



Recent R Measurement at BES

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BES Collaboration

QWG 2007

DESY

Hamburg, Germany

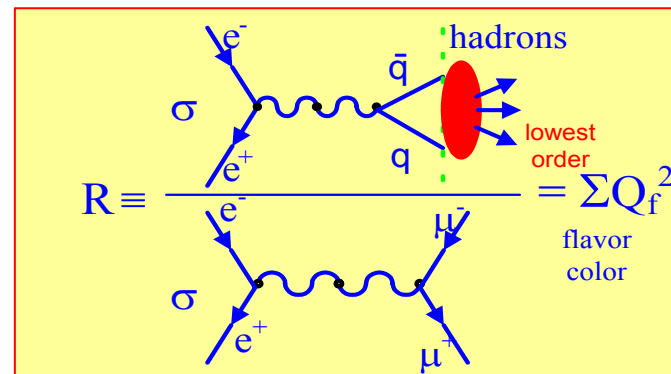
Outline

- **R value in continuous region**
- **High mass charmonium parameters**

What is R value

Definition

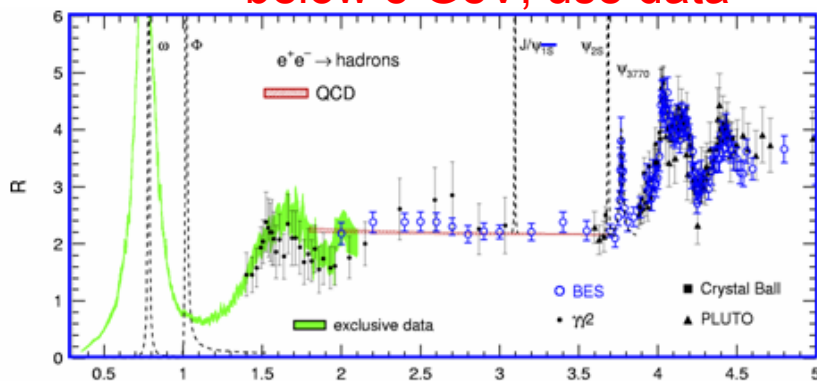
$$R = \frac{\sigma_{had}^0(e^+e^- \rightarrow \gamma^* \rightarrow \text{hadrons})}{\sigma_{\mu\mu}^0(e^+e^- \rightarrow \gamma^* \rightarrow \mu^+\mu^-)}$$



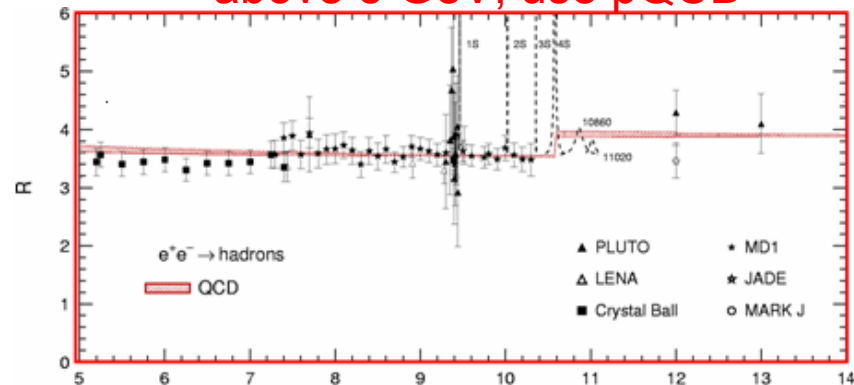
R value is the inclusive hadronic cross section in e^+e^- annihilation, and normalized by Born cross section of $\mu^+\mu^-$.

Significances: the precision of R value has important contributions to the precise test of the Standard Model (SM), such as the running electromagnetic coupling constant $\alpha(s)$, abnormal μ magnetic moment (g-2), the SM global fitting of the Higgs mass, direct test to the pQCD prediction on $R(s)$.

below 5 GeV, use data



above 5 GeV, use pQCD



R expression in experiment

R value is measured by

$$R_{exp} = \frac{N_{had}^{obs} - N_{bg}}{\sigma_{\mu\mu}^0 L \epsilon_{trg} \epsilon_{had}^0 (1 + \delta_{obs})}$$

N_{had}^{obs} : **observed number of hadronic events;**

N_{bg} : **number of background events;** L : **integrated luminosity;**

ϵ_{trg} : **trigger efficiency;** ϵ_{had}^0 : **hadronic events efficiency;**

$(1 + \delta_{obs})$: **effective initial state radiative correction factor.**

In expression R_{exp} , each quantity is obtained by

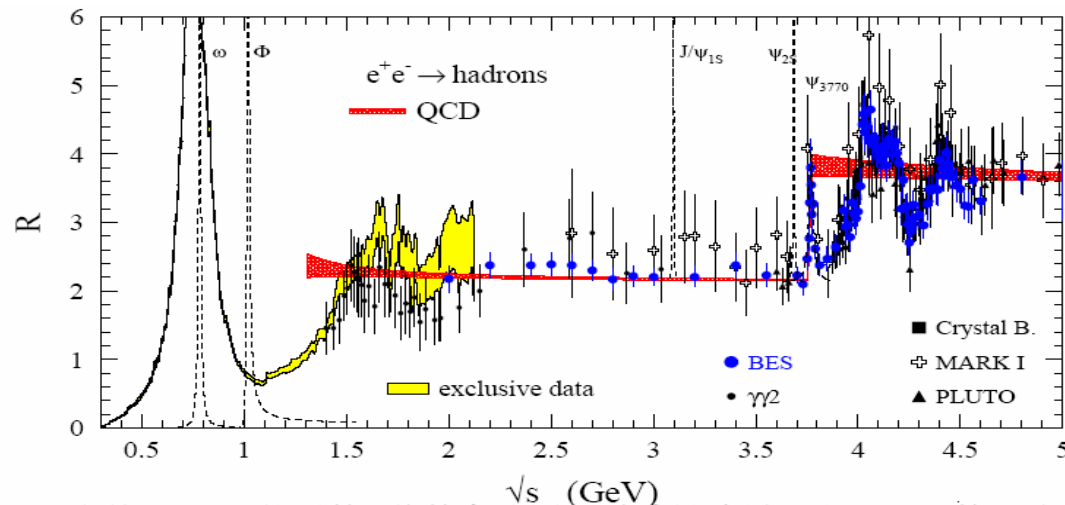
➤ **data analysis** $\longrightarrow N_{had}^{obs}, N_{bg}, L, \epsilon_{trg}$

➤ **theoretical calculations** $\longrightarrow (1 + \delta_{obs})$

➤ **Monte Carlo simulations** $\longrightarrow \epsilon_{had}^0$

Previous measurements

- In 1998 & 1999, scan data were taken between 2-5 GeV
- Energy step: in 3.7– 5.0 GeV are 10–20MeV, elsewhere is 100MeV
- Generation simulation: tuned LUARLW and JETSET
- Detector simulation: software based on EGS4
- Statistic error: about 2~3 %
- Systematic error: about 5~8 % (event selection and efficiency are dominant)
- Results: PRL 84 (2000) 594, PRL 88 (2002) 101802



Present measurement

- In 2004, data at 2.20, 2.60, 3.07 and 3.65 GeV were taken
- Statistical error: small, about 0.5%
- MC generation: LUARLW is retuned with improved ways
- Detector simulation: is updated as GEANT3 based software
- Event selection: improved, $N_{\text{had}} \geq 1$ -prong events are selected
- Systematical error: about 3.3%

Integral luminosity			Observed number of hadronic events	
$\sqrt{s}$ (GeV)	L (nb^{-1})	$\Delta L/L$ (%)	$N_{had}^{obs}(N_{gd} \geq 1\text{-prong})$	$N_{had}^{obs}(N_{gd} \geq 2\text{-prong})$
2.200	$121.50 \pm 1.10 \pm 4.00$	3.41	2862	2480
2.600	$1215.98 \pm 4.08 \pm 23.96$	2.00	24026	21511
3.070	$2272.20 \pm 6.64 \pm 30.00$	1.35	33933	31063
3.650	$6456.83 \pm 13.24 \pm 87.81$	1.38	83767	77660

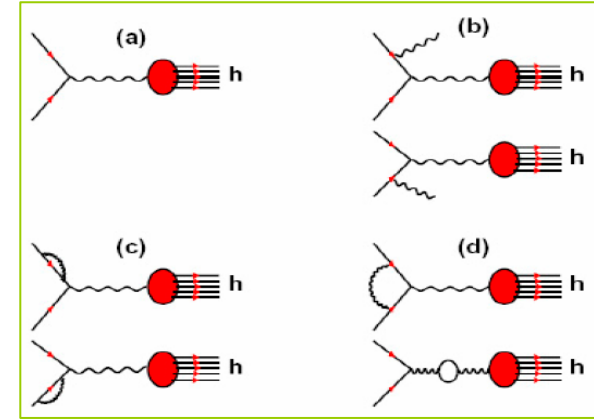
Effective ISR correction

Observed hadronic cross section: $\sigma_{obs}^T = \frac{N_{had}}{L}$

R value may be measured by

Procedure A: $\sigma^T = \frac{\sigma_{obs}^T}{\bar{\epsilon}} \Rightarrow R = \frac{N_{had}}{\sigma_{\mu\mu}^0 L \bar{\epsilon} (1 + \delta)}$

Procedure B: $\sigma_{obs}^0 = \frac{\sigma_{obs}^T}{1 + \delta_{obs}} \Rightarrow R = \frac{N_{had}}{\sigma_{\mu\mu}^0 L \epsilon(0) (1 + \delta_{obs})}$



G.Bonneau Nucl. Phys. B27, (1971) 281-397

F.A.Berends Nucl. Phys. B178, (1981)141-150

A.Osterfeld SLAC-PUB-4160(1986)

ISR precision is ~1% for $O(\alpha^3)$

where $(1 + \delta)$ and $(1 + \delta_{obs})$ are theoretical and effective ISR correction factors

$\bar{\epsilon}$ and $\epsilon(0)$ are efficiencies with all possible radiations and without radiation

Comparing above two expressions, they should be:

$$\bar{\epsilon}(1 + \delta) \approx \epsilon(0)(1 + \delta_{obs})$$

The product of ISR factor and hadronic efficiency must be treated as unique factor. In principle, the calculation of the ISR factor $(1 + \delta)$ and radiation simulation in MC must be consistent, and Lund area generator LUARLW needs this requirement.

Lund area law

Hadronization picture in e^+e^- annihilation:

In Lund model, n -hadrons production

$$e^+e^- \Rightarrow q\bar{q} \Rightarrow \text{string} \Rightarrow m_1 + m_2 + \cdots + m_n$$

The effective matrix element is (the gluon emission is neglected at low energies)

$$\mathcal{M} \equiv \mathcal{M}_{\text{QED}}(e^+e^- \rightarrow q\bar{q})\mathcal{M}_{\text{LUND}}(q\bar{q} \rightarrow m_1, m_2, \cdots m_n)$$

The hadronization is treated as string fragmentation:

$$\mathcal{M}_{\text{LUND}}(q\bar{q} \rightarrow m_1, m_2, \dots m_n) = C_n \mathcal{M}_{\perp} \mathcal{M}_{//}$$

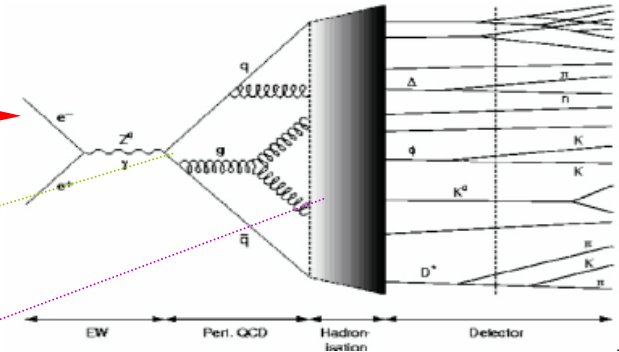
Transverse momentum: $\mathcal{M}_\perp = \exp(-\sum_{j=1}^n \vec{k}_j^2)$

Longitudinal momentum (area law):

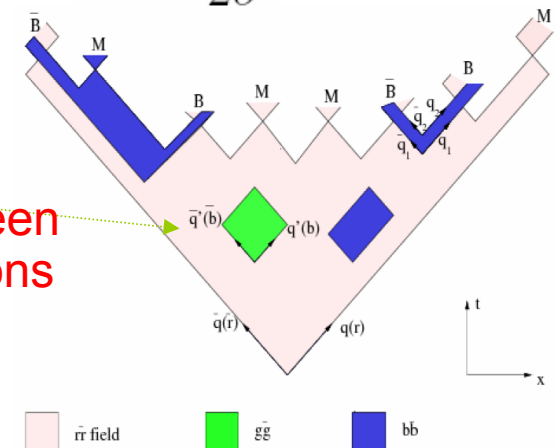
$$\mathcal{M}_{//} = \exp(i\xi \mathcal{A}_n) \quad \xi = \frac{1}{2k} + i\frac{b}{2}$$

Simulation of ISR based on Berends's calculations has been built in LUARLW with the angle and momentum distributions

$$d\sigma^{HB}(s) = \frac{\alpha}{\pi^2} \frac{\sin^2 \theta}{(1 - a^2 \cos^2 \theta)} \frac{dk d\Omega_\gamma}{k} (1 - k + \frac{k^2}{2}) d\sigma^0(s')$$



$$\vec{k}_j \equiv \frac{\vec{p}_{\perp j}}{2\sigma}$$



Tuning of LUARLW parameters

In data analysis, the fraction of the lost hadronic events is compensated by efficiency with Monte Carlo

$$\bar{\epsilon}_{had} = \frac{N_{obs}^{MC}}{N_{gen}^{MC}}$$

the reliability of efficiency depends on the consistency between data and Monte Carlo for all distributions related to hadronic criteria.

