Free energy of static quarks in lattice QCD

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in collaboration with

K. Petrov, P. Petreczky, F. Zantow

RBC-Bielefeld collaboration

O. Kaczmarek, arXiv:0710.0498 [hep-lat]

RBC-Bielefeld, arXiv:0710.0354 [hep-lat]

Heavy quark bound states above deconfinement

Strong interactions in the deconfined phase $T \gtrsim T_c$

Possibility of heavy quark bound states?

Suppression patterns of charmonium/bottomonium

Charmonium ($\chi_c, J/\psi$) as thermometer above T_c

$\Longrightarrow$ Potential models

$\rightarrow$ heavy quark potential (T=0)

$$V_1(r) = -\frac{4}{3}\frac{\alpha(r)}{r} + \sigma\, r$$

$\rightarrow$ heavy quark free energies ($T > T_c$)

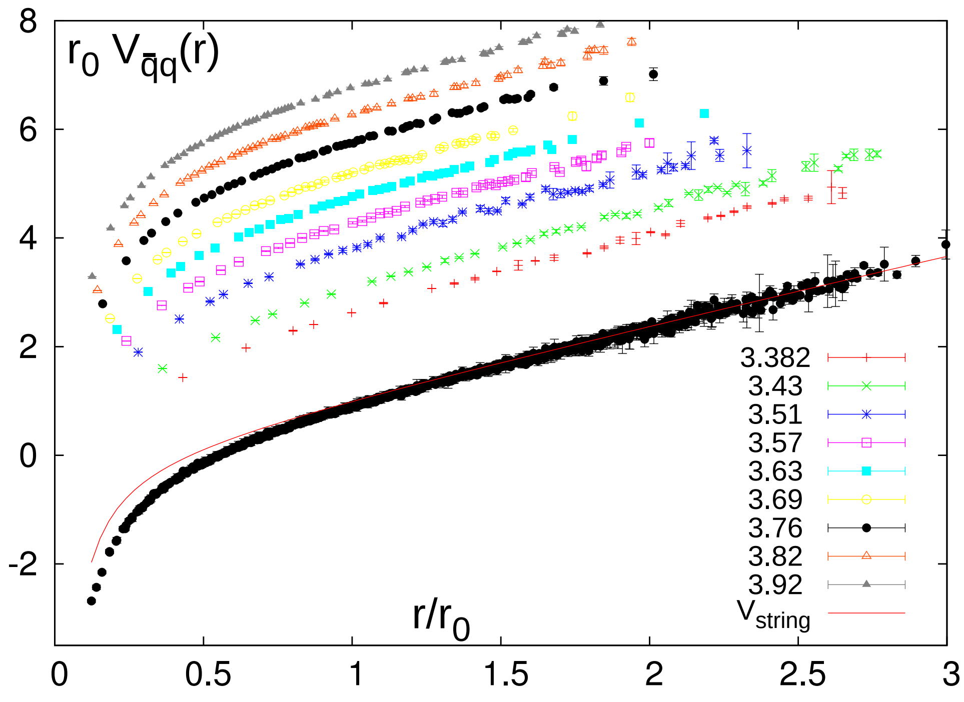
$$F_1(r,T) \simeq -\frac{4}{3}\frac{\alpha(r,T)}{r}\, e^{-m(T)r}$$

$\rightarrow$ heavy quark internal energies ($T \neq 0$)

$$F_1(r,T) = U_1(r,T) - T\, S_1(r,T)$$

$\Longrightarrow$ Charmonium correlation functions/spectral functions

Zero temperature potential - n_f=2+1



2+1 flavor QCD

highly improved p4-staggered

almost realistic quark masses

$m_\pi \simeq 220$ MeV, physical m_s

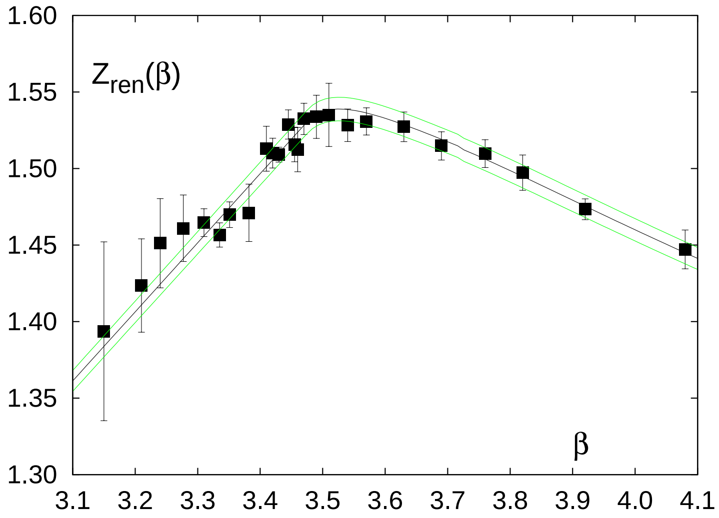
Large distance behaviour

consistent with string model prediction:

$$V(r) = -\frac{\pi}{12r} + \sigma r \quad , \text{ for large } r$$

$\longrightarrow$ used for renormalization

renormalization: $V_{T=0}(r) = -\log\left((Z_{ren}(\beta))^2 \frac{W(r,\tau)}{W(r,\tau+1)}\right)$

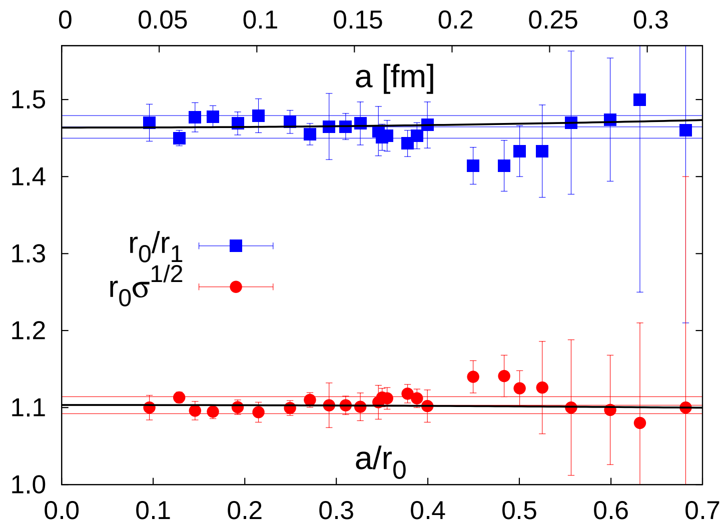


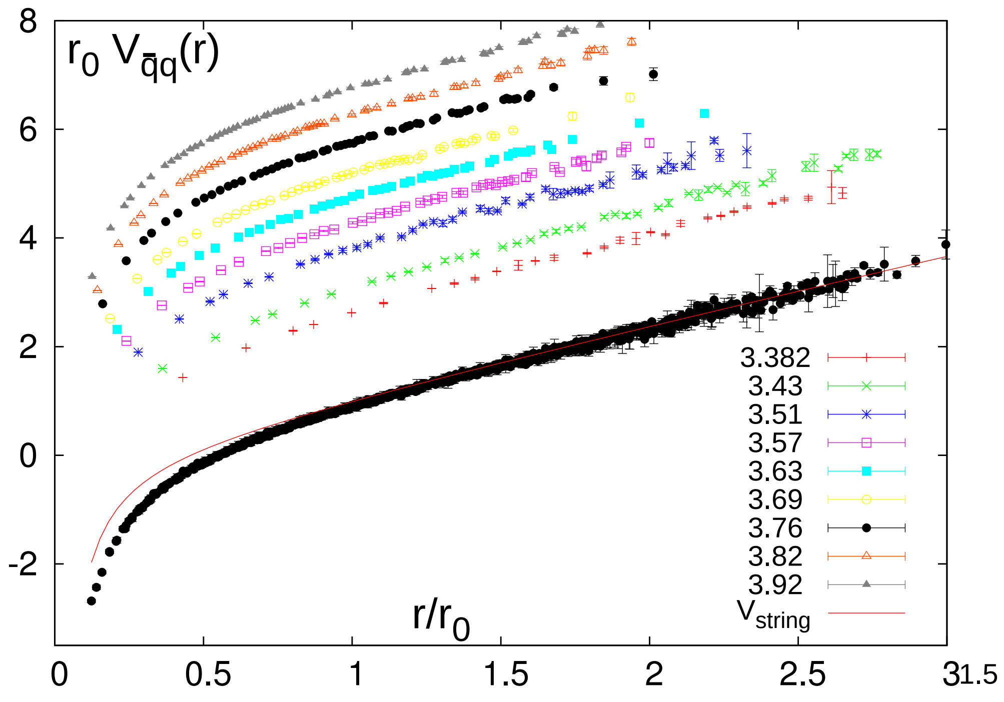
$$r^2 \frac{dV_{\bar{q}q}(r)}{dr}\bigg|_{r=r_0} = 1.65$$

$$r^2 \frac{dV_{\bar{q}q}(r)}{dr}\bigg|_{r=r_1} = 1.0$$

$$(r_0 = 0.469(7) \text{ fm})$$

cut-off effects are small





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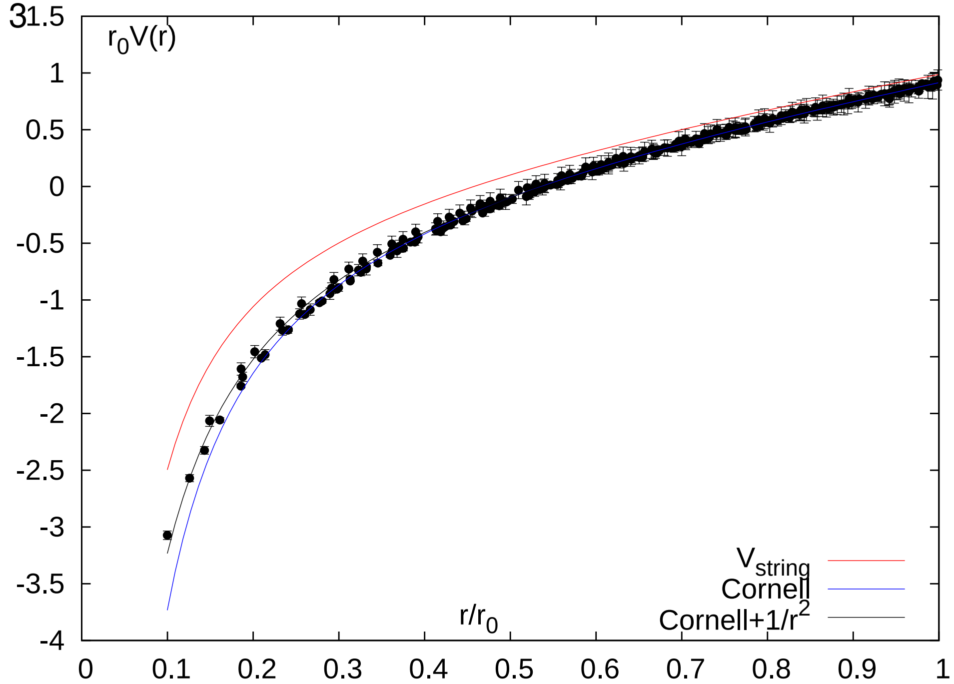
Short distance behaviour

deviations at small r

enhancement of the running coupling

fit: $V(r) = -0.392(6)/r + \sigma r$

r-dependent running coupling $\alpha(r)$

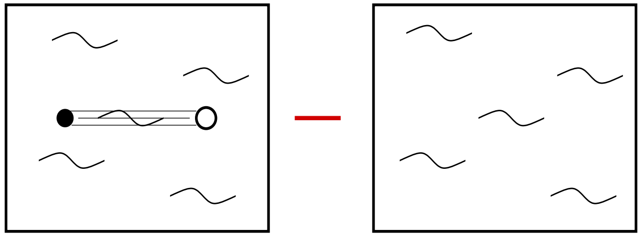


Polyakov loop correlation function and free energy:

