Inclusive Charm Production in Bottomonium Decays

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Outline

- Introduction
- $\chi_{bJ} \to$ c+X
- Υ(nS) $\to$ c+X
- Fragmentation into charmed hadrons
- Conclusions

Introduction

• Little work on open charm production in decay of bottomonium has been done.

$$\Gamma[\Upsilon \to ggg^* \to c\bar{c}gg] \quad \text{and} \quad \frac{d\Gamma}{dm_{c\bar{c}}}$$ Firtzsch, Streng, PLB77('78)

$$\Gamma[\chi \to c\bar{c}g] \quad \text{and} \quad \frac{d\Gamma}{dm_{c\bar{c}}}$$ Barbieri, Caffo, Remiddi, PLB83('79)

$\Rightarrow$ Infrared divergences.

•The problem of Infrared divergences was resolved by nonrelativistic QCD (NRQCD). Bodwin, Braaten, Lepage,PRD45('92);PRD51('95)

Why c+X?

- $\Gamma[\Upsilon, \chi_{bJ} \to \mathrm{LH}]$ is not easy to analyze.

- An ideal testing ground of the color-octet mechanism.

- Previous works on inclusive charm production concentrated on the invariant mass distribution of the charm-quark pair.

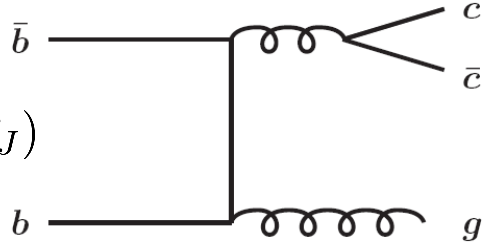
- Recent runs at CLEO-III and B-factories have accumulated large data at $\Upsilon(2S)$ and $\Upsilon(3S)$ resonances.

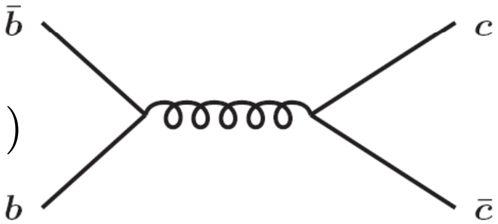
$\Rightarrow$ Ready for studying open charm production in bottomonium decays.

$$\chi_{bJ} \rightarrow c + X$$

Ref. Bodwin, Braaten, Kang, Lee, PRD76('07) [hep-ph/0704.2599]

Factorization formula for χ_{bJ} decay





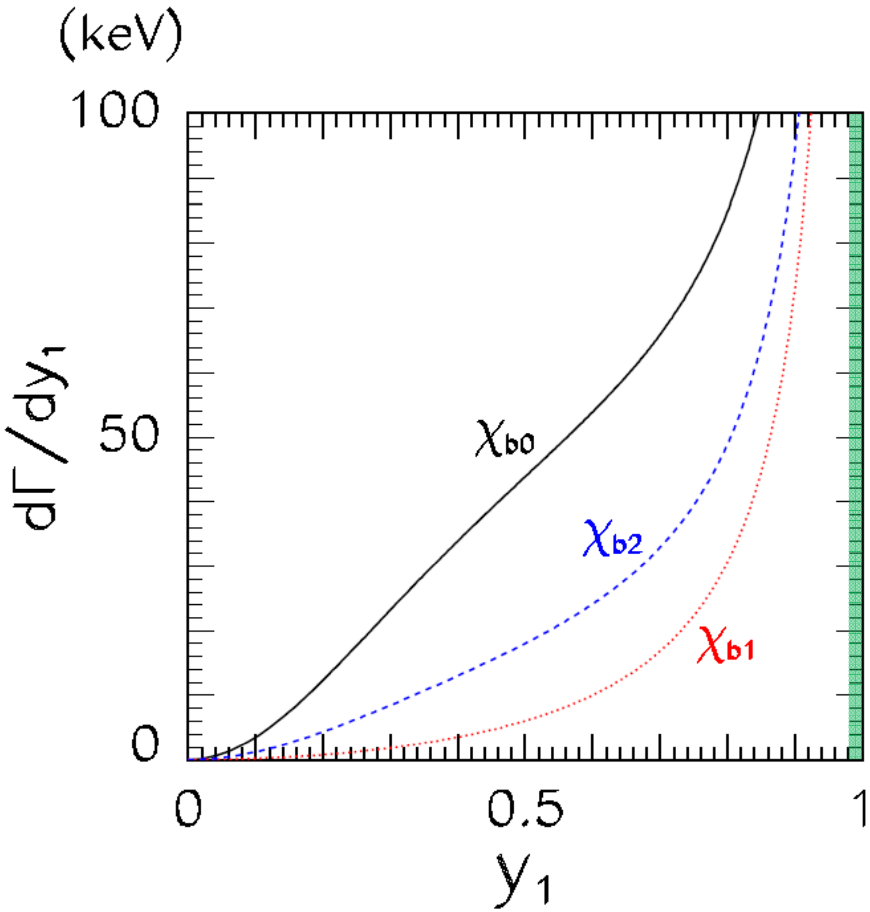
	4-fermion operator	Fock state	Scaling factor
$b\bar{b}_1(^3P_J)$	$\sim \mathcal{O}(v^2)$	$\sim \mathcal{O}(1)$	$\sim \mathcal{O}(\alpha_s^3 v^2)$
$b\bar{b}_8(^3S_1)$	$\sim \mathcal{O}(1)$	$\sim \mathcal{O}(v^2)$	$\sim \mathcal{O}(\alpha_s^2 v^2)$

