle\rangle_c\right]$$

▷ **We compute these corrections from the field strength correlators**

VELOCITY DEPENDENT POTENTIALS — RESULTS

▷ **V_b and V_d**

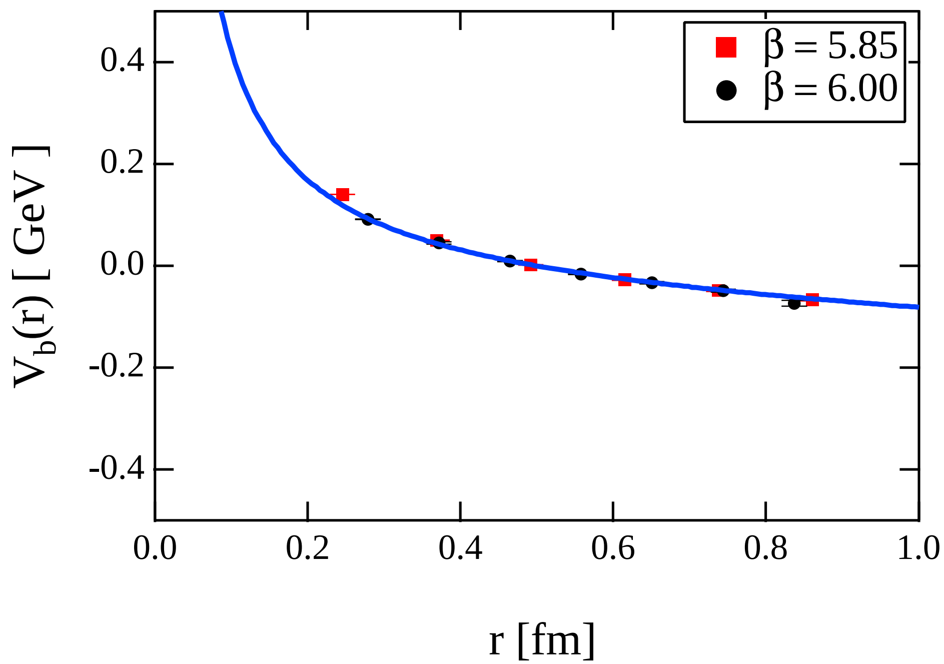


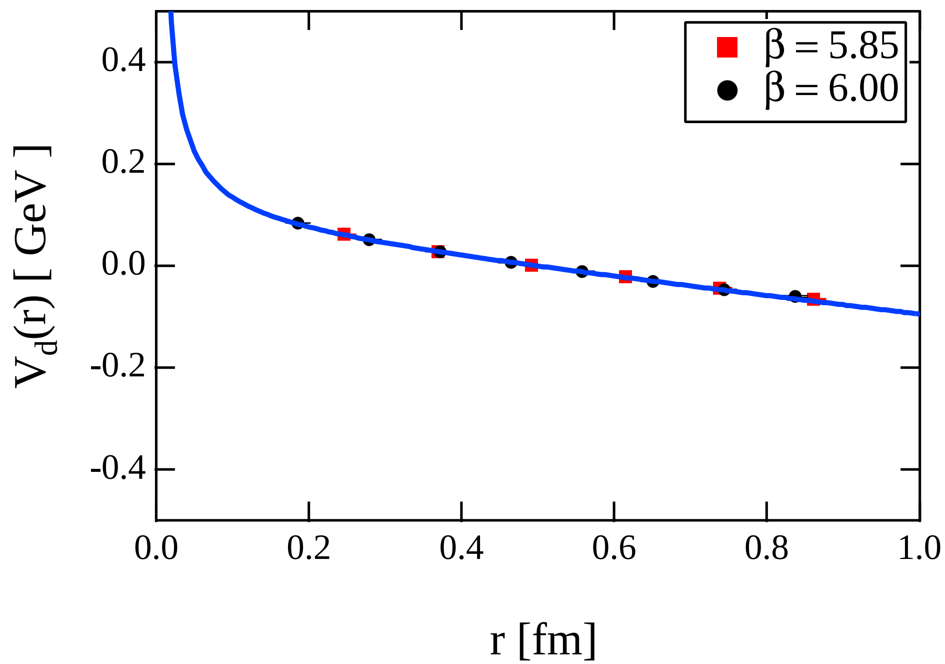


- **clean data up to 0.9 fm**
- **normalized at $r = 0.5$ fm**
- **good scaling behavior**

