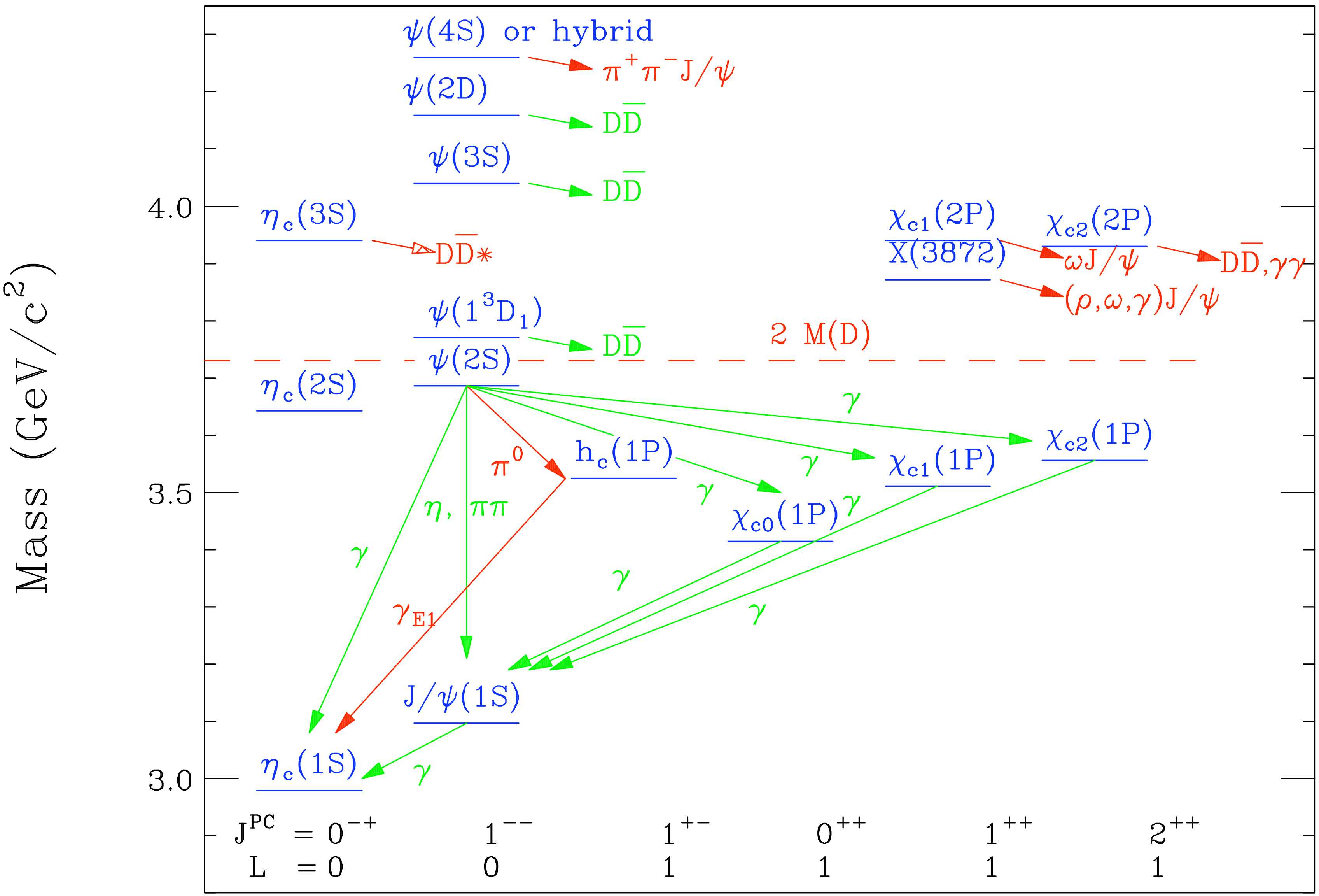
Quarkonium Transitions: Update

E. Eichten*

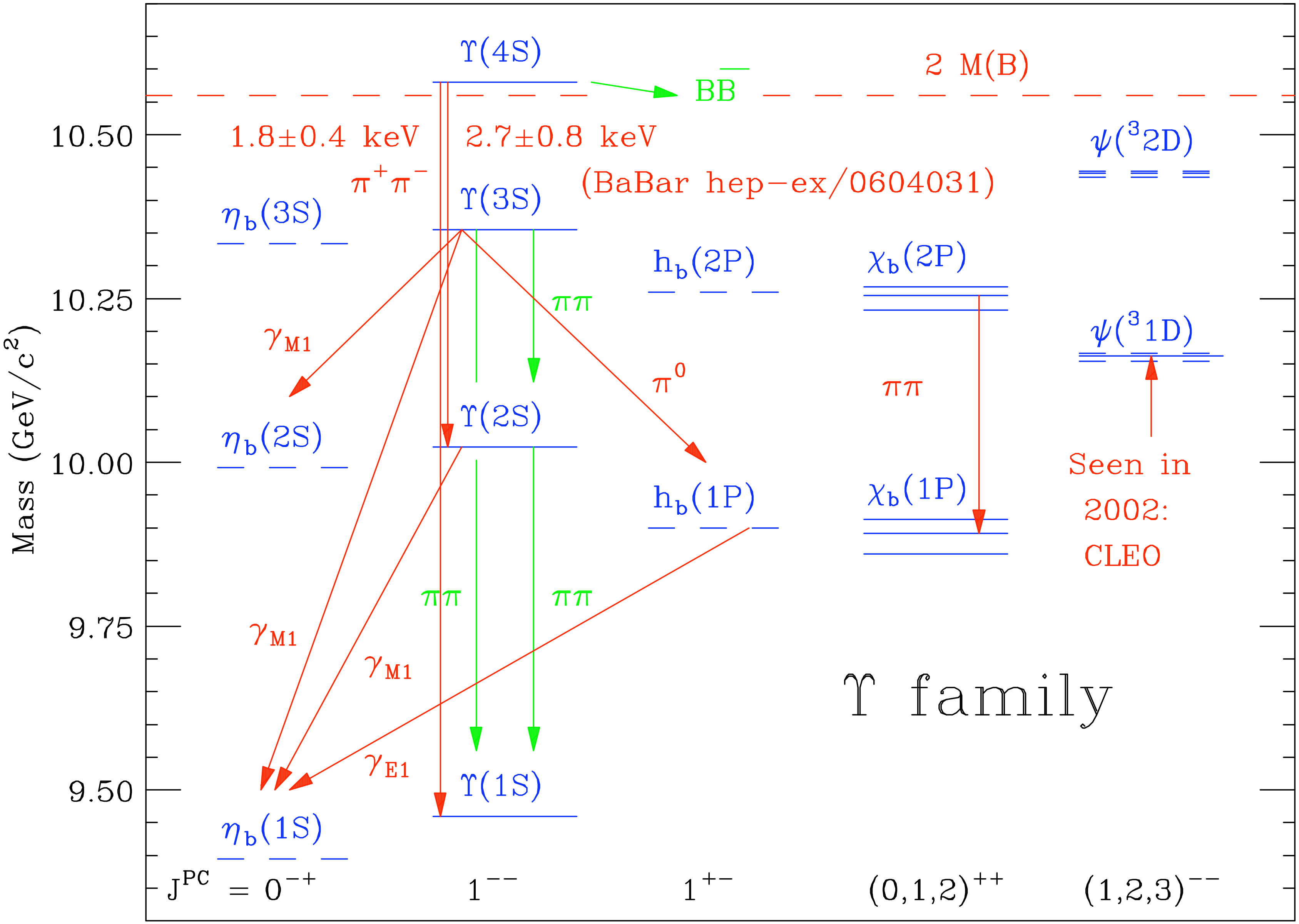
- QCD and the Multipole expansion
- Electromagnetic Transitions - E1, M1
  - Overlaps and Relativistic Corrections
- Hadronic Transitions
  - Two Pion Transitions
  - π, η and ω transitions
- XYZ transitions
- Summary

*E.E., S. Godfrey, Hanna Mahlke and J. Rosner [hep-ph/0701208]

$c\bar{c}$ Transitions



$b\bar{b}$ Transitions



Multipole Expansion

In QCD the effective interaction for heavy quarks is

HQET $\frac{\Lambda}{m_Q}$

NRQCD v

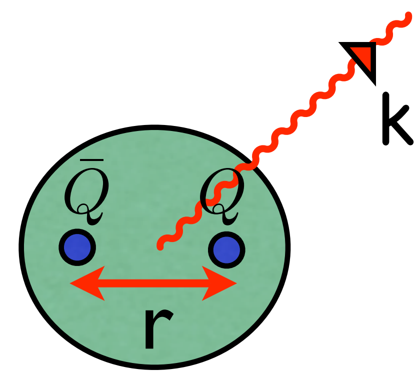
$$\mathcal{L}_{\rm NRQCD} = \psi^\dagger \left(iD_0 + \frac{\mathbf{D}^2}{2m_Q} \right) \psi + \frac{c_F}{2m_Q} \psi^\dagger \boldsymbol{\sigma} \cdot g\mathbf{B}\psi + o(\frac{1}{m_Q^2})$$
$$+[\psi \to i\sigma^2\chi^*, A_\mu \to -A_\mu^T]$$

Including EM interactions

$$\mathcal{H}_I = ie_Q\psi^\dagger \left(\frac{\mathbf{D}\cdot\mathbf{e}\mathbf{A} + \mathbf{e}\mathbf{A}\cdot\mathbf{D}}{2m_Q} \right) \psi + \frac{c_F e_Q}{2m_Q} \psi^\dagger \boldsymbol{\sigma} \cdot e\mathbf{B}\psi + \ldots$$

Electric Magnetic

Theory of quarkonium transitions relies on the multipole expansion



$$\mathbf{A}(R_{\rm cm}, r, t) = \mathbf{A}(R_{\rm cm}, t) + \mathbf{x}\cdot\nabla\mathbf{A}(R_{\rm cm}, t) + \ldots$$ expansion kr/2

Electric

$$\frac{1}{m_Q}\{\mathbf{p}, \mathbf{A}(R_{\rm cm}, t) + ...\} = \mathbf{r}\cdot\mathbf{E}(R_{\rm cm}, t) + ...$$

CM expand in kr/2

Siegert's theorem

using $i[\mathrm{H}, \mathbf{r}] = \frac{2\mathbf{p}}{m_Q}$ and $i[\mathrm{H}, \mathbf{A}] = \frac{\partial}{\partial t}\mathbf{A} = \mathbf{E}$

E1

$$e_Q\psi^\dagger \mathbf{r}\cdot e\mathbf{E}\psi + \cdots$$

power counting $\mathbf{p} \sim \frac{1}{\mathbf{r}} \sim m_Q \mathbf{v}$ $\quad k \sim \frac{m_Q v^2}{2}$

$$\frac{\mathrm{rk}}{2} \sim \mathrm{v}$$

Magnetic

M1

$$\frac{c_F e_Q}{2m_Q}\psi^\dagger \boldsymbol{\sigma}\cdot e\mathbf{B}\psi$$

$$c_F = 1 + \kappa_Q$$

anomalous magnetic moment

spin flip

power counting $\frac{\mathrm{k}}{m_Q} \sim \mathrm{v}^2$

Applying Multipole Expansion to Hadronic Transitions

For lowest order gluon emission:

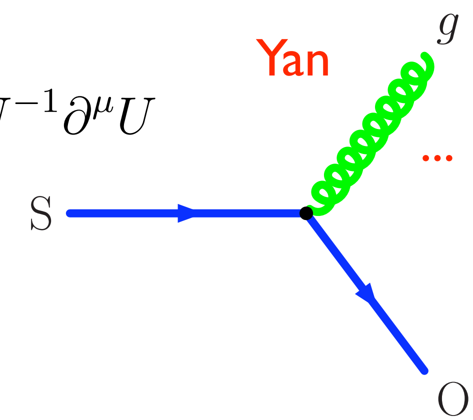
$$\mathcal{H}_I = i\psi^{\dagger\prime}\frac{\mathbf{r}}{\mathbf{2}}\cdot\mathbf{g}\mathbf{E}'_{\mathbf{a}}\mathbf{t}^{\mathbf{a}}\psi' + \frac{\mathbf{c_F}}{\mathbf{m_Q}}\psi^{\dagger\prime}\mathbf{s_Q}\cdot\mathbf{g}\mathbf{t}^{\mathbf{a}}\mathbf{B}'_{\mathbf{a}}\psi' + [\mathbf{Q}- > \bar{\mathbf{Q}}] + \cdots$$

Gottfried

Voloshin

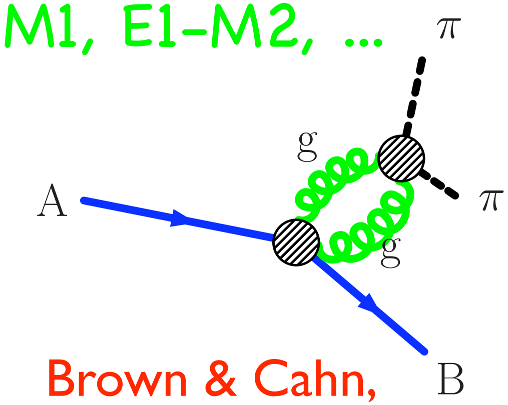
Yan

for dressed fields $\psi' = U^{-1}\psi$, $\mathbf{t}^{\mathbf{a}}\mathbf{A}'^{\mu}_{\mathbf{a}} = U^{-1}\mathbf{t}^{\mathbf{a}}\mathbf{A}^{\mu}_{\mathbf{a}}U - \frac{i}{g}U^{-1}\partial^{\mu}U$



But single emission takes color singlet state (S) to unphysical octet state (O).