There are many free parameters in LUARLW and JETSET, some of them are important for determining the hadronic efficiency, such as

✓ multiplicity distribution for fragmentation hadrons

$$P_n = \frac{\mu^n}{n!} \exp[c_0 + c_1(n - \mu) + c_2(n - \mu)^2] \quad , \quad \mu = \alpha + \beta \exp(\gamma \sqrt{s})$$

✓ ratios of **vector/pseudoscalar, strange/normal mesons, baryon/meson**

✓ dynamical parameters **b** and **σ_{pt}** in LUND area law

✓ PARJ(1-3), PARJ(11-17) in JETSET

✓

The values of the free parameters need to be tuned by comparing the sensitive distributions simulated by LUARLW with data at detector level, and make them to be consistent at all energy points.

Comparisons between data and LUARLW

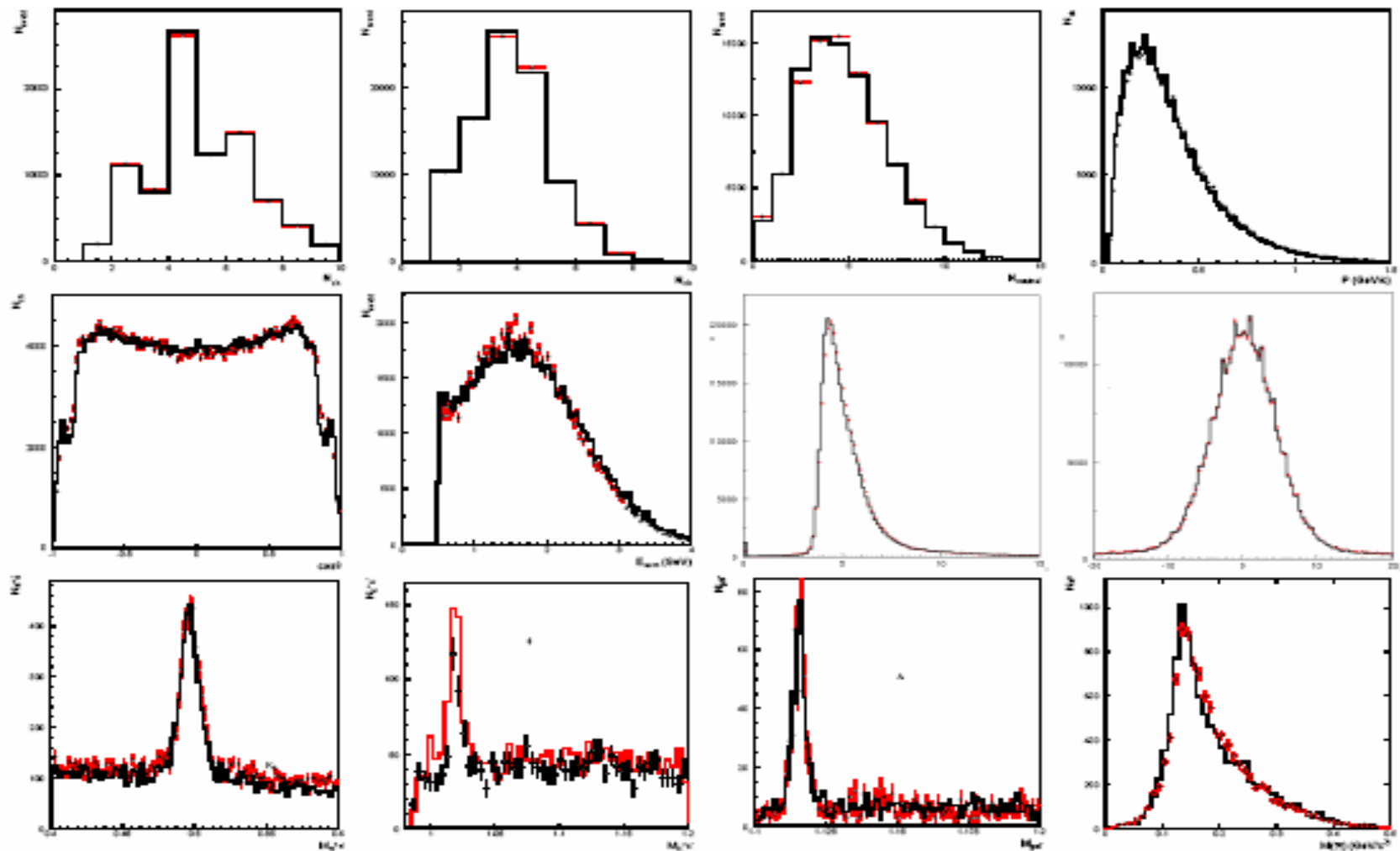


Figure 4: The comparisons of the distributions about multiplicity of the charged tracks, good charged tracks, neutral tracks, momentum p , polar-angle $\cos\theta$, deposit energy in BSC, time of flight and vertex of the charged tracks, invariant masses of $K_S \rightarrow \pi^+\pi^-$, $\phi \rightarrow K^+K^-$, $\Lambda \rightarrow p\pi^-$ and $\pi^0 \rightarrow \gamma\gamma$ between data (dots with error bars) and LUARLW (dots with error bars) at 3.65 GeV.

Error analysis

According to experimental expression

$$R_{exp} = \frac{N_{had}^{obs}}{\sigma_{\mu\mu}^0 L \epsilon_{trg} \bar{\epsilon}_{had} (1 + \delta)}$$

the effective number of hadronic events is defined

$$\tilde{N}_{had} = \frac{N_{had}}{\bar{\epsilon}_{had}}, \quad \text{where} \quad \bar{\epsilon}_{hd} = \frac{N_{obs}^{MC}}{N_{gen}^{MC}}$$

Main error estimation:

$$\frac{\Delta R}{R} \cong \sqrt{\left(\frac{\Delta \tilde{N}_{had}}{\tilde{N}_{had}}\right)^2 + \left(\frac{\Delta L}{L}\right)^2 + \left(\frac{\Delta \epsilon_{trg}}{\epsilon_{trg}}\right)^2 + \left(\frac{\Delta(1+\delta)}{(1+\delta)}\right)^2}$$

Error from the track reconstruction

$$\Delta \epsilon_{trk} = \sum_{n_{gd}} P(n_{gd}) B(n_{er}; n_{gd}, \sigma_{trk})$$

Error $\Delta \epsilon_{trk}$ due to tracking efficiency (%)

$\sqrt{s}(\text{GeV})$	$N_{gd} \geq 1\text{-prong}$	$N_{gd} \geq 2\text{-prong}$
2.200	0.35	0.60
2.600	0.32	0.62
3.070	0.29	0.61
3.650	0.26	0.63

P : multiplicity distribution

B : binominal distribution

σ_{trk} : tracking error (2%)

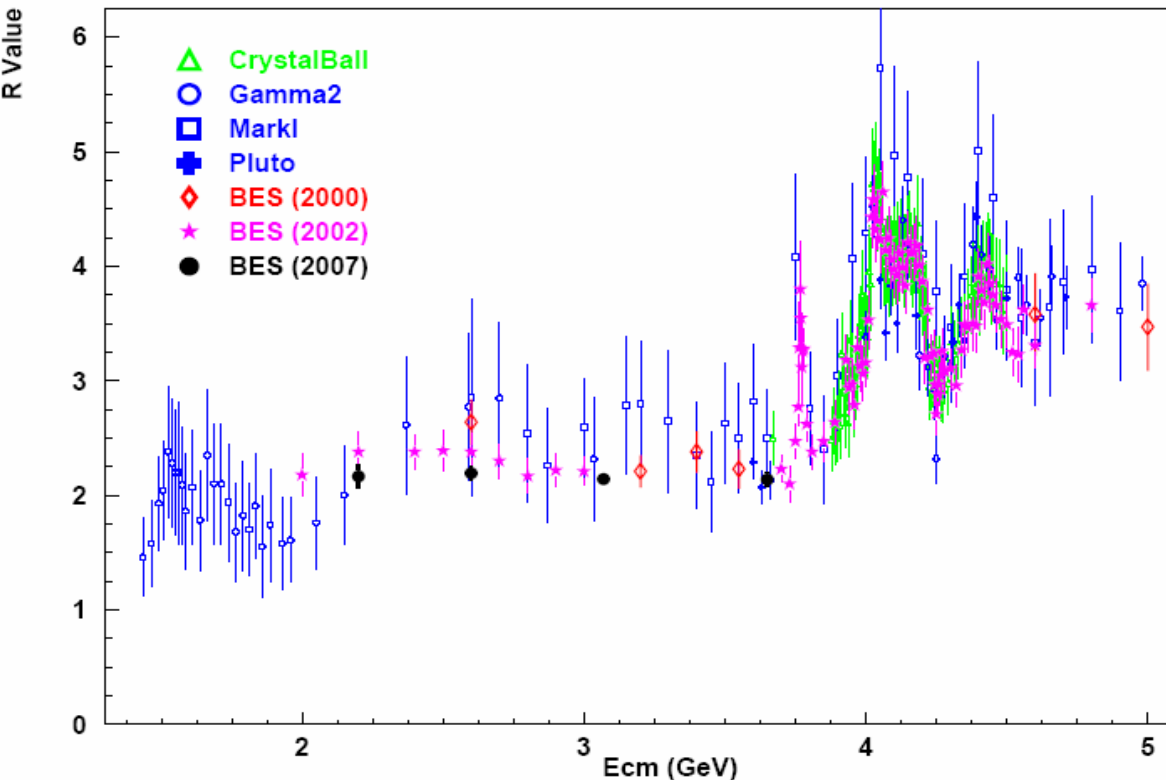
n_{gd} : number of good track in one event

n_{er} : number of wrongly reconstructed tracks in one event

Errors of the effective number of hadronic events (including event selection and hadronic efficiency)

Selection type		Cut condition	Systematic error of $\Delta N_{had}/N_{had}$ (%)			
			2.2 GeV	2.6 GeV	3.07 GeV	3.65 GeV
track level	charged	Mfit=2	0.20	0.24	0.36	0.37
		$V_{xy} < 2$ (cm)	0.45	0.15	0.40	0.90
		$ \cos\theta < 0.84$	1.13	0.75	0.78	1.21
		$p < E_b(1 + 0.1\sqrt{1 + E_b^2})$	0.05	0.02	0.02	0.01
		$t_{tof} \leq t_{proton} + 2(\text{ns})$	0.06	0.05	0.09	0.13
		$E_{BSC} \leq \text{Min}(1, 0.6E_b)$	0.67	1.01	0.92	0.64
	neutral	—	—	—	—	—
event level	all tracks	$E_{BSC}^{sum} > \text{Max}(0.5, 0.28E_b)$	0.96	0.50	0.74	1.10
	≥ 3 -prong	$N_{good} \geq 3$	—	—	—	—
	$=2$ -prong	$N_{good} = 2$	—	—	—	—
		$\theta_{12} < 165^\circ$	0.87	0.90	0.62	0.54
		$ \Delta R \geq 34$ or $ \Delta Z \geq 60$ (cm)	0.92	0.83	0.87	0.65
	new $=1$ -prong	$N_{good} = 1$	—	—	—	—
		if $X_{rate} > 0.5 \rightarrow e$	0.63	0.18	0.03	0.15
		if $p > 1$ GeV, $\mu_{hit} \geq 1 \rightarrow \mu$	0.19	0.04	0.02	0.14
		$N_\gamma \geq 2$	0.12	0.00	0.00	0.18
		$E_\gamma > 0.1$ GeV	0.97	0.18	0.13	0.40
		$N_{hit\ layer} \geq 2$	0.51	0.14	0.11	0.07
		$\theta_{\gamma chgtrk} > 25^\circ$	0.62	0.68	0.20	0.66
		1-C fit, $\text{Prob}(\chi^2) > 0.01$ for π^0	0.40	0.50	0.12	0.09
		Total error for $\Delta N_{had}/N_{had}$	2.58	2.05	1.88	2.33

Preliminary results (2007)



Results show:

- Precisions of new measurements are about 4.7% at 2.20 GeV, and 3.3% at 2.60, 3.07 and 3.65 GeV.
- Values of R for selecting more than 1-prong and 2-prong events are in good consistency.
- The cross checks of R values measured with the procedure A and B are done, their differences are about 1%.

R values measured with procedure B

$\sqrt{s}(\text{GeV})$	$R(N_{gd} \geq 1\text{-prong})$	$R(N_{gd} \geq 2\text{-prong})$
2.200	$2.17 \pm 0.04 \pm 0.10$	$2.17 \pm 0.04 \pm 0.10$
2.600	$2.19 \pm 0.01 \pm 0.07$	$2.20 \pm 0.02 \pm 0.07$
3.070	$2.14 \pm 0.01 \pm 0.05$	$2.15 \pm 0.01 \pm 0.06$
3.650	$2.14 \pm 0.01 \pm 0.06$	$2.16 \pm 0.01 \pm 0.06$

Procedure A:

$$R = \frac{N_{had}}{\sigma_{\mu\mu}^0 L \bar{\epsilon}(1 + \delta)}$$

Procedure B:

$$R = \frac{N_{had}}{\sigma_{\mu\mu}^0 L \epsilon(0)(1 + \delta_{obs})}$$

Fitting of heavy charmonium parameters

The 4 heavy charmoniums with $J^{PC} = 1^{--}$ are

$$\psi(3770) \quad \psi(4040) \quad \psi(4160) \quad \psi(4415)$$

Their properties of production and decay are characterized by the Breit-Wigner amplitude and resonant parameters:

$$\mathcal{T} = \frac{M\sqrt{\Gamma^e\Gamma^h}}{W^2 - M^2 + iM\Gamma_{tot}} e^{i\delta}$$

- nominal mass M
- total width Γ_{tot}
- electronic width Γ_{ee}
- initial phase angle δ

According to Eichten's model, there are following decay channels (f)

$$\psi(3770) \Rightarrow D\bar{D};$$

$$\psi(4040) \Rightarrow D\bar{D}, D^*\bar{D}^*, D\bar{D}^*, \bar{D}D^*, D_s\bar{D}_s;$$

$$\psi(4160) \Rightarrow D\bar{D}, D^*\bar{D}^*, D\bar{D}^*, \bar{D}D^*, D_s\bar{D}_s, D_s\bar{D}_s^*;$$

$$\psi(4415) \Rightarrow D\bar{D}, D^*\bar{D}^*, D\bar{D}^*, \bar{D}D^*, D_s\bar{D}_s, D_s\bar{D}_s^*, D_s^*\bar{D}_s^*.$$

K.Seth's results (hep-ex/0405007)

K.Seth fitted R values measured by CB and BES, and obtained the resonant parameters of $\psi(4040)$, $\psi(4160)$ and $\psi(4415)$. The values of parameters of them were updated in PDG2006 mainly based on the K. Seth's evaluation.