VELOCITY DEPENDENT POTENTIALS — RESULTS

▷ V_b and V_d with fit function $V(r) = -\dfrac{A}{r} + Br + C$





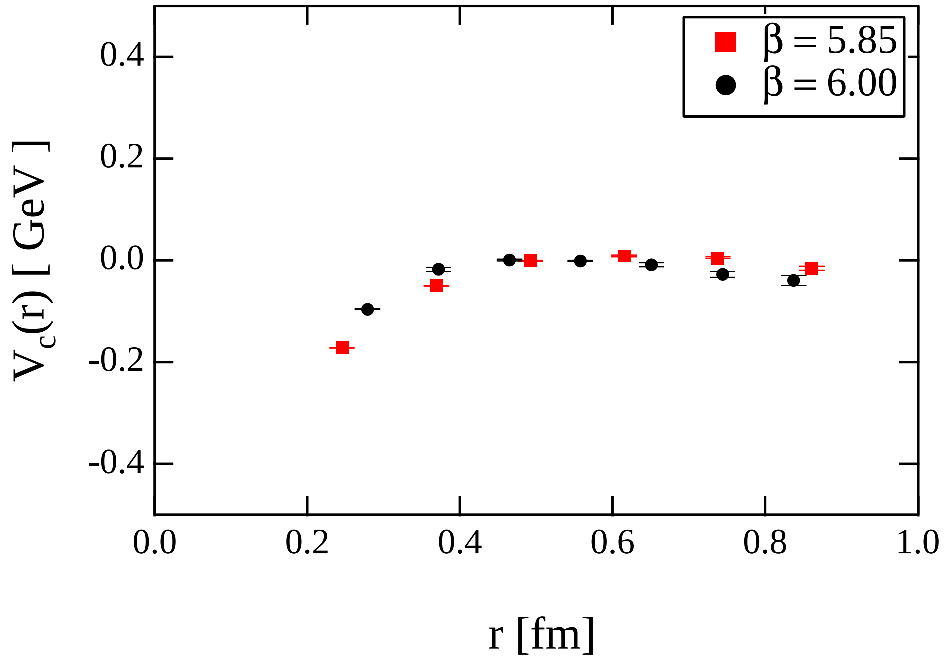
- clean data up to 0.9 fm
- normalized at $r = 0.5$ fm
- good scaling behavior

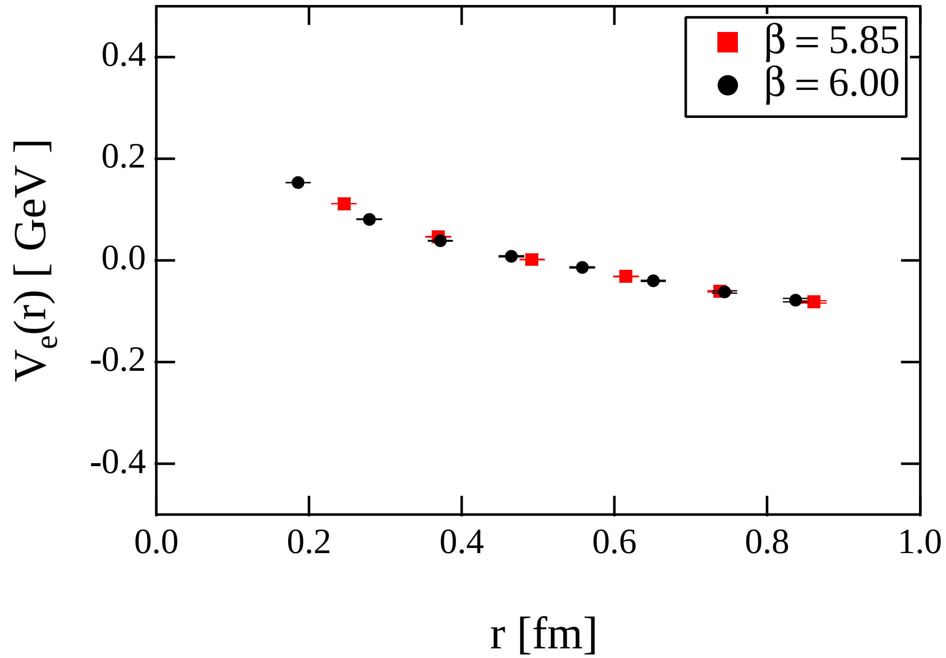
Preliminary fit result (β=6.00)

	A	Ba^2	Ca
V_b :	$-0.25(2)$	$-0.003(1)$	$-0.08(1)$
V_d :	$-0.042(4)$	$-0.0076(3)$	$-0.187(2)$