L. McLerran, B. Svetitsky (1981)

$$\frac{\mathcal{Z}_{\mathbf{Q\bar{Q}}}}{\mathcal{Z}(\mathbf{T})} \simeq \frac{1}{\mathcal{Z}(\mathbf{T})}\int \mathcal{D}\mathbf{A}\ldots\,\mathbf{L}(\mathbf{x})\,\mathbf{L}^{\dagger}(\mathbf{y})\,\exp\left(-\int_{\mathbf{0}}^{\mathbf{1/T}}\mathbf{dt}\int \mathbf{d^3x}\mathcal{L}\left[\mathbf{A},\ldots\right]\right)$$

$\log() \Longrightarrow$



$$= \; -\frac{\mathbf{F_{Q\bar{Q}}(r,T)}}{\mathbf{T}}$$

$$\mathbf{Q\bar{Q}} = \mathbf{1},\ \mathbf{8},\ \mathbf{av}$$

Lattice data used in our analysis:

$\mathbf{N_f = 0}$:
$32^3 \times 4, 8, 16$-lattices
(*Symanzik*)

O. Kaczmarek,
F. Karsch,
P. Petreczky,
F. Zantow (2002, 2004)

$\mathbf{N_f = 2}$:
$16^3 \times 4$-lattices
(*Symanzik, p4-stagg.*)
hybrid-R

$m_\pi/m_\rho \simeq 0.7$ $(m/T = 0.4)$

O. Kaczmarek, F. Zantow (2005),
O. Kaczmarek et al. (2003)

$\mathbf{N_f = 3}$:
$16^3 \times 4$-lattices
(*stagg., Asqtad*)
hybrid-R

$m_\pi/m_\rho \simeq 0.4$

P. Petreczky,
K. Petrov (2004)

$\mathbf{N_f = 2+1}$:
$24^4 \times 6$-lattices
(*Symanzik, p4fat3*)
RHMC

$m_\pi \simeq 220$ **MeV, phys.** m_s

OK, RBC-Bielefeld
arXiv:0710.0498,0710.03

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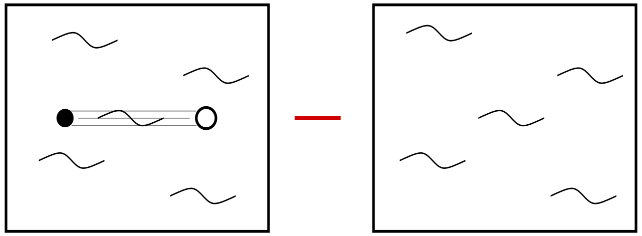
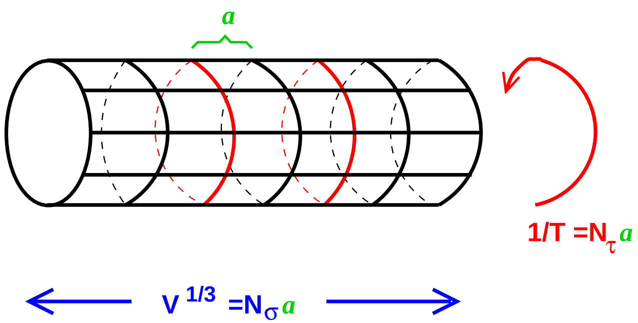


$= \; -\frac{\mathbf{F_{Q\bar{Q}}(r,T)}}{\mathbf{T}}$

$\mathbf{Q\bar{Q}} = \mathbf{1},\ \mathbf{8},\ \mathbf{av}$

O. Philipsen (2002)
O. Jahn, O. Philipsen (2004)

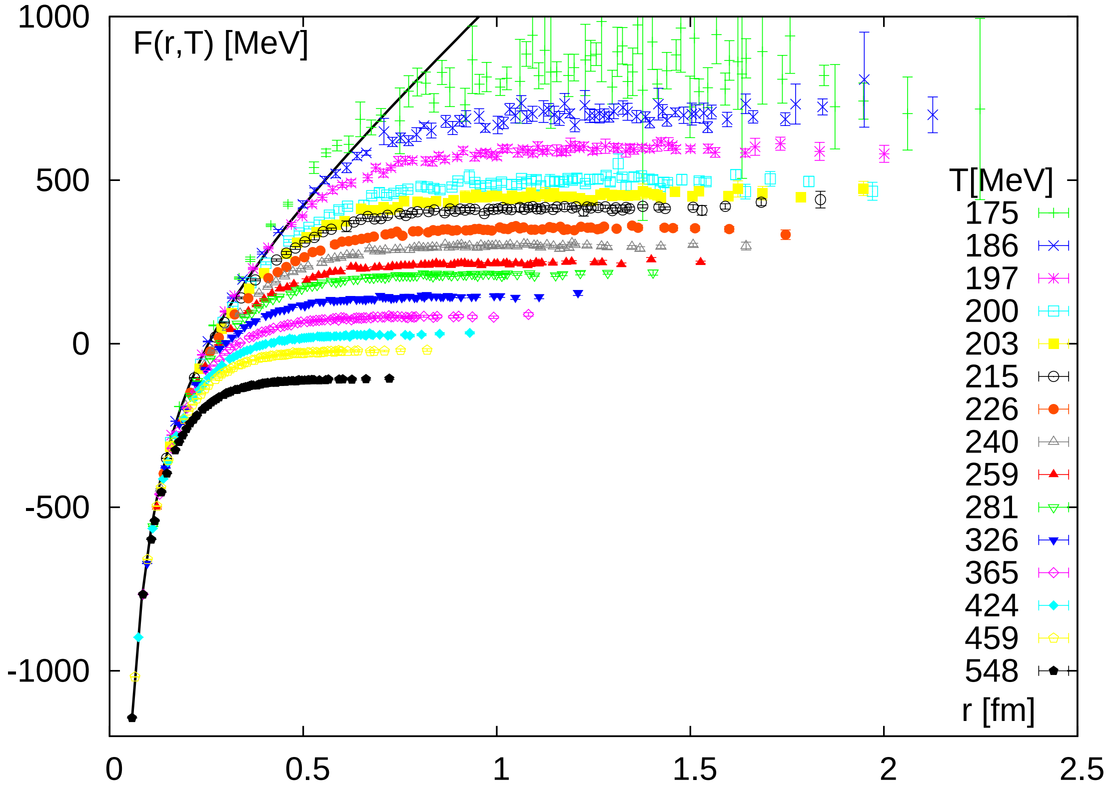


$$-\ln\left(\langle\tilde{\mathrm{Tr}}L(\mathbf{x})\tilde{\mathrm{Tr}}L^{\dagger}(\mathbf{y})\rangle\right) = \frac{F_{\bar{q}q}(r,T)}{T}$$

$$-\ln\left(\langle\tilde{\mathrm{Tr}}L(\mathbf{x})L^{\dagger}(\mathbf{y})\rangle\right)\Big|_{GF} = \frac{F_1(r,T)}{T}$$

$$-\ln\left(\frac{9}{8}\langle\tilde{\mathrm{Tr}}L(\mathbf{x})\tilde{\mathrm{Tr}}L^{\dagger}(\mathbf{y})\rangle - \frac{1}{8}\langle\tilde{\mathrm{Tr}}L(\mathbf{x})L^{\dagger}(\mathbf{y})\rangle\Big|_{GF}\right) = \frac{F_8(r,T)}{T}$$

Heavy quark free energy - 2+1-flavors



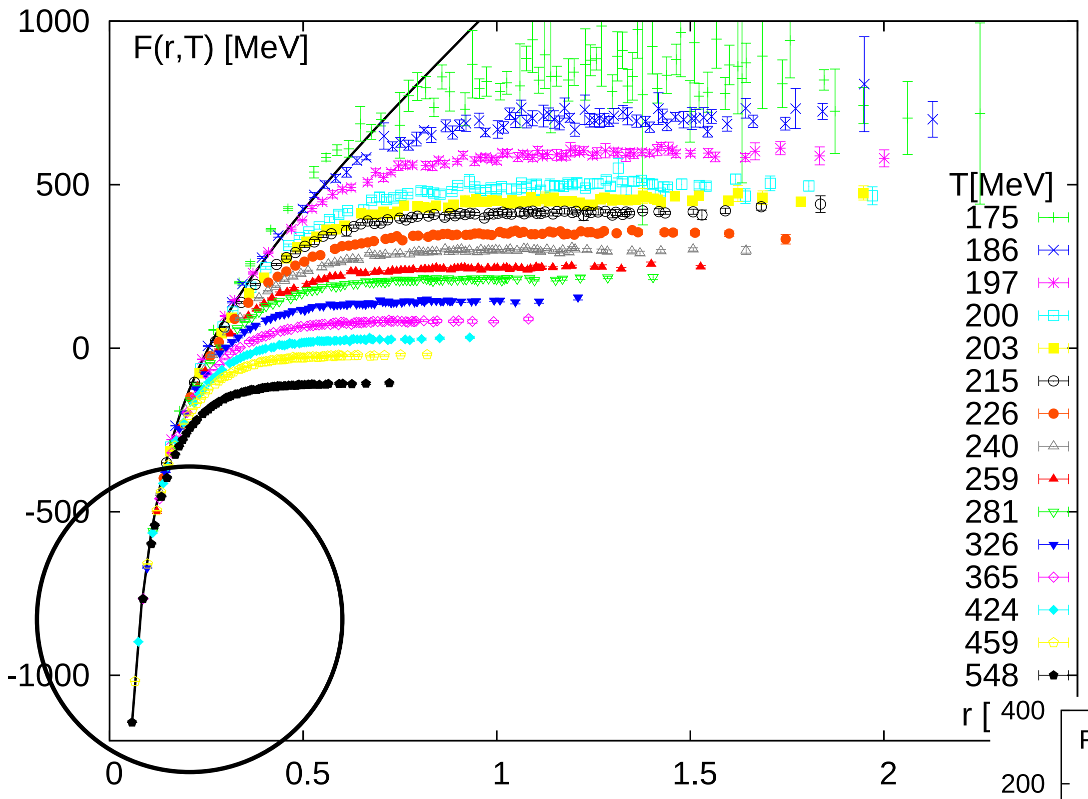
Renormalization of $F(r,T)$

using $Z_{ren}(g^2)$ obtained at T=0

$$e^{-F_1(r,T)/T} = \left(Z_r(g^2)\right)^{2N_\tau} \langle \mathrm{Tr}\ (L_x L_y^\dagger)\rangle$$

alternative renormalization procedures *(PLB543(2002)41, PoS(Lattice2007)195)* all equivalent

Heavy quark free energy - 2+1-flavors



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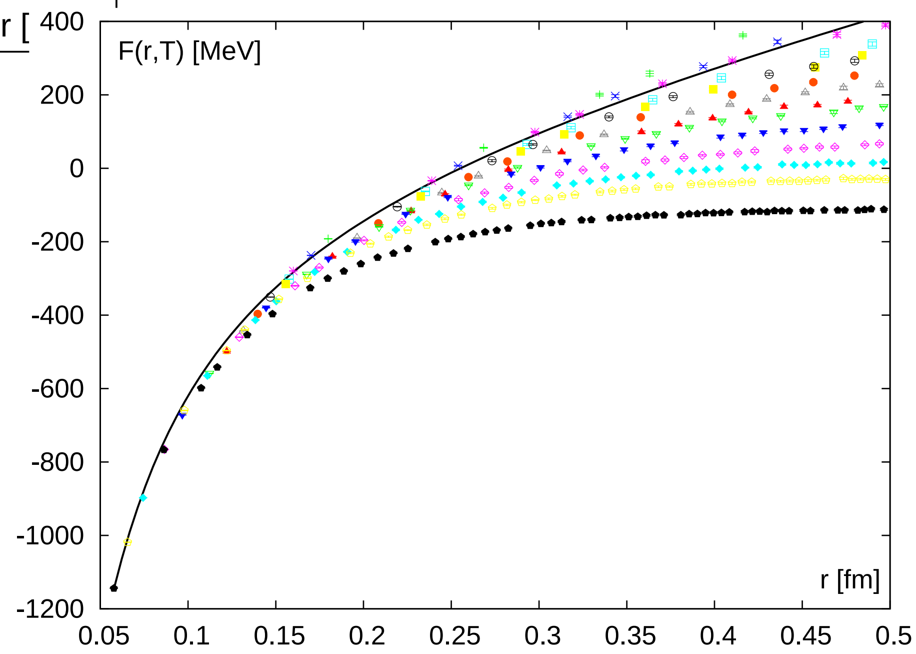
alternative renormalization procedures
(PLB543(2002)41, PoS(Lattice2007)195)
all equivalent