$\psi(4040)$

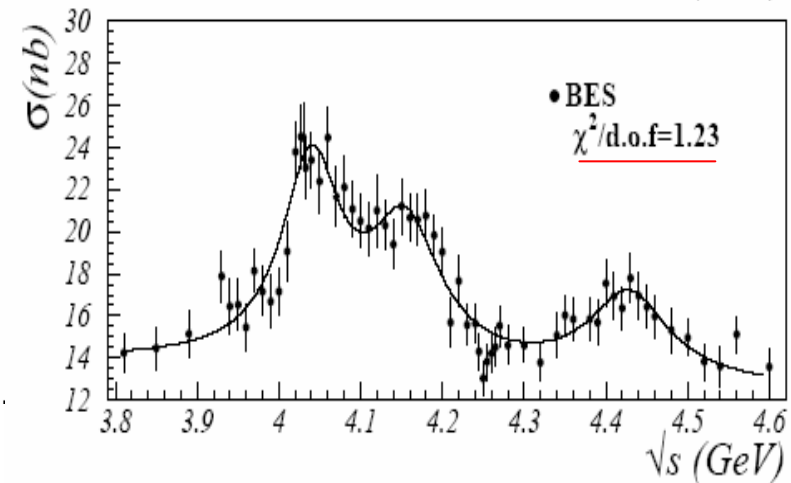
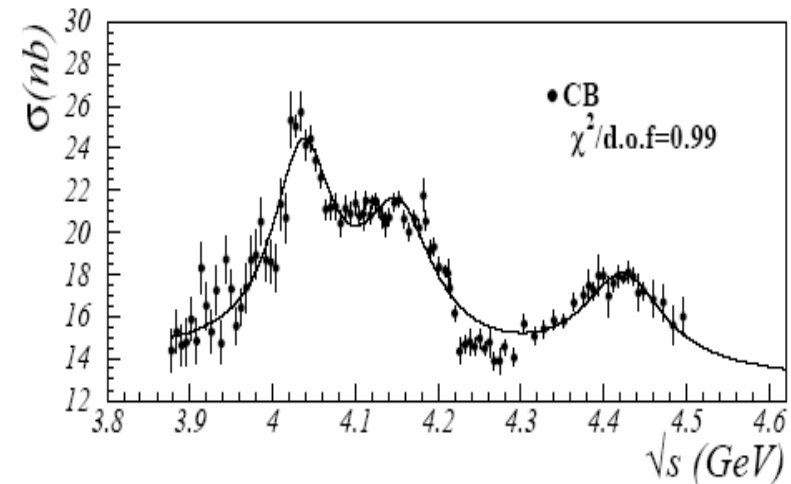
	$M^{(1)}$ (MeV)	$\Gamma_{tot}^{(1)}$ (MeV)	$\Gamma_{ee}^{(1)}$ (keV)
PDG[1]	4040 ± 10	52 ± 10	0.75 ± 0.15
CB[6]	4037 ± 2	85 ± 10	0.88 ± 0.11
BES[7]	4040 ± 1	89 ± 6	0.91 ± 0.13
CB+BES	4039.4 ± 0.9	88 ± 5	0.89 ± 0.08

$\psi(4160)$

	$M^{(2)}$	$\Gamma_{tot}^{(2)}$	$\Gamma_{ee}^{(2)}$
PDG[1]	4159 ± 20	78 ± 20	0.77 ± 0.23
CB[6]	4151 ± 4	107 ± 10	0.83 ± 0.08
BES[7]	4155 ± 5	107 ± 16	0.84 ± 0.13
CB+BES	4153 ± 3	107 ± 8	0.83 ± 0.07

$\psi(4415)$

	$M^{(3)}$	$\Gamma_{tot}^{(3)}$	$\Gamma_{ee}^{(3)}$
PDG[1]	4415 ± 6	43 ± 15	0.47 ± 0.10
CB[6]	4425 ± 6	119 ± 16	0.72 ± 0.11
BES[7]	4429 ± 9	118 ± 35	0.64 ± 0.23
CB+BES	4426 ± 5	119 ± 15	0.71 ± 0.10



Conclusion:

CB and BES measurements are in excellent agreement

Breit-Wigner amplitude

- In quantum mechanics, the wave function for a unstable particle (r)

$$\psi_r(t) = \hat{\psi}_r(0) e^{i\delta_r} \cdot e^{-i\omega_r t} \cdot e^{-t/2\tau} = \hat{\psi}_r(0) e^{i\delta_r} \cdot e^{-it(M_r - i\Gamma_r/2)}$$

- The amplitude as the function of energy W is

$$\chi_r(W) = \int \psi_r(t) e^{iWt} dt = \frac{K_r \cdot e^{i\delta_r}}{(W - M_r) - i\Gamma_r/2}$$

initial phase factor
at moment of production

- Usually, the Breit-Wigner amplitude is written as

$$T_r = \frac{\frac{1}{2}\Gamma_r e^{i\delta_r}}{W - M_r - i\Gamma_r/2} = \frac{\frac{1}{2}\sqrt{\Gamma_r \cdot \Gamma_r} e^{i\delta_r}}{W - M_r - i\Gamma_r/2}$$

- If we consider the special process with initial state e^+e^- and decay final state f

$$T_r^f = \frac{\frac{1}{2}\sqrt{B_r^e \Gamma_r \cdot B_r^f \Gamma_r} e^{i\delta_r}}{W - M_r - i\Gamma_r/2} = \frac{\frac{1}{2}\sqrt{\Gamma_r^e \cdot \Gamma_r^f} e^{i\delta_r}}{W - M_r - i\Gamma_r/2}$$

- In relativistic case, the negative energy state ($W \rightarrow -W$) should be included

$$T_r^f = \left[\frac{\frac{1}{2}\sqrt{\Gamma_r^e \cdot \Gamma_r^f}}{W - M_r + i\Gamma_r/2} + \frac{\frac{1}{2}\sqrt{\Gamma_r^e \cdot \Gamma_r^f}}{-W - M_r + i\Gamma_r/2} \right] = \frac{M_r \sqrt{\Gamma_r^e \Gamma_r^f}}{W^2 - M_r^2 + iM_r \Gamma_r} e^{i\delta_r}$$

Problems in fitting

➤ Interference

- the interferential summation of the amplitudes with the same decay channel f

$$\left. \vphantom{\sum_r} \right\} \mathcal{T}_{res}^f = \sum_r \mathcal{T}_r^f$$

- the non-interferential summation of the different decay channels f

$$\left. \vphantom{\sum_f} \right\} |\mathcal{T}_{res}|^2 = \sum_f |\mathcal{T}_{res}^f|^2$$

- inclusive resonant cross section expressed by the form of R value

$$\left. \vphantom{\frac{\sigma_{res}}{\sigma_{\mu\mu}^0}} \right\} R_{res} = \frac{\sigma_{res}}{\sigma_{\mu\mu}^0} = \frac{12\pi}{s} |\mathcal{T}_{res}|^2$$

➤ Continuous background (polynomial)

$$R_{con} = C_0 + C_1(W - 2M_{D^\pm}) + C_2(W - 2M_{D^\pm})^2$$

➤ Energy-dependent hadronic width (potential model in quantum mechanics)

$$\Gamma_r^{had}(W) = \frac{2M_r}{M_r + W} \sum_f \Gamma_r^f(W) \quad \Gamma_r^f(W) = \hat{\Gamma}_r \sum_L \frac{Z_f^{2L+1}}{B_L}$$

➤ Objective function in fitting

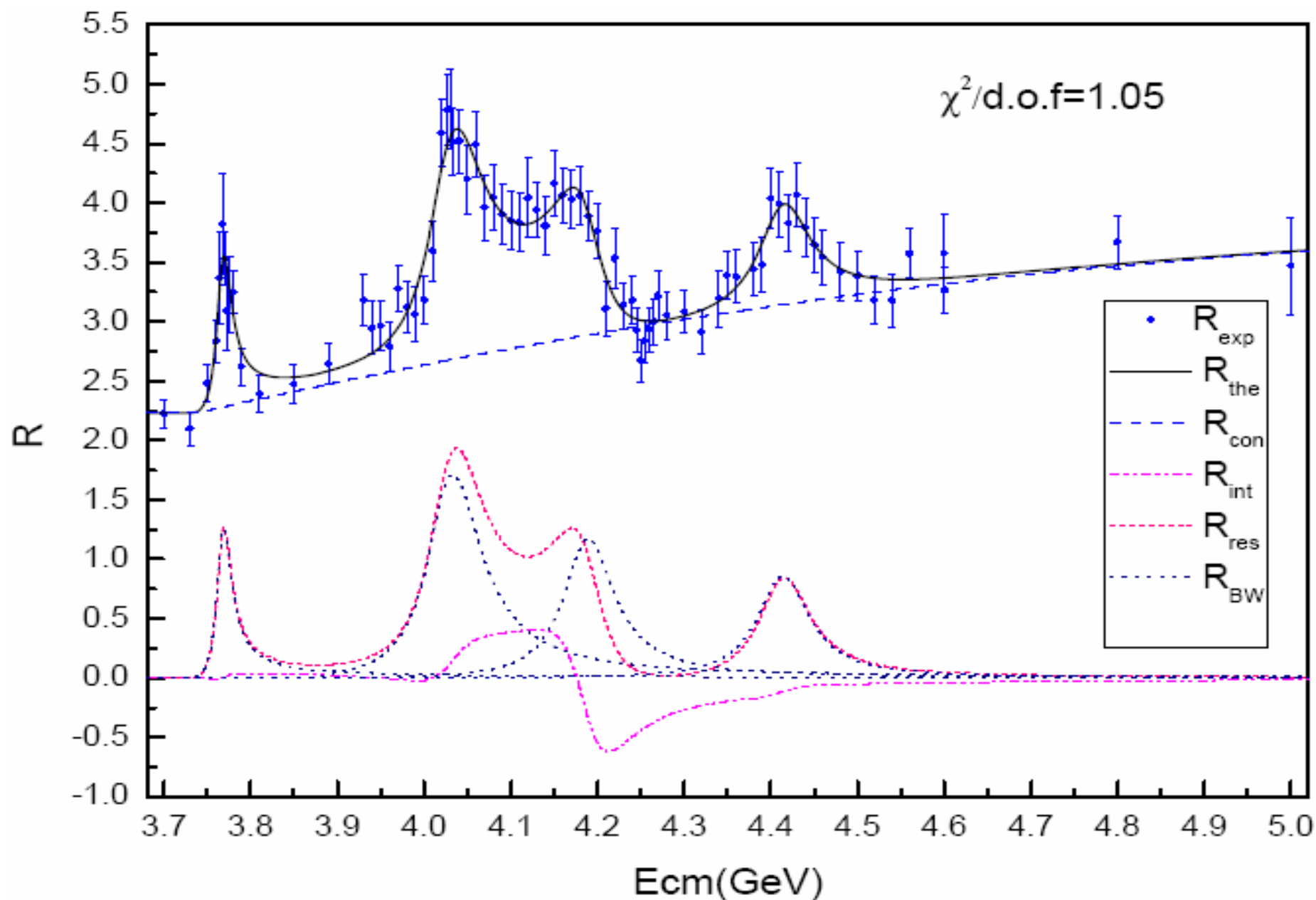
$$\chi^2 = \sum_i \frac{[f_c \tilde{R}_{exp}(W_i) - \tilde{R}_{the}(W_i)]^2}{[f_c \Delta \tilde{R}_{exp}^{(i)}]^2} + \frac{(f_c - 1)^2}{\sigma_c^2} \xrightarrow{\text{convergence}}$$

updated
 M Γ_{ee} Γ_{tot} δ
 R values

$$\tilde{R}_{exp} = \frac{N_{had}^{obs} - N_{bg}}{\sigma_{\mu\mu}^0 L \epsilon_{trg} \epsilon_{had}}$$