VELOCITY DEPENDENT POTENTIALS — RESULTS

▷ V_c and V_e





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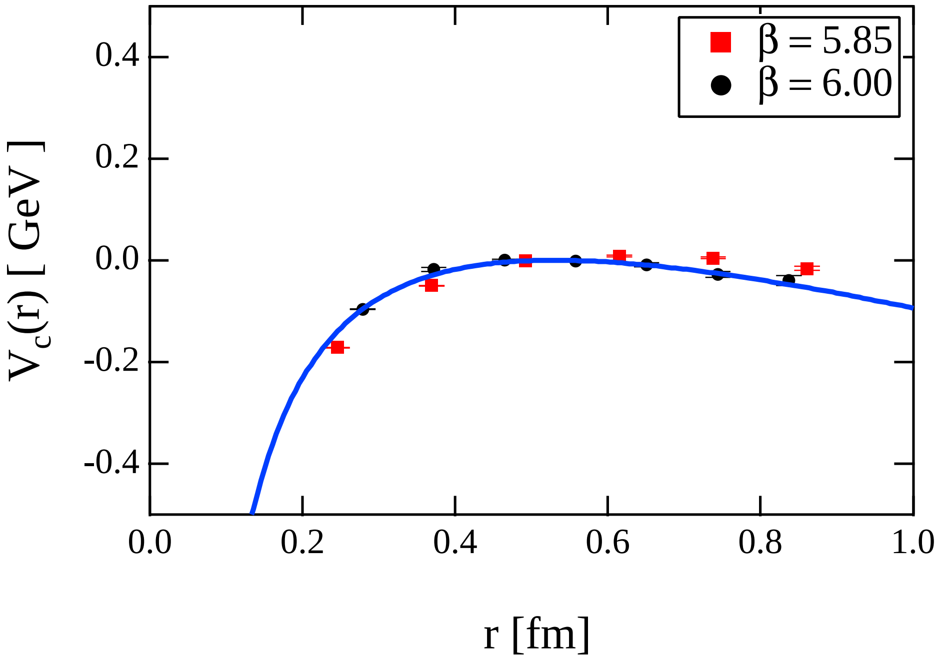
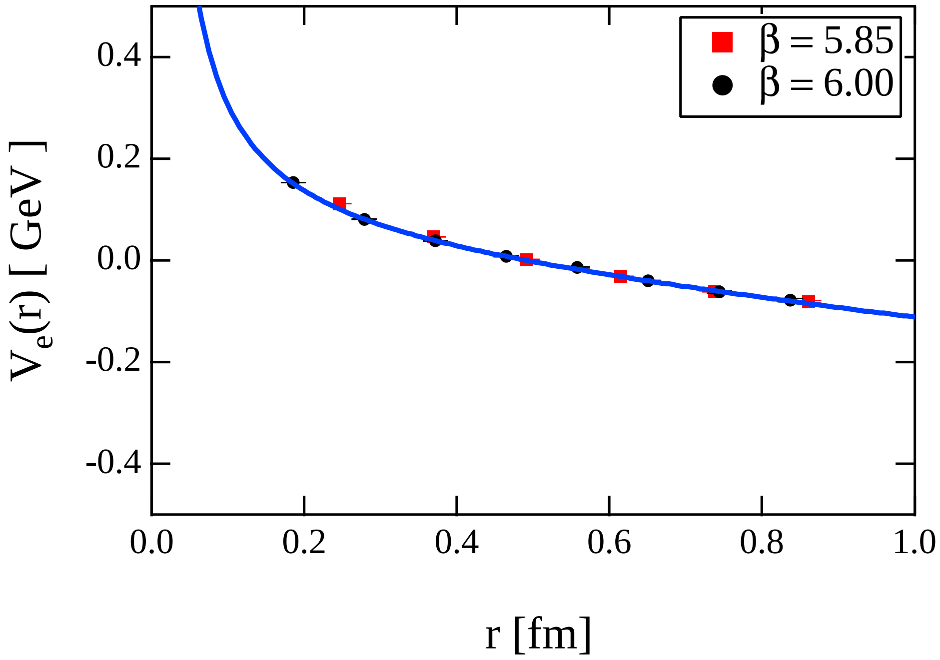