The NRQCD factorization formula is expressed as

$$\Gamma[\chi_{bJ} \to c + X] = A_J(\Lambda)\frac{\langle \mathcal{O}_1 \rangle_{\chi_b}}{m_b^4} + A_8 \frac{\langle \mathcal{O}_8 \rangle_{\chi_b}^{(\Lambda)}}{m_b^2}.$$

Bodwin, Braaten, Kang, Lee, PRD76('07)

Distribution of charm-quark momentum



Color-octet contributions

y_1 : scaled momentum of the charm quark

$$r = m_c^2/m_b^2 = 4m_D^2/m_{\chi_{bJ}}^2,$$
$$x_1 = E_1/m_b,$$
$$y_1 = \sqrt{\frac{x_1^2 - r}{1 - r}}.$$

Singular at the end point.

Bodwin, Braaten, Kang, Lee, PRD76('07)

Short-distance coefficients

$$A_0^{(c)}(\Lambda) = \frac{C_F\alpha_s^3}{N_c}\left\{\left[\frac{2(2+r)}{9}\log\frac{8(1-r)m_b}{r\Lambda} - \frac{58+23r}{27}\right]\sqrt{1-r} + \frac{5}{9}\log\frac{1+\sqrt{1-r}}{1-\sqrt{1-r}}\right\},$$

$$A_1^{(c)}(\Lambda) = \frac{C_F\alpha_s^3}{N_c}\left\{\left[\frac{2(2+r)}{9}\log\frac{8(1-r)m_b}{r\Lambda} - \frac{16+11r}{27}\right]\sqrt{1-r} - \frac{4}{9}\log\frac{1+\sqrt{1-r}}{1-\sqrt{1-r}}\right\},$$

$$A_2^{(c)}(\Lambda) = \frac{C_F\alpha_s^3}{N_c}\left\{\left[\frac{2(2+r)}{9}\log\frac{8(1-r)m_b}{r\Lambda} - \frac{116+91r}{135}\right]\sqrt{1-r} - \frac{8}{45}\log\frac{1+\sqrt{1-r}}{1-\sqrt{1-r}}\right\},$$

$$A_8^{(c)} = \frac{(1+r/2)\sqrt{1-r}}{3}\pi\alpha_s^2.$$

$$\frac{A_0^{(c)}(\Lambda)}{A_8^{(c)}} \sim 1.6, \qquad \frac{A_1^{(c)}(\Lambda)}{A_8^{(c)}} \sim 0.075, \qquad \frac{A_2^{(c)}(\Lambda)}{A_8^{(c)}} \sim 0.49.$$

Bodwin, Braaten, Kang, Lee, PRD76('07)