Double transitions dominate: E1-E1, E1-M1, M1-M1, E1-M2, ...



Factorization:

E1-E1 $\frac{g_E^2}{8} < B|\mathbf{r_i}gt^a\mathcal{G}\mathbf{r_j}gt^b|A >$ $\quad \delta_{ab}$ $< \pi\pi|\mathbf{E}^{\mathbf{i}}_{\mathbf{a}}\mathbf{E}^{\mathbf{i}}_{\mathbf{b}}|\mathbf{0} >$

electric polarizability chiral methods

Brown & Cahn, ...

model $\mathcal{G} = (\mathrm{E_A} - \mathcal{H}^0_{\mathrm{NR}})^{-1} = \sum_{\mathrm{KL}} \frac{|\mathrm{KL}><\mathrm{KL}|}{\mathrm{E_A} - \mathrm{E_{KL}}}$ $(Q\bar{Q}$ octet$)$

Kuang & Yan

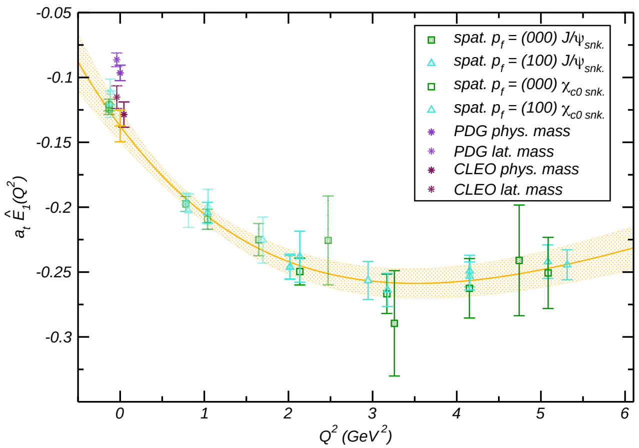
quark confining string

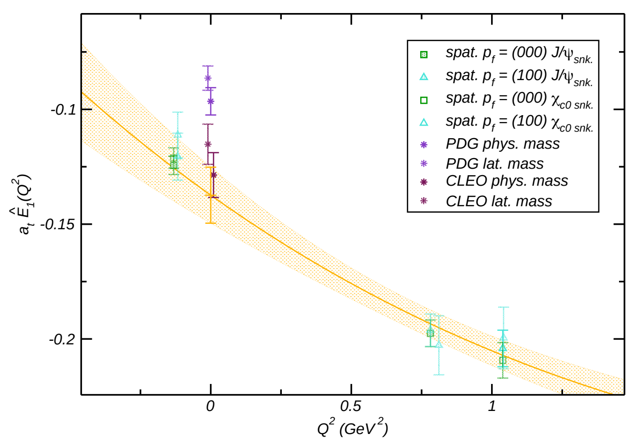
Other Approaches

- **Lattice**

  Direct calculation - Extrapolate to Q^2=0.

  Dudek, Edwards, Richards
  [PR D73:074507 (2007)]





- **pNRQCD**

  (Vairo's talk)

  Systematic Effective Lagrangian approach.

  Higher states an issue

- **HQET+ChET**

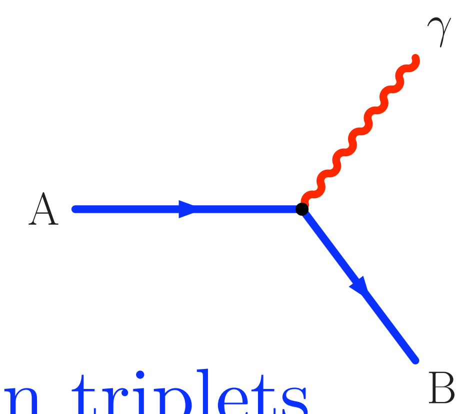
  Model independent symmetry relations.

  Incorporated in MPE approach.

  See review:
  Heavy Quarkonium Physics
  Cern-2005-005

Photon Transitions

E1 Transitions



$$\Gamma(i \xrightarrow{\mathrm{E1}} f + \gamma) = \frac{4\alpha e_Q^2}{3}(2J_f + 1)\mathrm{S}^{\mathrm{E}}_{if} k^3 |\mathcal{E}_{if}|^2$$

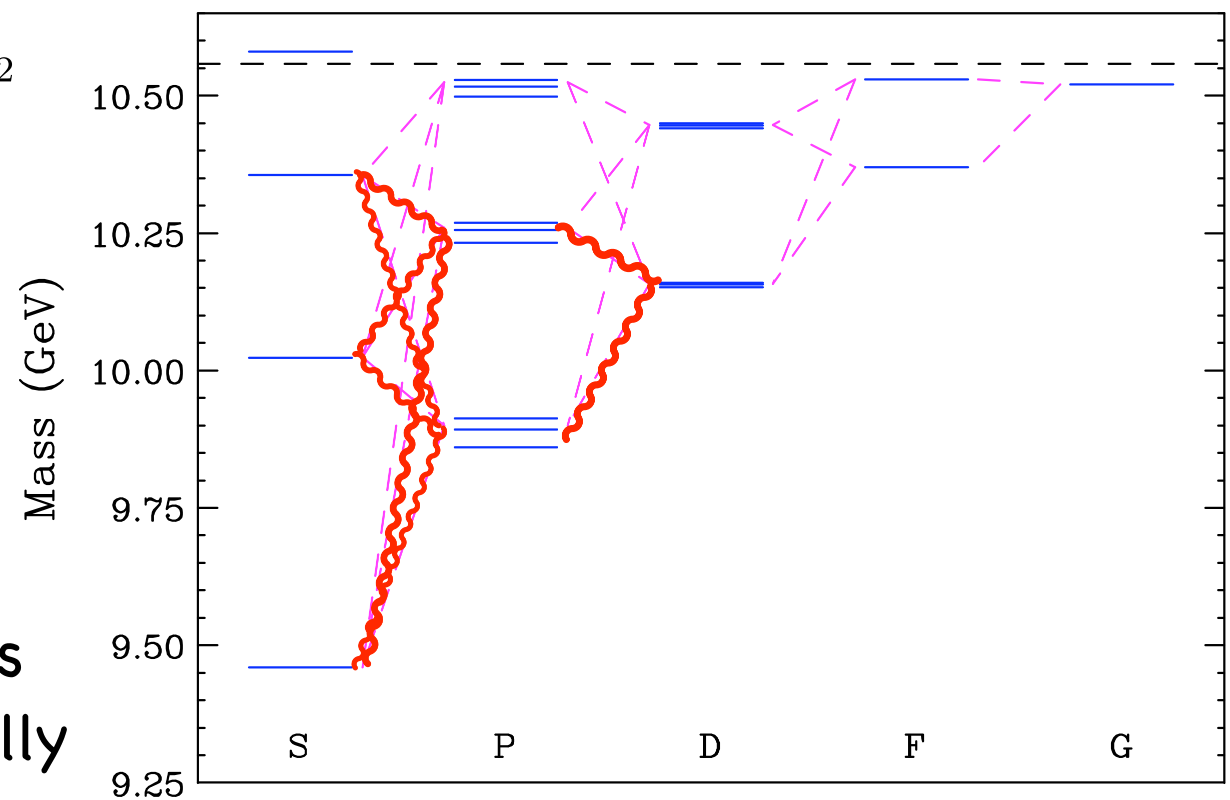
CG factor

$$\mathrm{S}^{\mathrm{E}}_{if} = \max(L_i, L_f) \begin{Bmatrix} J_i & 1 & J_f \\ L_f & S & L_i \end{Bmatrix}^2$$

Overlap

$$\mathcal{E}_{if} = \int r^2 dr\, R_{n_i \mathrm{L}_i}(r) r R_{n_f \mathrm{L}_f}(r)$$

$b\bar{b}$ spin triplets



Sensitive to detailed dynamics for transitions involving radially excited states

S states -> P states

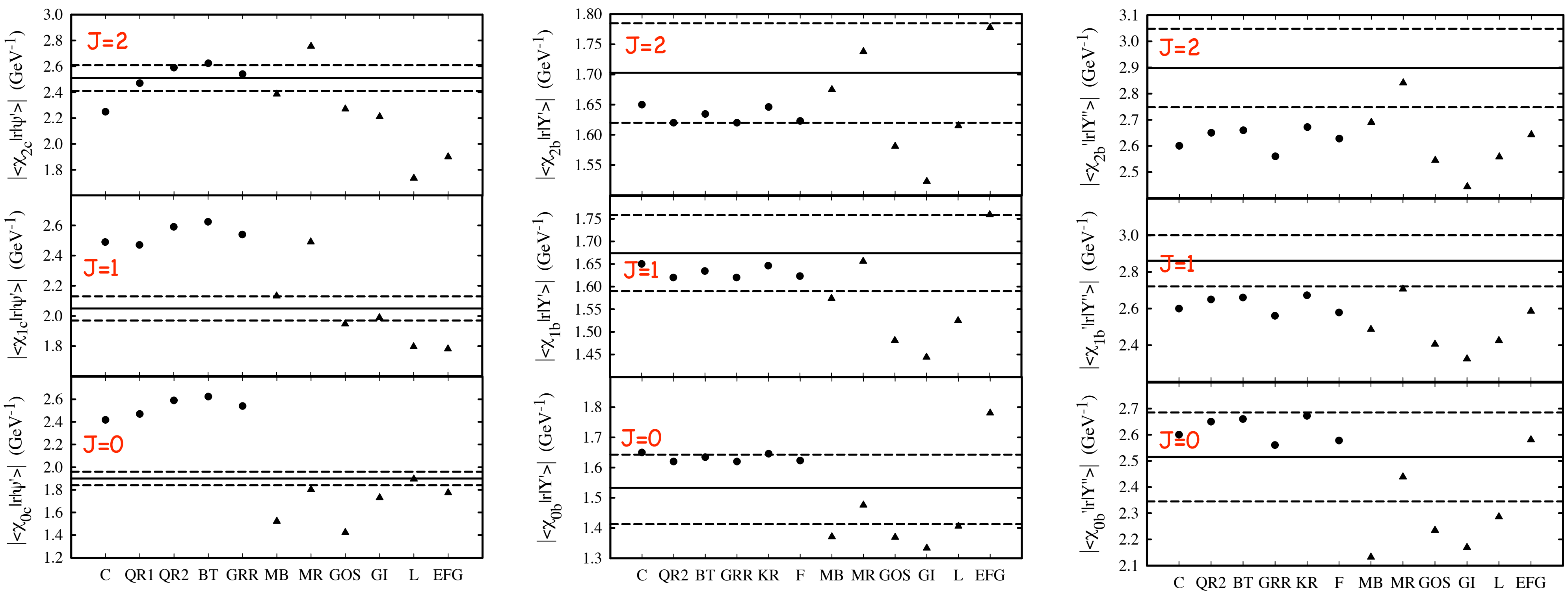
$\mathcal{E}_{if}$

- Generally good agreement with NRMPE
- Relativistic corrections 10%-20% effects in cc system.
- Need better theoretical guidance.