- clean data up to 0.9 fm
- normalized at $r = 0.5$ fm
- good scaling behavior

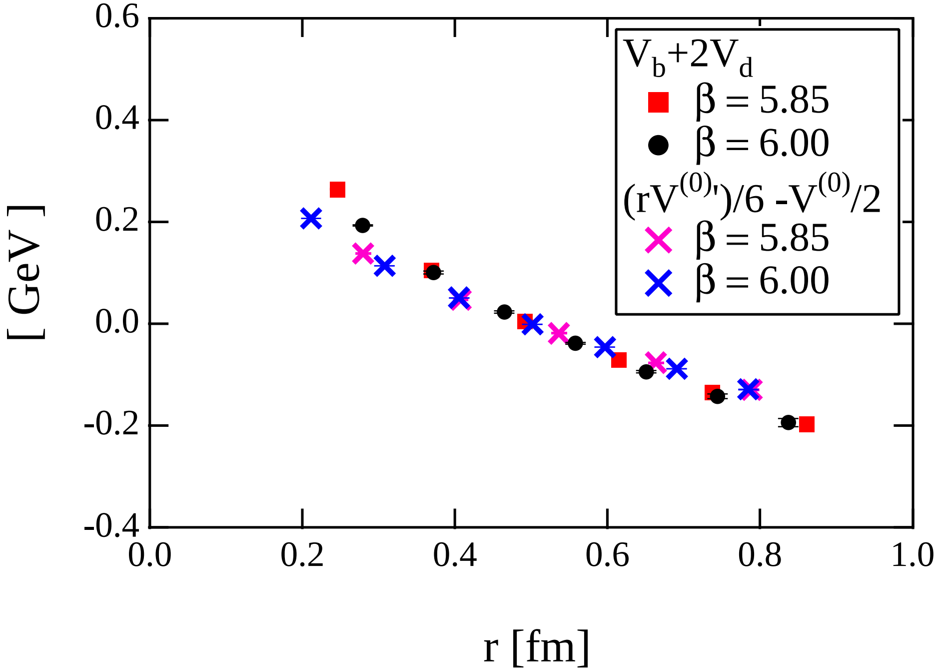
Preliminary fit result (β=6.00)

	A	Ba^2	Ca
V_c :	$0.61(4)$	$-0.019(2)$	$0.08(1)$
V_e :	$-0.156(4)$	$-0.0069(3)$	$-0.017(2)$

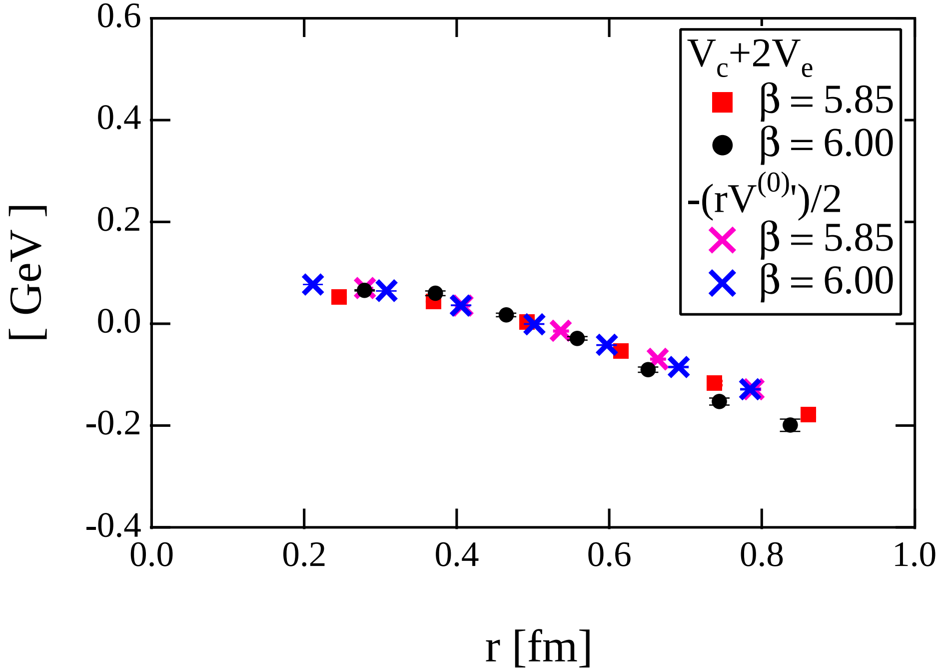
BBMP RELATIONS

▷ **BBMP relation is derived from Poincaré invariance of field strength correlators (in the continuum limit)** [Barchielli,Brambilla,Montaldi&Prosperi('88,90)]

$$V_b + 2V_d = -\frac{1}{2}V^{(0)} + \frac{r}{6}V^{(0)\prime},$$



$$V_c + 2V_e = -\frac{r}{2}V^{(0)\prime}$$



- **normalized at $r = 0.5$ fm**

SUMMARY

▷ **We have investigated the relativistic corrections to the heavy quark potential at $O(1/m)$ and $O(1/m^2)$**

Current observation. . .