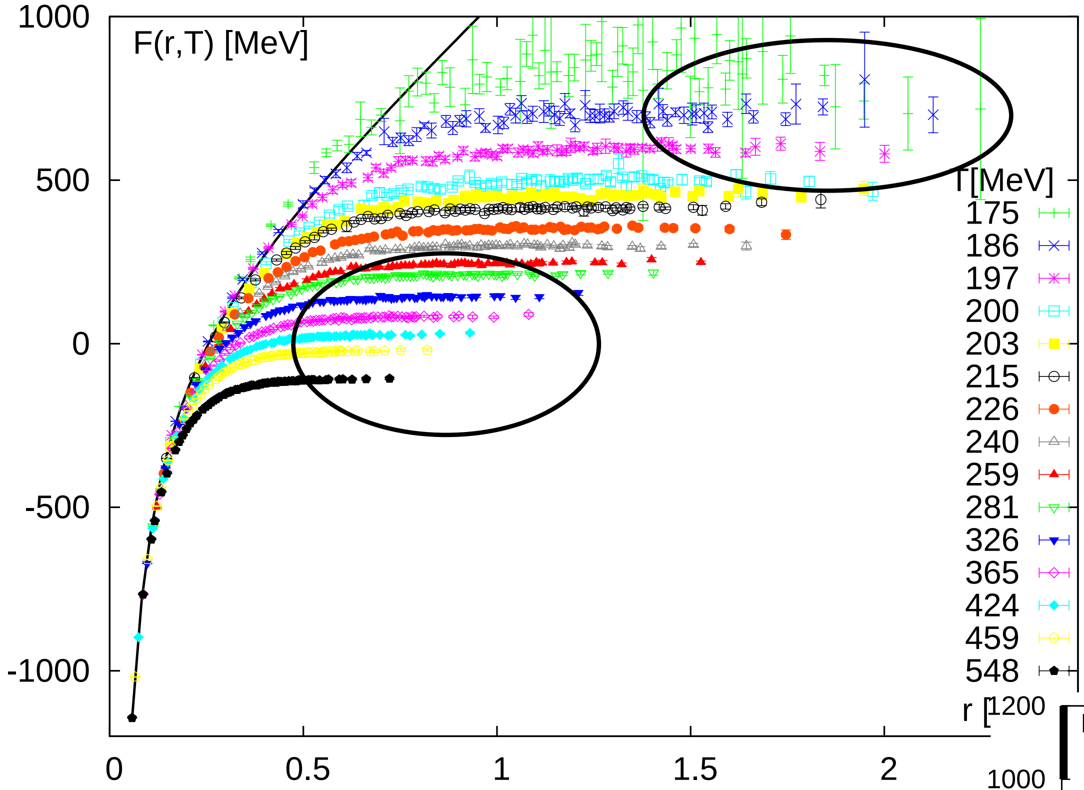
T-independent

$r \ll 1/\sqrt{\sigma}$

$F(r,T) \sim g^2(r)/r$



Heavy quark free energy - 2+1-flavors



String breaking

$T < T_c$

$F(r\sqrt{\sigma} \gg 1, T) < \infty$

high-T physics

$rT \gg 1$; screening

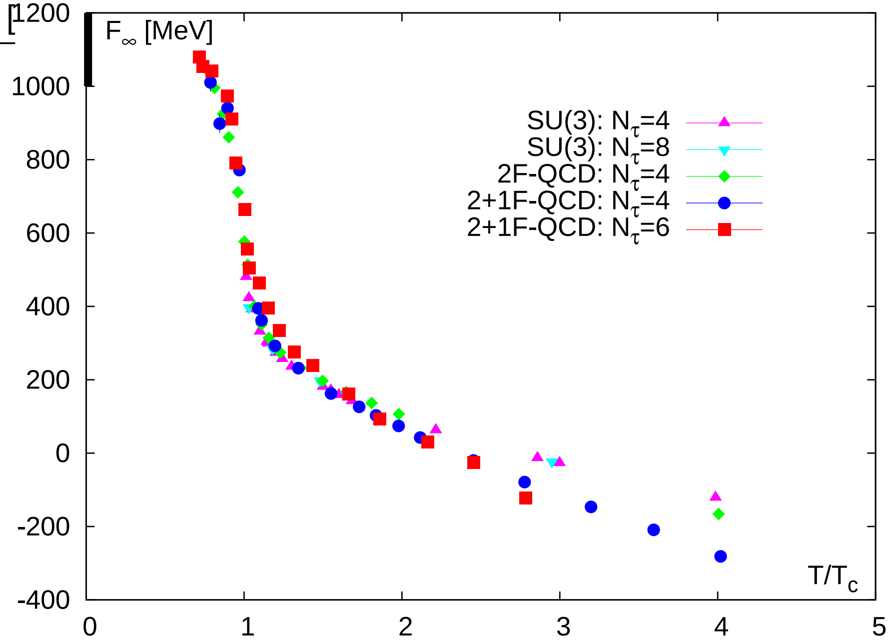
$\mu(T) \sim g(T)T$

$F(\infty, T) \sim -T$

T-independent

$r \ll 1/\sqrt{\sigma}$

$F(r,T) \sim g^2(r)/r$



Using short distance behaviour of free energies

Renormalization of $F(r,T)$ at short distances

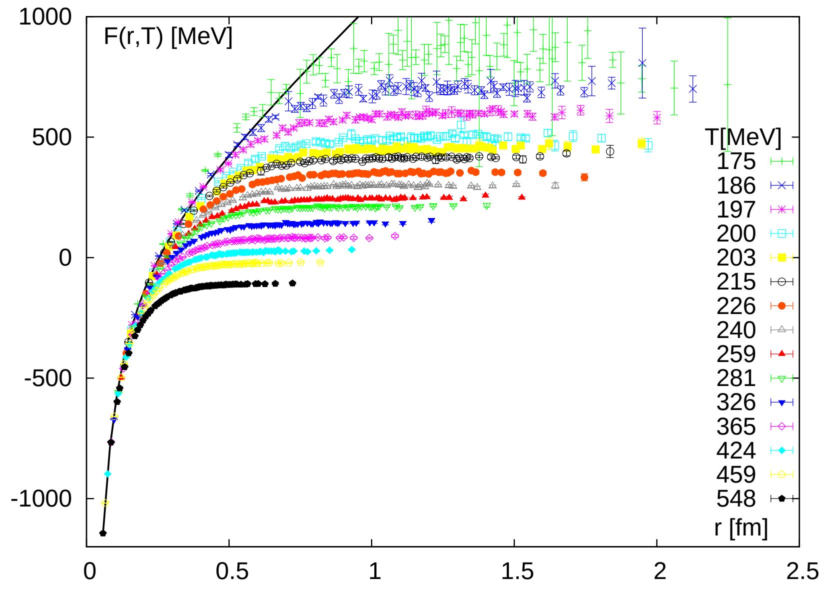
$$e^{-F_1(r,T)/T} = \left(Z_r(g^2)\right)^{2N_\tau} \langle \mathrm{Tr}\ (L_x L_y^\dagger)\rangle$$

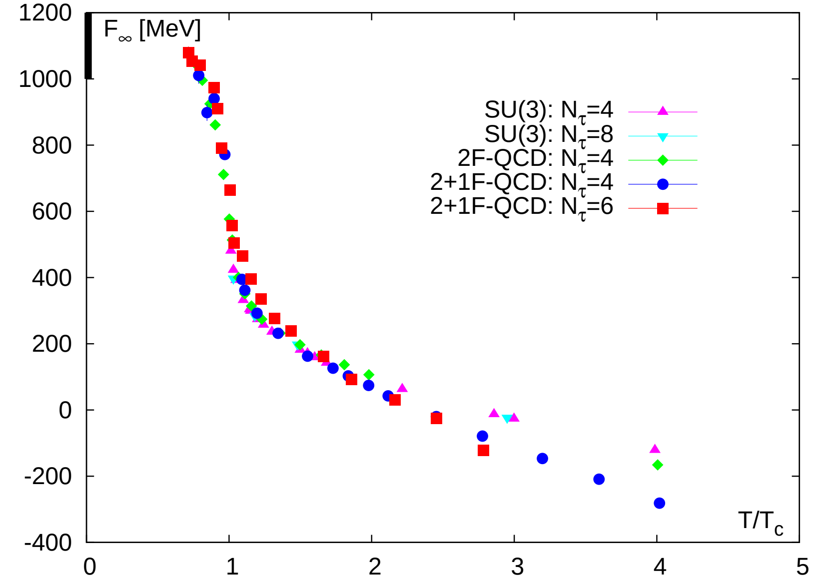
Renormalization of the Polyakov loop

$$L_{ren} = \left(Z_R(g^2)\right)^{N_t} L_{lattice}$$

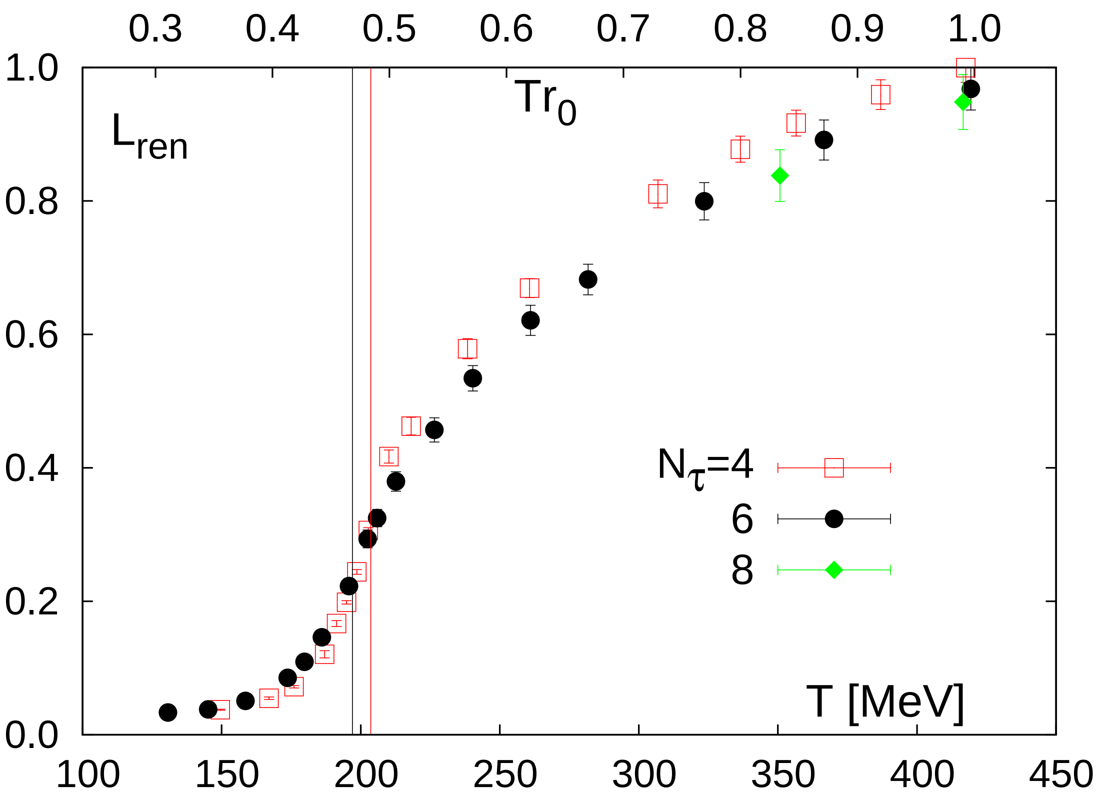
L_{ren} defined by long distance behaviour of $F(r,T)$