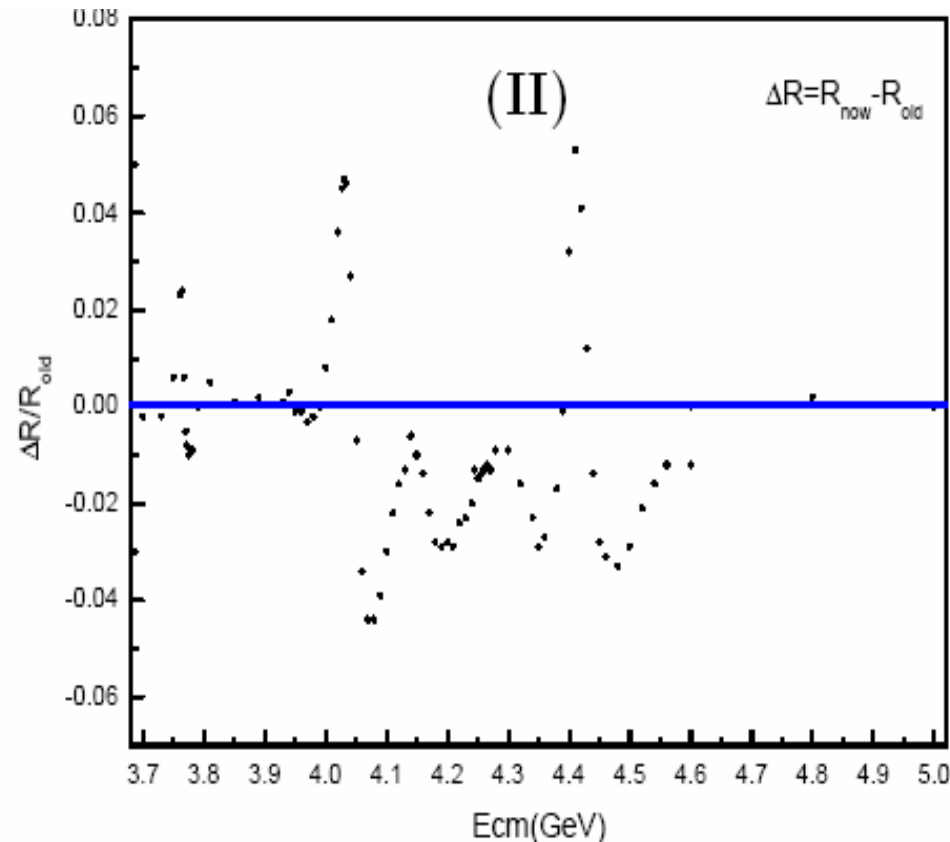
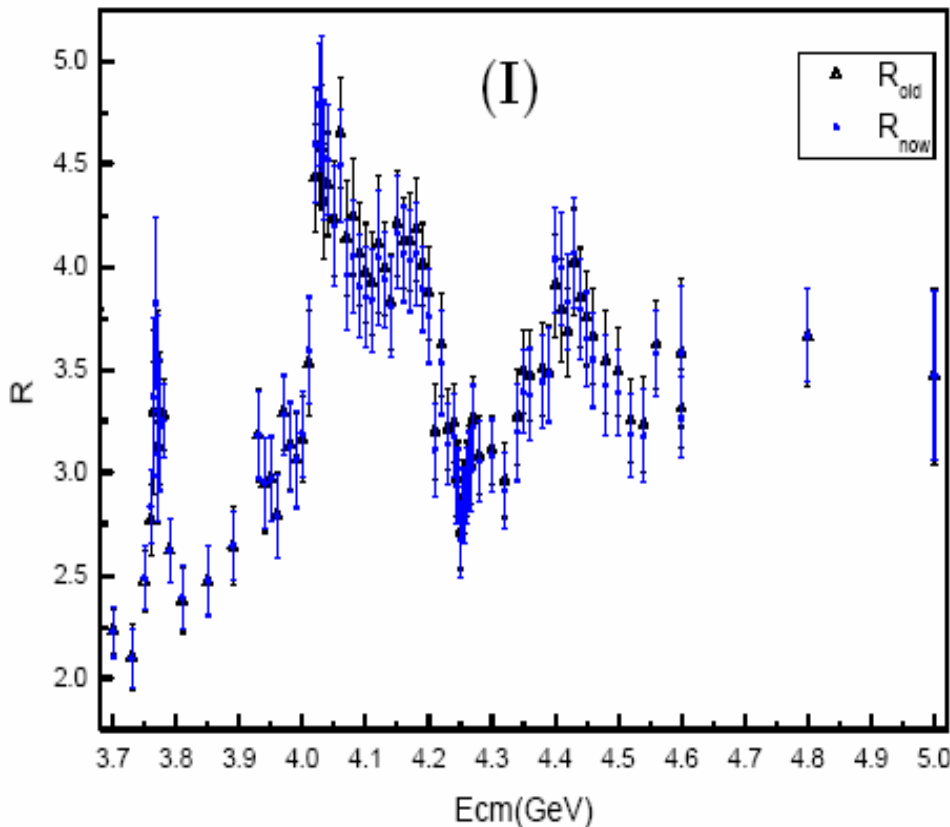
$$\tilde{R}_{the} = (1 + \delta_{obs}) R_{the}$$

Updated R value



Comparisons of old and new results

The updated R value in resonant region are different from the previous results in **Phys. Rev. Lett. 88 (2002)101802** due to the changes of the resonant parameters and the initial state radiative correction factor $(1+\delta_{\text{obs}})$ in expression R_{exp} .

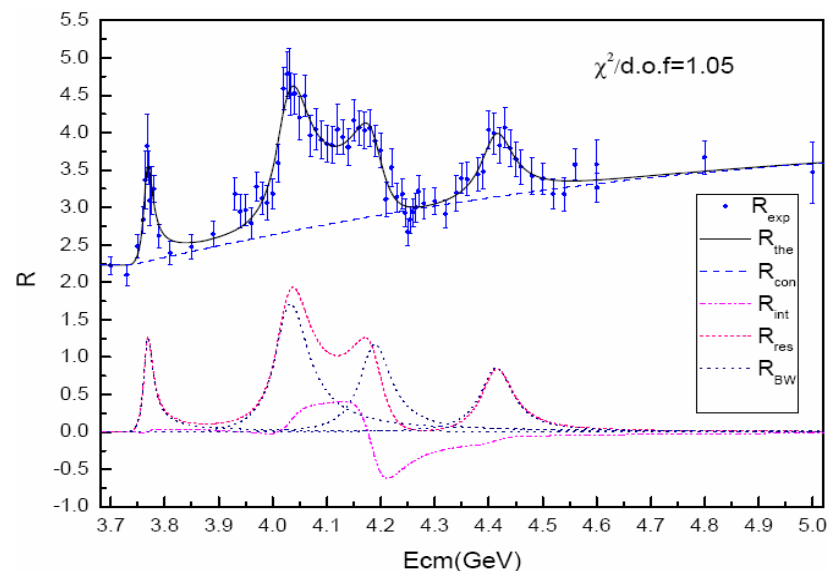
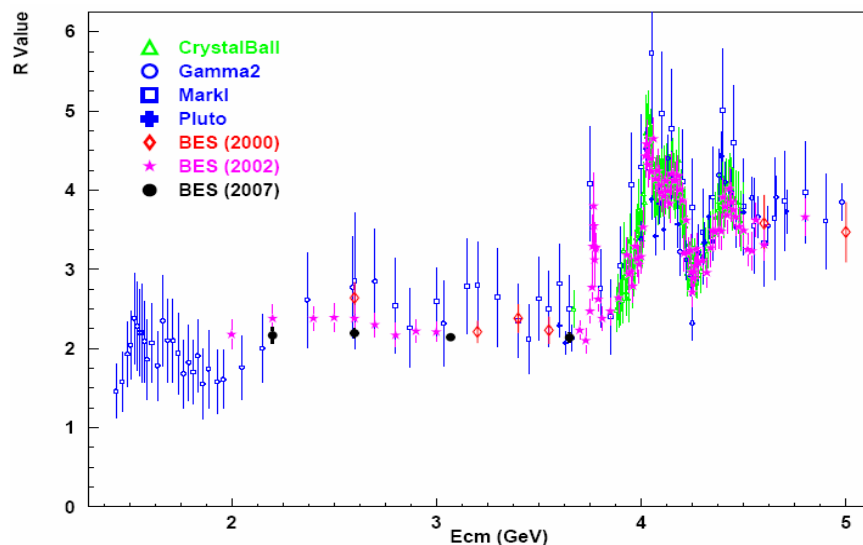


Resonant parameters

		$\psi(3770)$	$\psi(4040)$	$\psi(4160)$	$\psi(4415)$
M (MeV/ c^2)	PDG2004	3769.9 ± 2.5	4040 ± 10	4159 ± 20	4415 ± 6
	PDG2006	3771.1 ± 2.4	4039 ± 1.0	4153 ± 3	4421 ± 4
	CB(Seth)	-	4037 ± 2	4151 ± 4	4425 ± 6
	BES(Seth)	-	4040 ± 1	4155 ± 5	4455 ± 6
	BES(this work)	3771.3 ± 1.7	4042.7 ± 5.8	4193.2 ± 6.7	4417.2 ± 9.1
Γ_{tot} (MeV)	PDG2004	23.6 ± 2.7	52 ± 10	78 ± 20	43 ± 15
	PDG2006	23.0 ± 2.7	80 ± 10	103 ± 8	62 ± 20
	CB(Seth)	-	85 ± 10	107 ± 10	119 ± 16
	BES(Seth)	-	89 ± 6	107 ± 16	118 ± 35
	BES(this work)	25.6 ± 6.3	88.9 ± 12.4	78.8 ± 16.1	80.4 ± 24.7
Γ_{ee} (keV)	PDG2004	0.26 ± 0.04	0.75 ± 0.15	0.77 ± 0.23	0.47 ± 0.10
	PDG2006	0.24 ± 0.03	0.86 ± 0.08	0.83 ± 0.07	0.58 ± 0.07
	CB(Seth)	-	0.88 ± 0.11	0.83 ± 0.08	0.72 ± 0.11
	BES(Seth)	-	0.91 ± 0.13	0.84 ± 0.13	0.64 ± 0.23
	BES(this work)	0.19 ± 0.04	0.94 ± 0.19	0.50 ± 0.28	0.40 ± 0.15
δ (degree)	BES(this work)	0	135 ± 58	306 ± 40	247 ± 75

Summary

- The R values in energy region 2.20 – 3.65 GeV are measured with the improved events selection criteria and the retuned LUARLW. The errors of measurements are about 4.7% at 2.2 GeV, and 3.3% at 2.60, 3.07 and 3.65 GeV. The new results agree with the prediction of pQCD well.
- The resonant parameters of high mass charmonium are determined. In the fitting, the interference and energy-dependent hadronic width are considered. The R values in resonant region are updated due to the new values of the resonant parameters and ISR factors.



THANK YOU

R value at 3.65 GeV measured by another group in BES

Phys. Rev. Lett. 97, (2006)121801

Fit fine scanned hadronic cross section between 3.660 – 3.872 GeV:

$$R_{uds} = 2.262 \pm 0.054 \pm 0.109$$

$$M_{\psi(3770)} = 3772.2 \pm 0.7 \pm 0.3$$

$$\Gamma_{\psi(3770)}^{\text{tot}} = 26.9 \pm 2.4 \pm 0.3 \text{ MeV} \quad \Gamma_{\psi(3770)}^{ee} = 251 \pm 26 \pm 11 \text{ eV}$$

Phys. Lett. B652, (2007)238

Direct measurement with another independent way:

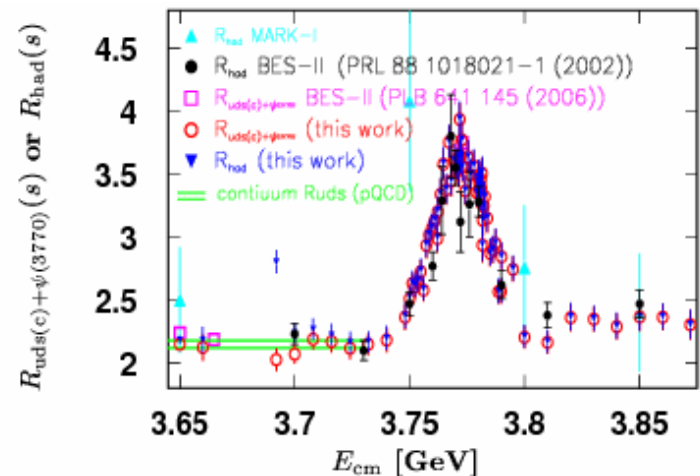
$$R_{uds} = 2.141 \pm 0.025 \pm 0.085$$

ArXiv:hep-xp/0605105

Direct measurement and the weighted average:

$$\bar{R}_{uds} = 2.224 \pm 0.019 \pm 0.089$$

$$BF(\psi(3770) \rightarrow \text{non} - D\bar{D}) = (16.1 \pm 1.6 \pm 5.7)\%$$



Fitting schemes

Two experimental quantities: **R value** or **R-like value**

Scheme A: fitting true R value

$$R_{exp} = \frac{N_{had}^{obs} - N_{bg}}{\sigma_{\mu\mu}^0 L \epsilon_{trg} \epsilon_{had}^0 (1 + \delta_{obs})}$$

$$R_{the}$$

$$\chi^2 = \sum_i \frac{(f R_{exp}(s_i) - R_{the}(s_i))^2}{(f \Delta \hat{R}_{exp}^{(i)})^2} + \frac{(f - 1)^2}{\sigma_f^2}$$

Errors are not constant in iterative fitting, but they can not correctly update in fitting

Scheme B: fitting R -like value

$$\tilde{R}_{exp} = \frac{N_{had}^{obs} - N_{bg}}{\sigma_{\mu\mu}^0 L \epsilon_{trg} \epsilon_{had}^0}$$

$$\tilde{R}_{the} = (1 + \delta_{obs}) R_{the}$$

$$\chi^2 = \sum_i \frac{(f \tilde{R}_{exp}(s_i) - \tilde{R}_{the}(s_i))^2}{(f \Delta \tilde{R}_{exp}^{(i)})^2} + \frac{(f - 1)^2}{\sigma_f^2}$$

Errors are independent of fitting, and they keep constant in iterative fitting

It is noticed that the errors of the experimental quantities will affect the convergence condition and then the fitting results. Therefore the correct input of the error is important. **Errors in scheme B are correct.**