$c\bar{c}$		
State	$<\lvert r\rvert>$ (fm)	$<v^2>$
J/ψ	0.32	0.26
$\chi_c(1P)$	0.57	0.24
$\psi(2S)$	0.70	0.29
$\psi(3770)$	0.78	0.28

$b\bar{b}$		
State	$<\lvert r\rvert>$ (fm)	$<v^2>$
$\Upsilon(1S)$	0.19	0.091
$\chi_b(1P)$	0.35	0.072
$\Upsilon(2S)$	0.44	0.086
$\Upsilon(1D)$	0.50	0.080
$\chi_b(2P)$	0.56	0.089
$\Upsilon(3S)$	0.63	0.100
$\Upsilon(4S)$	0.80	0.116

$2^3S_1 \to 1^3P_J$ ($c\bar{c}$) $2^3S_1 \to 1^3P_J$ ($b\bar{b}$) $3^3S_1 \to 2^3P_J$ ($b\bar{b}$)



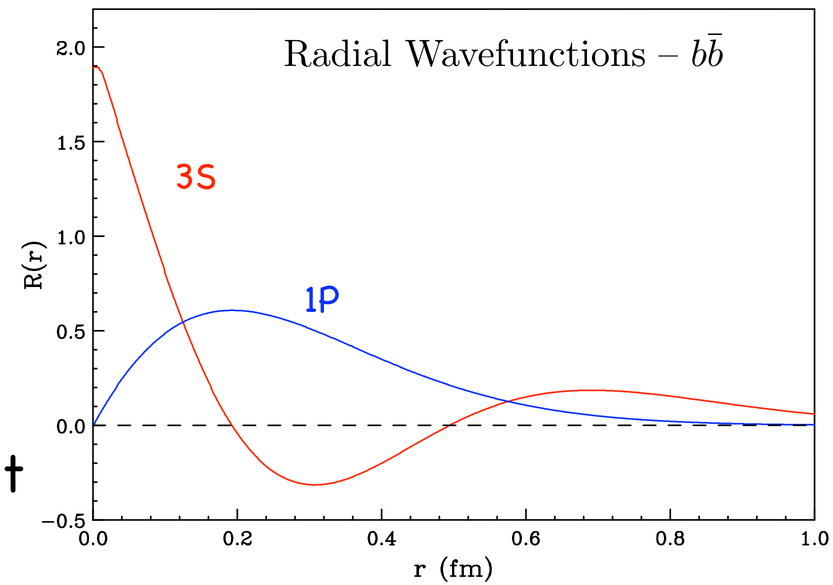
E.E., S. Godfrey, H. Mahlke and J. Rosner [hep-ph/0701208]

- $3^3S_1 \to 1^3P_J$ transition dynamically suppressed. Rate very sensitive to relativistic corrections.

$$\begin{aligned}
\mathcal{E}(3^3S_1, 1^3P_0) &= 0.067 \pm 0.012 \text{ GeV}^{-1} \\
< \mathcal{E}(3^3S_1, 1^3P_J) >_J &= 0.050 \pm 0.006 \text{ GeV}^{-1} \\
J = (2,1,0) & \quad (0.097, 0.045, -0.015)
\end{aligned}$$

Exp

GI Model

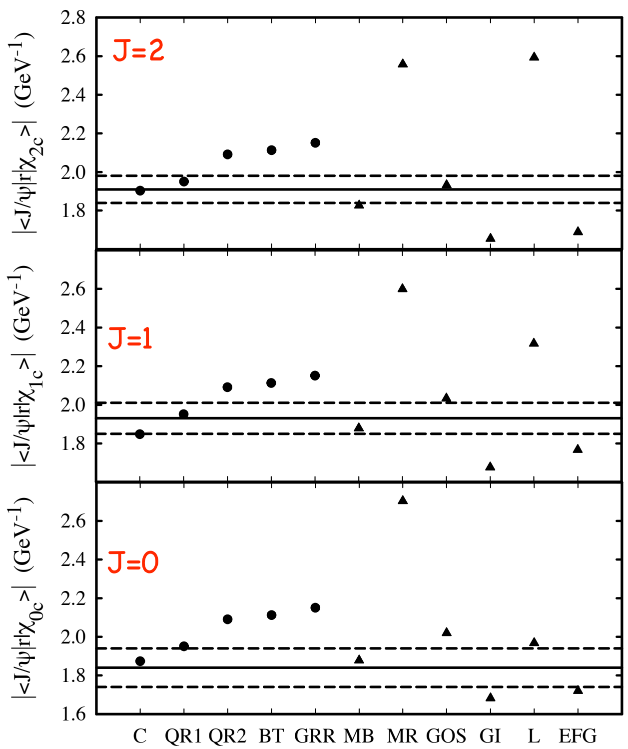


- nP -> mS transitions. Generally good agreement with NR predictions. Again better theoretical control for relativistic corrections needed

Table 1: Cancellations in $\mathcal{E}_{if}$ by node regions.

$b\bar{b}$	initial state node			
Transition	< 1	1 to 2	2 to 3	total
$2S \to 1P$	0.07	-1.68		-1.61
$3S \to 2P$	0.04	-0.12	-2.43	-2.51
$3S \to 1P$	0.04	-0.63	0.65	0.06

$1^3P_J \to 1^3S_1$ $(c\bar{c})$



Level	Final state	Predicted $\mathcal{B}$ (%) (2)	Measured $\mathcal{B}$ (%) (12)
2^3P_0	$\gamma + 1S$	0.96	0.9 ± 0.6
	$\gamma + 2S$	1.27	4.6 ± 2.1
2^3P_1	$\gamma + 1S$	11.8	8.5 ± 1.3
	$\gamma + 2S$	20.2	21 ± 4
2^3P_2	$\gamma + 1S$	5.3	7.1 ± 1.0
	$\gamma + 2S$	18.9	16.2 ± 2.4

- $\psi(3770) \to 1^3P_J$ transitions:
  Can study relativistic effects including coupling to decay channels.

	$\Gamma(\psi(3770) \to \gamma\chi_{cJ})$ in keV		
	$J=2$	$J=1$	$J=0$
Our results CLEO [PR D74 (2006) 031106]	< 21	70 ± 17	172 ± 30
Rosner (non-relativistic) [7]	24 ± 4	73 ± 9	523 ± 12
Ding-Qin-Chao [6]			
non-relativistic	3.6	95	312
relativistic	3.0	72	199
Eichten-Lane-Quigg [8]			
non-relativistic	3.2	183	254
with coupled-channels corrections	3.9	59	225
Barnes-Godfrey-Swanson [9]			
non-relativistic	4.9	125	403
relativistic	3.3	77	213

- $\psi'(2S) \to 1^3P_J \to J/\psi$ transitions:
  Can study size of higher multipole terms M2 and E3.

	$\chi_{cJ} \to J/\psi + \gamma$		
J	theory	E835	PDG
2	$a_2 \approx -\frac{\sqrt{5}}{3}\frac{k}{4m_c}(1+\kappa_c)$	$-0.093^{+0.039}_{-0.041} \pm 0.006$	-0.140 ± 0.006
2	$a_3 \approx 0$	$0.020^{+0.055}_{-0.044} \pm 0.009$	$0.011^{+0.041}_{-0.033}$
1	$a_2 \approx -\frac{k}{4m_c}(1+\kappa_c)$	$0.002 \pm 0.032 \pm 0.004$	$-0.002^{+0.008}_{-0.017}$

J	$\psi' \to \chi_{cJ} + \gamma$ theory
2	$a_2 \approx -\frac{\sqrt{3}}{2\sqrt{10}}\frac{k}{m_c}[(1+\kappa_c)(1+\frac{\sqrt{2}}{5}X) - i\frac{1}{5}X]/[1-\frac{1}{5\sqrt{2}}X]$
2	$a_3 \approx -\frac{12\sqrt{2}}{175}\frac{k}{m_c}X[1+\frac{3}{8}Y]/[1-\frac{1}{5\sqrt{2}}X]$
1	$a_2 \approx -\frac{k}{4m_c}[(1+\kappa_c)(1+\frac{2\sqrt{2}}{5}X) + i\frac{3}{10}X]/[1+\frac{1}{\sqrt{2}}X]$

M1 Transitions - S States

- Basics

$$\Gamma(i \xrightarrow{\mathrm{M1}} f+\gamma) = \frac{4\alpha e_Q^2}{3m_Q^2}(2J_f+1)k^3[\mathcal{M}_{if}|]^2$$

$$\mathcal{M}_{if} = \int r^2 dr\, R_{n_i\mathrm{L}_i}(r) j_0(\frac{rk}{2}) R_{n_f\mathrm{L}_f}(r)$$

$j_0 = 1 - (kr)^2/24 + \ldots$, so in NR limit

$$\begin{aligned} k=0: \quad \mathcal{M}_{if} &= 1 \quad n_i = n_f; L_i = L_f \\ &= 0 \quad \text{otherwise} \end{aligned}$$

$$\Gamma(J/\psi \to \eta_c\gamma) = \frac{16}{3}\alpha e_c^2 \frac{k_\gamma^3}{M_{J/\psi}^2}(1+\kappa_c)[1+o(v^2)]$$

1.19 ± 0.33 keV Exp [CUSB]

- LQCD $\Gamma(J/\psi \to \eta_c+\gamma) = 2.0 \pm 0.1 \pm 0.4$

Dudek, Edwards, Richards [PR D73:074507 (2007)]

- pNRQCD

Model independent - completely accessible by perturbation theory to $o(v^2)$

$$\Gamma(J/\psi \to \eta_c\gamma) = \frac{16}{3}\alpha e_c^2 \frac{k_\gamma^3}{M_{J/\psi}^2}\left[1 + C_F\frac{\alpha_s(M_{j/\psi}/2)}{\pi} + \frac{2}{3}(C_F\alpha_s(p_{J/\psi}))^2\right]$$

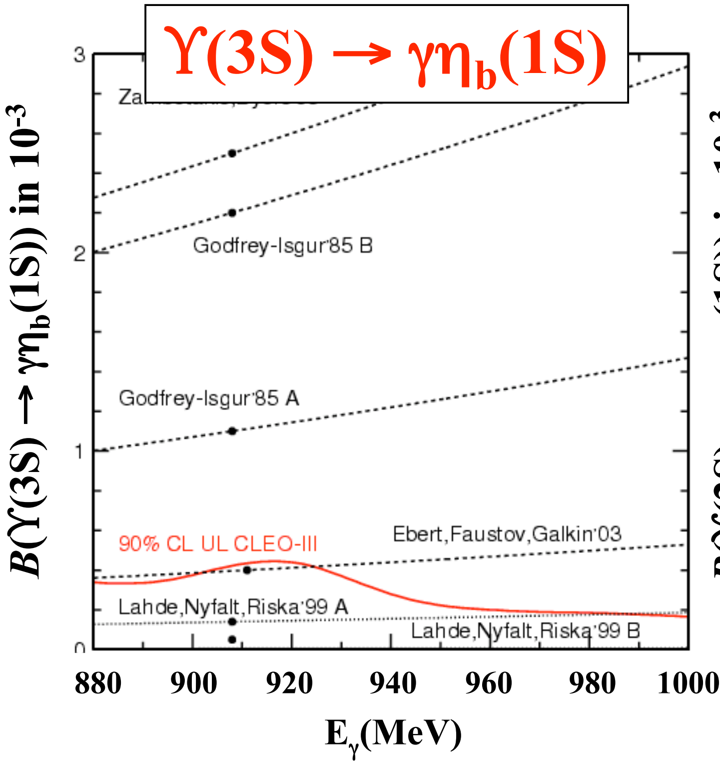
Brambilla, Jia & Vairo [PR D73:054005 (2006)]

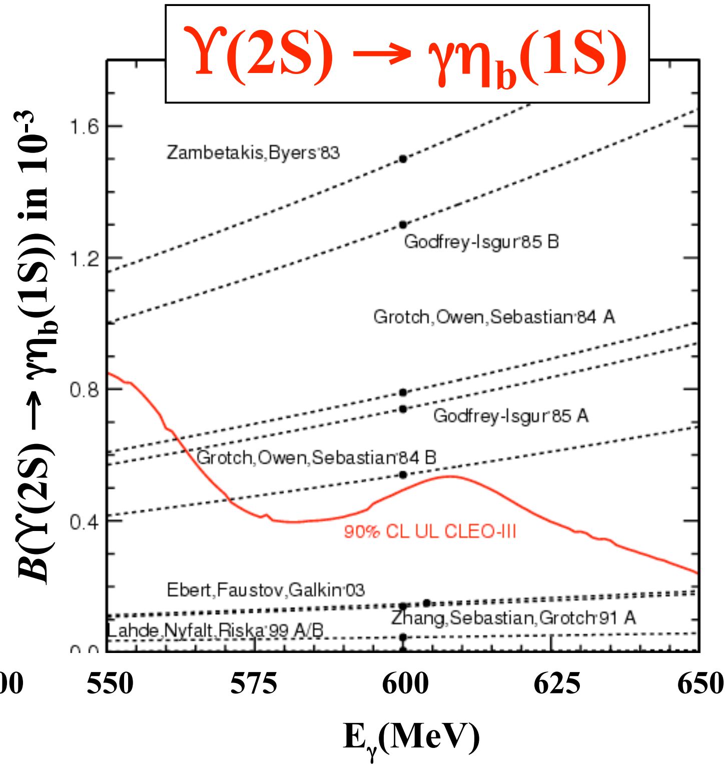
No large anomalous magnetic moment

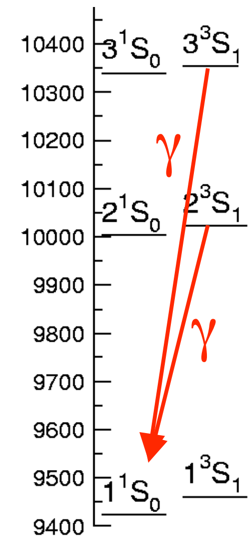
No scalar long range interaction

$\Gamma(J/\psi \to \eta_c\gamma) = (1.5 \pm 1.0)$ keV.