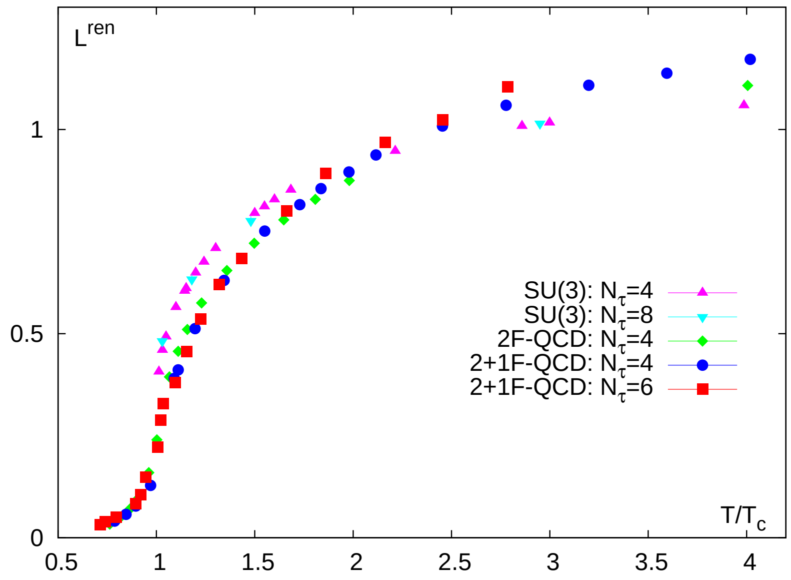
$$L_{ren} = \exp\left(-\frac{F(r=\infty,T)}{2T}\right)$$

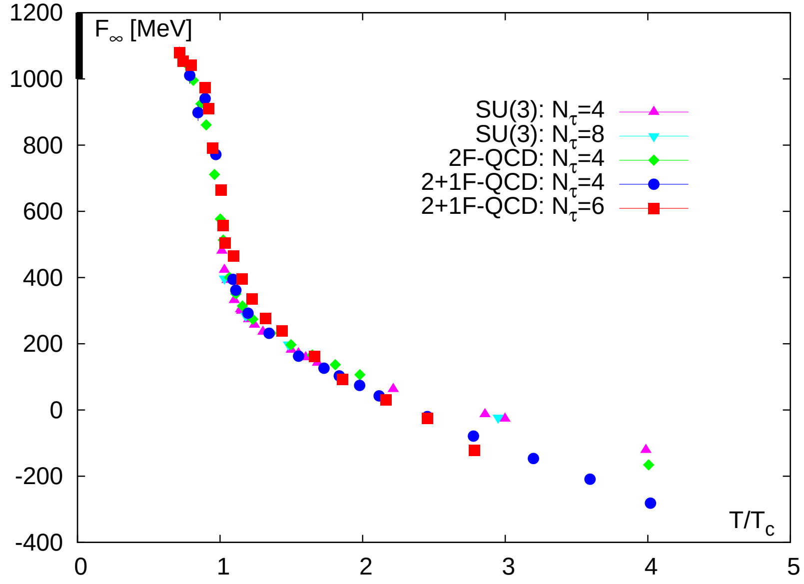




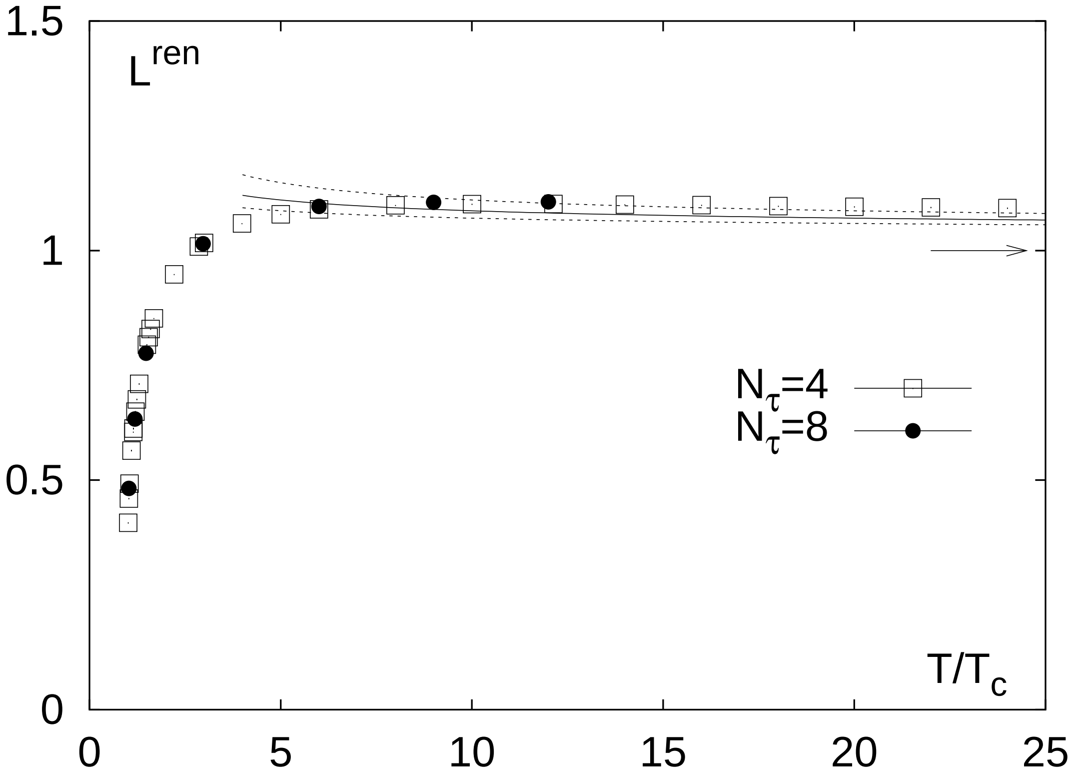
Renormalized Polyakov loop







$$L_{ren} = \exp\left(-\frac{F(r=\infty,T)}{2T}\right)$$

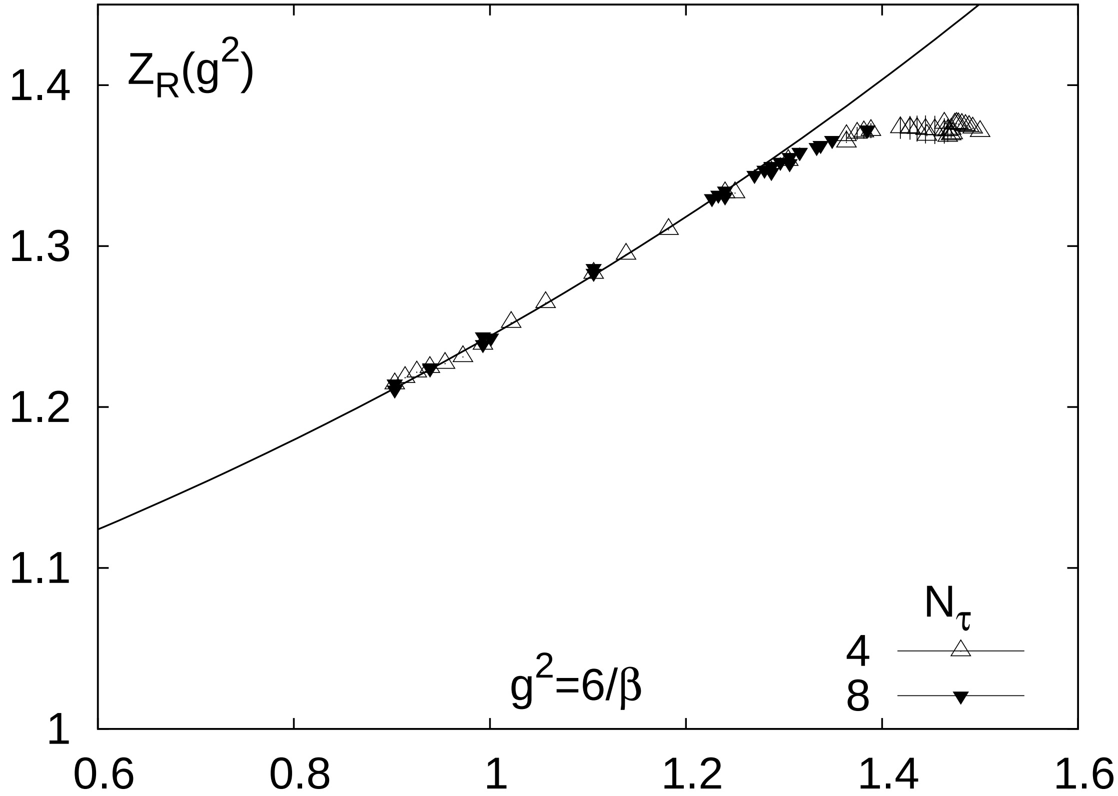


High temperature limit, $L^{ren} = 1$, reached from above as expected from PT

Clearly non-perturbative effects below $5T_c$

$$L_{ren} = \exp\left(-\frac{F(r=\infty, T)}{2T}\right)$$

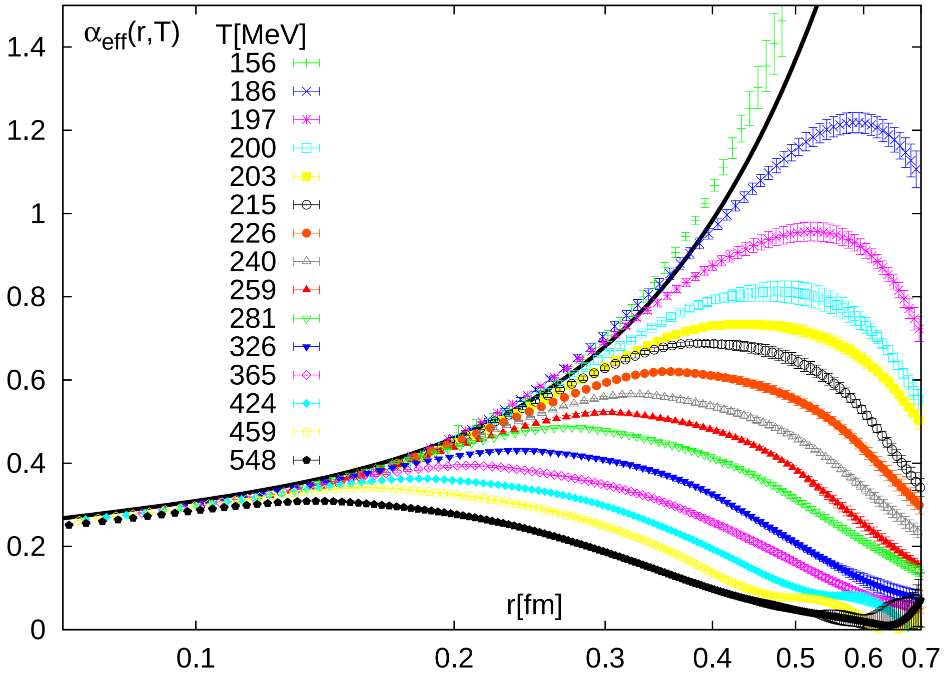
Renormalization constants obtained from heavy quark free energies



The renormalization constants depend on the bare coupling, i.e. $Z_R(g^2)$

$$Z_R(g^2) \simeq \exp\left(g^2(N^2-1)/NQ^{(2)} + g^4 Q^{(4)} + o(g^6)\right)$$

with $Q^{(2)} = 0.0597(13)$ consistent with lattice perturbation theory (Heller + Karsch, 1985)



Free energy in perturbation theory:

$$F_1(r,T) \equiv V(r) \simeq -\frac{4}{3}\frac{\alpha(r)}{r} \quad \text{for} \quad r\Lambda_{QCD} \ll 1$$

$$F_1(r,T) \simeq -\frac{4}{3}\frac{\alpha(T)}{r}e^{-m_D(T)r} \quad \text{for} \quad rT \gg 1$$

