Charmonium spectrum including higher spin and exotic states

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Outline

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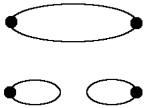
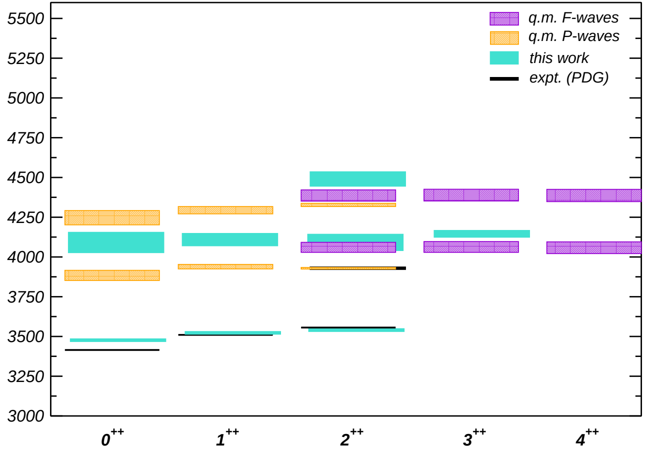
Challenges

- calculation of the spectra with high precision
- hyperfine splitting
- configuration mixing
- disconnected contributions



Lattice results from JLab



Dudek et al.

arXiv:0707.4162

Fermion action

- crucial question: Which fermion action to choose?
- chiral symmetry (mostly) plays minor role in charm systems: chiral actions like overlap or domain wall are overkill
- effective actions like NRQCD:
  - no true continuum limit
  - large radiative and relativistic corrections ($\langle v^2 \rangle \approxeq 0.5$)
- happy medium: (Clover-)Wilson action with fine lattices

Outline

Simulation details

- valence & sea quark action: Clover-Wilson
- gluon action: plaquette
- QCDSF configurations in use:

β	κ	volume	m_π[GeV]	a[fm]	L[fm]
5.20	0.13420	$16^3 \times 32$	1.007(2)	0.1145	1.8
5.25	0.13460	$16^3 \times 32$	0.987(2)	0.0986	1.6
5.29	0.13500	$16^3 \times 32$	0.929(2)	0.0893	1.4
5.40	0.13500	$24^3 \times 48$	1.037(1)	0.0767	1.7

A. Ali Khan *et al.*, Phys.Lett.B564:235-240

- charm quark mass parameter set by tuning $\frac{1}{4}m_{\eta_c} + \frac{3}{4}m_{J/\Psi}$
- runs performed on 16 node partitions on the local QCDOC using the Chroma software library (see arXiv:hep-lat/0409003)

Variational method

- Choose basis of operators and construct a cross correlator matrix:

$$C_{ij}(t) = \langle O_i(t) O_j^\dagger(0) \rangle = \sum_n v_i^n v_j^{n*} e^{-t\,E_n}$$

- Solve symmetric generalized eigenvalue problem:

$$C(t_0)^{-1/2}\, C(t)\, C(t_0)^{-1/2}\, \vec{\psi}^\alpha = \lambda^\alpha(t, t_0)\, \vec{\psi}^\alpha$$

- Eigenvalues behave like:

$$\lambda^\alpha(t, t_0) \propto e^{-(t-t_0)\,E_\alpha} [1 + \mathcal{O}(e^{-(t-t_0)\,\Delta E_\alpha})]$$