▷ **Measured at 0.2 $\lesssim r \lesssim$ 0.9 fm**

▷ **Good scaling behavior**

▷ **For $V^{(1)}$**

- **Linearly rising behavior at $r \gtrsim$ 0.6 fm**
- **A few to 17 percent correction to the string tension $\Longrightarrow$ flavor dependent**

▷ **For velocity-dependent potentials**

- **Parametrization with "1/r + linear + constant" function seems to work**
- **BBMP relation, satisfied**

OUTLOOK

- ▷ **Simulation with a finer lattice, ongoing**
- ▷ **Update of spin-dependent potentials, ongoing**
- ▷ **Comparison with models and phenomenology, to be done**
- ▷ **Renormalization procedure for the field strength operator, to be improved for better scaling behavior**

$V^{(1)}(R)$

fit result

- $V(r) = V^{(0)}(r) + \frac{2}{m}V^{(1)}(r) + O(\frac{1}{m^2})$

  $V^{(0)}_{\text{fit}}(r) = -\frac{c}{r} + \sigma r + \mu \quad \Rightarrow \quad c = 0.297(1)$

- $V^{(1)}_{\text{fit}}(r) = -\frac{c'}{r} + \mu'$

  $\Rightarrow ac' = 0.081(4),\ a^2\mu' = 0.417(1)$

  For $m_c = 1.3$ GeV

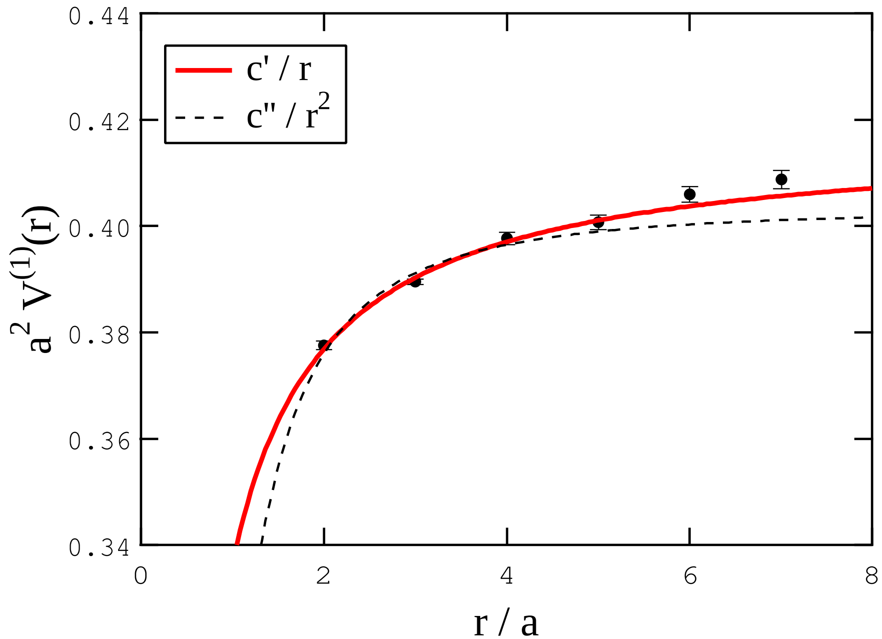
  $\Rightarrow 2c'/m_c = 0.26(1)$

  For $m_b = 4.7$ GeV

  $\Rightarrow 2c'/m_c = 0.073(4)$

  cf. perturbation theory $\propto 1/r^2$

  [Melnikov etal('98), Hoang('99), Brambilla etal('01)]



[Koma,Koma&Wittig,PRL97('06)]

$O(1/m)$ POTENTIAL — DEFINITIONS

▷ **Non-perturbative expression** [Brambilla,Pineda,Soto&Vairo('01)]

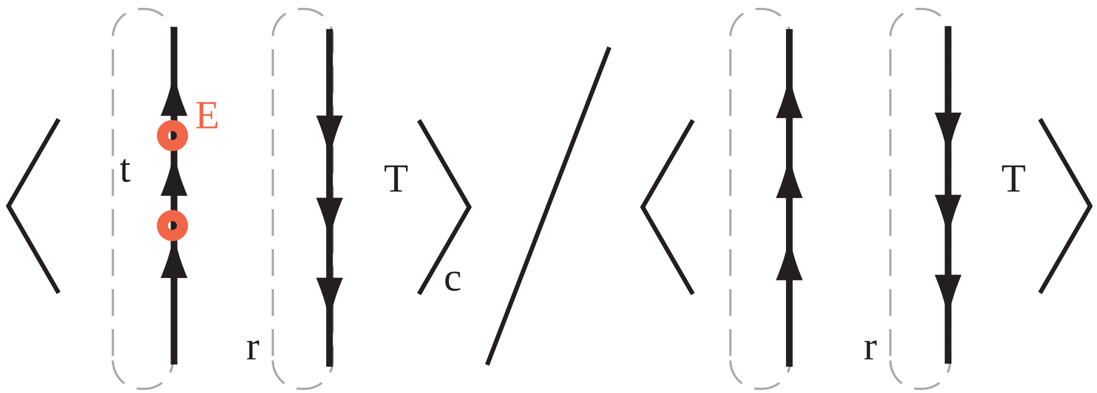
$$V^{(1)}(r) = -\frac{1}{2}\lim_{\tau'\to\infty}\int_0^{\tau'} dt\; t\langle\langle g\vec{E}(0,0)\cdot g\vec{E}(0,t)\rangle\rangle_c$$