QCD running coupling in the qq-scheme

$$\alpha_{qq}(r,T) = \frac{3}{4}r^2\frac{dF_1(r,T)}{dr}$$

non-perturbative confining part for $r \gtrsim 0.4$ fm

$$\alpha_{qq}(r) \simeq 3/4r^2\sigma$$

present below and just above T_c

remnants of confinement at $T \gtrsim T_c$

temperature effects set in at smaller r with increasing T

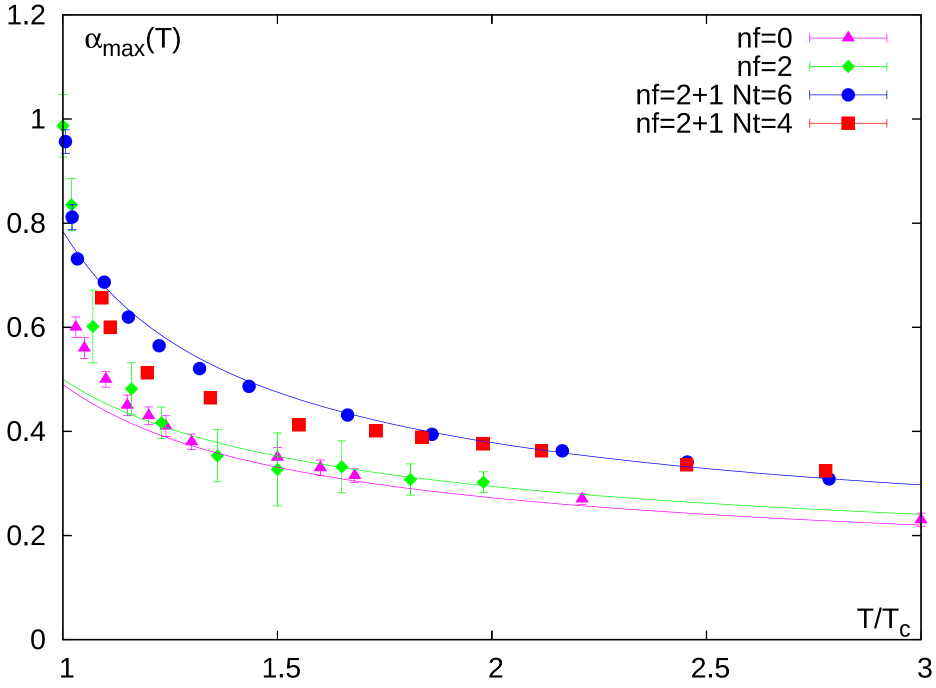
maximum due to screening

$\Longrightarrow$ At which distance do T-effects set in ?

$\Longrightarrow$ definition of the screening radius/mass

$\Longrightarrow$ definition of the T-dependent coupling

Temperature depending running coupling



define $\tilde{\alpha}_{qq}(T)$ by maximum of $\alpha_{qq}(r,T)$:

$$\tilde{\alpha}_{qq}(T) \equiv \alpha_{qq}(r_{max}, T)$$

perturbative behaviour at high T:

$$g^{-2}_{\text{2-loop}}(T) = 2\beta_0 \ln\left(\frac{\mu T}{\Lambda_{\overline{MS}}}\right) + \frac{\beta_1}{\beta_0} \ln\left(2\ln\left(\frac{\mu T}{\Lambda_{\overline{MS}}}\right)\right),$$

non-perturbative large values near T_c

not a large Coulombic coupling

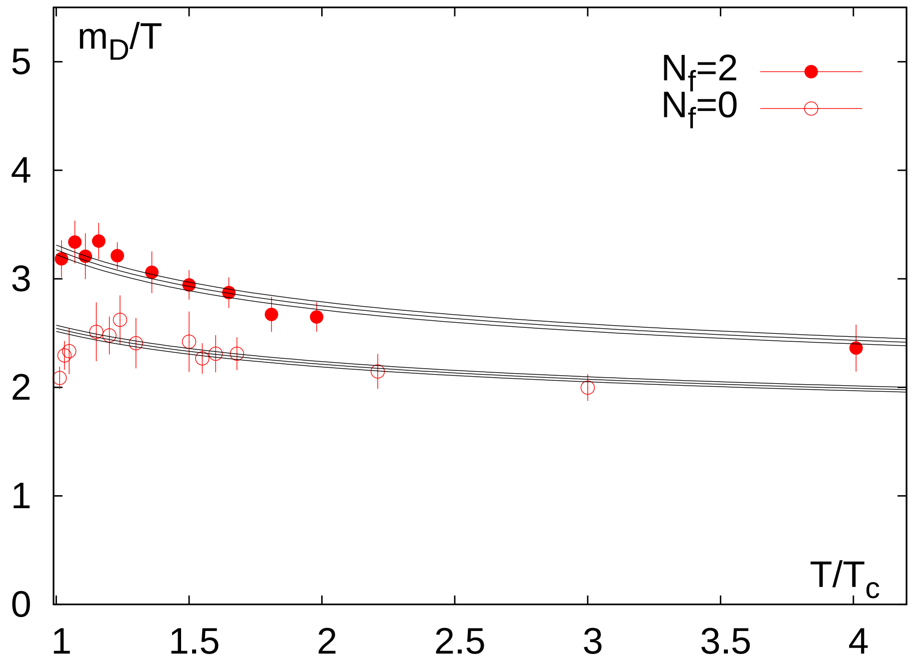
remnants of confinement at $T \gtrsim T_c$

string breaking and screening difficult to separate

slope at high T well described by perturbation theory

$\Longrightarrow$ At which distance do T-effects set in ?

$\Longrightarrow$ calculation of the screening mass/radius



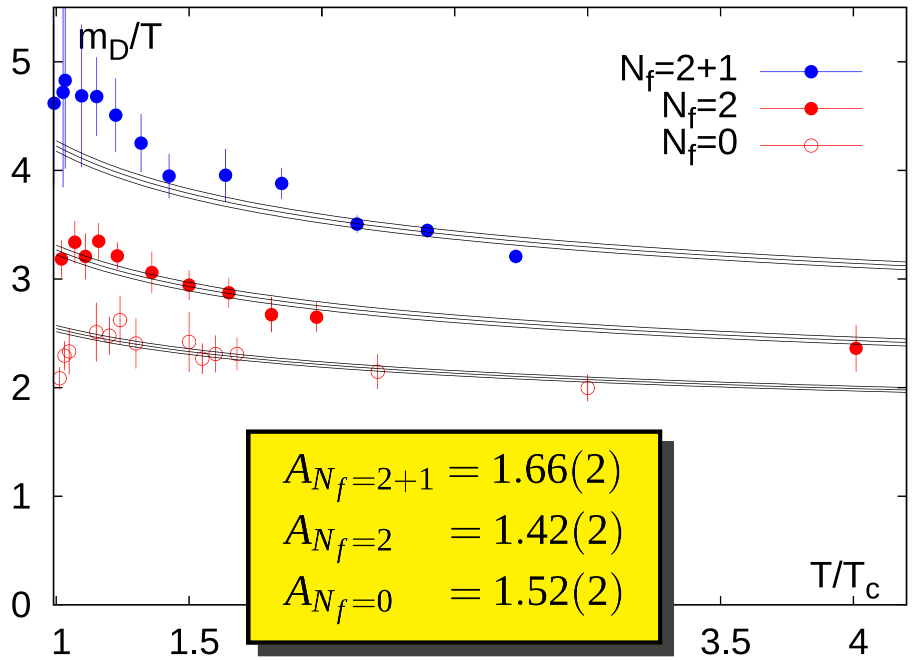
Screening masses obtained from fits to:

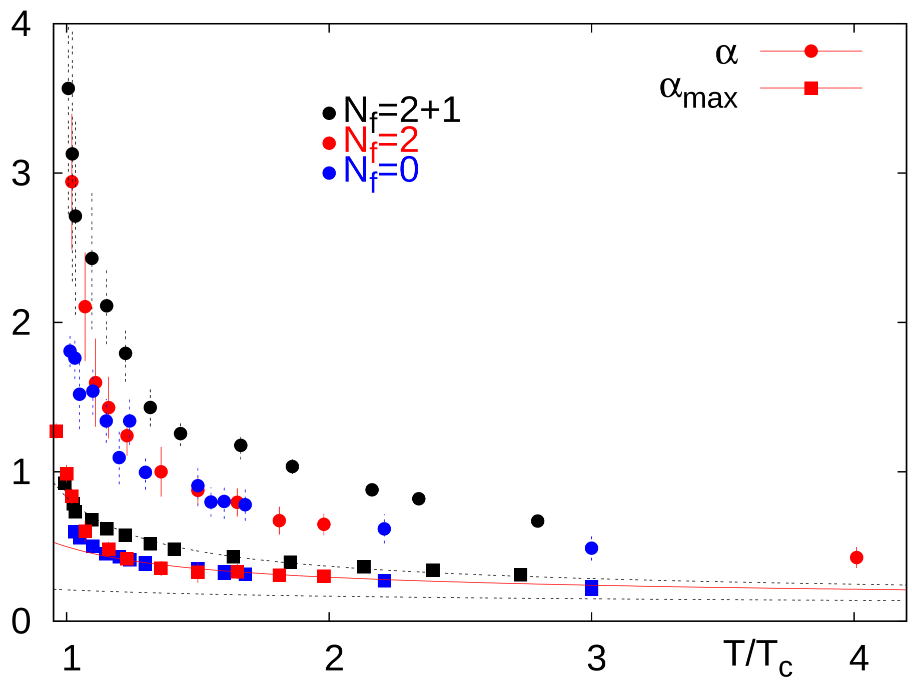
$$F_1(r,T) - F_1(r=\infty,T) = -\frac{4\alpha(T)}{3r} e^{-m_D(T)r}$$

at large distances $rT \gtrsim 1$

leading order perturbation theory:

$$\frac{m_D(T)}{T} = \left(1 + \frac{N_f}{6}\right)^{1/2} g(T)$$





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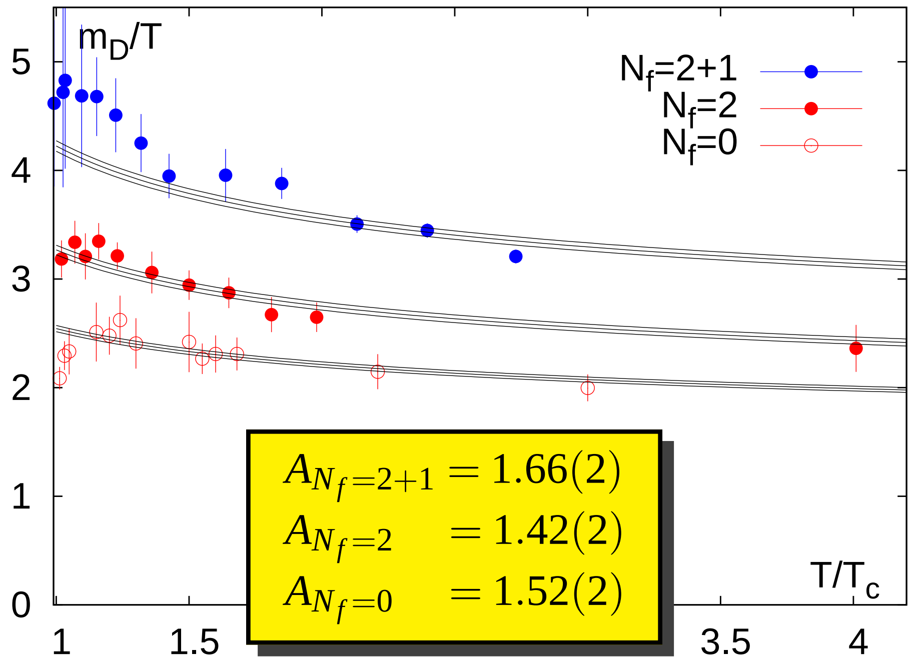
at large distances $rT \gtrsim 1$

leading order perturbation theory:

$$\frac{m_D(T)}{T} = A\left(1 + \frac{N_f}{6}\right)^{1/2} g(T)$$

perturbative limit reached very slowly



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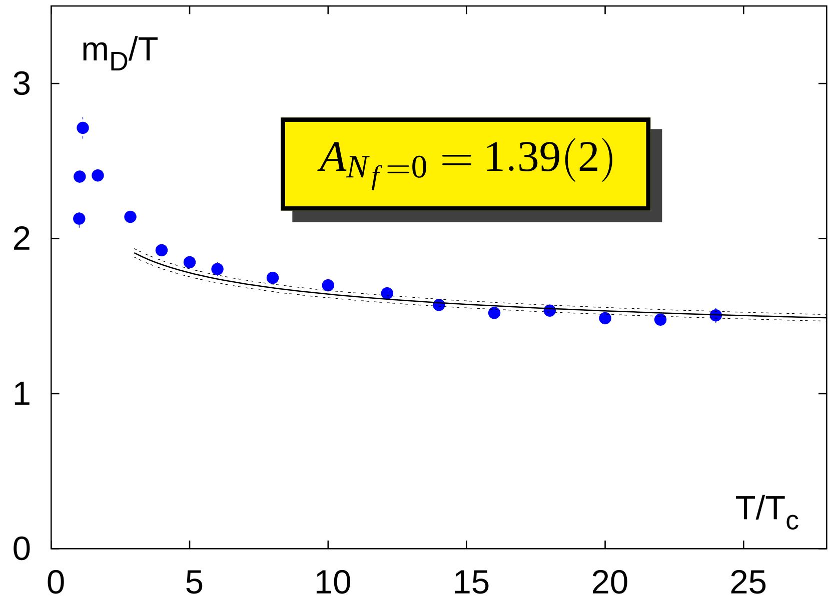