Matrix elements for χ_{bJ}

Lattice simulation

Bodwin, Sinclair, Kim, PRD65('02)

$$\langle \mathcal{O}_1 \rangle_{\chi_b(1P)} = 3.2 \pm 0.7 \text{ GeV}^5,$$

$$\frac{\langle \mathcal{O}_8 \rangle^{(\Lambda)}_{\chi_b(1P)}}{\langle \mathcal{O}_1 \rangle_{\chi_b(1P)}} = 0.0021 \pm 0.0007 \text{ GeV}^{-2}.$$

$$\rho_8 \equiv \frac{m_b^2 \langle \mathcal{O}_8 \rangle^{(m_b)}_{\chi_b}}{\langle \mathcal{O}_1 \rangle_{\chi_b}} = 0.044 \pm 0.015.$$

Potential model (Buchmüller-Tye potential)

$$\langle \mathcal{O}_1 \rangle_{\chi_b(1P)} \approx 2.03 \text{ GeV}^5,$$

$$\langle \mathcal{O}_1 \rangle_{\chi_b(2P)} \approx 2.37 \text{ GeV}^5.$$

Bodwin, Braaten, Kang, Lee, PRD76('07)

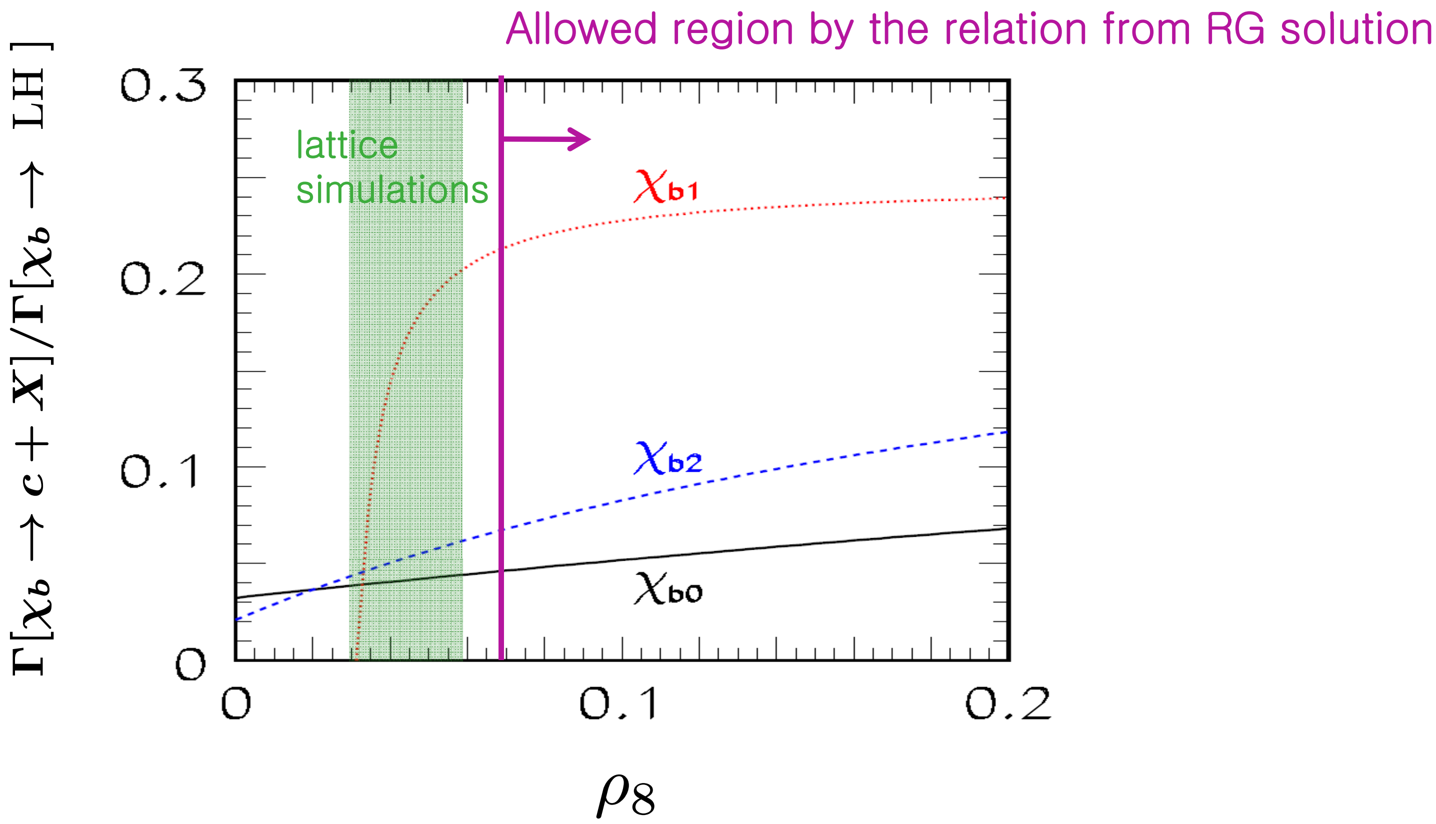
From the solution to the RG equation

$$\langle \mathcal{O}_8 \rangle^{(m_b)}_{\chi_b} = \langle \mathcal{O}_8 \rangle^{(\Lambda)}_{\chi_b} + \frac{4C_F}{3N_c\beta_0} \log\left(\frac{\alpha_s(\Lambda)}{\alpha_s(m_b)}\right) \frac{\langle \mathcal{O}_1 \rangle_{\chi_b}}{m_b^2}.$$