- Hindered M1 transitions
  - $\Upsilon(3S) \to \eta_b$ and $\Upsilon(2S) \to \eta_b$

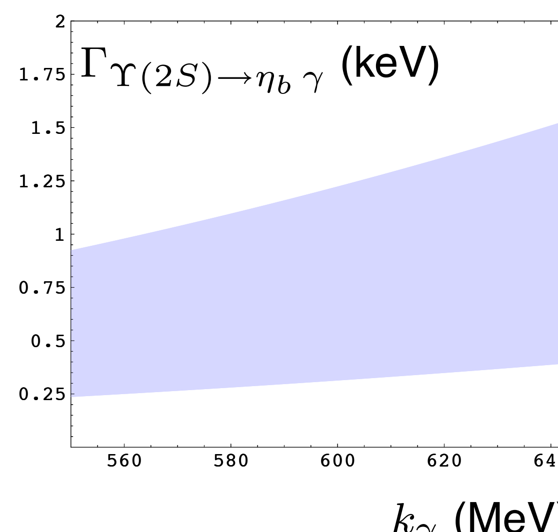






- Phenomenological model results vary greatly due to poorly understood relativistic corrections.

- pNRQCD expectation



CLEO < 0.14 keV (90%c.l.)

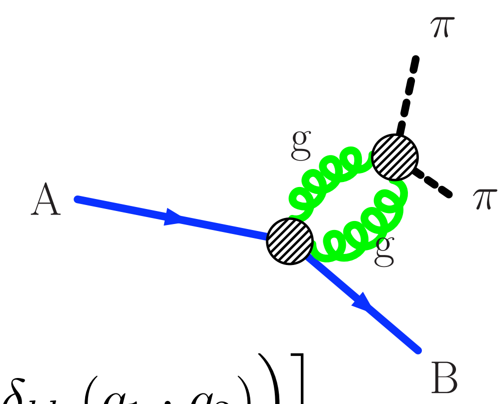
Hadronic Transitions

Two pion transitions

$$\mathcal{M}_{if}^{gg} = \frac{1}{16} < B|\mathbf{r_i}\xi^a \mathcal{G}\mathbf{r_j}\xi^a|A > \quad \frac{g_{\mathrm{E}}^2}{6} < \pi_\alpha\pi_\beta|Tr(\mathbf{E^i E^j})|0 >$$

α_{AB}^{EE}

Factorization



Model: Kuang & Yan

[PR D24, 2874 (1981)]

Hadronize

S-wave

D-wave

$$\frac{\delta_{\alpha\beta}}{\sqrt{(2\omega_1)(2\omega_2)}}\left[C_1\delta_{kl}q_1^\mu q_{2\mu} + C_2\left(q_{1k}q_{2l}+q_{1l}q_{2k}-\frac{2}{3}\delta_{kl}\ (q_1\cdot q_2)\right)\right]$$

S state -> S state

$$d\Gamma \sim K\sqrt{1-\frac{4m_\pi^2}{M_{\pi\pi}^2}}(M_{\pi\pi}^2-2m_\pi^2)^2\ dM_{\pi\pi}^2 \qquad K \equiv \frac{\sqrt{(M_A+M_B)^2-M_{\pi\pi}^2}\sqrt{(M_A-M_B)^2-M_{\pi\pi}^2}}{2M_A}$$

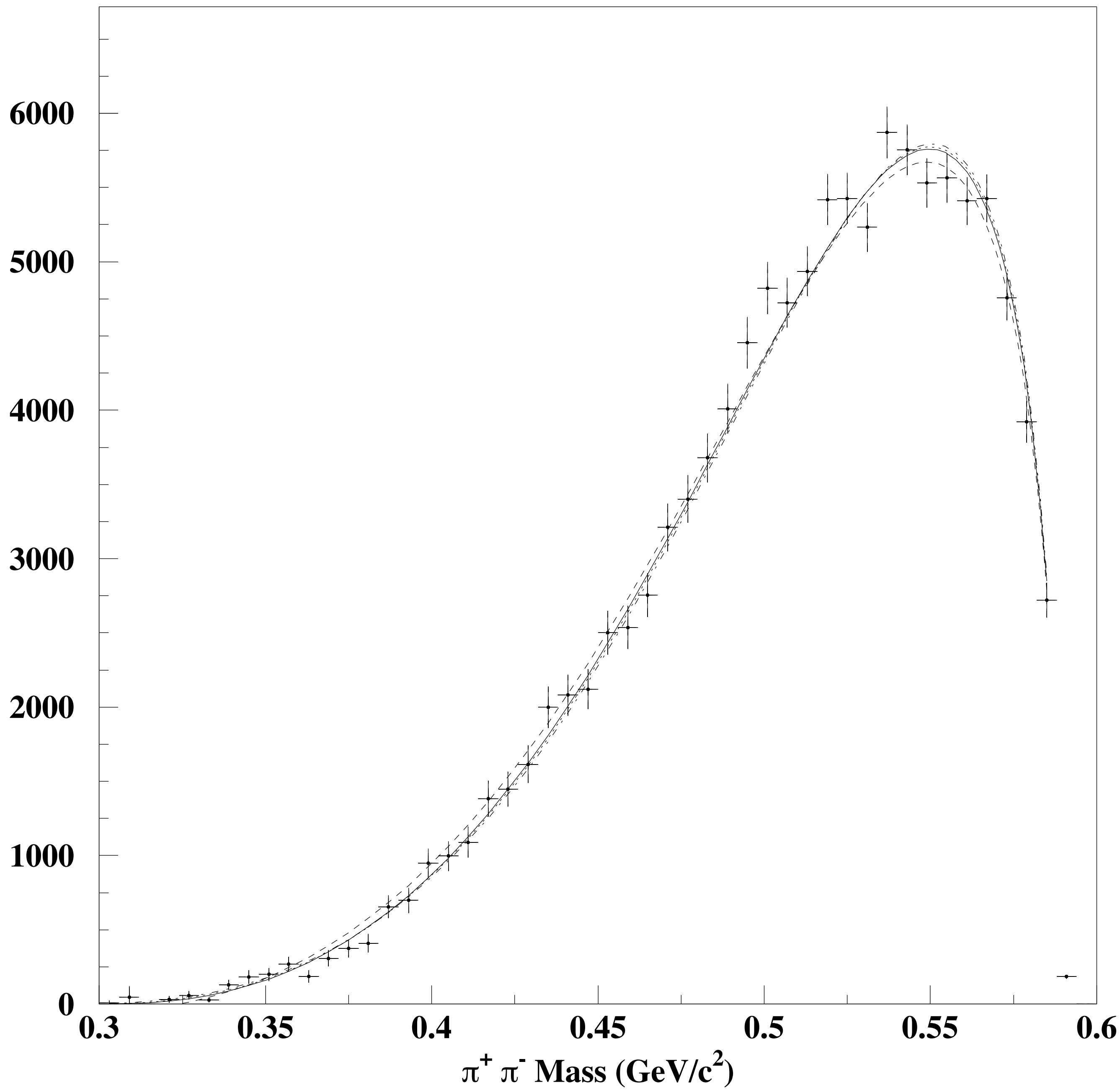
$$\Gamma = \mathrm{G}\ |\alpha_{\mathrm{AB}}^{\mathrm{EE}}\ \mathrm{C}_1|^2$$

Phase Space

Overlap - Vibrating String Model

BES

$$\psi' \to J/\psi + \pi^+\pi^-$$



D state -> S state

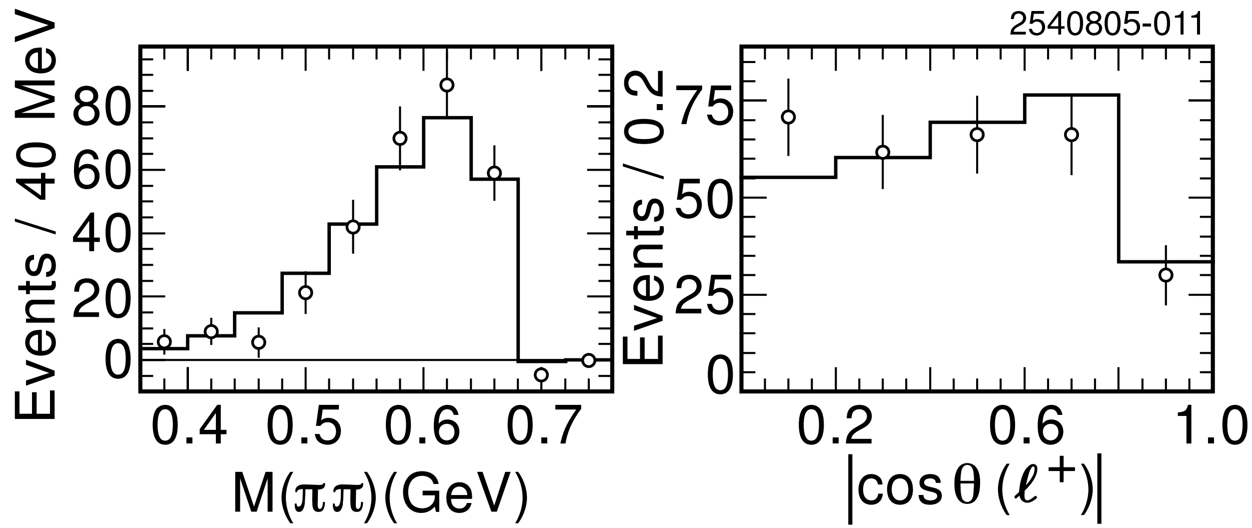


FIG. 4: Distributions in $\pi^+\pi^-\ell^+\ell^-$ events of the $\pi^+\pi^-$ mass (left) and polar angle (right) of the positively charged lepton from data (open circles) and MC (solid line line).

Determines

$$C_2/C_1 = 1.52^{+0.35}_{-0.45}$$ CLEO

P state -> P state

Assume only S wave term => J = J′

$$\Gamma_{\pi\pi} = (0.83 \pm 0.22 \pm 0.08 \pm 0.19)\ \text{keV}$$

CLEO [PR D73, 012003 (2006)]

$2P_J \to 1P_{J'} + 2\pi$ - First observation[CLEO]

Results agree with Kuang and Yan (1988)

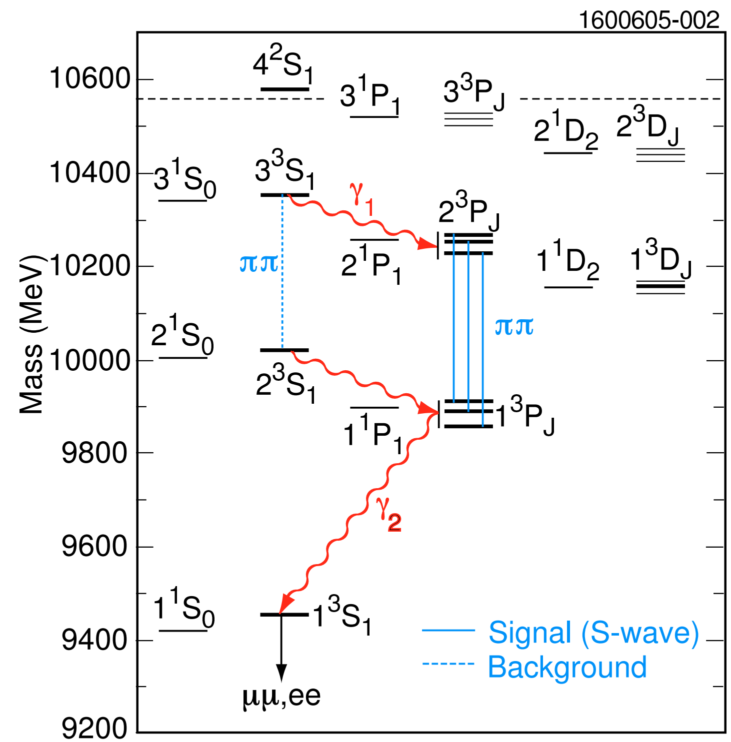


Table 4: Two pion transitions observed in the $c\bar{c}$ system.

Transition $i \to f$	$+\ X$	$m_{\pi\pi}^{(\max)}$ (MeV)	Branching Fraction (%)	Partial Width [1] (keV)
$\psi(2S) \to J/\psi$	$\pi^+\pi^-$	589	$33.54 \pm 0.14 \pm 1.10$	113.0 ± 8.4
	$\pi^0\pi^0$		$16.52 \pm 0.14 \pm 0.58$	55.7 ± 4.1
$\psi(3770) \to J/\psi$	$\pi^+\pi^-$	676	$(1.89 \pm 0.20 \pm 0.20) \times 10^{-1}$	43.5 ± 11.5
	$\pi^0\pi^0$		$(0.80 \pm 0.25 \pm 0.16) \times 10^{-1}$	18.4 ± 9.8

=> $|C_1| = 8.87\times10^{-3}$
(BT potential)

=> $|C_2|/|C_1| = 1.52^{+0.35}_{-0.45}$

Table 5: Two pion transitions observed in the $b\bar{b}$ system.