$$= -\frac{1}{2}\sum_{n=1}^{\infty}\frac{|\langle 0(r)|g\vec{E}(0)|n(r)\rangle|^2}{(\Delta E_{n0}(r))^2}\,,\qquad \textbf{(Spectral representation)}$$

where $\hat{T}|n(r)\rangle = e^{-aE_n(r)}|n(r)\rangle$, $\Delta E_{n0}(r) = E_n(r) - E_0(r)$, $\quad E_0(r) = V_0^{(0)}(r)$

▷ **Field strength correlator on the L^3T lattice**

$$\langle\langle gE^i(0,0)\,gE^i(0,t)\rangle\rangle_c =$$



We measure this quantity accurately by utilizing the multilevel algorithm

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- ▷ **Field strength correlator on the L^3T lattice**

$$\langle\langle gE^i(0,0)\, gE^i(0,t)\rangle\rangle_c =$$

$$\sum_{n>0}\left(2|\langle 0(r)|gE^i(0)|n(r)\rangle|^2 e^{-\Delta E_{n0}(r)\frac{T}{2}}\cosh\left(\Delta E_{n0}(r)(\frac{T}{2}-t)\right)\right) + O(e^{-\Delta E_{10}(r)T})$$

We measure this quantity very accurately by utilizing the multilevel algorithm and fit it with its spectral representation

MULTILEVEL ALGORITHM

▷Modified version for PLCF with two field strength operators

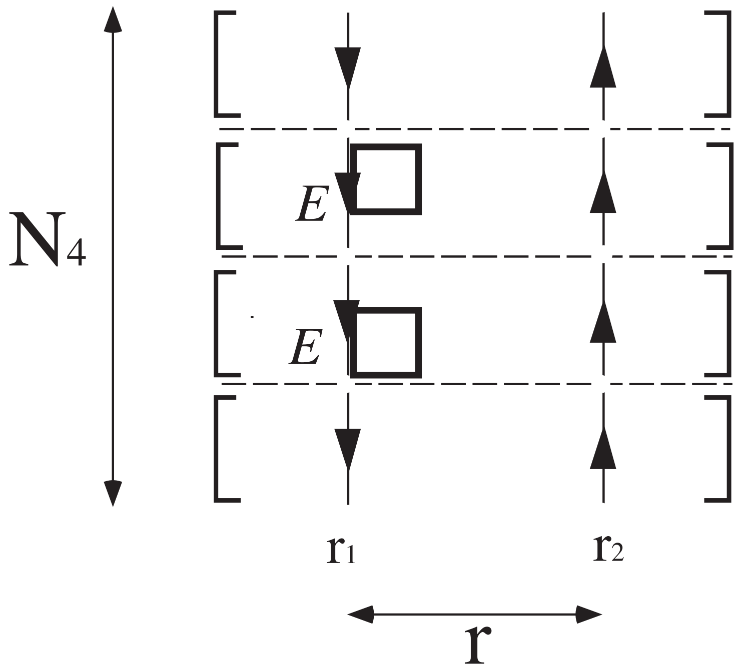
(1) Compute the component of the Polyakov loops with the field strength insertion in each time slice

(2) Compute sublattice correlators

(3) Take average of sublattice correlators through internal update (iupd) (Large memory is required)

(4) Construct correlation functions from sublattice correlators.
(Average over all spatial points, all possible combinations of two fields insertion for given τ)