T depencence qualitatively described by perturbation theory

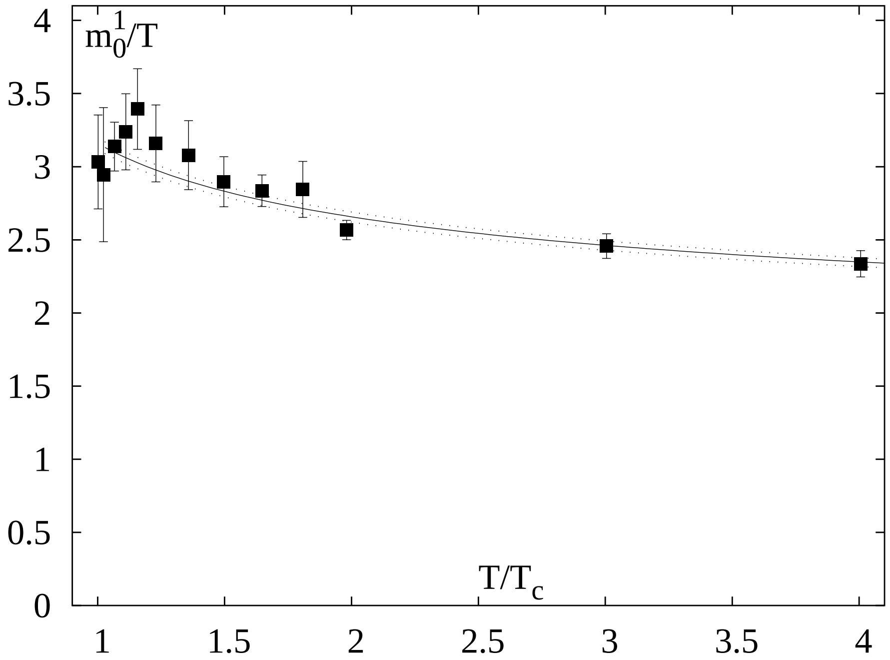
But $A \approx 1.4$ - $1.5 \implies$ non-perturbative effects

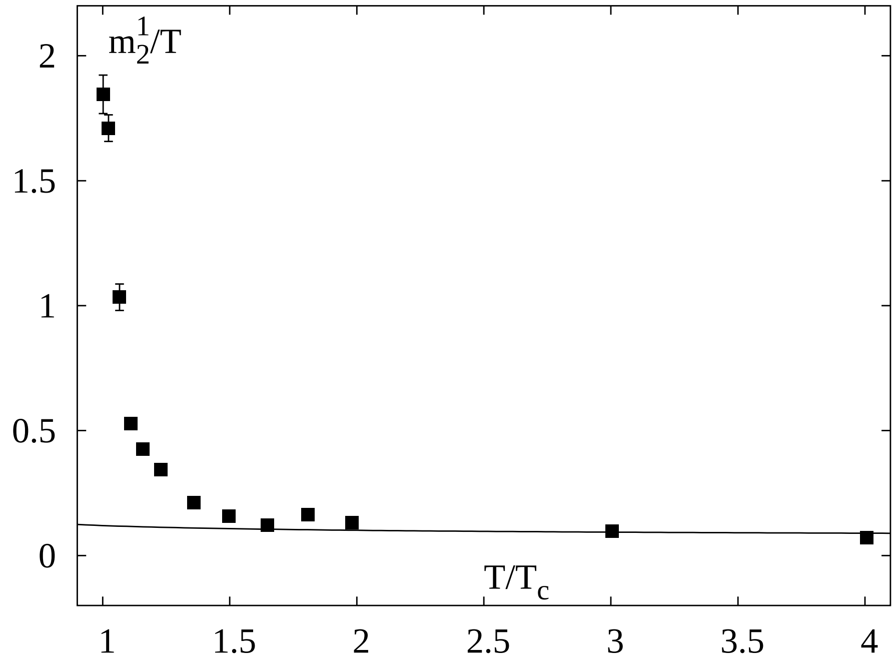
$A \to 1$ in the (very) high temperature limit leading order perturbation theory:

$$\frac{m_D(T,\mu_q)}{T} = g(T)\sqrt{1+\frac{N_f}{6}+\frac{N_f}{2\pi^2}\left(\frac{\mu_q}{T}\right)^2}$$

Taylor expansion:

$$m_D(T) = m_0(T) + m_2(T)\left(\frac{\mu_q}{T}\right)^2 + \mathcal{O}(\mu_q^4)$$

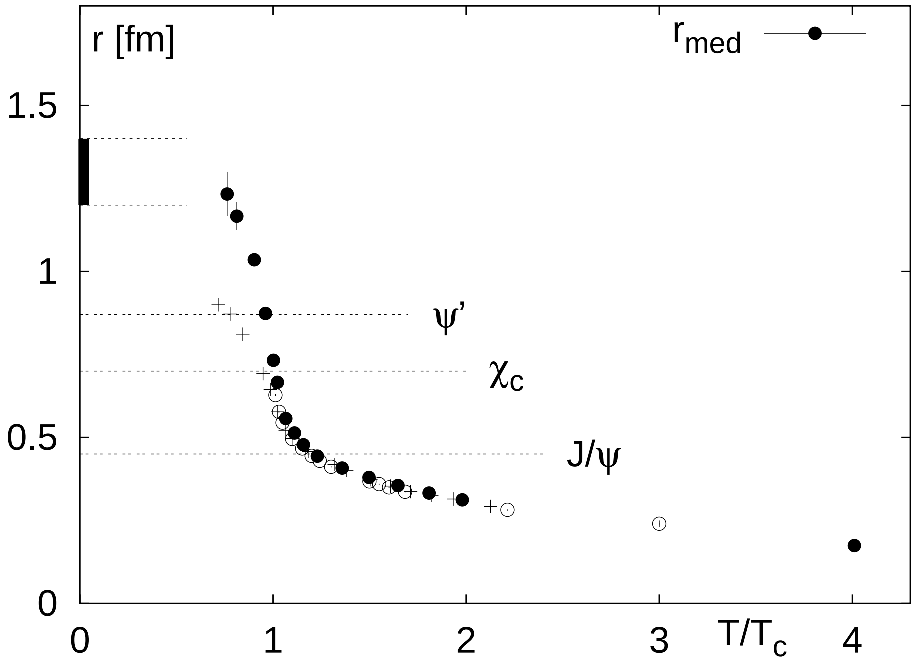




$m_2(T)$ agrees with perturbation theory for $T \gtrsim 1.5 T_c$

non-perturbative effects dominated by gluonic sector

Heavy quark bound states above T_c?



bound states above deconfinement?

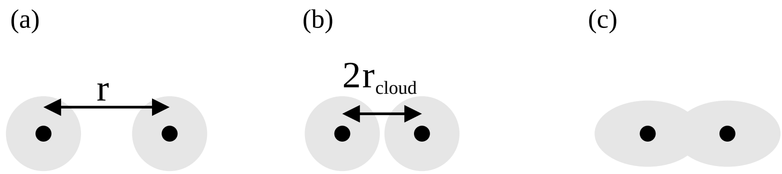
first estimate:

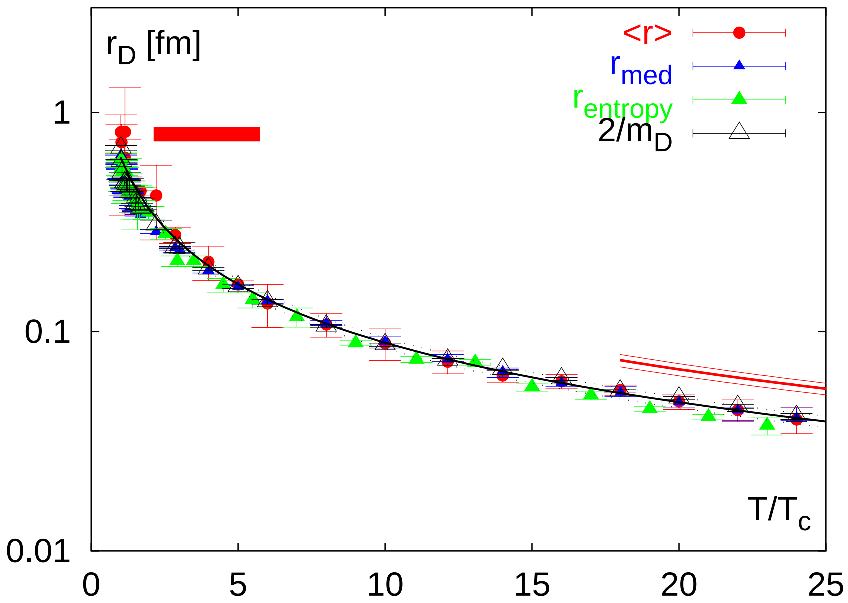
mean charge radii of charmonium states compared to screening radius

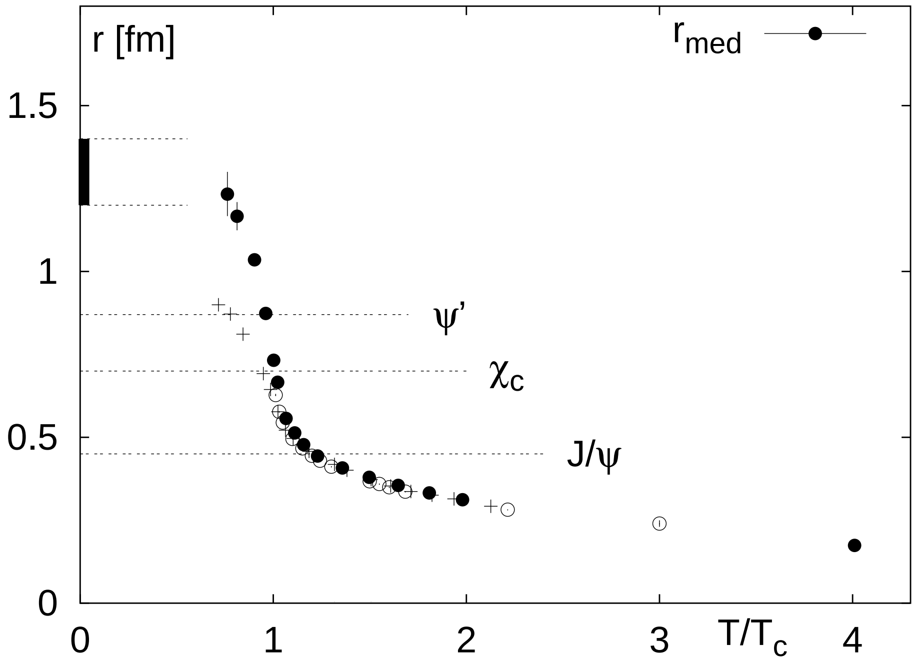
thermal modifications on ψ' and χ_c already at T_c