Transition $i \to f$	$+\ X$	$m_{\pi\pi}^{(\max)}$ (MeV)	Branching Fraction (%)	Partial Width [2] (keV)
$\Upsilon(2S) \to \Upsilon(1S)$	$\pi^+\pi^-$	563	18.8 ± 0.6	6.0 ± 0.5
	$\pi^0\pi^0$		9.0 ± 0.8	2.6 ± 0.2
$\Upsilon(3S) \to \Upsilon(1S)$	$\pi^+\pi^-$	895	4.48 ± 0.21	0.77 ± 0.06
	$\pi^0\pi^0$		2.06 ± 0.28	0.36 ± 0.06
$\Upsilon(3S) \to \Upsilon(2S)$	$\pi^+\pi^-$	332	2.8 ± 0.6	0.48 ± 0.12
	$\pi^0\pi^0$		2.00 ± 0.32	0.35 ± 0.07
$\Upsilon(4S) \to \Upsilon(1S)$	$\pi^+\pi^-$	1120	$(0.90 \pm 0.15) \times 10^{-2}$	1.8 ± 0.4
$\Upsilon(4S) \to \Upsilon(2S)$	$\pi^+\pi^-$	557	$(0.83 \pm 0.16) \times 10^{-2}$	1.7 ± 0.5
$\chi_{b2}(2P) \to \chi_{b2}(1P)$	$\pi^+\pi^-$	356	$(6.0 \pm 2.1) \times 10^{-1}$	0.83 ± 0.32
$\chi_{b1}(2P) \to \chi_{b1}(1P)$	$\pi^+\pi^-$	363	$(8.6 \pm 3.1) \times 10^{-1}$	0.83 ± 0.32

Rescaled Kuang &Yan model

} 9.4

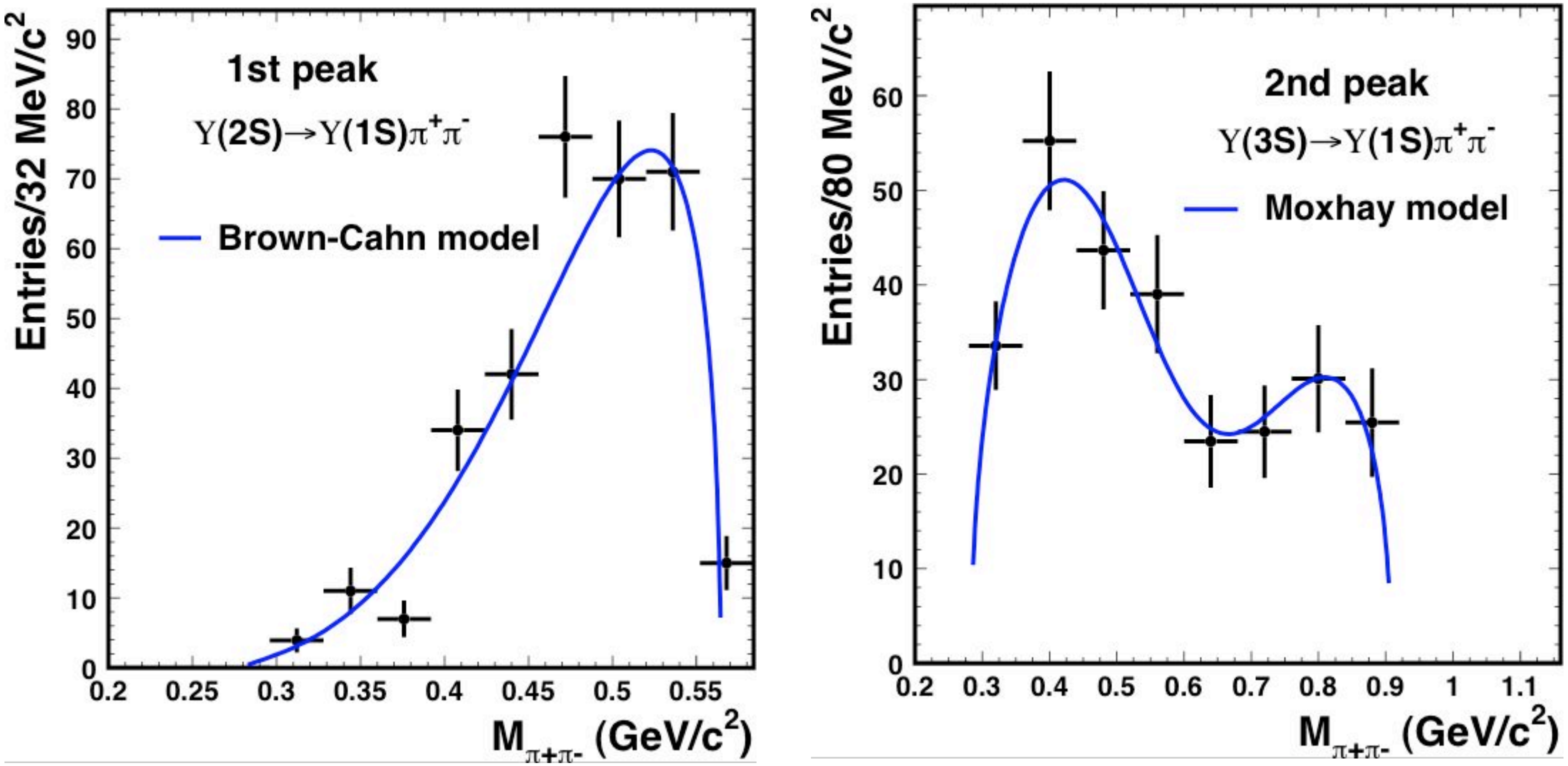
} 1.4

} 0.6

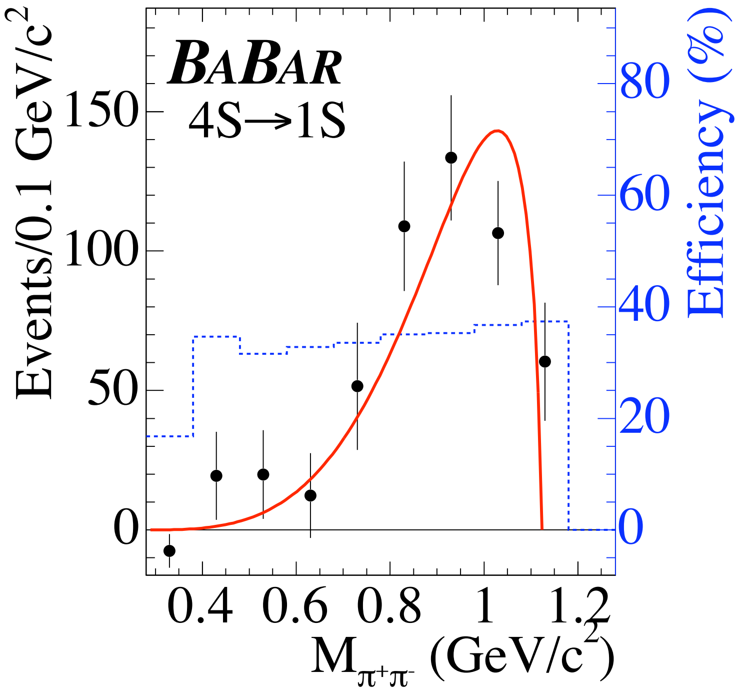
0.6
0.6

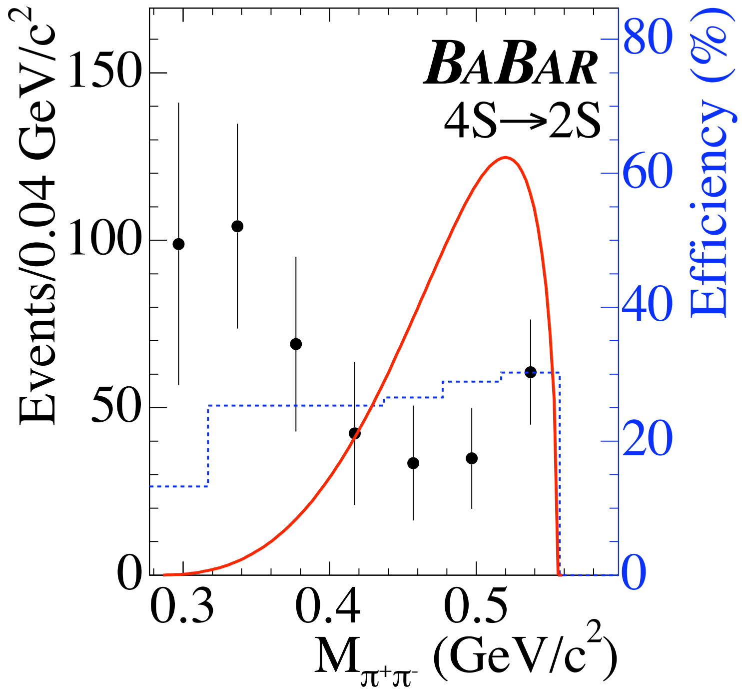
Model generally in good agreement with experiment

$M_{\pi\pi}$ distributions

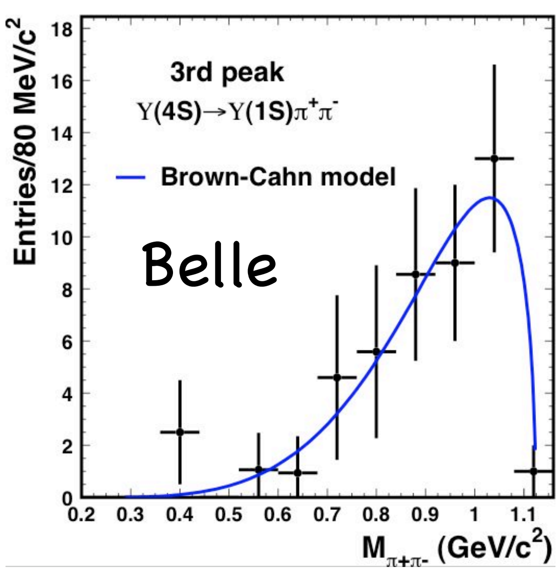


$\Upsilon(2S)$ and $\Upsilon(3S) \rightarrow \Upsilon(1S)$ are well described by Brown-Cahn and Moxhay models. → consistent with CLEO measurement(PRD58 052004, PRD49 40).





$M_{\pi\pi}$ distributions(cont.)



$\Upsilon(4S) \to \Upsilon(1S)$ is consistent with Brown-Cahn model.

Puzzle:

$$\Upsilon(3S) \rightarrow \Upsilon + \pi\pi$$
$$\Upsilon(4S) \rightarrow \Upsilon(2S) + \pi\pi$$

don't show leading (S-wave) two pion invariant mass distribution

Many proposals for explaining the ϒ(3S)->ϒ transition but most don't survive results for ϒ(4S):

- Final State Interactions

  Problem: Compare ϒ(4S)->ϒ(2S), ϒ(2S)->ϒ(1S) and ψ(2S) -> J/ψ essentially the same phase space but different distributions.

- coupling to decay channels

  Problem: Compare ϒ(3S)->ϒ(1S) to ψ(2S)->J/ψ, ϒ(4S)->ϒ(1S)

  Coupled channel effects should be larger in second set.

- 4 quark intermediate state

  Problem: Compare ϒ(4S)->ϒ(2S), ϒ(3S)->ϒ(1S)

  similiar distributions but shifted masses

- dynamical accident - suppress the leading E1 E1 term

  Worth a closer look.

Hybrid States and Lattice QCD

Heavy quark limit:

Born-Oppenheimer approximation

$$\Psi_{Q\bar{Q}}(\vec{r}) = \frac{u_{nl}(r)}{r} \mathrm{Y}_{\mathrm{lm}}(\theta, \phi)$$

$$-\frac{1}{2\mu}\frac{d^2u(r)}{dr^2} + \left\{ \frac{\langle \boldsymbol{L}^2_{Q\bar{Q}} \rangle}{2\mu r^2} + V_{Q\bar{Q}}(r) \right\} u(r) = E\, u(r)$$

$$\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}, \quad \boldsymbol{S} = \boldsymbol{s}_Q + \boldsymbol{s}_{\bar{Q}}, \quad \boldsymbol{L} = \boldsymbol{L}_{Q\bar{Q}} + \boldsymbol{J}_g$$

$$\langle L_r J_{gr} \rangle = \langle J^2_{gr} \rangle = \Lambda^2$$

$$\langle \boldsymbol{L}^2_{Q\bar{Q}} \rangle = L(L+1) - 2\Lambda^2 + \langle \boldsymbol{J}^2_g \rangle.$$

$$< J_g^2 >= 0, 2, 6, \ldots$$

naively 0, 1, 2, ... valence gluons

Spectroscopic notation of diatomic molecules

$$P = \varepsilon(-1)^{L+\Lambda+1}, \qquad C = \eta\,\varepsilon(-1)^{L+S+\Lambda}.$$

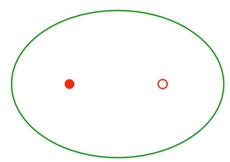
Λ = 0, 1, 2, ... denoted Σ, Π, Δ, ...

η= ±1 (symmetry under combined charge conjugation and spatial inversion)
denoted g(+1) or u(-1)

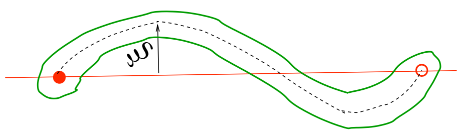
$|LSJM;\lambda\eta\rangle + \varepsilon|LSJM;-\lambda\eta\rangle$ with ε =+1 for Σ^+ and ε =-1 for Σ^- both signs for Λ>0.