- see C. Michael, NPB259, 58; M. Lüscher and U. Wolff, NPB339, 222

Operators

name	O_h repr.	J^{PC}	state	operator
a_0	A_1	0^{++}	χ_{c0}	1
π	A_1	0^{-+}	η_c	γ_5
ρ	T_1	1^{--}	J/ψ	γ_i
a_1	T_1	1^{++}	χ_{c1}	$\gamma_5\gamma_i$
b_1	T_1	1^{+-}	h_c	$\gamma_i\gamma_j$
$\pi \times \nabla$	T_1	1^{+-}	h_c	$\gamma_5\nabla_i$
$(\rho \times \nabla)_{\mathrm{T}_1}$	T_1	1^{++}	χ_{c1}	$\epsilon_{ijk}\gamma_j\nabla_k$
$(a_1 \times \nabla)_{\mathrm{T}_2}$	T_2	2^{--}		$\gamma_5 s_{ijk}\gamma_j\nabla_k$
$(b_1 \times \nabla)_{\mathrm{T}_1}$	T_1	1^{-+}	exotic	$\gamma_4\gamma_5\epsilon_{ijk}\gamma_j\nabla_k$
$(\pi \times D)_{\mathrm{T}_2}$	T_2	2^{-+}		$\gamma_4\gamma_5 D_i$
$(\rho \times D)_{\mathrm{A}_2}$	A_2	3^{--}		$\gamma_i D_i$
$(a_1 \times D)_{\mathrm{A}_2}$	A_2	3^{++}		$\gamma_5\gamma_i D_i$
$(b_1 \times D)_{\mathrm{A}_2}$	A_2	3^{+-}		$\gamma_4\gamma_5\gamma_i D_i$
$(a_1 \times B)_{\mathrm{T}_2}$	T_2	2^{+-}	exotic	$\gamma_5 s_{ijk}\gamma_j B_k$

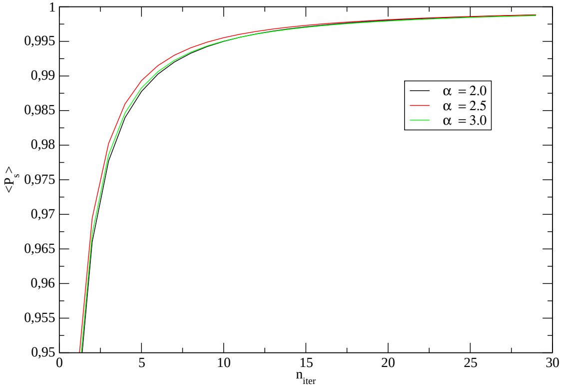
$$A_1 \rightarrow J = 0, 4$$
$$A_2 \rightarrow J = 3$$
$$T_1 \rightarrow J = 1, 3, 4$$
$$T_2 \rightarrow J = 2, 3, 4$$

$$D_i = s_{ijk}\nabla_j\nabla_k$$
$$B_i = \epsilon_{ijk}\nabla_j\nabla_k$$

see X. Liao and T. Manke hep-lat/0210030

APE Link Smearing

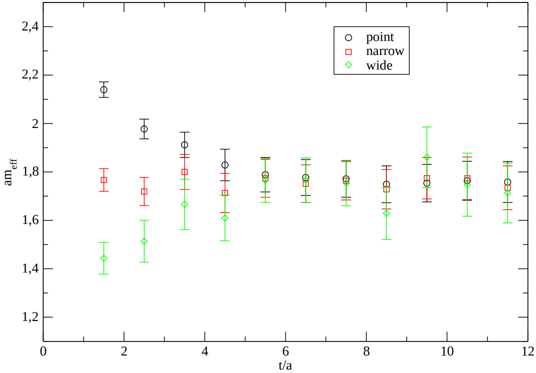
measure for effect of APE Smearing: $\langle P_s \rangle$



APE smearing quite expensive
$\Rightarrow$ we run at $\alpha = 2.5$ and $n_{iter} = 15$