$$\Lambda = m_b v.$$

$$\rho_8 \gtrsim 0.068.$$

Branching fractions



Bodwin, Braaten, Kang, Lee, PRD76('07)

$\Upsilon(nS) \to c + X$

Kang, Kim, Lee, Yu, arXiv:0707.4056 [hep-ph] (To appear in PRD)

Factorization formula for Υ(nS) decay

The inclusive charm production rate in Υ decay is

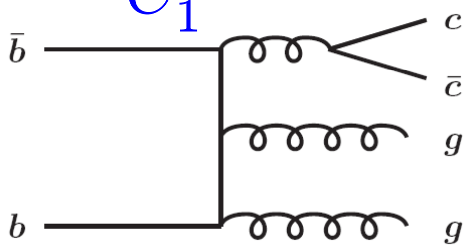
$$\Gamma[\Upsilon \to c + X] = C_1^{(c)} \frac{\langle \mathcal{O}_1(^3S_1) \rangle_\Upsilon}{m_b^2}.$$

At leading order in v, the color-octet terms do not contribute to the decay rate.

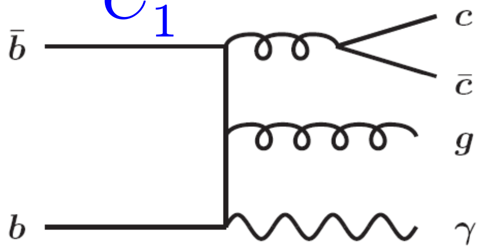
Color-singlet contributions

QCD contributions

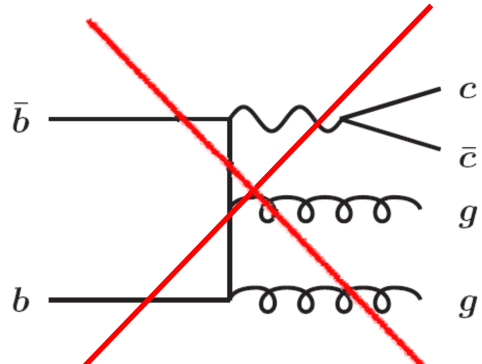
$C_1^{(c\bar{c}gg)}$



$C_1^{(c\bar{c}g\gamma)}$



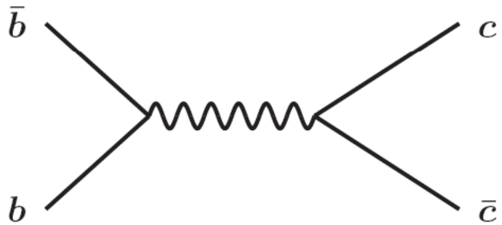
$\sim 2\%$



$\sim 0.1\%$

QED contribution

$C_1^{(c/\gamma)}$



The QED contribution can be estimated as

$$\mathrm{Br}[\Upsilon \to \gamma^* \to c\bar{c}] \approx N_c e_c^2 \mathrm{Br}[\Upsilon \to e^+ e^-] \approx 3\%.$$