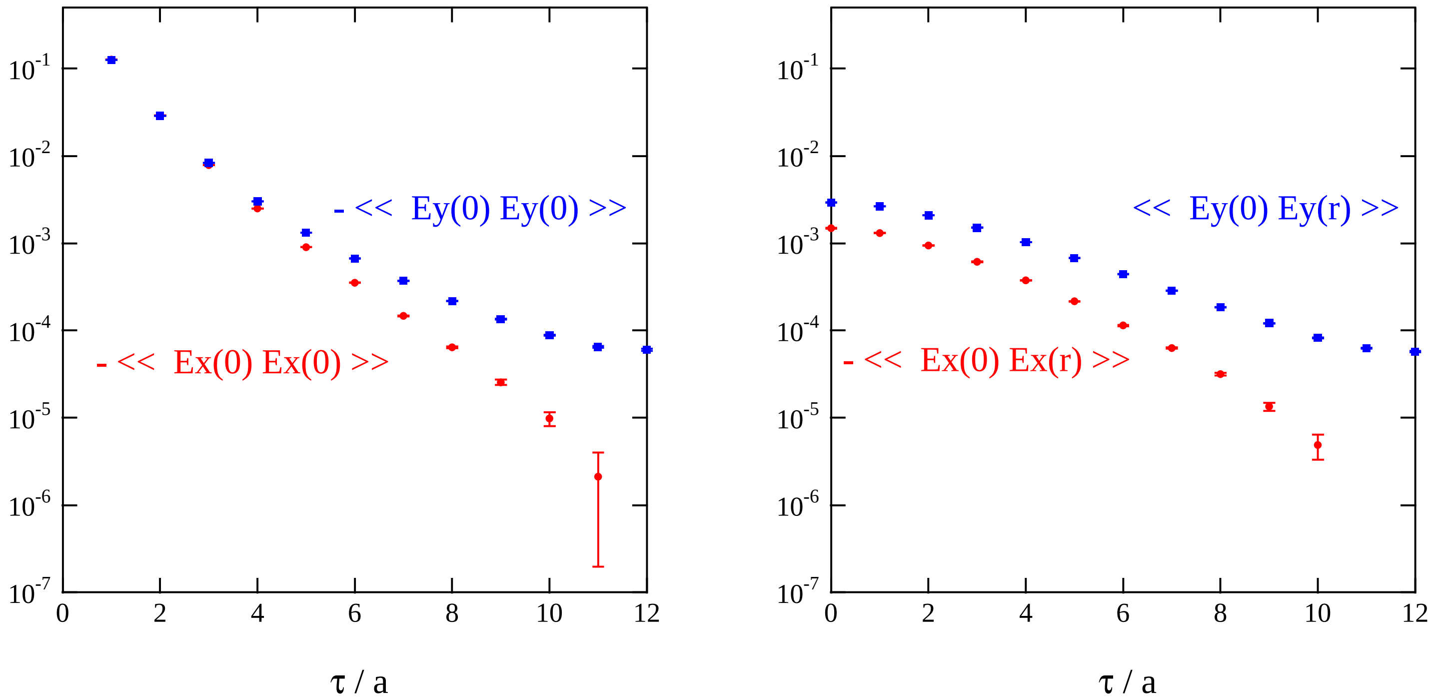
$\Longrightarrow$ **Measurement from 1 conf.**



FSC can be measured with high accuracy through the product of stabilized sublattice correlators