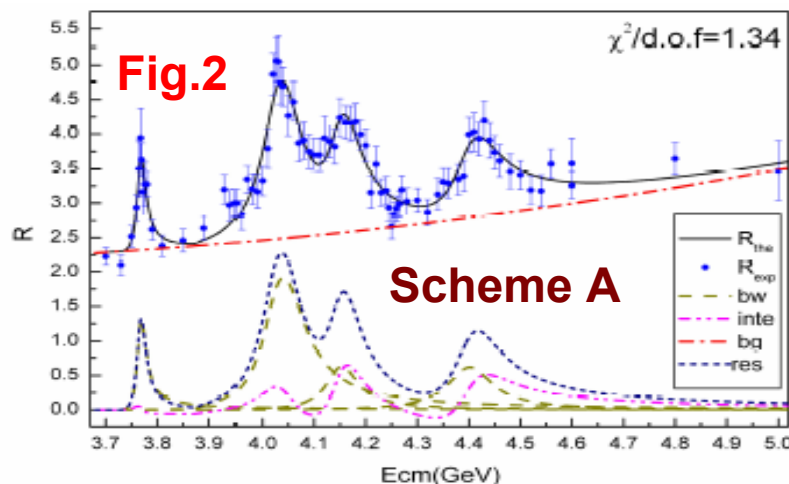
scheme dependence

Phase angle $\delta \neq 0$ and $\delta = 0$

scheme A and scheme B

total width energy dependence in QM and polynomial of degree 2 for charm BG

$\delta = 0$

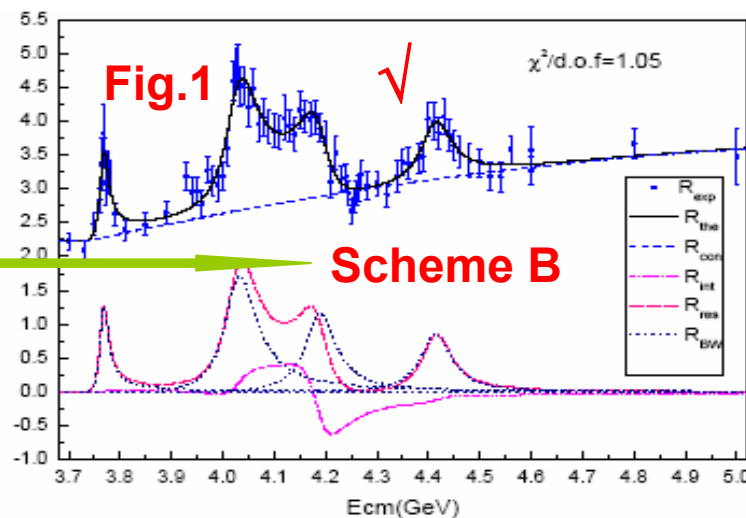
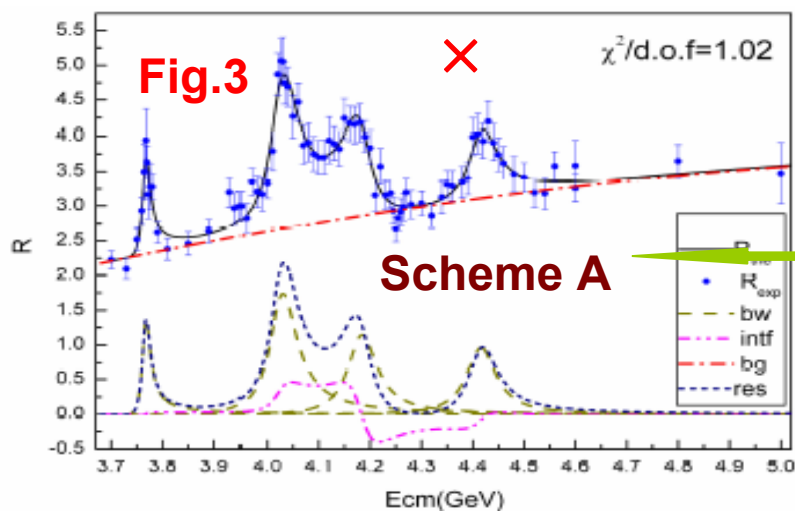


Interference
are different for
 $\delta = 0$ or $\delta \neq 0$

It is noticed that the peak of $\psi(4040)$ in scheme A is clearer than in scheme B.

But scheme A is incorrect !!!

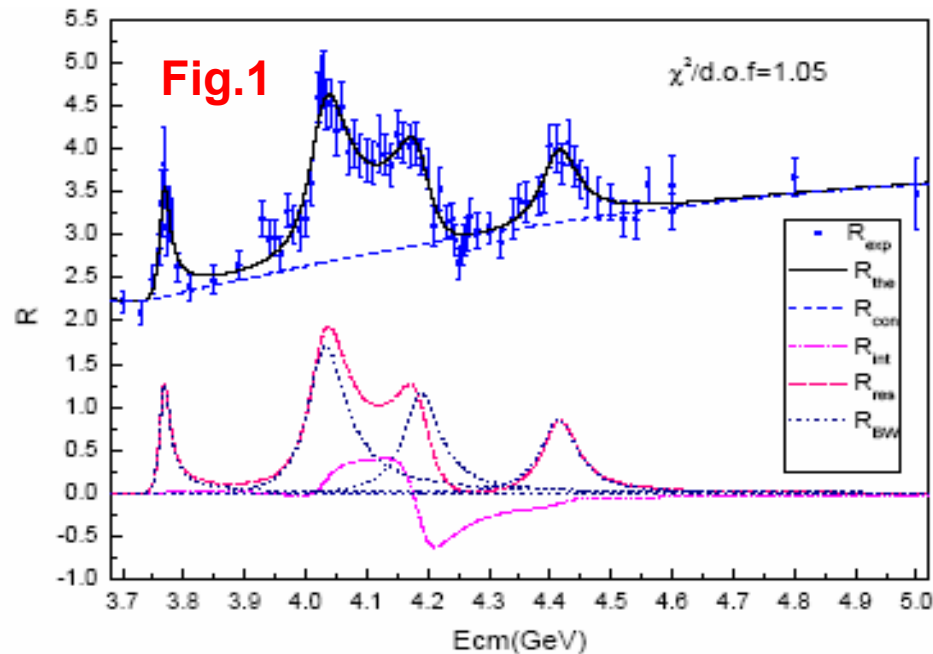
$\delta \neq 0$



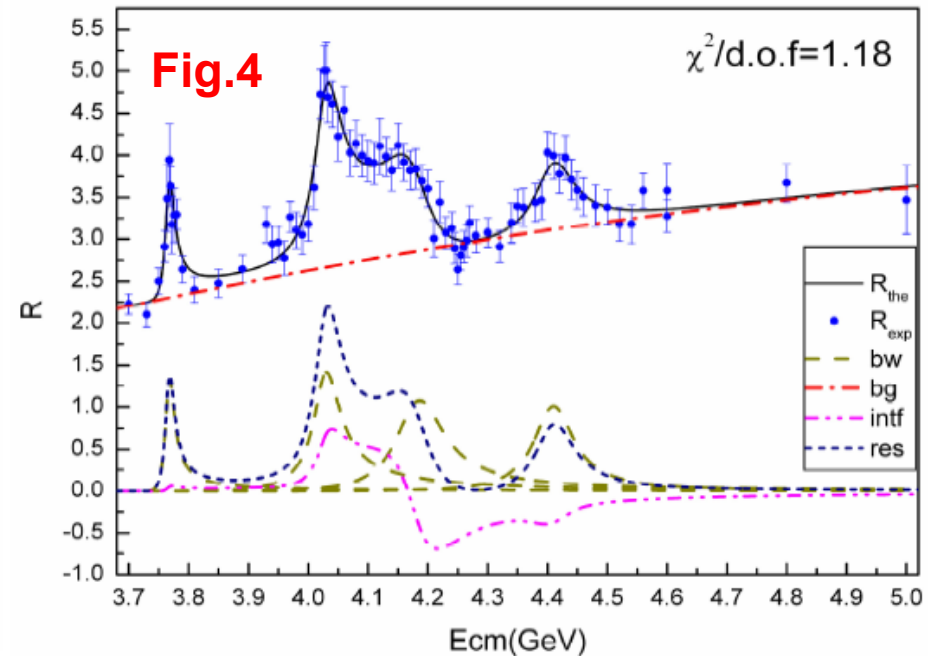
model dependence

Energy dependence for total width: **QM and EIT**

- Breit-Wigner with non-zero phase angle
- Polynomial of degree 2 for the charmed continuous BG
- Objective function B for χ^2



Energy dependence of total width in
quantum mechanics



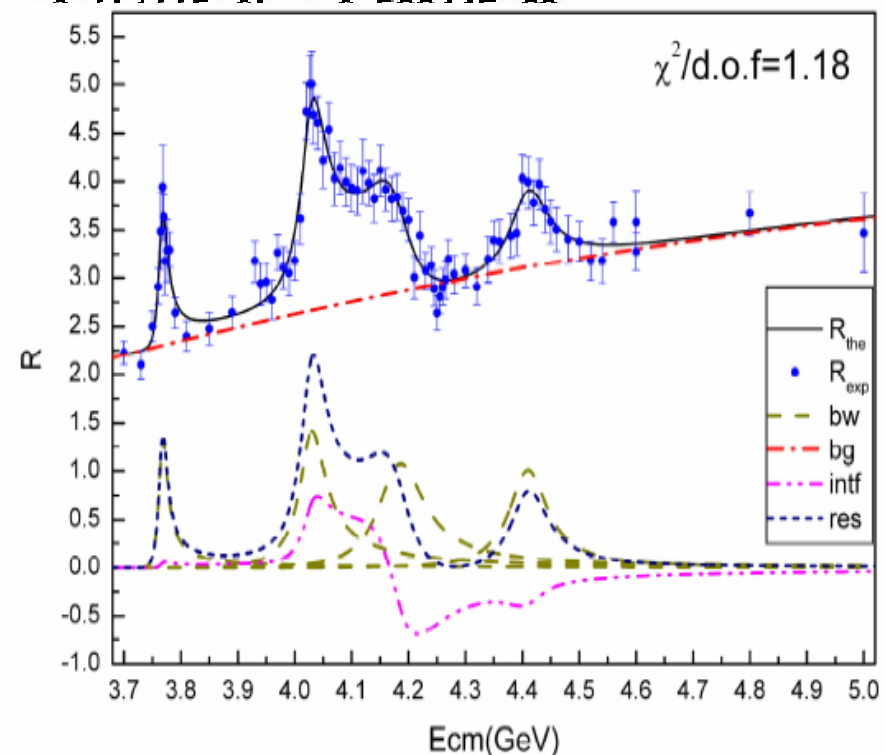
Energy-dependence of total width in
effective interaction theory

Appendix: MINUIT's report for EIT

MIGRAD MINIMIZATION HAS CONVERGED.

FCN= 62.36267 FROM MIGRAD STATUS=CONVERGED 111 CALLS 5112 TOTAL
EDM= 0.39E-05 STRATEGY=1 ERROR MATRIX UNCERTAINTY= 6.6%

EXT NO.	PARAMETER NAME	VALUE	ERROR	STEP SIZE	FIRST DERIVATIVE
1	WE3770	0.18042	0.34017E-01	-0.48003E-05	0.24214E-02
2	N3770	3770.9	1.6142	-0.97994E-04	-0.87933E-02
3	W3770	24.325	5.6910		
4	WE4040	0.61797	0.11060		
5	N4040	4040.0	0.11829E-02		
6	WPP4040	69.976	11.706		
7	WE4160	0.66593	0.24537		
8	N4160	4192.7	0.7443		
9	WPP4160	82.774	30.824		
10	WE4415	0.49702	0.16830		
11	N4415	4412.3	0.6644		
12	WPP4415	77.105	21.659		
13	coeff1	2.2019	0.98127E-01		
14	coeff2	0.15204E-02	0.30413E-03		
15	coeff3	-0.33287E-06	0.31450E-06		
16	scale	1.0000	0.32102E-01		
17	theta1	1.0641	0.52322		
18	theta2	3.4804	0.60458		
19	theta3	-10.232	0.94722		
20	WUP4040	0.74594E-04	13.105		
21	WUU4040	17.220	9.4186		
22	WUP4160	0.76064E-05	13.766		
23	WUU4160	23.152	25.977		
24	WUP4415	0.88733E-04	15.709		
25	WUU4415	6.5102	15.939		

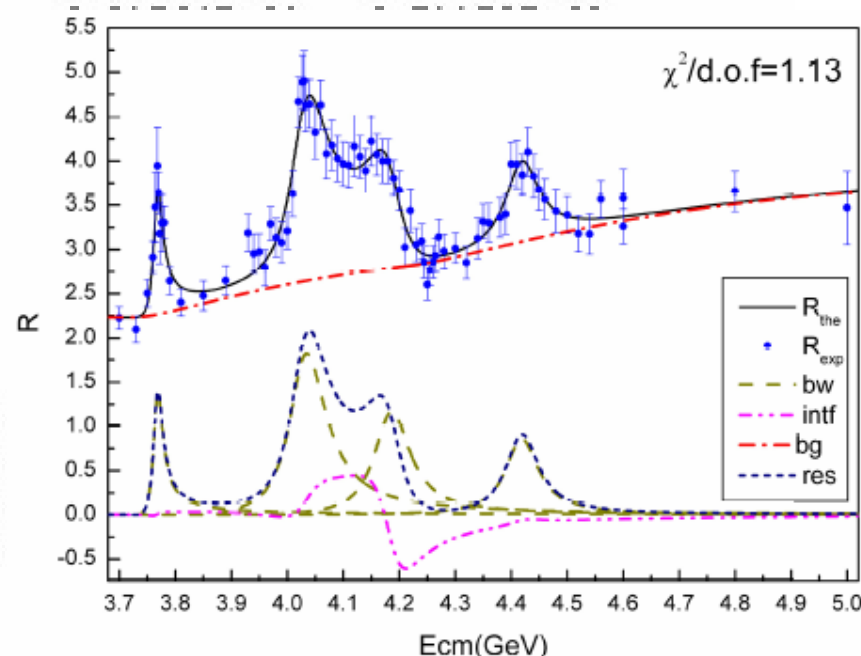


0.36723E-03	-0.37111E-02
0.76062E-03	0.34982E-02
0.67111E-03	-0.13739E-01

Appendix: MINUIT's report for DASP

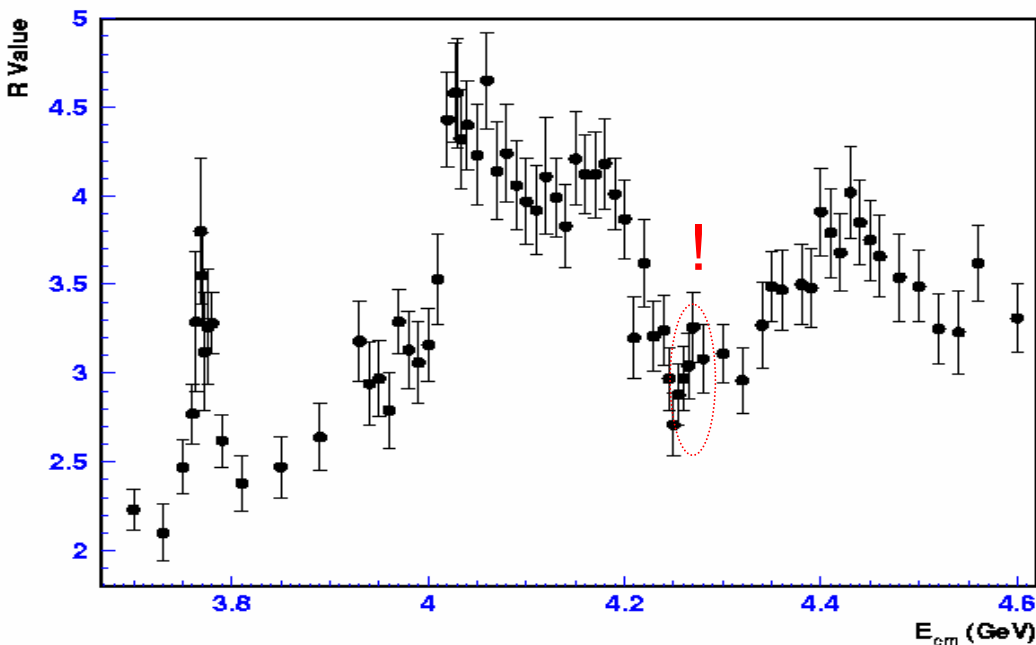
FCN= 62.42465 FROM HIGRAD STATUS=CONVERGED 1936 CALLS 15021 TOTAL
EDH= 0.66E-05 STRATEGY=1 ERROR MATRIX UNCERTAINTY= 7.0%