Color-singlet matrix elements for Υ

state	Phenomenology[1]	Lattice[2]	Potential models[3]	BKL[4]
$\Upsilon(1S)$	3.6 ± 0.5	3.95 ± 0.43 $\sim 1.84\sigma$	3.6 ± 1.8	$3.07^{+0.21}_{-0.19}$
$\Upsilon(2S)$	1.5 ± 0.2	-	1.7 ± 0.6	$1.62^{+0.11}_{-0.10}$
$\Upsilon(3S)$	1.4 ± 0.3	-	1.2 ± 0.5	$1.28^{+0.09}_{-0.08}$

in units of GeV^3.

Phenomenology : Braaten, Fleming, Leibovich, PRD'01. $\langle v^2 \rangle = \frac{M_{\Upsilon(nS)} - 2m_b}{2m_b}$

Lattice : Bodwin, Sinclair, Kim, PRD'02.

Potential models : Eichten, Quigg, PRD'95. (averaged by Braaten, Fleming, Leibovich).

BKL : see the talk by Bodwin. $\langle v^2 \rangle_{1S} = -0.009, \langle v^2 \rangle_{2S} = 0.090, \langle v^2 \rangle_{3S} = 0.155.$

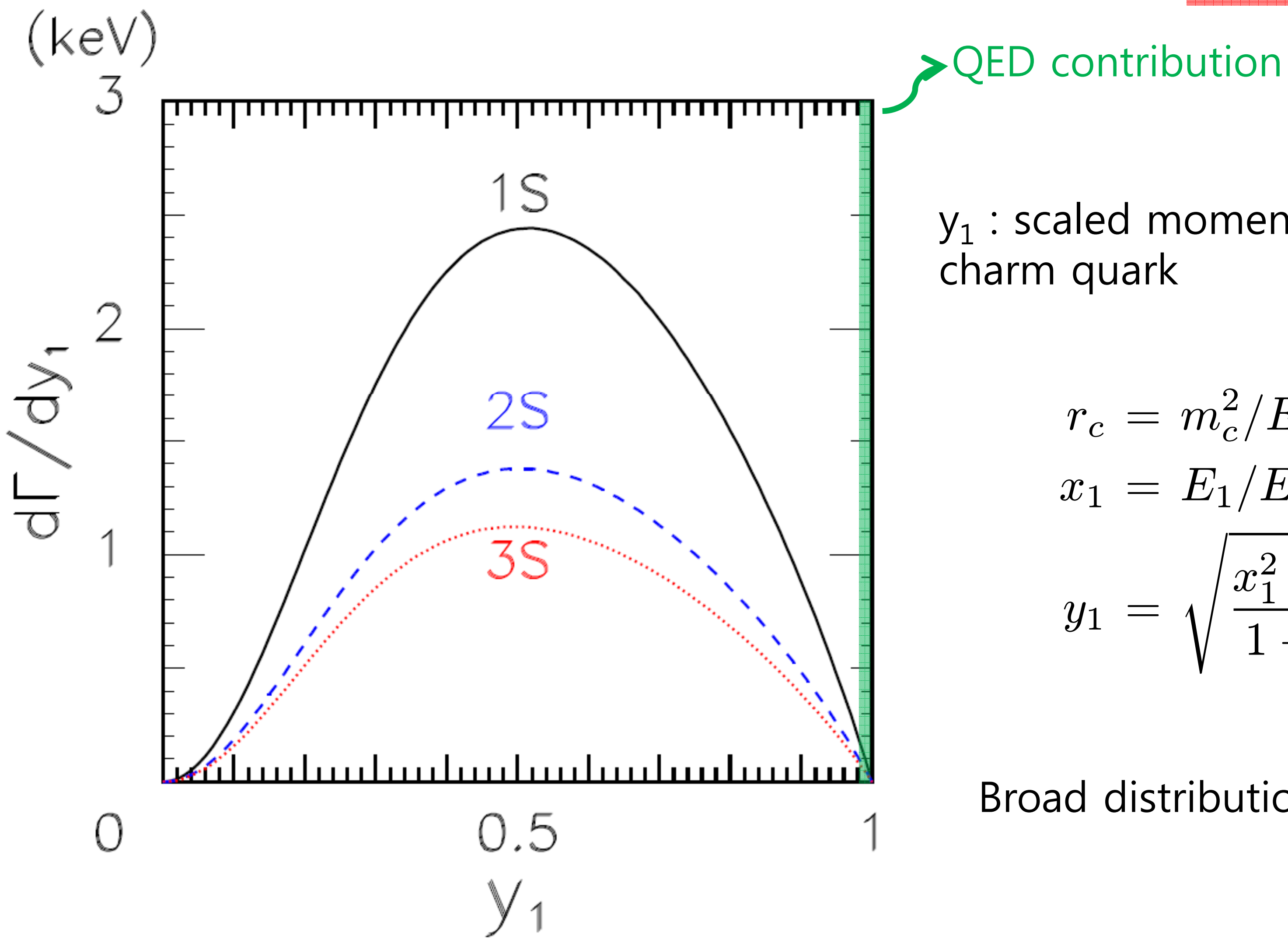
Color-singlet matrix elements for Υ

	m_b	decay formula	$\langle v^2 \rangle$
BFL	4.77 GeV	$\mathcal{O}(v^2)$	Gremm-Kapustin
BKL	4.6 GeV	resummed to all orders in v	Generalized Gremm-Kapustin + Cornell potential

	$\langle v^2 \rangle_{\Upsilon(1S)}$	$\langle v^2 \rangle_{\Upsilon(2S)}$	$\langle v^2 \rangle_{\Upsilon(3S)}$
BFL	-0.0084	0.051	0.085
BKL	$-0.009^{+0.003}_{-0.003}$	$0.090^{+0.011}_{-0.011}$	$0.155^{+0.018}_{-0.018}$