J/ψ may survive above deconfinement

$$\mathbf{r_{med}} \;:\; \mathbf{V(r_{med}) \equiv F_1(r \to \infty, T)}$$







bound states above deconfinement?

first estimate:

mean charge radii of charmonium states compared to screening radius

thermal modifications on ψ' and χ_c already at T_c

J/ψ may survive above deconfinement

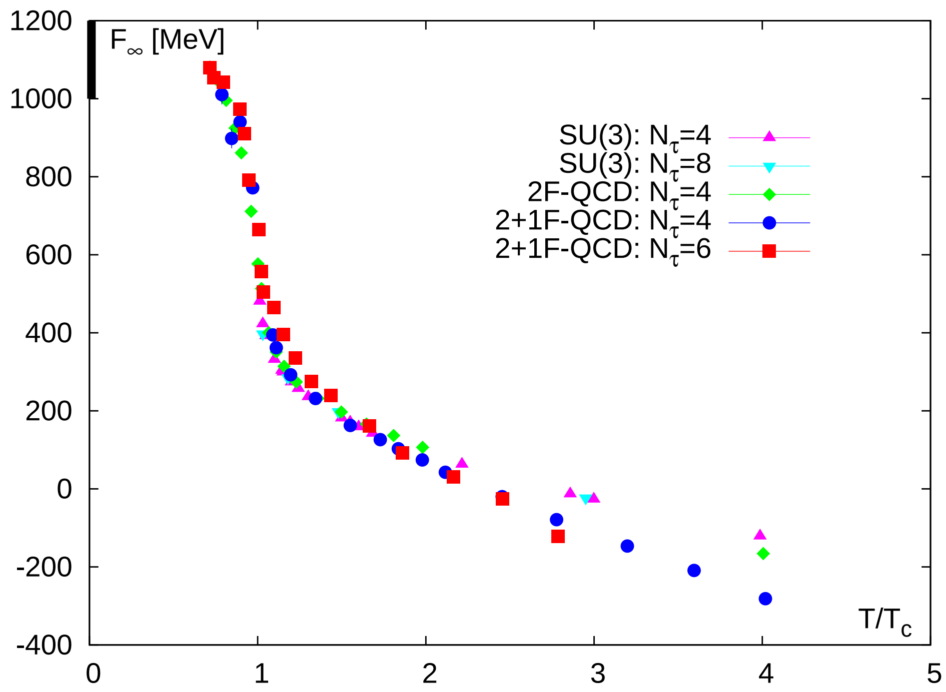
Better estimates:

effective potentials in Schrödinger Equation — Potential models, effective potential $V_{eff}(r,T)$

But: Free energies vs. internal energies $F(r,T) = U(r,T) - TS(r,T)$

direct calculation using correlation functions — Maximum entropy method $\rightarrow$ spectral function

Free energy vs. Entropy at large separations



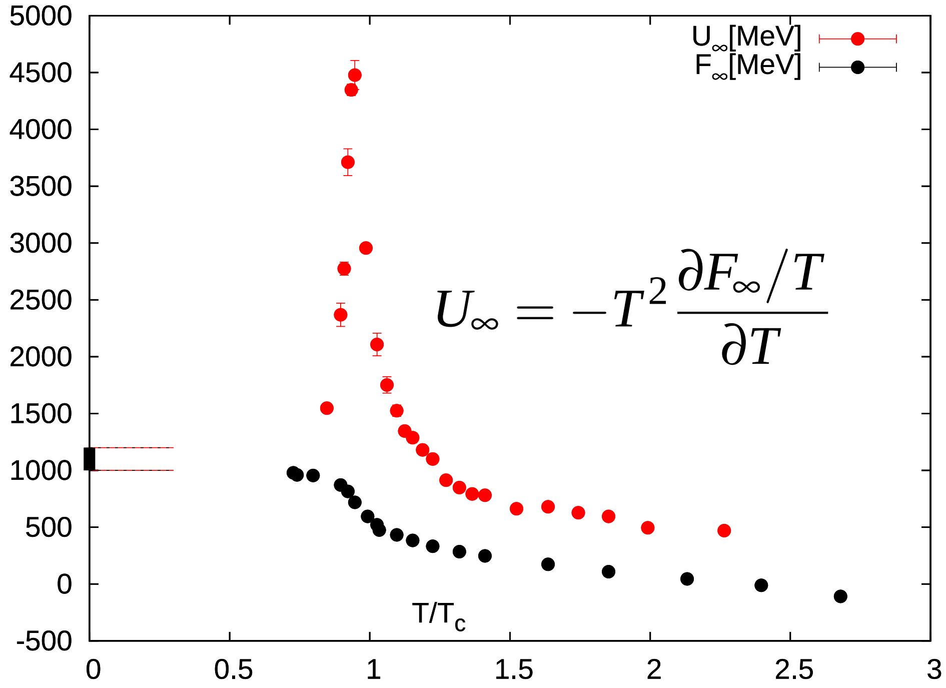
Free energies not only determined by potential energy

$$F_\infty = U_\infty - TS_\infty$$

Entropy contributions play a role at finite T

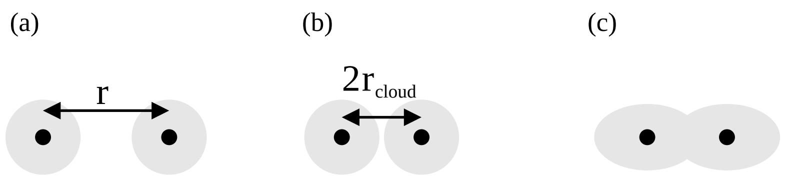
$$S_\infty = -\frac{\partial F_\infty}{\partial T}$$

Free energy vs. Entropy at large separations

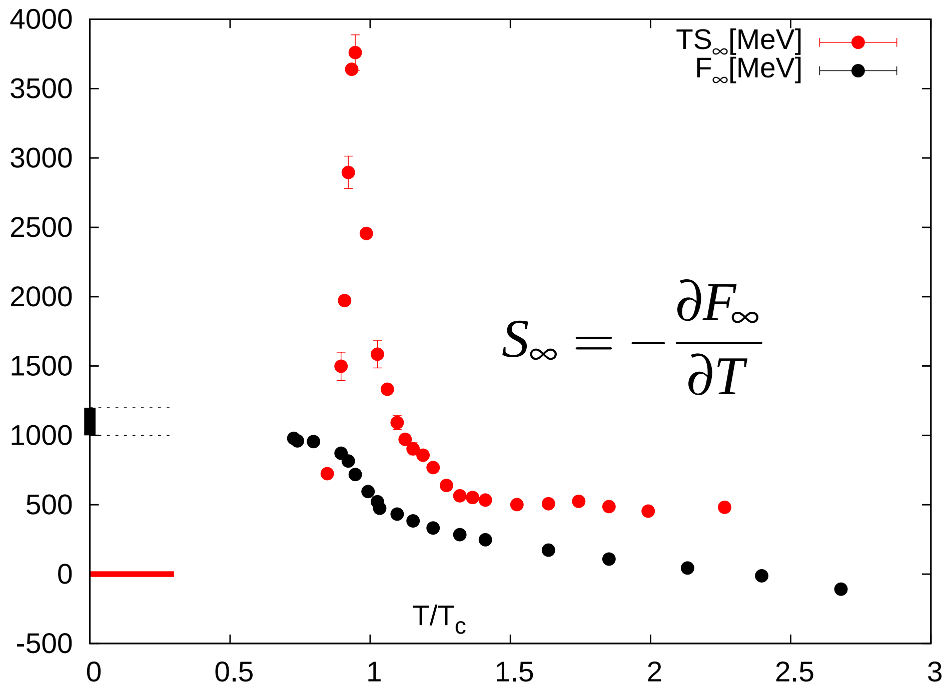


High temperatures:

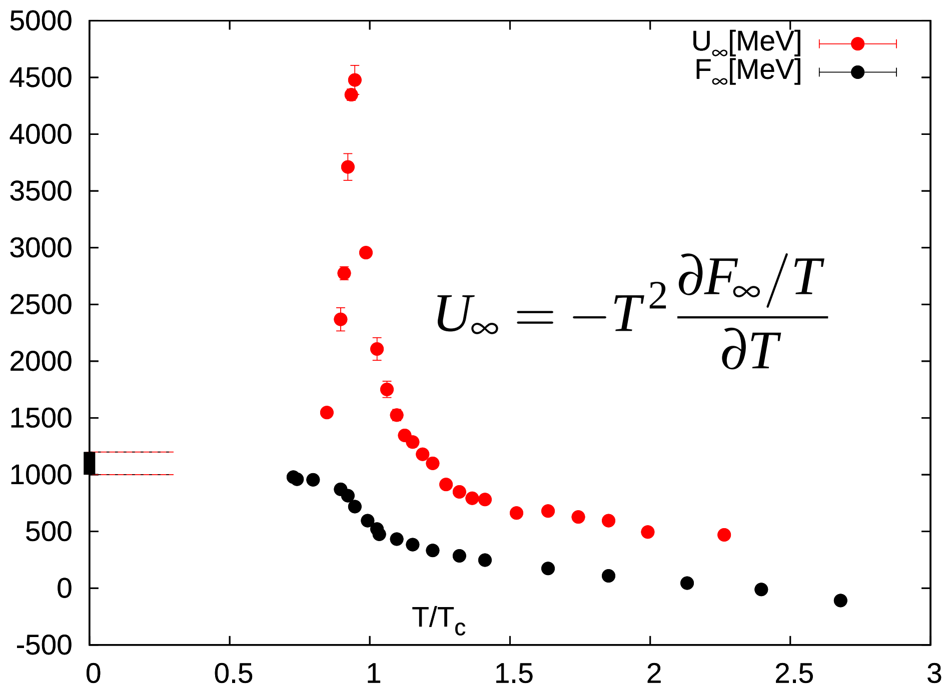
$$
\begin{aligned}
\mathbf{F}_\infty(\mathbf{T}) &\simeq -\frac{\mathbf{4}}{\mathbf{3}}\mathbf{m_D}(\mathbf{T})\alpha(\mathbf{T}) \simeq -\mathcal{O}(g^3 T) \\
\mathbf{TS}_\infty(\mathbf{T}) &\simeq +\frac{\mathbf{4}}{\mathbf{3}}\mathbf{m_D}(\mathbf{T})\alpha(\mathbf{T}) \\
\mathbf{U}_\infty(\mathbf{T}) &\simeq -\mathbf{4m_D}(\mathbf{T})\alpha(\mathbf{T})\frac{\beta(\mathbf{g})}{\mathbf{g}} \\
&\simeq -\mathcal{O}(g^5 T)
\end{aligned}
$$

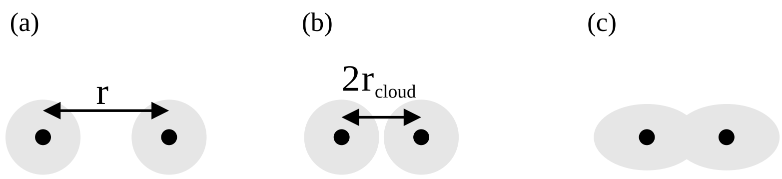


The large distance behavior of the finite temperature energies is rather related to screening than to t he temperature dependence of masses of corresponding heavy-light mesons!