FIELD STRENGTH CORRELATORS

▷ **e.g.)** $r/a = 5$ **at** $\beta = 5.85$ **on the** $18^3 24$ **lattice**



- **statistical errors are quite small**

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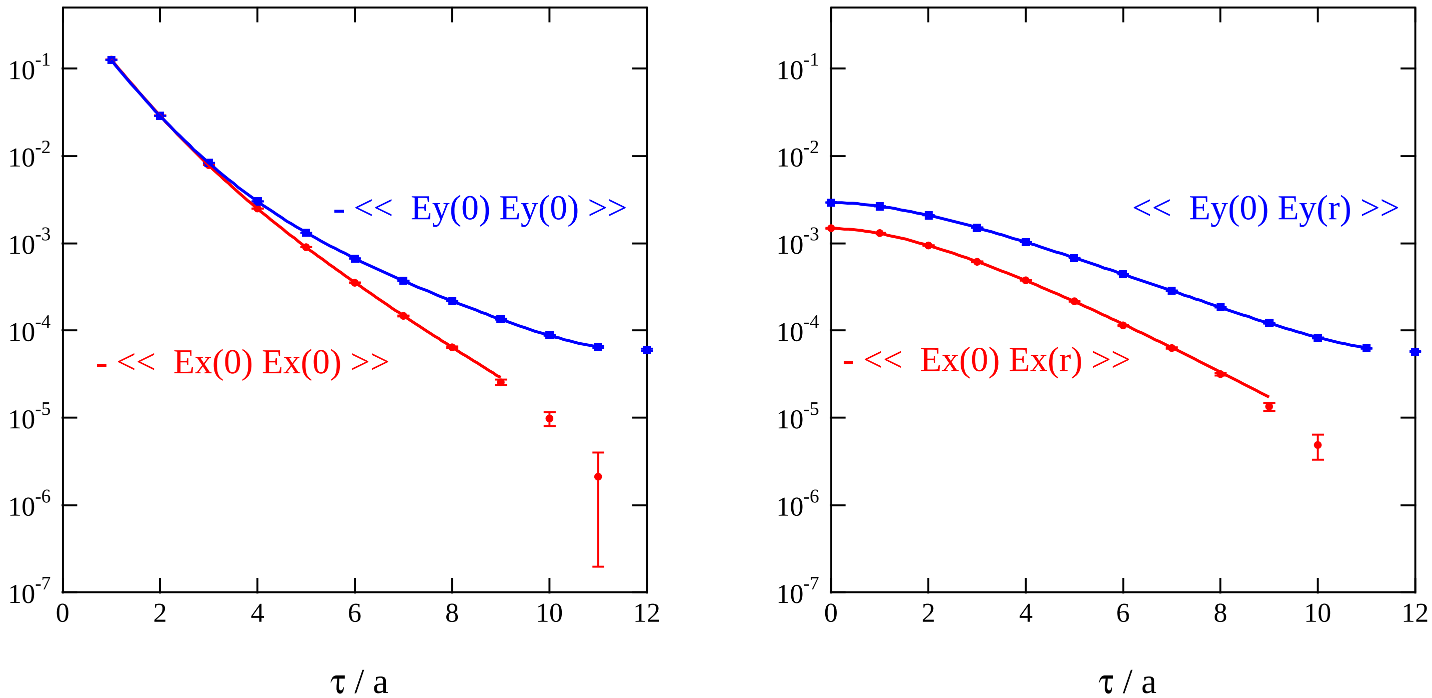
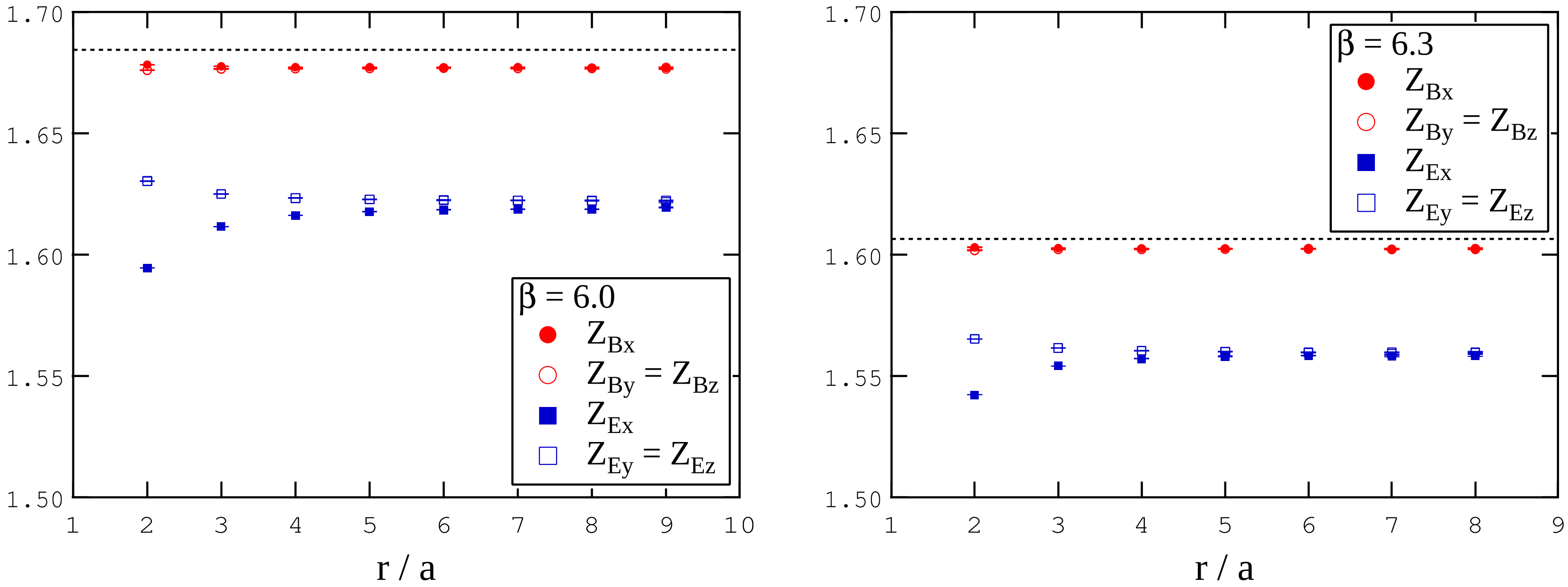


- **statistical errors are quite small**
- **fitting to the spectral rep. of the FSC works nicely**

HUNTLEY-MICHAEL FACTOR

▷ **Huntley-Michael factor** $Z_{F_{\mu\nu}} = \langle PP^* \rangle / \langle \mathrm{Re}U_{\mu\nu} \rangle_{PP^*}$



- **dependence on r and relative orientation to the q-$\bar{q}$ axis,** $\vec{r} = (r, 0, 0)$