EXT NO.	PARAMETER NAME	VALUE	ERROR	STEP SIZE	FIRST DERIVATIVE
1	VE3770	0.20225	0.40136E-01	0.90606E-04	0.66576E-02
2	H3770	3771.2	1.6498	0.90421E-04	-0.52118E-02
3	U3770	25.988	6.2678	0.21723E-03	0.63405E-02
4	VE4040	0.91024	0.18895		
5	H4040	4040.6	3.7875		
6	U4040	86.188	11.443		
7	VE4160	0.54063	0.19170		
8	H4160	4189.2	6.6293		
9	U4160	78.403	14.464		
10	VE4415	0.36782	0.10256		
11	H4415	4420.2	9.3570		
12	U4415	72.762	20.411		
13	coeff1	0.22236	0.33391E-01		
14	coeff2	0.33084E-09	0.78589E-01		
15	coeff3	0.30812E-09	0.61956E-01		
16	coeff4	0.20649E-09	0.76427E-01		
17	coeff5	0.58067E-09	0.10792		
18	coeff6	0.65915	0.22801		
19	const	2.2344	0.10136	0.76288E-06	-0.20247E-02
20	scale	1.0000	0.32998E-01	0.36410E-04	0.32186E-01
21	theta1	2.2974	0.66299	-0.59792E-04	0.11575E-02
22	theta2	5.1798	0.59059	-0.13589E-03	0.11593E-02
23	theta3	4.1630	1.4160	-0.15280E-02	-0.57808E-03

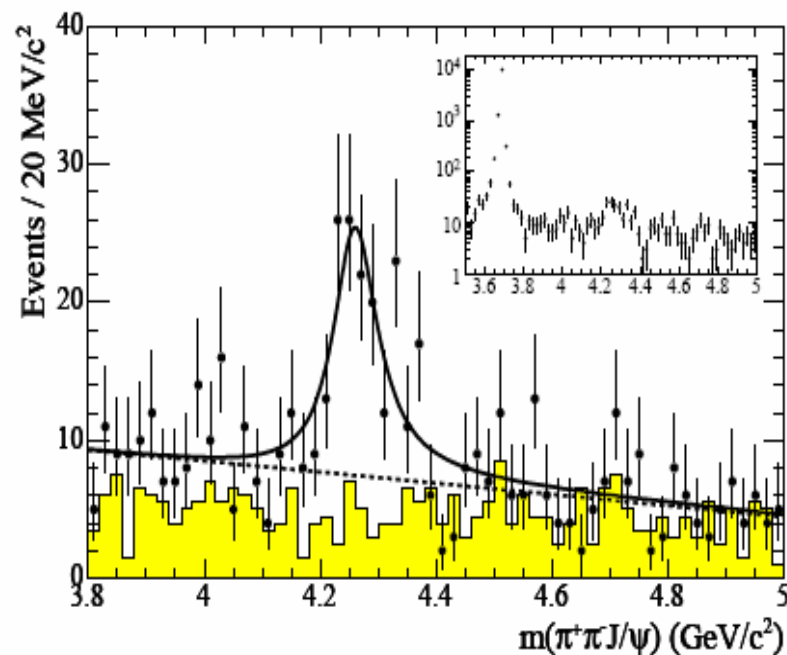


Upper limit of electronic width of Y(4260)

Scanned resonant structure of the higher charmonia by BES



BABAR discovered Y(4260)



Based on the published R value measured at BES, the upper limit of the electronic width of Y(4260) was estimated:

$$\Gamma_{ee} < 580 \text{ eV}/c^2 \text{ at } 90\% \text{ CL}$$

See the detail descriptions in *Phys. Lett. B* 640, (2006) 182-187

Summary of the previous fitting

Some works have measured the resonant parameters of the higher charmonia. The methods of these works may be summarized as:

- Fit the published R values
- Did not consider the phase angle of the Breit-Wigner amplitude
- Neglected the interference effects
- Assumed the total width is energy independent

Fitting

Experimental quantity

$$R_{exp} = \frac{N_{had}^{obs} - N_{bg}}{\sigma_{\mu\mu}^0 L \epsilon_{trg} \epsilon_{had} (1 + \delta_{obs})}$$

Theoretical quantity

$$R_{the} = R_{con} + R_{res}$$

Resonant parameters

Problems in Fitting

If we inspect the previous fittings, the following questions should be reviewed

Physical

- Breit-Wigner amplitude with δ or not?
- energy dependence of total width ?
- form of the continuous charm BG ?
- interference among the 4ψ ?

Definition of χ^2 in fitting

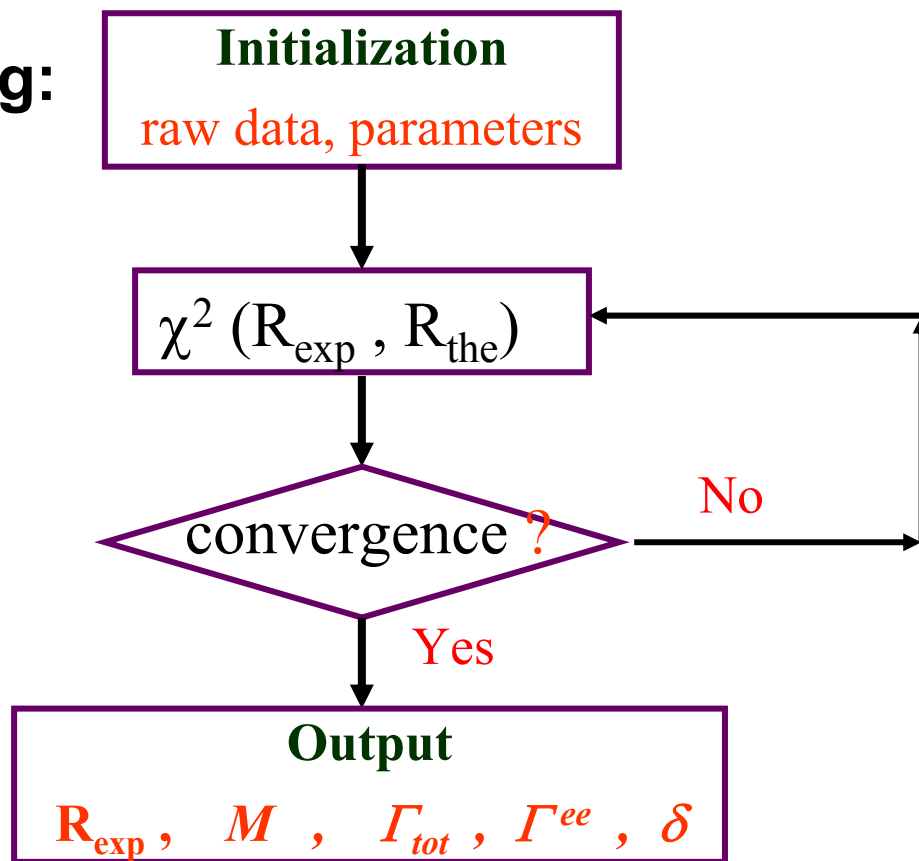
- objective function A: fitting true R value
- objective function B: fitting R-like value

All of these physical problems and fitting schemes will influence the values of the resonant parameters

Fitting procedures

The values of the resonant parameters will influence $(1+\delta)$ and then R_{exp} value, so the **measurement of R value and the determination of the resonant parameters should be done in iterative way and in same procedure with the MINUIT.** But no one did so before. 🙄

Follow chart for fitting:



Comparisons between data and LUARLW

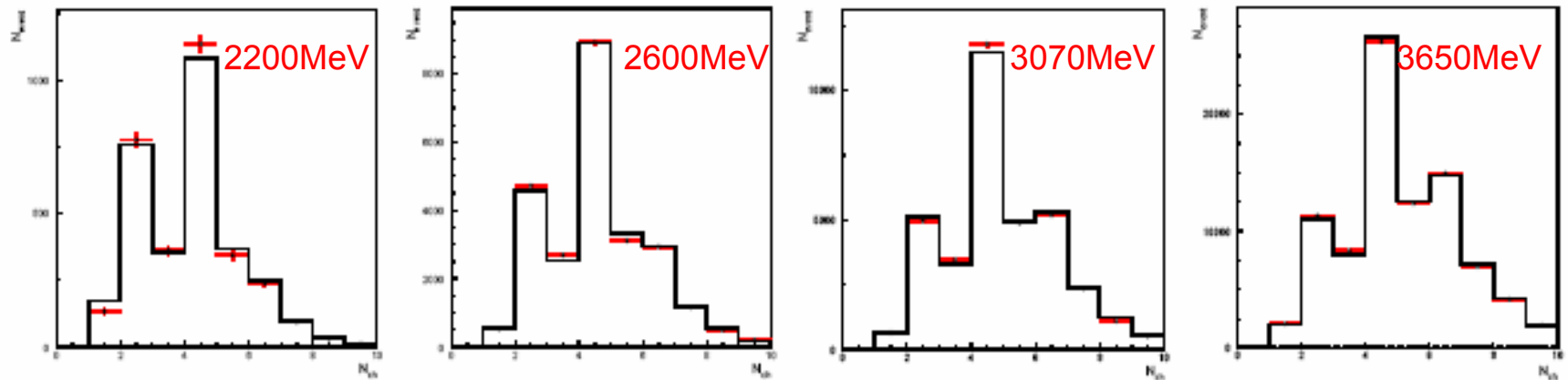


Figure 1: Distribution of the total charged multiplicity.

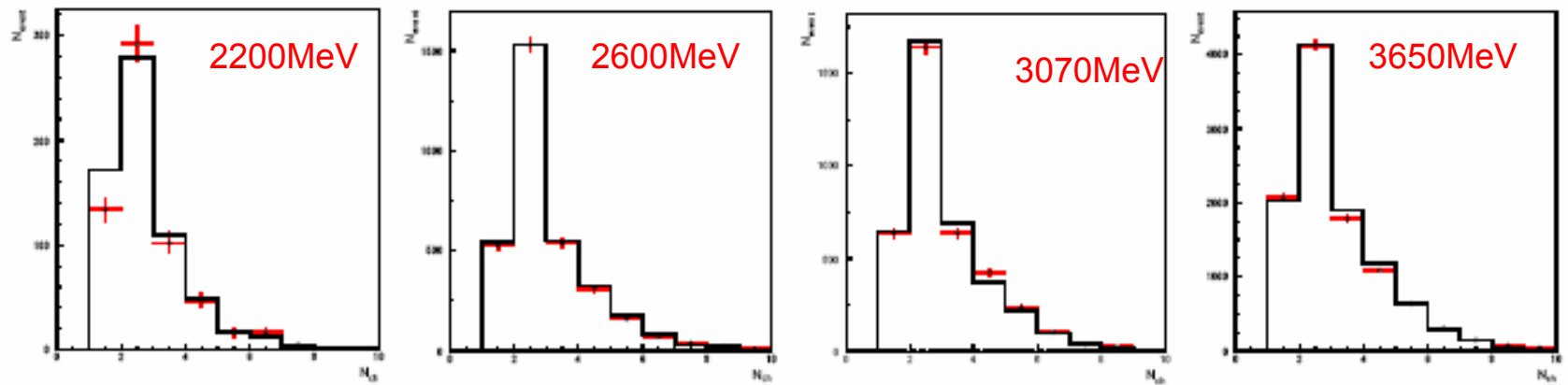


Figure 2: Distribution of charged multiplicity of the events with one good charged track.

Comparisons between data and LUARLW

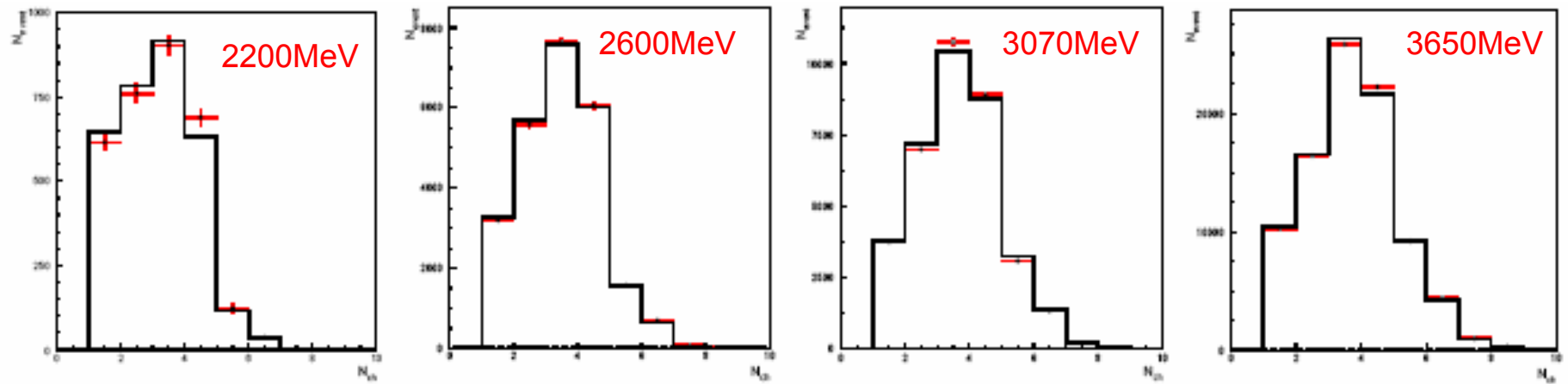


Figure 3: Good charged multiplicity distributions.