Free energy vs. Entropy at large separations





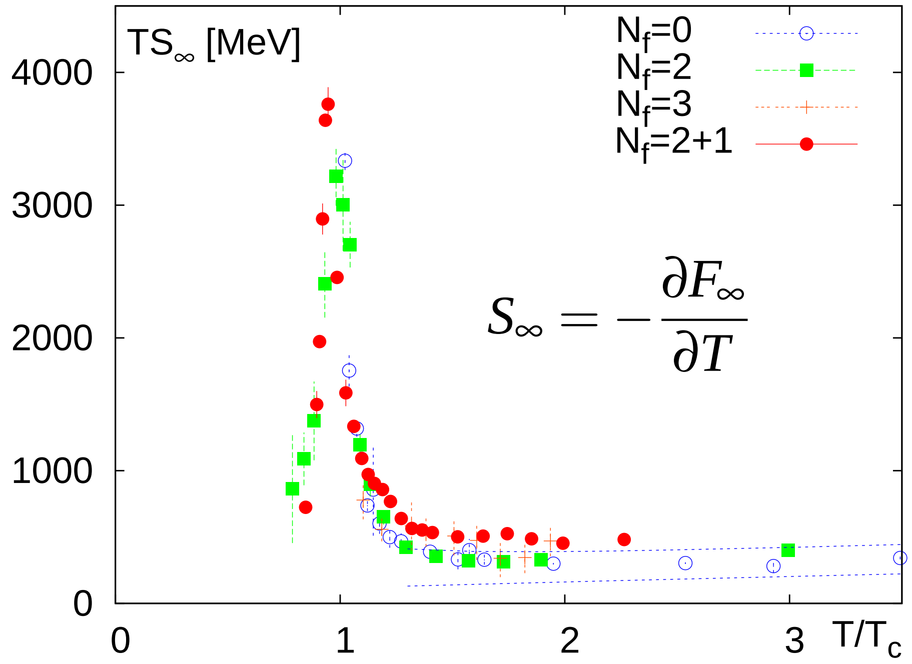
The large distance behavior of the finite temperature energies is rather related to screening than to t he temperature dependence of masses of corresponding heavy-light mesons!

High temperatures:

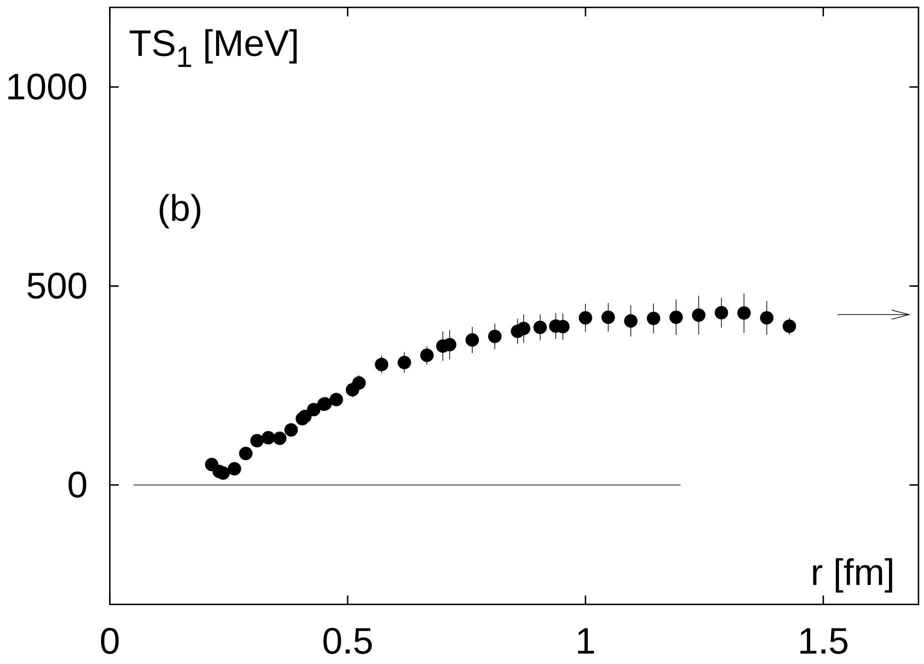
$$\mathbf{F}_\infty(\mathbf{T}) \simeq -\frac{4}{3}\mathbf{m_D}(\mathbf{T})\alpha(\mathbf{T}) \simeq -\mathcal{O}(g^3 T)$$

$$\mathbf{TS}_\infty(\mathbf{T}) \simeq +\frac{4}{3}\mathbf{m_D}(\mathbf{T})\alpha(\mathbf{T})$$

$$\mathbf{U}_\infty(\mathbf{T}) \simeq -4\mathbf{m_D}(\mathbf{T})\alpha(\mathbf{T})\frac{\beta(\mathbf{g})}{\mathbf{g}} \simeq -\mathcal{O}(g^5 T)$$



r-dependence of internal energies (so far only $N_f = 2$)



$$F_1(r,T) = U_1(r,T) - TS_1(r,T)$$

$$S_1(r,T) = \frac{\partial F_1(r,T)}{\partial T}$$

$$U_1(r,T) = -T^2 \frac{\partial F_1(r,T)/T}{\partial T}$$

Entropy contributions vanish in the limit $r \to 0$

$$F_1(r \ll 1, T) = U_1(r \ll 1, T) \equiv V_1(r)$$

important at intermediate/large distances

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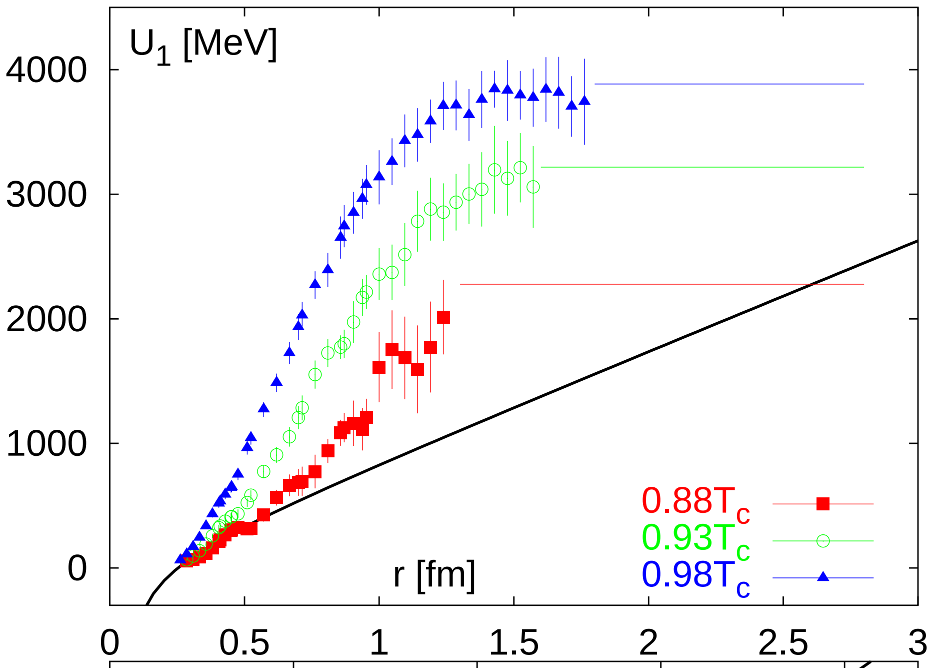

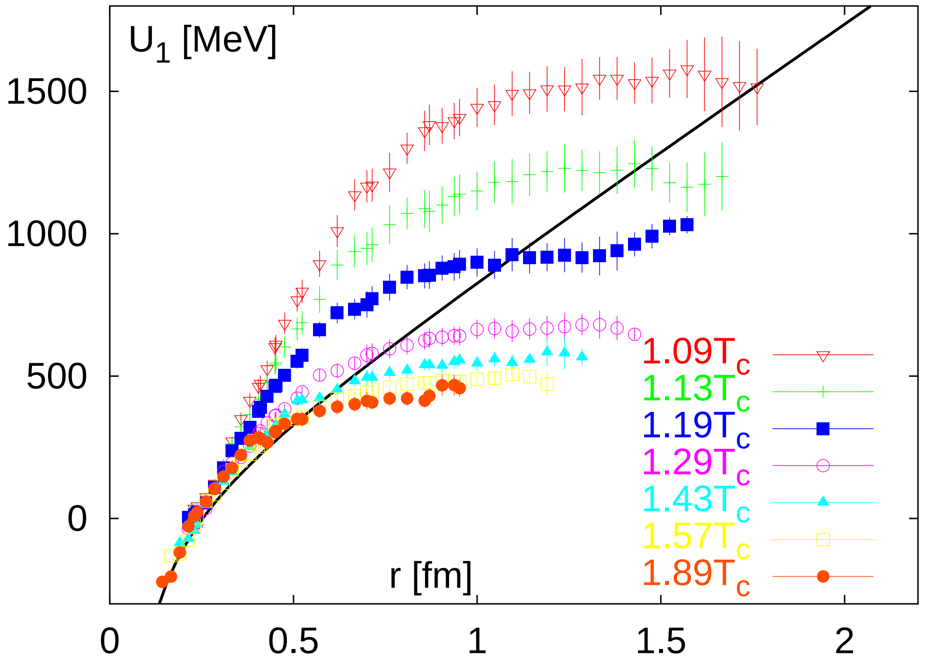

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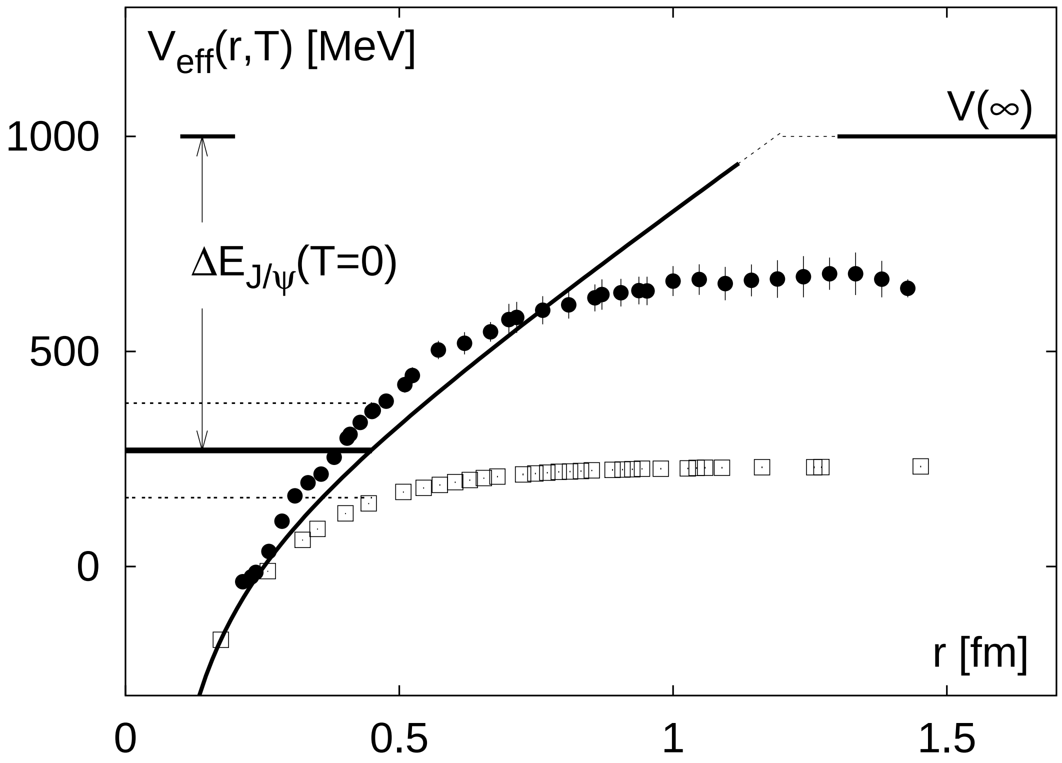
$\Longrightarrow$ Implications on heavy quark bound states?

$\Longrightarrow$ What is the correct $V_{eff}(r,T)$?

Heavy quark bound states



steeper slope of $V_{eff}(r,T) = U_1(r,T)$

$\Longrightarrow$ J/ψ stronger bound using $V_{eff} = U_1(r,T)$

$\Longrightarrow$ dissociation at higher temperatures compared to $V_{eff}(r,T) = F_1(r,T)$

Conclusions

- Heavy quark free energies, internal energies and entropy
  - Complex r and T dependence
  - Running coupling shows remnants of confinenement above T_c
  - Entropy contributions play a role at finite T
  - Non-perturbative effects in m_D up to high T
  - Non-perturbative effects dominated by gluonic sector
- Bound states in the quark gluon plasma
  - Estimates from potential models
  - Higher dissociation temperature using V_1
  - (directly produced) J/ψ may exist well above T_c
  - Full QCD calculations of correlation/spectral functions needed
  - What are relevant processes for charmonium?