(my estimates from BFL's paper)

Distribution of charm-quark momentum



y_1 : scaled momentum of the charm quark

$$r_c = m_c^2/E_b^2,$$
$$x_1 = E_1/E_b,$$
$$y_1 = \sqrt{\frac{x_1^2 - r_c}{1 - r_c}}.$$

Broad distribution.

Branching fractions

$\mathbf{Br}(\%)$ \ State	$\mathbf{Br}^{(c/g^*)}$	$\mathbf{Br}^{(c/\gamma^*)}$	$\mathbf{Br}^{(c)}$
$\boldsymbol{\Upsilon}(\mathbf{1S})$	2.71 ± 0.67	4.79 ± 1.21	7.50 ± 1.39
$\boldsymbol{\Upsilon}(\mathbf{2S})$	2.77 ± 0.72	4.32 ± 1.15	7.09 ± 1.41
$\boldsymbol{\Upsilon}(\mathbf{3S})$	3.67 ± 0.97	5.35 ± 1.43	9.02 ± 1.82

The branching fractions from QED contributions are 1.5 ~ 1.7 times larger than those from QCD contributions.

Fragmentation into charmed hadron

The charm quark hadronizes into one of charmed hadrons, such as D^0, D^+, D_s^+, or Λ_c^+ or their excited states with a probability of almost 100%.

The hadronization can be expressed in terms of the fragmentation function $D_{c\to h}$

$$\frac{d\Gamma}{dy_h} = \frac{dz_h}{dy_h}\int_{z_h}^{1}\frac{dz_1}{z_1}D_{c\to h}(z_h/z_1)\frac{dy_1}{dz_1}\frac{d\Gamma}{dy_1},$$

where z_1 is the scaled light-cone momentum of the charm and z_h is for the charmed hadron.