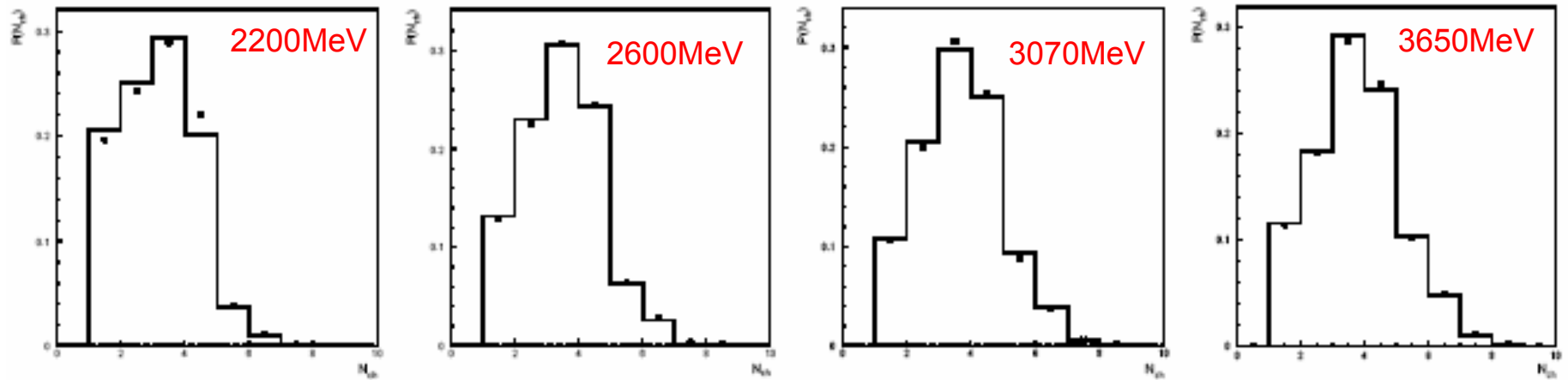


Figure 4: Normalized good charged multiplicity distributions.

Comparisons between data and LUARLW

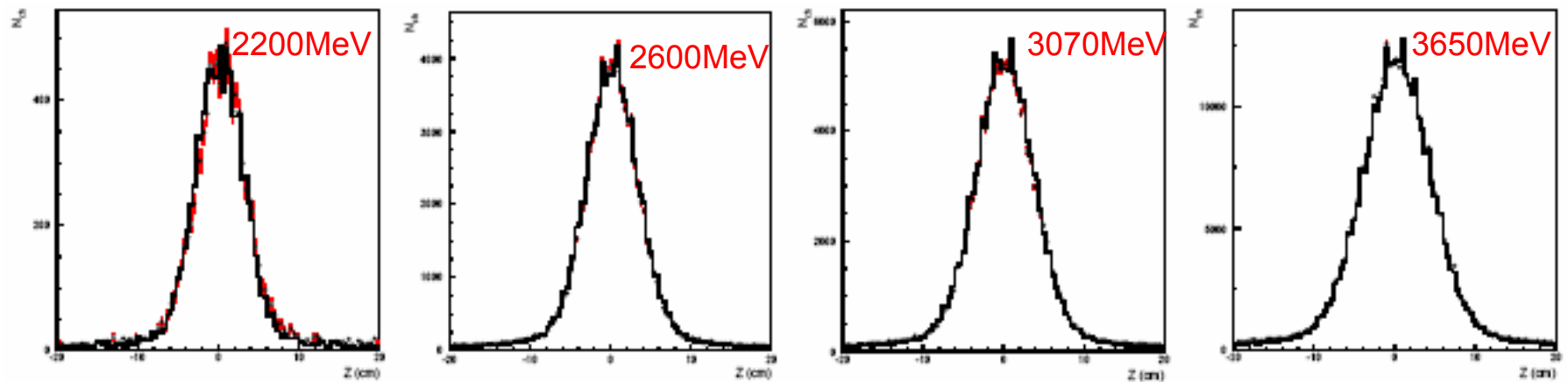


Figure 5: Vertex distribution in z direction for good charged track.

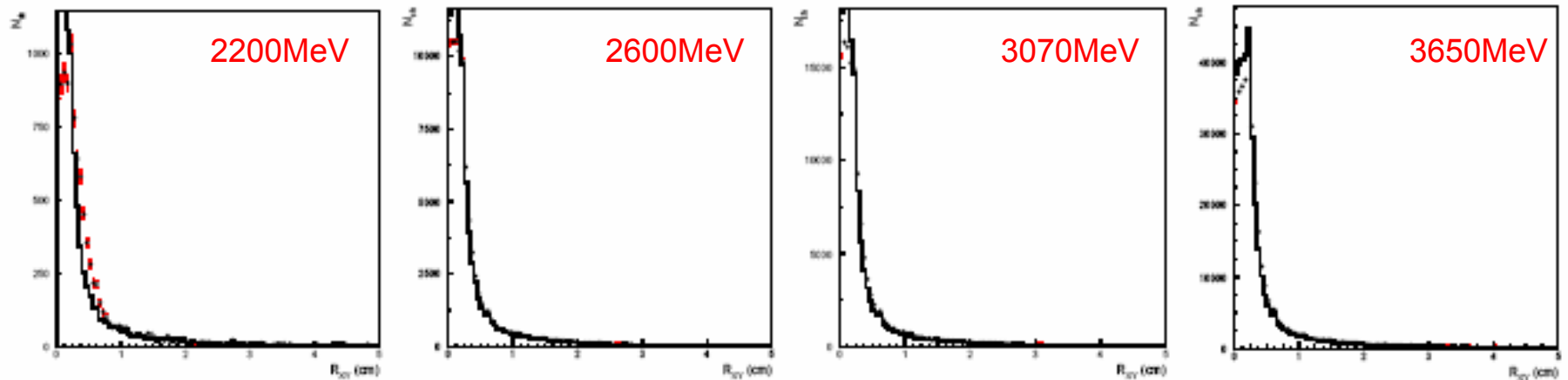


Figure 6: Vertex distribution in $x-y$ plane for good charged track.

Comparisons between data and LUARLW

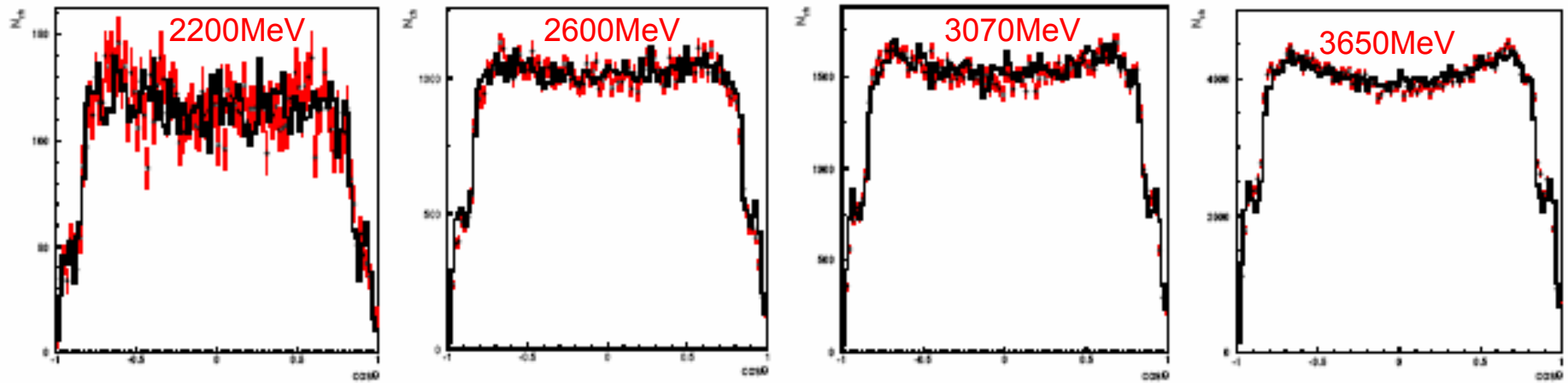


Figure 7: Polar angle $\cos \theta$ in MDC distribution.

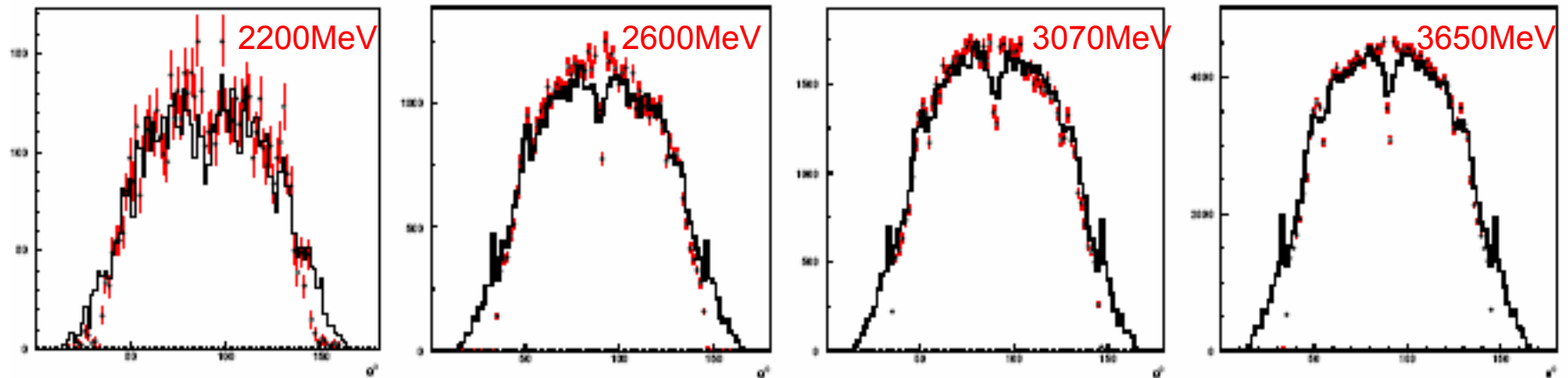


Figure 8: Polar angle $\cos \theta$ in BSC distribution.

Comparisons between data and LUARLW

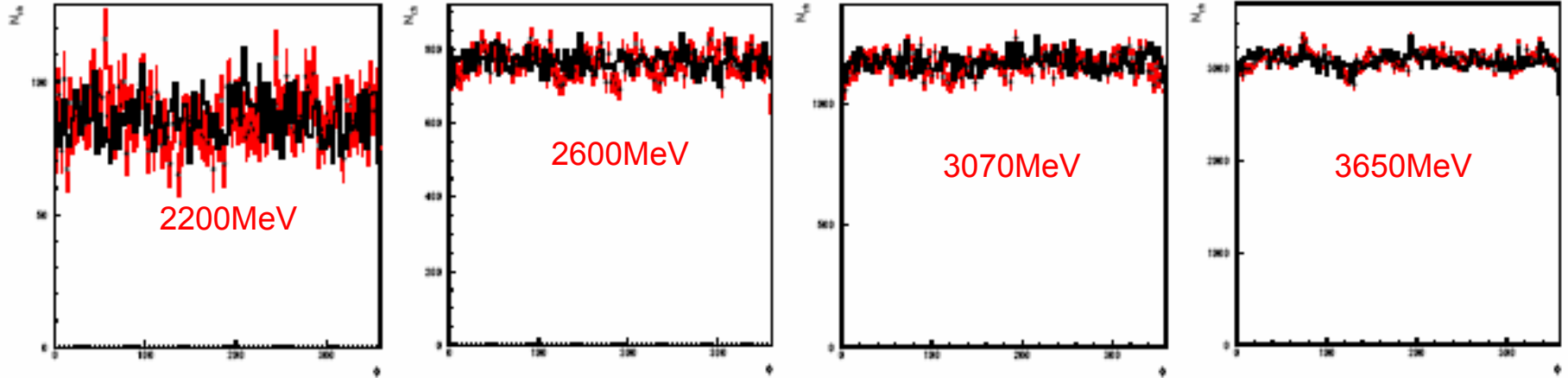


Figure 9: Azimuthal angle ϕ distribution.

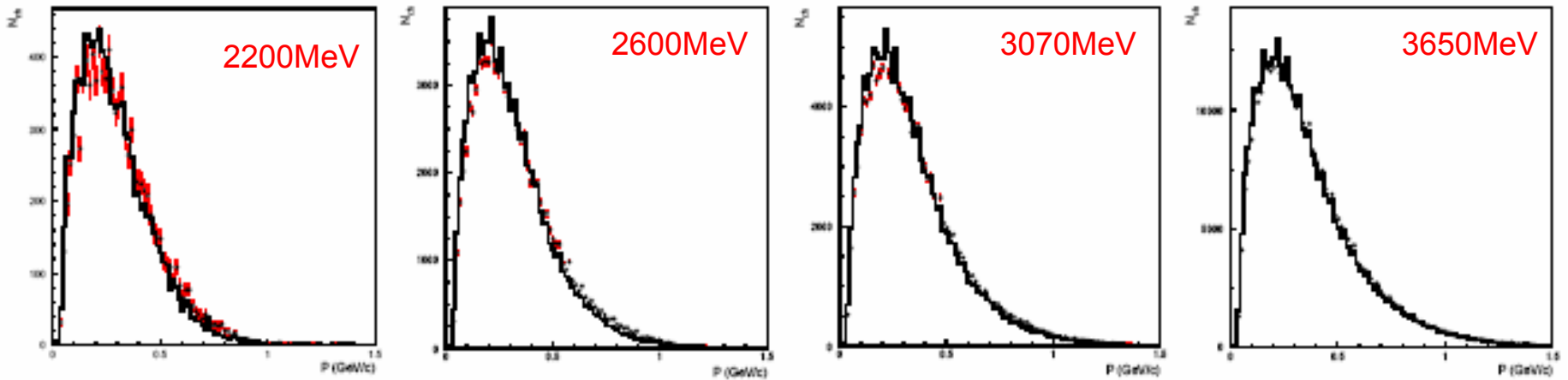


Figure 10: Track momentum p distribution.