$V_{Q\bar{Q}}(r)$

Short distance: Perturbative QCD, pNRQCD
singlet: $-4/3\ \alpha_s\ /r$
octet : $2/3\ \alpha_s\ /r$ gluelumps



Large distance: String
$\sigma\, r + \pi N/r$ NG string behavour



For cc and bb systems neither is adequate.

$V_{QQ}(r)$ determined by direct lattice calculations

Operators for excited gluon states

TABLE I: Operators to create excited gluon states for small $q\bar{q}$ separation R are listed. $\mathbf{E}$ and $\mathbf{B}$ denote the electric and magnetic operators, respectively. The covariant derivative $\mathbf{D}$ is defined in the adjoint representation [10].

gluon state	J	operator
$\Sigma_g^{+\prime}$	1	$\mathbf{R}\cdot\mathbf{E}$, $\mathbf{R}\cdot(\mathbf{D}\times\mathbf{B})$
Π_g	1	$\mathbf{R}\times\mathbf{E}$, $\mathbf{R}\times(\mathbf{D}\times\mathbf{B})$
Σ_u^-	1	$\mathbf{R}\cdot\mathbf{B}$, $\mathbf{R}\cdot(\mathbf{D}\times\mathbf{E})$
Π_u	1	$\mathbf{R}\times\mathbf{B}$, $\mathbf{R}\times(\mathbf{D}\times\mathbf{E})$
Σ_g^-	2	$(\mathbf{R}\cdot\mathbf{D})(\mathbf{R}\cdot\mathbf{B})$
Π_g'	2	$\mathbf{R}\times((\mathbf{R}\cdot\mathbf{D})\mathbf{B}+\mathbf{D}(\mathbf{R}\cdot\mathbf{B}))$
Δ_g	2	$(\mathbf{R}\times\mathbf{D})^i(\mathbf{R}\times\mathbf{B})^j+(\mathbf{R}\times\mathbf{D})^j(\mathbf{R}\times\mathbf{B})^i$
Σ_u^+	2	$(\mathbf{R}\cdot\mathbf{D})(\mathbf{R}\cdot\mathbf{E})$
Π_u'	2	$\mathbf{R}\times((\mathbf{R}\cdot\mathbf{D})\mathbf{E}+\mathbf{D}(\mathbf{R}\cdot\mathbf{E}))$
Δ_u	2	$(\mathbf{R}\times\mathbf{D})^i(\mathbf{R}\times\mathbf{E})^j+(\mathbf{R}\times\mathbf{D})^j(\mathbf{R}\times\mathbf{E})^i$

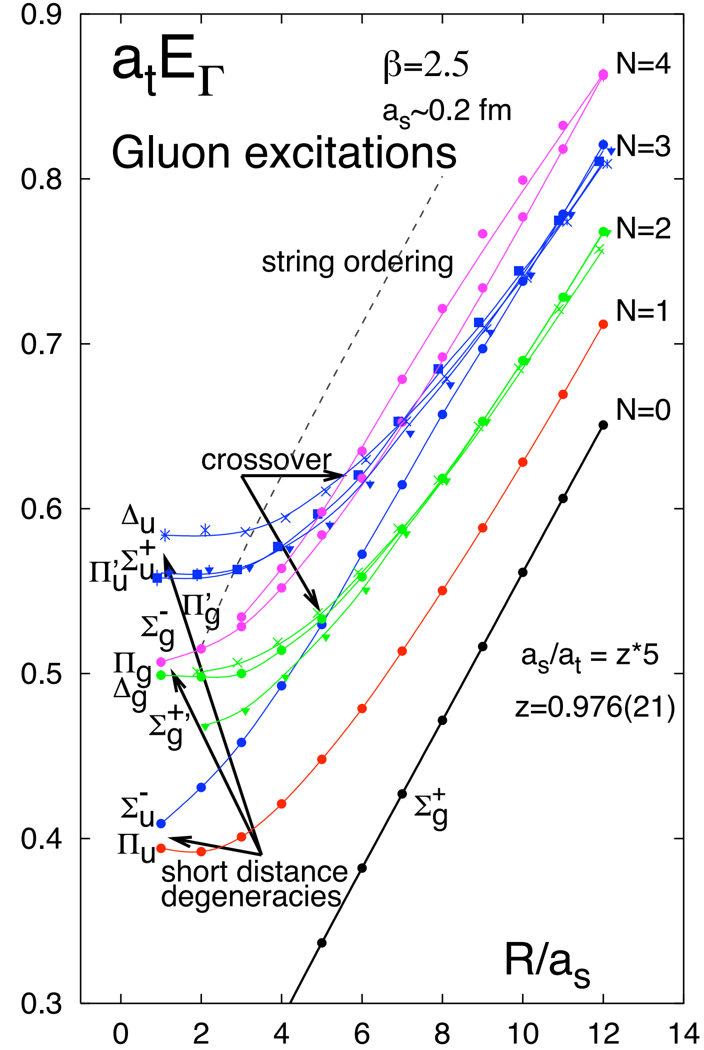


FIG. 2: Short-distance degeneracies and crossover in the spectrum. The solid curves are only shown for visualization. The dashed line marks a lower bound for the onset of mixing effects with glueball states which requires careful interpretation.

K.J. Juge, J. Kuti and C. Morningstar [PRL 90, 161601 (2003)]

Like the E1 case ?

Δn =2 overlap suppressed.

Predicted for Υ(3S)->Υ(1S)

3. The rate for $\Upsilon'' \to \Upsilon\pi\pi$ is surprisingly small. If we compare the phase-space integrals (2.4) for the two transitions $\Upsilon'' \to \Upsilon\pi\pi$ and $\Upsilon' \to \Upsilon\pi\pi$, their ratio is large,

$$\frac{G(\Upsilon'' \to \Upsilon\pi\pi)}{G(\Upsilon' \to \Upsilon\pi\pi)} \approx 33 \,. \tag{2.24}$$

The matrix element for $\Upsilon'' \to \Upsilon\pi\pi$ is tremendously suppressed:

$$\left|\frac{f^1_{if}(\Upsilon'' \to \Upsilon\pi\pi)}{f^1_{if}(\Upsilon' \to \Upsilon\pi\pi)}\right|^2 \approx (2-4)\times 10^{-3} \,. \tag{2.25}$$

The large suppression is due to two effects. First, there is a great deal of cancellation among different terms in the series for $f^1_{if}(\Upsilon'' \to \Upsilon\pi\pi)$. Second, many high vibrational levels contribute, so the mean distance from these levels to Υ'' is large. Because of the delicate cancellations, we cannot expect our results to be very reliable.

Kuang &Yan (1981)

Below lowest intermediate state threshold

$$\sum_{nl} \frac{|\Psi_{nl}><\Psi_{nl}|}{E_i - E_{nl}} \sim \frac{1}{E_i - E^{\rm TH}_{\rm string}} + \cdots$$

Hence transition rates fairly insensitive to intermediate states details

Transition	G (GeV^7)	$\lvert<i\lvert r^2\rvert f>\rvert$ (GeV^{-2})	$G<i\lvert r^2\rvert f>^2$ $\times 10^2$
$\psi(2S) \to J/\psi$	3.56×10^{-2}	3.36	40.2
$\Upsilon(2S) \to \Upsilon(1S)$	2.87×10^{-2}	1.19	4.06
$\Upsilon(3S) \to \Upsilon(1S)$	1.09	2.37×10^{-1}	0.61
$\Upsilon(3S) \to \Upsilon(2S)$	9.09×10^{-5}	3.70	0.12
$\Upsilon(4S) \to \Upsilon(1S)$	5.58	9.74×10^{-2}	0.48
$\Upsilon(4S) \to \Upsilon(2S)$	2.61×10^{-2}	4.64×10^{-1}	0.56

Note the large variations in phase space and overlaps for the various Υ states.

Model results:

$M(\Sigma_g^{+\prime}(1P)) \approx 4.55\ (c\bar{c})$
$10.80\ (b\bar{b})$

$\Sigma_g^{+\prime}(nP)$ n	$(M(n)-M(n-1))$ (MeV)	$<\|r\|>$ (fm)	$<v^2>$
$c\bar{c}$ 1	-	0.85	0.37
2	360	1.20	0.74
$b\bar{b}$ 1	-	0.45	0.09
2	300	0.64	0.18
3	265	0.80	0.25
4	240	0.96	0.31
5	225	1.09	0.37

Full overlap calculations gives:

$$\mathcal{F}(\text{full}) = \sum_n < i|r|X(n) >< X(n)|r|f > \frac{E_i - E_{X(0)}}{E_i - E_{X(n)}}$$

Transition	$\|\mathcal{F}\|$(full) (GeV^{-2})
$\psi(2S) \to J/\psi$	3.82
$\Upsilon(2S) \to \Upsilon(1S)$	1.37
$\Upsilon(3S) \to \Upsilon(1S)$	1.33×10^{-1}
$\Upsilon(3S) \to \Upsilon(2S)$	3.70
$\Upsilon(4S) \to \Upsilon(1S)$	1.17×10^{-1}
$\Upsilon(4S) \to \Upsilon(2S)$	2.71×10^{-1}

	$< \Sigma_g^{+\prime}(mP)\|r\|\Upsilon(nS) >$ (GeV^{-1})				
n	$m=1$	$m=2$	$m=3$	$m=4$	$m=5$
1	0.874	0.460	0.283	0.196	0.142
2	−2.12	0.871	0.481	0.291	0.196
3	0.811	−3.14	0.99	0.531	0.314
4	0.082	1.23	−3.98	1.14	0.585

If leading <E1-E1> suppressed, can the <M1-M1> significant?

Detailed study: S-wave

Voloshin [PR D74:054022(2006)]

$$\mathcal{M} = S\,(\epsilon_1 \cdot \epsilon_2) + D_1\,\ell_{\mu\nu}\,\frac{P^\mu P^\nu}{P^2}\,(\epsilon_1 \cdot \epsilon_2) + D_2\,q_\mu\,q_\nu\,\epsilon^{\mu\nu} + D_3\,\ell_{\mu\nu}\,\epsilon^{\mu\nu}\ .$$

S-wave

$$S(\psi_2 \to \pi^+\pi^-\psi_1) = \qquad (25)$$
$$-\frac{4\pi^2}{b}\,\alpha_0^{(12)}\left[(1-\chi_M)\,(q^2+m^2) - (1+\chi_M)\,\kappa\left(1+\frac{2m^2}{q^2}\right)\left(\frac{(q\cdot P)^2}{P^2} - \frac{1}{2}\,q^2\right)\right](\psi_1\cdot\psi_2)\ ,$$

and three D-waves

$$P_\mu = M_A \delta^0_\mu \qquad r_\mu = (k_{1\mu} - k_{2\mu})$$

$$D_1(\psi_2 \to \pi^+\pi^-\psi_1) = -\frac{4\pi^2}{b}\,\alpha_0^{(12)}\,(1+\chi_M)\,\frac{3\kappa}{2}\,\frac{\ell_{\mu\nu}P^\mu P^\nu}{P^2}\,(\psi_1\cdot\psi_2)\ ,$$

spin independent

$$D_2(\psi_2 \to \pi^+\pi^-\psi_1) = \frac{4\pi^2}{b}\,\alpha_0^{(12)}\left(\chi_2 + \frac{3}{2}\,\chi_M\right)\frac{\kappa}{2}\left(1+\frac{2m^2}{q^2}\right)q_\mu q_\nu \psi^{\mu\nu}$$

$$D_3(\psi_2 \to \pi^+\pi^-\psi_1) = \frac{4\pi^2}{b}\,\alpha_0^{(12)}\left(\chi_2 + \frac{3}{2}\,\chi_M\right)\frac{3\kappa}{4}\,\ell_{\mu\nu}\psi^{\mu\nu}$$

spin dependent

$$\psi^{\mu\nu} = \psi_1^\mu \psi_2^\nu + \psi_1^\nu \psi_2^\mu - (2/3)\,(\psi_1\cdot\psi_2)\,(P^\mu P^\nu/P^2 - g^{\mu\nu})$$

$$\ell_{\mu\nu} = r_\mu r_\nu + \frac{1}{3}\left(1 - \frac{4m^2}{q^2}\right)(q^2\,g_{\mu\nu} - q_\mu q_\nu)$$

magnetic: $\chi_M = \dfrac{\alpha_M}{\alpha_0}$, $O(v^2)$

S-D mixing: $\chi_2 = \dfrac{\alpha_2}{\alpha_0}$, $O(v^2)$

If <M1-M1> term significant,
expect noticeable presence of D2 and D3 in $\Upsilon(3S) \to \Upsilon + \pi\pi$

BUT - In addition to the suppression of the M1-M1 term by $\langle v^2 \rangle$ relative to the dominate E1-E1 term:

Radial overlap amplitude:

$$\sum_{n,l} \frac{< f|\Psi_{nl}) >< \Psi_{nl}|i >}{E_i - E_{X(nl)}}$$

with the hybrid states

$$\Psi_{nl} = \Pi_u^+(nP)$$

Again below lowest intermediate state threshold

$$\sum_{nl} \frac{|\Psi_{nl} >< \Psi_{nl}|}{E_i - E_{nl}} \sim \frac{1}{E_i - E^{\rm TH}_{\rm string}} + \cdots$$

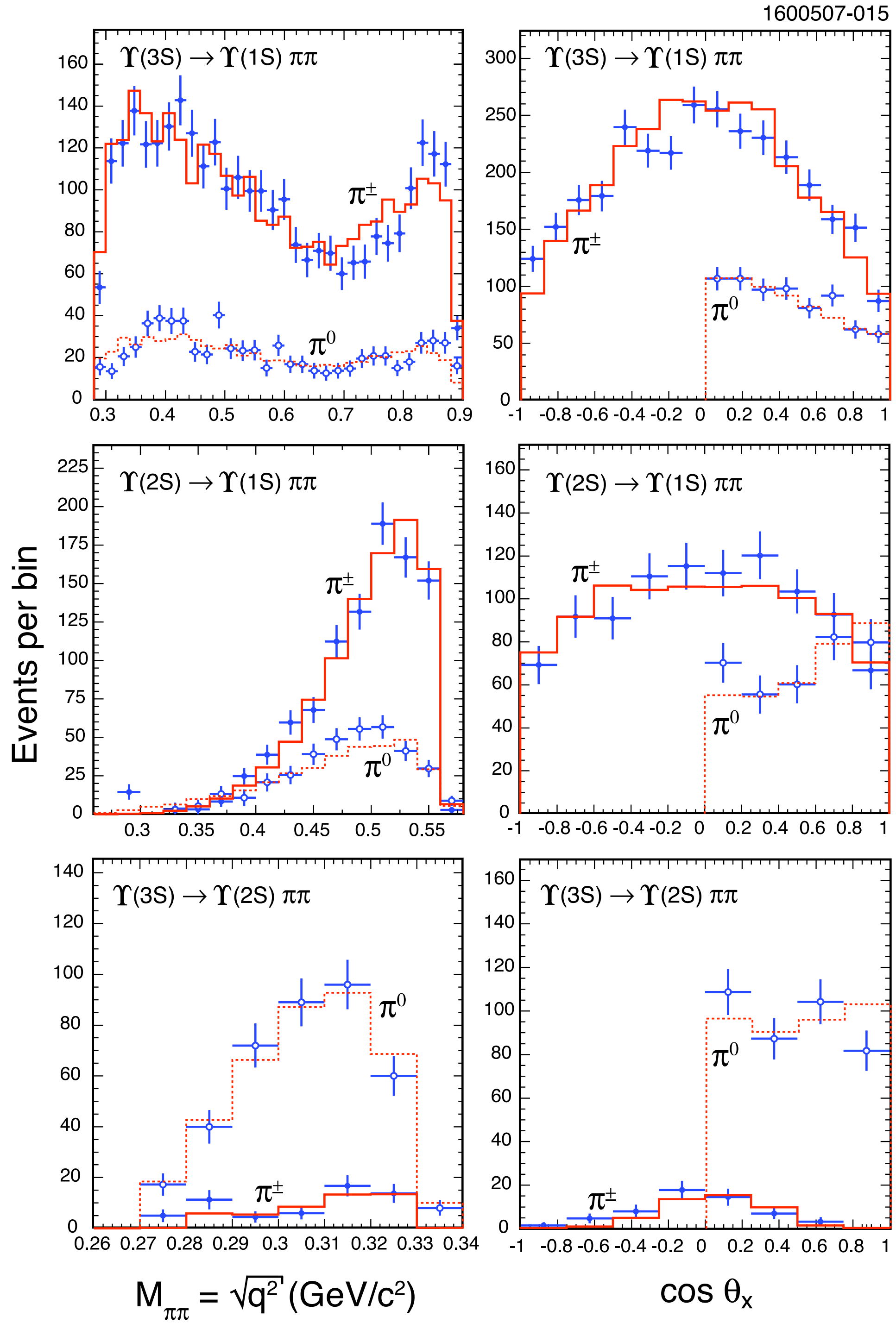
In this limit the overlap vanishes since $\langle f|i \rangle = 0$ ($i \neq f$)

A complete calculation yields:

Transition	$\lvert\mathcal{F}\rvert$(full) (GeV^{-2})
$\psi(2S) \to J/\psi$	1.81×10^{-1}
$\Upsilon(2S) \to \Upsilon(1S)$	3.04×10^{-1}
$\Upsilon(3S) \to \Upsilon(1S)$	1.70×10^{-1}
$\Upsilon(3S) \to \Upsilon(2S)$	1.74×10^{-1}
$\Upsilon(4S) \to \Upsilon(1S)$	1.06×10^{-1}
$\Upsilon(4S) \to \Upsilon(2S)$	0.92×10^{-1}

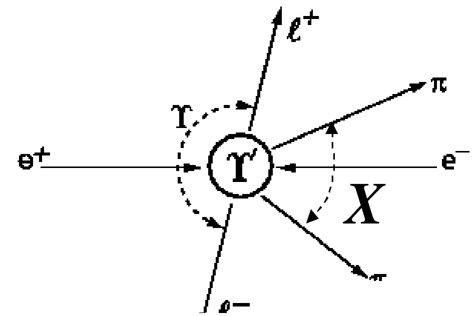
	$< \Pi_u^+(mP)\lvert r\rvert\Upsilon(nS) >$ (GeV^{-1})				
n	$m = 1$	$m = 2$	$m = 3$	$m = 4$	$m = 5$
1	0.705	0.470	0.346	0.274	0.226
2	−0.851	0.358	0.306	0.239	0.200
3	: 0.027	−0.934	0.263	0.254	0.199
4	−0.006	0.024	−0.968	0.220	0.227

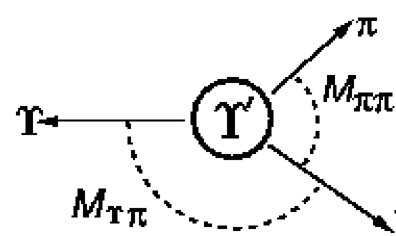
The M1-M1 term is highly suppressed !

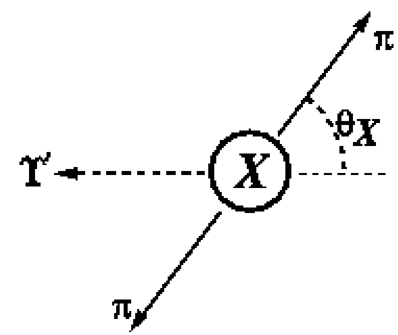


CLEO

[hep-ex/0706.2317]







$$q^2 = M_{\pi\pi}^{\ 2}$$

$$r^2 = M_{\Upsilon\pi}^{\ 2}$$

$$\cos\theta_X = \frac{M_{\Upsilon(2S)}^{\ 2} + M_{\Upsilon(1S)}^{\ 2} + 2m_\pi^{\ 2} - q^2 - 2r^2}{\sqrt{\Lambda_3(M_{\Upsilon(2S)}^{\ 2}, M_{\Upsilon(1S)}^{\ 2}, q^2)\,\frac{q^2 - 4m_\pi^{\ 2}}{q^2}}}$$

$Q\bar{Q}(n^3S_1) \to Q\bar{Q}(m^3S_1) + \pi^+\pi^-$

$$M = \mathrm{A}(\varepsilon'\cdot\varepsilon)(q^2 - 2m_\pi^{\ 2}) + \mathrm{B}(\varepsilon'\cdot\varepsilon)E_1E_2 + \mathrm{C}\left[(\varepsilon'\cdot q_1)(\varepsilon\cdot q_2) + (\varepsilon'\cdot q_2)(\varepsilon\cdot q_1)\right]$$

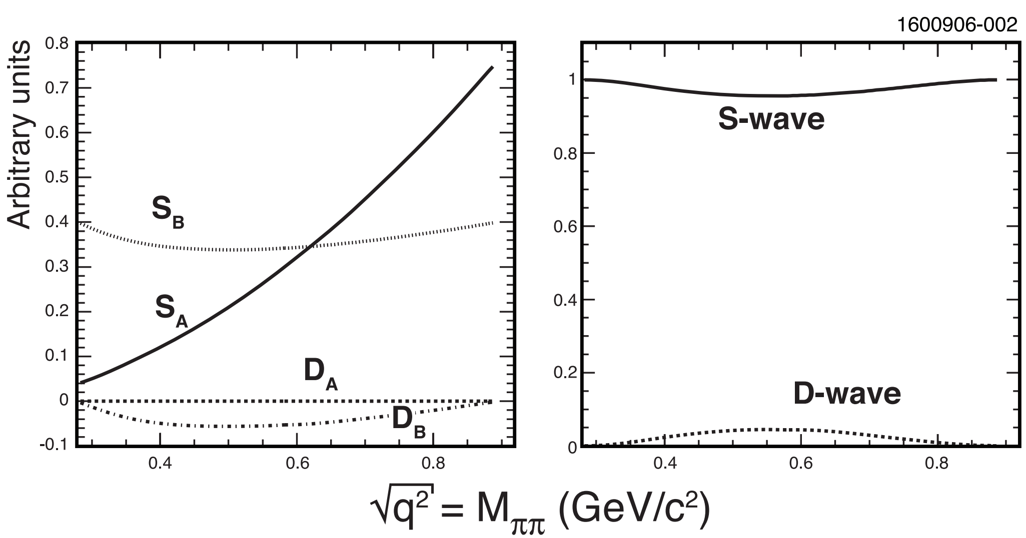
- Hindered M1-M1 term => C≈0. Consistent with CLEO results.
- Small D-wave contributions
- Useful to look at polarization info.

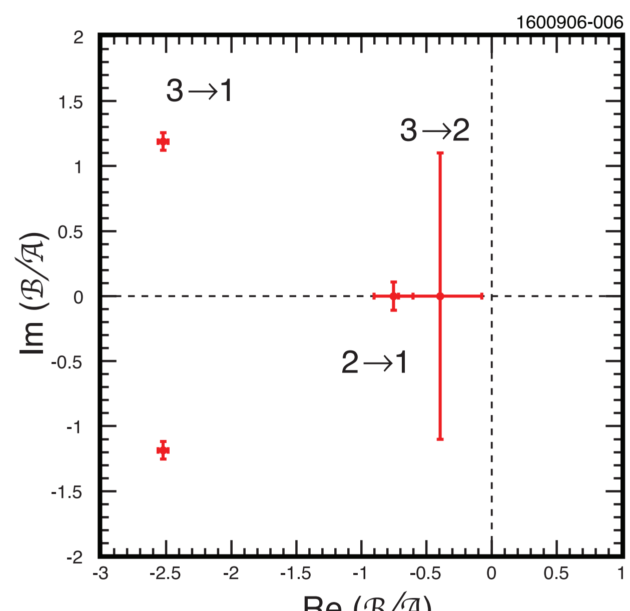
Dubynskiy & Voloshin [hep-ph/0707.1272]

CLEO [hep-ex/0706.2317]

Fit, No $\mathcal{C}$			stat.	effcy. ($\pi^\pm$)	effcy.(π^0)	bg. sub.
$\Upsilon(3S)\to\Upsilon(1S)\pi\pi$	$\Re(\mathcal{B}/\mathcal{A})$	-2.523	±0.031	±0.019	±0.011	±0.001
	$\Im(\mathcal{B}/\mathcal{A})$	±1.189	±0.051	±0.026	±0.018	±0.015
$\Upsilon(2S)\to\Upsilon(1S)\pi\pi$	$\Re(\mathcal{B}/\mathcal{A})$	-0.753	±0.064	±0.059	±0.035	±0.112
	$\Im(\mathcal{B}/\mathcal{A})$	0.000	±0.108	±0.036	±0.012	±0.001
$\Upsilon(3S)\to\Upsilon(2S)\pi\pi$	$\Re(\mathcal{B}/\mathcal{A})$	-0.395	±0.295		±0.025	±0.120
	$\Im(\mathcal{B}/\mathcal{A})$	±0.001	±1.053		±0.180	±0.001
Fit, float $\mathcal{C}$			**stat.**	**effcy. ($\pi^\pm$)**	**effcy.(π^0)**	**bg. sub.**
$\Upsilon(3S)\to\Upsilon(1S)\pi\pi$	$\lvert\mathcal{B}/\mathcal{A}\rvert$	2.89	±0.11	±0.19	±0.11	±0.027
	$\lvert\mathcal{C}/\mathcal{A}\rvert$	0.45	±0.18	±0.28	±0.20	±0.093