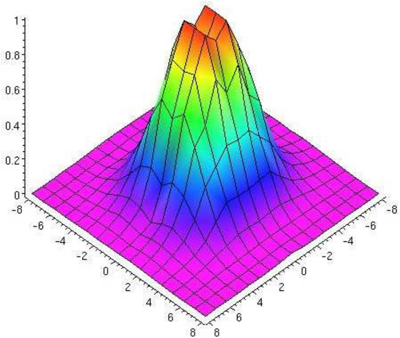
Jacobi Smearing

- optimize each operator separately

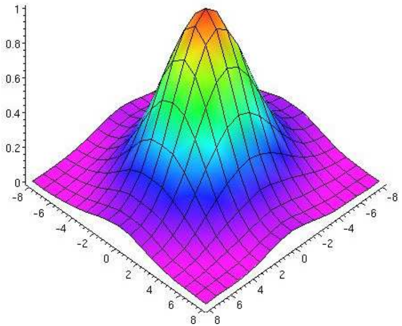


APE + Jacobi Smearing

What APE smearing does to the trial wavefunctions: Plot of $|\psi|^2$
$\kappa = 0.3, n_{Jacobi} = 100; \quad \alpha = 2.5, n_{APE} = 15$



ordinary gauge field
without APE smearing



ordinary gauge field
with APE smearing

All to all propagators

- create N random noise vectors

$$\chi^i_{\alpha,a,x} = \frac{1}{\sqrt{2}}(v+iw) \qquad v,w \in \{\pm 1\}, i = 1,\ldots,N$$

- define the *random contraction*

$$\frac{1}{N}\sum_i \chi^i_{\alpha,a,x}\chi^{i\,*}_{\beta,b,y} = \delta_{x,y}\delta_{a,b}\delta_{\alpha,\beta} + O(\frac{1}{\sqrt{N}})$$

- by inverting the Dirac Operator M on these sources we obtain N solution vectors

$$\eta^i = M^{-1}\chi^i, \quad i = 1,\ldots,N.$$

- naive estimate for A2AP

$$\sum_i \eta^i \chi^{i\,\dagger} = \sum_i M^{-1}\chi^i\chi^{i\,\dagger} = M^{-1}\left(1 + O(\frac{1}{\sqrt{N}})\right)$$

Improvements

- dilution (time, spin, color, even-odd, ...)
- hybrid method
- maximal variance reduction / domain decomposition
- truncation
- hopping parameter acceleration

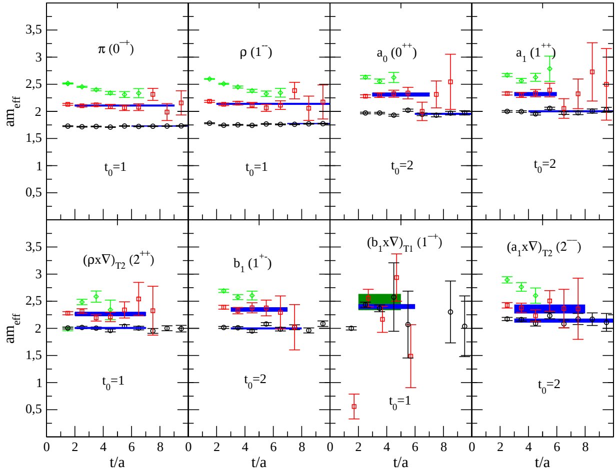
$$M_W^{-1} = (1 - \kappa D)^{-1} = 1 + \kappa D + \ldots + (\kappa D)^{n-1} + \sum_{i=n}^{\infty} (\kappa D)^i$$

$$M_W^{-1} = 1 + \kappa D + \ldots + (\kappa D)^{n-1} + (\kappa D)^n M_W^{-1}$$

$$\Rightarrow (\kappa D)^n M_W^{-1} = M_W^{-1} - (1 + \kappa D + \ldots + (\kappa D)^{n-1})$$