Comparisons between data and LUARLW

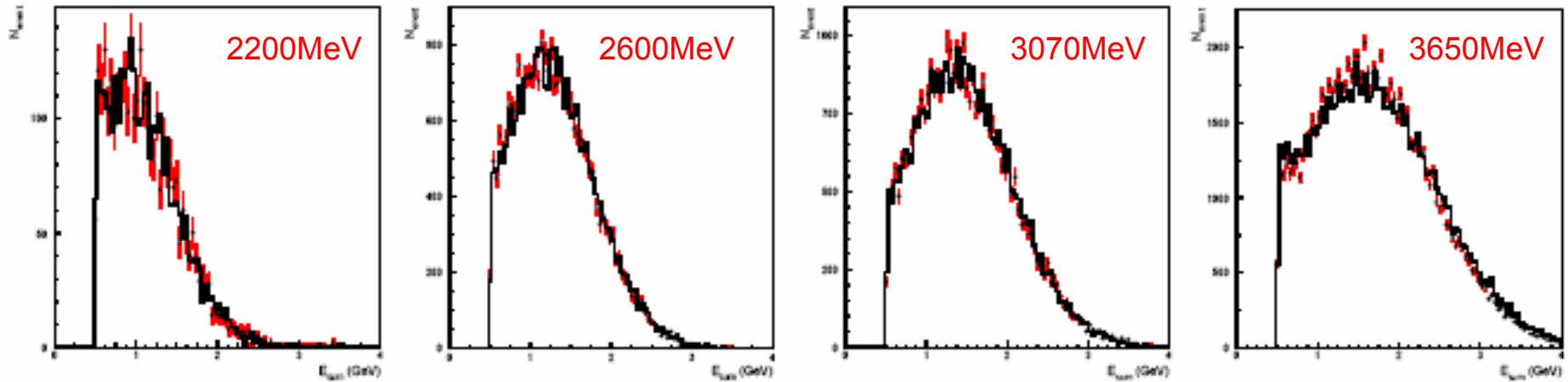


Figure 11: Distribution of deposit energy in BSC.

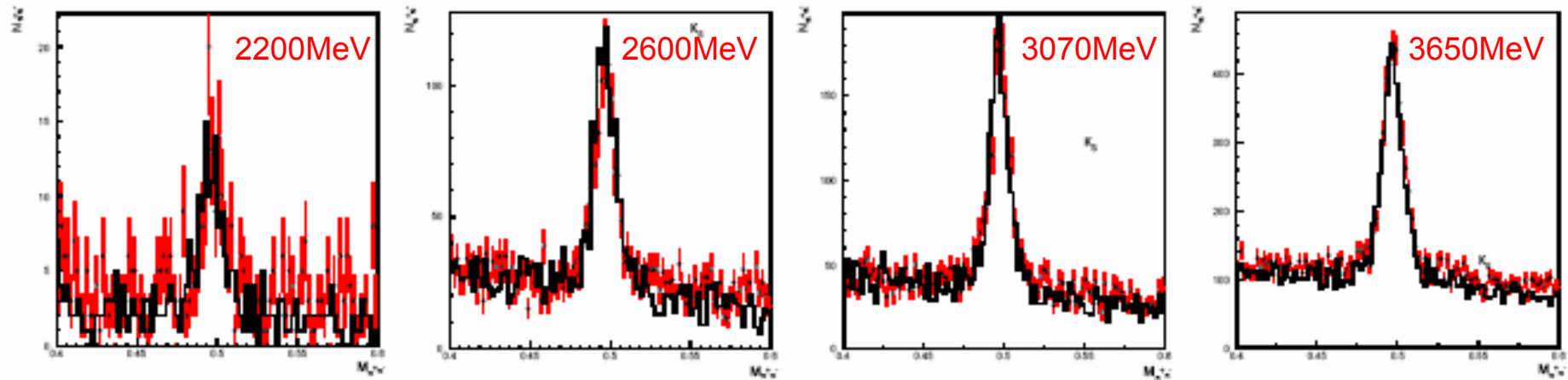


Figure 12: Distribution of invariant mass of $K_S \rightarrow \pi^+\pi^-$.

Comparisons between data and LUARLW

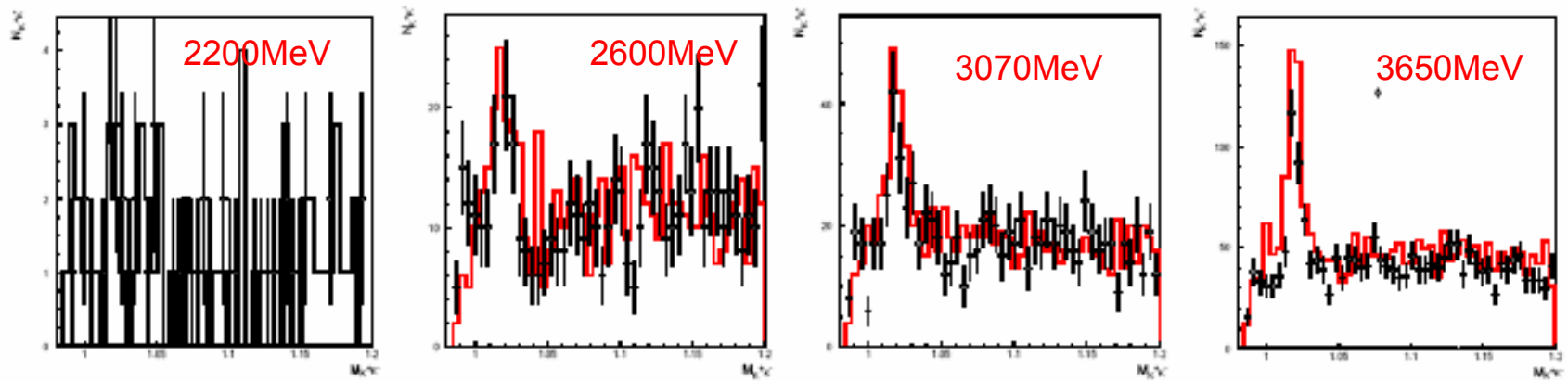


Figure 13: Distribution of invariant mass of $\phi \rightarrow K^+K^-$.

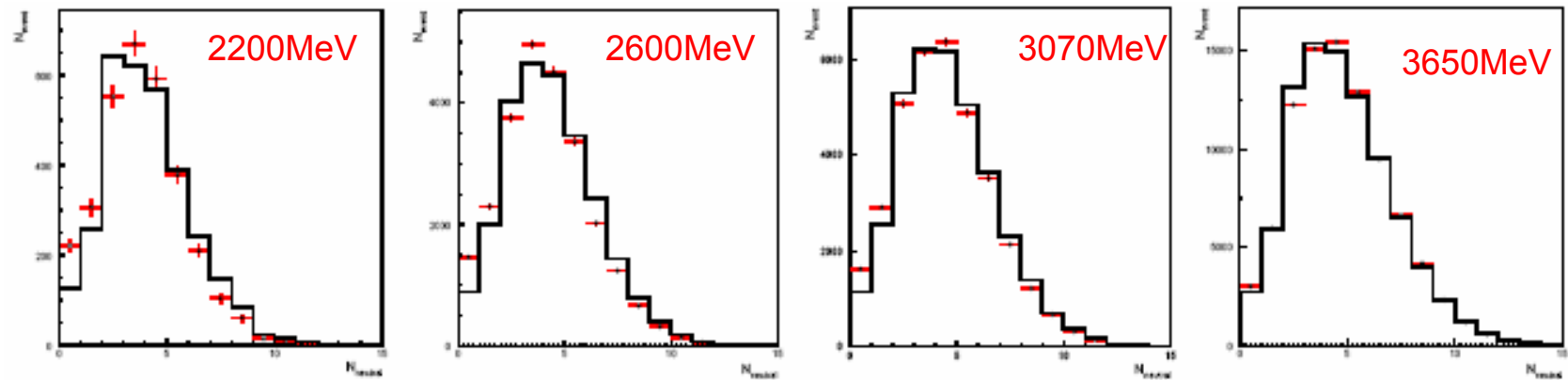


Figure 14: Distribution of the neutral multiplicity.

Comparisons between data and LUARLW

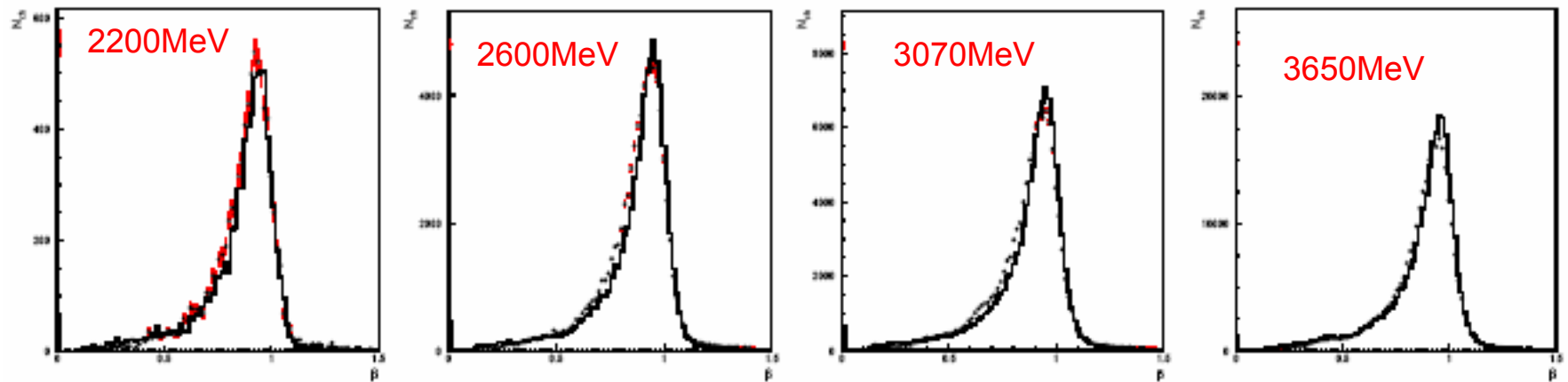


Figure 15: Distribution of the speed of the good charged track.

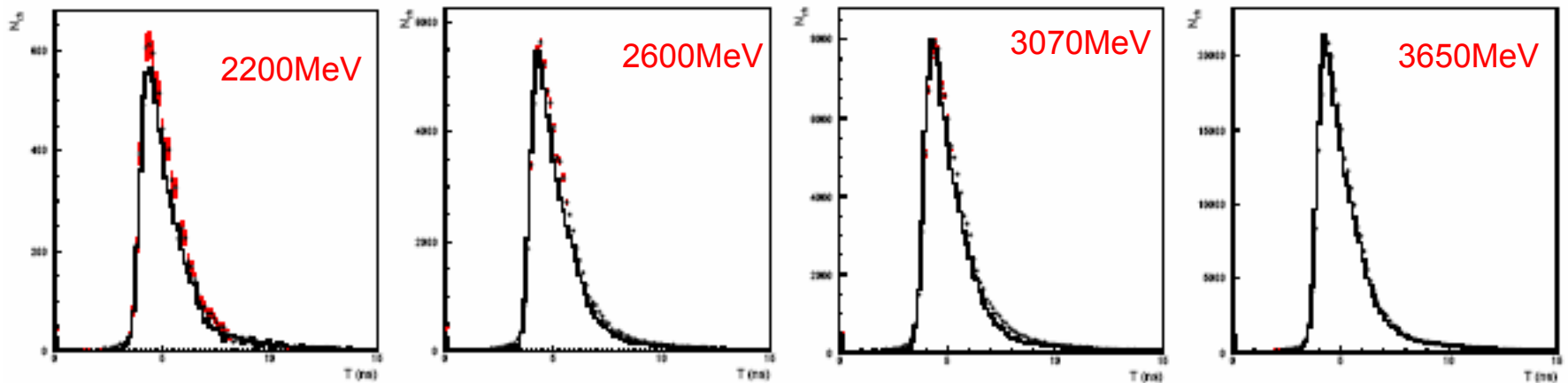


Figure 16: Distribution of the time of flight.

Determination of α_s

$\sqrt{s}(\text{GeV})$	R_{exp}	$\alpha_s(s)$	$\alpha_s(5\text{GeV})$
2.200	$2.1662 \pm 0.0405 \pm 0.0991$	$0.2451^{+0.0606+0.1548}_{-0.0590-0.1439}$	$0.2220^{+0.0328+0.0755}_{-0.0365-0.0985}$
2.600	$2.1916 \pm 0.0141 \pm 0.0669$	$0.2828^{+0.0214+0.1056}_{-0.0211-0.0981}$	$0.2217^{+0.0125+0.0568}_{-0.0128-0.0643}$
3.070	$2.1406 \pm 0.0116 \pm 0.0544$	$0.2077^{+0.0169+0.0802}_{-0.0168-0.0788}$	$0.1813^{+0.0125+0.0569}_{-0.0128-0.0627}$
3.650	$2.1388 \pm 0.0074 \pm 0.0619$	$0.2051^{+0.0108+0.0915}_{-0.0107-0.0896}$	$0.1878^{+0.0089+0.0727}_{-0.0090-0.0788}$

$\sqrt{s}(\text{GeV})$	$\alpha_s(5\text{GeV})$	area S	$\overline{\alpha_s}(5\text{GeV})$	$\alpha_s(M_Z)$
2.200	$0.2220^{+0.0823}_{-0.1050}$	0.4013	$0.2020^{+0.0329}_{-0.0370}$	$0.1147^{+0.0095}_{-0.0126}$
2.600	$0.2217^{+0.0581}_{-0.0655}$	0.2655		
3.070	$0.1813^{+0.0582}_{-0.0640}$	0.2591		
3.650	$0.1878^{+0.0733}_{-0.0784}$	0.3222		

PDG2006

$$\alpha_s(M_Z^2) = 0.1170 \pm 0.0012$$