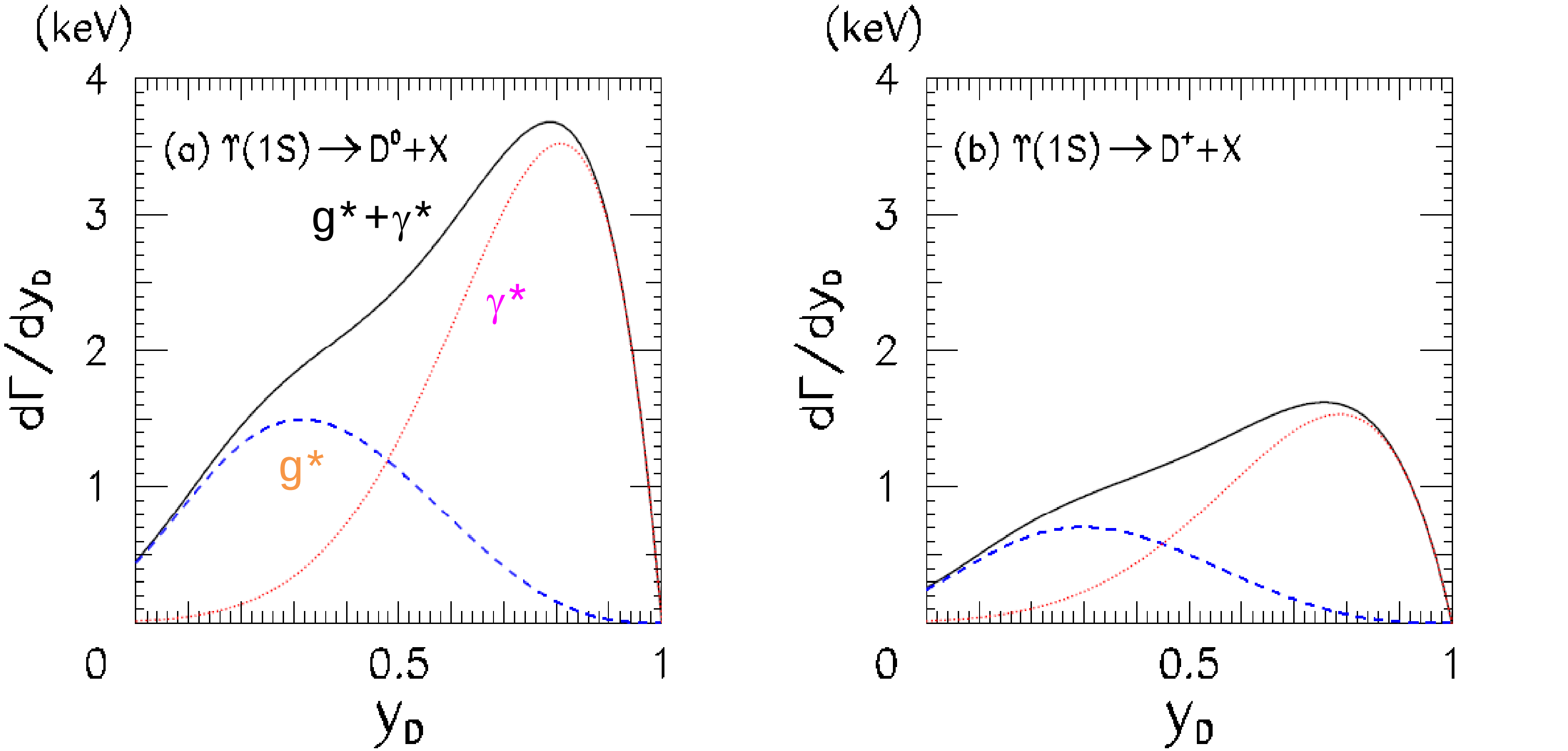
Fragmentation function

Belle,PRD73,032002(2006)