3S->1S





Single light hadron transitions

higher order <E1 M1>; <M1 M1>, <E1 M2>

$$\mathrm{C}_i\mathrm{C}_f = -1 \qquad +1$$

$$O(v) \qquad O(v^2)$$

symmetry breaking: π; η, ω

$$\tilde{\pi}^0 = \pi^0 + \epsilon\eta + \epsilon'\eta'$$

$$\tilde{\eta} = \eta - \epsilon\pi^0 + \theta\eta'$$

$$\tilde{\eta}' = \eta' - \theta\eta - \epsilon'\pi^0,$$

Transition $i \to f$	$+\ X$	Branching Fraction [3] (%)	Partial Width (keV)
$\psi(2S) \to J/\psi$	η	$3.25 \pm 0.06 \pm 0.11$	11.0 ± 0.84
	π^0	$0.13 \pm 0.01 \pm 0.01$	0.44 ± 0.06
$\psi(2S) \to h_c(1P)$	π^0	$(1.0 \pm 0.2 \pm 0.18) \times 10^{-1}$	0.34 ± 0.10
$\psi(3770) \to J/\psi$	η	$(0.87 \pm 0.33 \pm 0.22) \times 10^{-1}$	20 ± 11

Transition $i \to f$	$+\ X$	Branching Fraction (%)	Partial Width [4] (keV)
$\Upsilon(2S) \to \Upsilon(1S)$	η	$(2.5 \pm 0.7 \pm 0.5) \times 10^{-2}$	$(7.2 \pm 2.3) \times 10^{-3}$
$\chi_{b1}(2P) \to \Upsilon(1S)$	ω	$1.63 \pm 0.33 \pm 0.16$	1.56 ± 0.59
$\chi_{b2}(2P) \to \Upsilon(1S)$	ω	$1.10 \pm 0.30 \pm 0.11$	1.52 ± 0.64

chiral effective theory:

$$\epsilon = \frac{(m_d - m_u)\sqrt{3}}{4(m_s - \frac{m_u + m_d}{2})}, \qquad \epsilon' = \frac{\tilde{\lambda}(m_d - m_u)}{\sqrt{2}(m^2_{\eta'} - m^2_{\pi^0})}, \qquad \theta = \sqrt{\frac{2}{3}}\ \frac{\tilde{\lambda}\left(m_s - \frac{m_u + m_d}{2}\right)}{m^2_{\eta'} - m^2_{\eta}}.$$

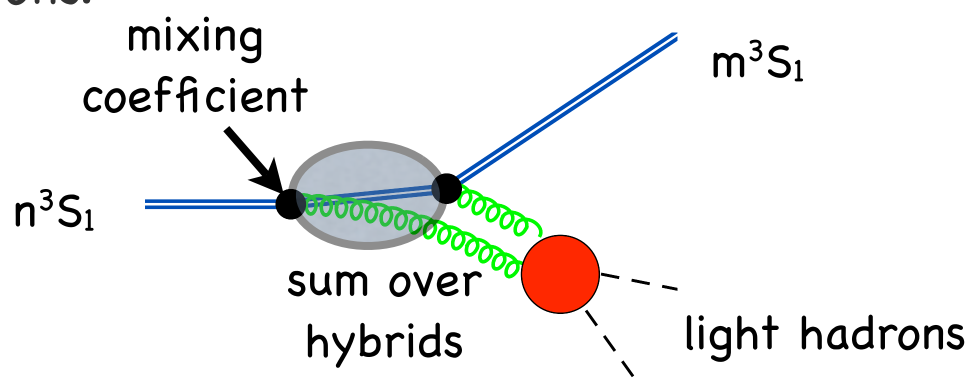
XYZ hadronic transitions

- Many new states : (Round table Friday 17:35)

State	EXP	M + i Γ (MeV)	J^{PC}	Decay Modes Observed	Production Modes Observed
X(3872)	Belle, CDF, D0, Cleo, BaBar	3871.2 ± 0.5 + i(<2.3)	1^{++}	$\pi^+\pi^- J/\psi$, $\pi^+\pi^-\pi^0 J/\psi$, $\Upsilon J/\psi$	B decays, ppbar
	Belle BaBar	$3875.4 \pm 0.7^{+1.2}_{-2.0}$ $3875.6 \pm 0.7^{+1.4}_{-1.5}$		$D^0 D^0 \pi^0$	B decays
Z(3930)	Belle	$3929 \pm 5 \pm 2$ + i($29 \pm 10 \pm 2$)	2^{++}	$D^0 D^0$, $D^+ D^-$	$\Upsilon\Upsilon$
Y(3940)	Belle BaBar	$3943 \pm 11 \pm 13$ + i($87 \pm 22 \pm 26$) $3914.3^{+3.8}_{-3.4} \pm 1.6$+ i($33^{+12}_{-8} \pm 0.60$)	1^{--}	$\omega J/\psi$	B decays
X(3940)	Belle	$3942^{+7}_{-6} \pm 6$ + i($37^{+26}_{-15} \pm 8$)	J^{P+}	DD*	e^+e^- (recoil against J/ψ)
Y(4008)	Belle	$4008 \pm 40^{+72}_{-28}$ + i($226 \pm 44^{+87}_{-79}$)	1^{--}	$\pi^+\pi^- J/\psi$	e^+e^- (ISR)
X(4160)	Belle	$4156^{+25}_{-20} \pm 15$+ i($139^{+111}_{-61} \pm 21$)	J^{P+}	D*D*	B decays
Y(4260)	BaBar Cleo Belle	$4259 \pm 8^{+8}_{-6}$ + i($88 \pm 23^{+6}_{-4}$) $4284^{+17}_{-16} \pm 4$ + i($73^{+39}_{-25} \pm 5$) $4247 \pm 12^{+17}_{-32}$ + i($108 \pm 19 \pm 10$)	1^{--}	$\pi^+\pi^- J/\psi$, $\pi^0\pi^0 J/\psi$, $K^+K^- J/\psi$	e^+e^- (ISR), e^+e^-
Y(4350)	BaBar Belle	4324 ± 24 + i(172 ± 33) $4361 \pm 9 \pm 9$ + i($74 \pm 15 \pm 10$)	1^{--}	$\pi^+\pi^- \psi(2S)$	e^+e^- (ISR)
$Z^+(4430)$	Belle	$4433 \pm 4 \pm 1$+ i($44^{+17}_{-13}{}^{+30}_{-11}$)	J^P	$\pi^+ \psi(2S)$	B decays
Y(4620)	Belle	$4664 \pm 11 \pm 5$ + i($48 \pm 15 \pm 3$)	1^{--}	$\pi^+\pi^- \psi(2S)$	e^+e^- (ISR)

Comments for Hybrid Interpretations

- Information from hadronic transitions can be used to estimate decay rates for a hybrid 1^{--} state (H) to a (QQ) state ($\psi(nS)$) + light hadrons.



- Branching ratios: $BR(H \to \psi' + \pi^+\pi^-)/BR(H \to J/\psi + \pi^+\pi^-)$ calculable.

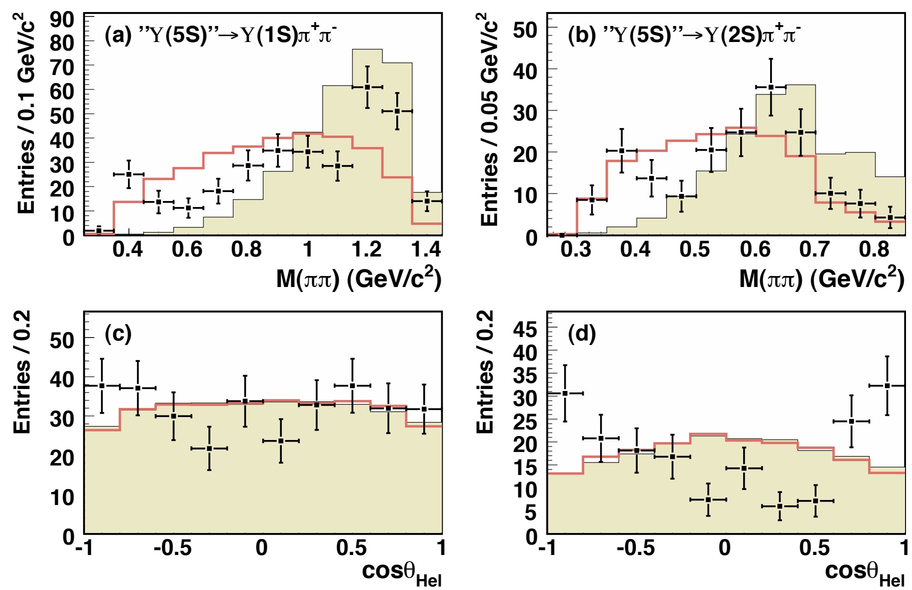
- Mixing between (QQ) states and hybrid (QQg) states can be calculated using Lattice QCD.

New Belle Measurements - [hep-ex/0710.2577]

$\Upsilon(5S) \to \pi^+\pi^- + \Upsilon(nS)$ (n=1,2,3)

Process	N_s	Σ	Eff.(%)	σ(pb)	$\mathcal{B}$(%)	Γ(MeV)
$\Upsilon(1S)\pi^+\pi^-$	325^{+20}_{-19}	20σ	37.4	$1.61 \pm 0.10 \pm 0.12$	$0.53 \pm 0.03 \pm 0.05$	$0.59 \pm 0.04 \pm 0.09$
$\Upsilon(2S)\pi^+\pi^-$	186 ± 15	14σ	18.9	$2.35 \pm 0.19 \pm 0.32$	$0.78 \pm 0.06 \pm 0.11$	$0.85 \pm 0.07 \pm 0.16$
$\Upsilon(3S)\pi^+\pi^-$	$10.5^{+4.0}_{-3.3}$	3.2σ	1.5	$1.44^{+0.55}_{-0.45} \pm 0.19$	$0.48^{+0.18}_{-0.15} \pm 0.07$	$0.52^{+0.20}_{-0.17} \pm 0.10$
$\Upsilon(1S)K^+K^-$	$20.2^{+5.2}_{-4.5}$	4.9σ	20.3	$0.185^{+0.048}_{-0.041} \pm 0.028$	$0.061^{+0.016}_{-0.014} \pm 0.010$	$0.067^{+0.017}_{-0.015} \pm 0.013$

- Large partial rates. Continuum $e^+e^- \to \pi\pi\Upsilon(nS)$ background not subtracted.
- $M(\pi\pi)$ and angular distribution. Compare to $\Upsilon(4S)$.



Transition Ratio	Belle
$R(2,1)$	$1.47 \pm 0.15 \pm 0.20$
$R(3,1)$	$0.91 \pm 0.35 \pm 0.15$

$$R(n,m) \equiv \frac{\Gamma(\Upsilon(5S) \to \pi^+\pi^- + \Upsilon(nS))}{\Gamma(\Upsilon(5S) \to \pi^+\pi^- + \Upsilon(mS))}$$

$$\Gamma(\Upsilon(5S) \to \pi^+\pi^- + \Upsilon(nS)) \propto G(n)|f(n)|^2$$

$$\text{with } f(n) = \sum_l \frac{<\Upsilon(5S)|r|\Sigma_g^{+'}(lP)><\Sigma_g^{+'}(lP)|r|\Upsilon(nS)>}{M_{\Upsilon(5S)} - E_l(\Sigma) + i\Gamma_l(\Sigma)}|^2$$

phase space (GeV^{-7})

$$G(n) = 28.7,\ 0.729,\ 1.33 \times 10^{-2}$$
$$\text{for } n = 1, 2, 3$$

theory - hadronic transition rates

- If lowest hybrid mass near ϒ(5S) a few states dominate sum. Results sensitive to mass value.
- If hybrid mass 10.75 + i(0.1) (GeV), obtain R(2,1)≈1.1 and R(3,1)≈0.08.
- Overall scale of transitions more than an order of magnitude larger than theory expects.

Summary

- In the presence of much more accurate data, multipole expansions for both electromagnetic and hadronic transitions hold up well.
- Significant relativistic corrections for the cc system. Reduced corrections for the bb system. Generally consistent with velocity scaling expectations:
- In puzzling transitions (below threshold), the leading order expansion coefficient is dynamically suppressed:
  - $\Upsilon(3S) \to \chi_b(1P)$ E1 rate - Cancellations in overlap amplitude for states with nodes in their radial wavefunctions. Here nearly complete.
  - $\Upsilon(3S) \to \eta_b(1S)$, ... M1 transitions - Hindered, so overlap is zero in leading order.

- $\Upsilon(3S) \to \Upsilon(1S) + 2\pi$, ... E1-E1 leading term dynamically suppressed. Small D-wave component in the two pion invariant mass distribution.
- For all $\Upsilon(nS) \to \Upsilon(mS) + 2\pi$, M1-M1 terms are dynamically suppressed relative to their natural $O(v^2)$ strength.

- The situation above threshold is not yet clear:
  - For any XYZ state that is a hybrid, its decays to quarkonium states may be related to the standard hadronic transitions.
  - The new $\Upsilon(5S) \to \Upsilon(nS) + 2\pi$ (n=1,2,3) transitions reported by Belle are much larger than expected.
- Modern theoretical tools (effective theories and nonperturbative LQCD) combined with more detailed high statistics experimental data will help resolve these issues.