Outline

Effective masses

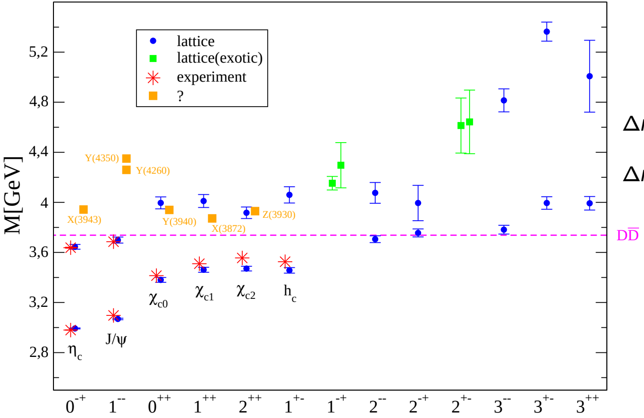


$\beta = 5.20$

$a^{-1} \approx 1730\text{MeV}$

$\#\text{conf} = 100$

Spectrum

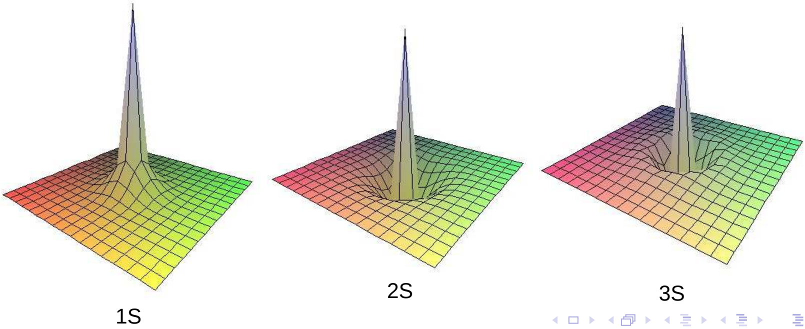


$$\Delta m_{1S} = 73(2)\text{MeV}$$

$$\Delta m_{2S} = 47(6)\text{MeV}$$

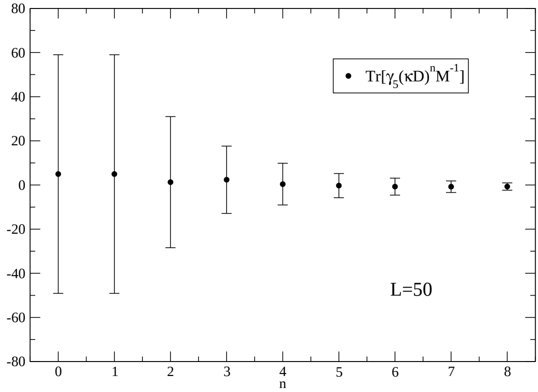
"Wavefunctions"

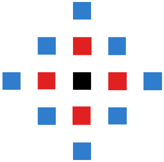
- fix to coulomb gauge
- reconstruct the wavefunction from the eigenvector components: $\Psi^{\alpha}(x) = \sum_j \psi_j^{\alpha} S_j(x)$
- $\mathrm{rms}(1S) \approx 0.39\mathrm{fm}$



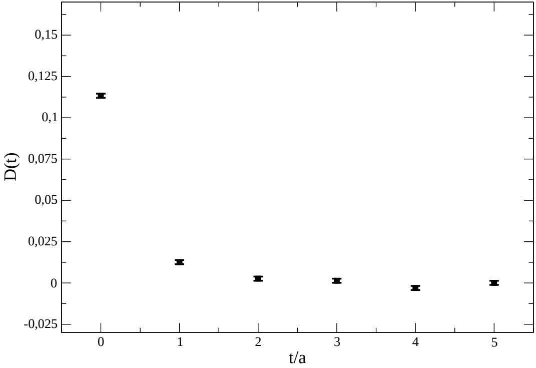
HPA for the Wilson Operator

$$\mathrm{Tr}(\gamma_5 M^{-1}) = \mathrm{Tr}\,\gamma_5 + \kappa \mathrm{Tr}(\gamma_5 D) + \kappa^2 \mathrm{Tr}(\gamma_5 D^2) + \dots$$





Disconnected contribution to η_c



Clover action:
$\mathrm{Tr}(\gamma_5 D_{cl}^2) \propto F\tilde{F}$

$\Rightarrow n = 2$

$L = 100$

$\frac{\sigma_{stoch}}{\sigma_{gauge}} \approx 0.85$

Outline

Outlook

- continuum limit
- four quark/molecule operators
- wavefunctions optimized for excited states in CCM
- further a2a propagator improvements
- heavy-light system: spectrum, matrix elements, ...