Fragmentation function	Form	Comments
Bowler[1]	$N\frac{1}{z^{1+bm^2}}(1-z)^a \exp\left(-\frac{bm_\perp^2}{z}\right)$	best fit to the data
Lund[2]	$N\frac{1}{z}(1-z)^a \exp\left(-\frac{bm_\perp^2}{z}\right)$	
Kartvelishvili[3]	$Nz^{\alpha_c}(1-z)$	in our analysis
Collins-Spiller[4]	$N\left(\frac{1-z}{z}+\frac{(2-z)\varepsilon_c'}{1-z}\right)(1+z^2)\left(1-\frac{1}{z}-\frac{\varepsilon_c'}{1-z}\right)^{-2}$	
Peterson[5]	$N\frac{1}{z}\left(1-\frac{1}{z}-\frac{\varepsilon_c}{1-z}\right)^{-2}$	widely used, but worst agreement

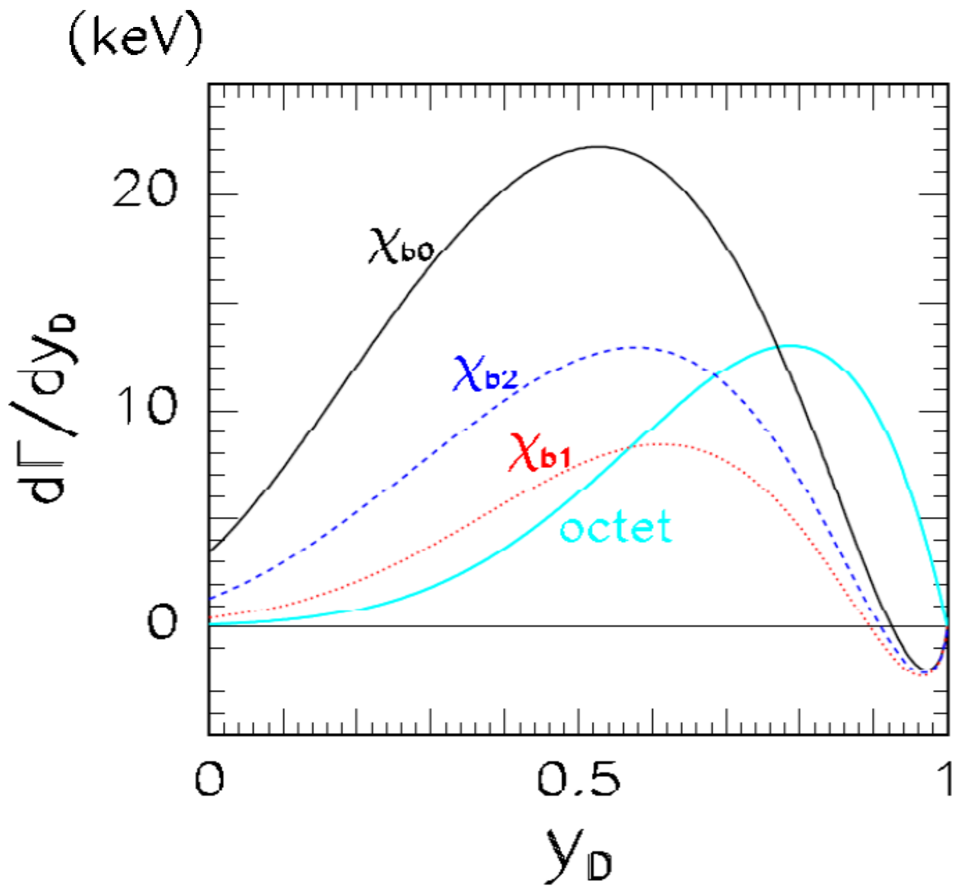
1. Bowler, Z.Phys.C11('81).
2. Andersson, Gustafson, Soderberg, Z.Phys.C20('83).
3. Kartvelishvili, Likhoded, Petrov, PLB78('78).
4. Collins, Spiller, J.Phys.G11('85).
5. Peterson, Schlatter, Schmitt, Zerwas, PRD27('83).

Momentum distributions for hadrons



Include feed-down from D*.

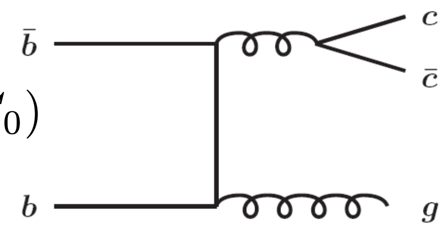
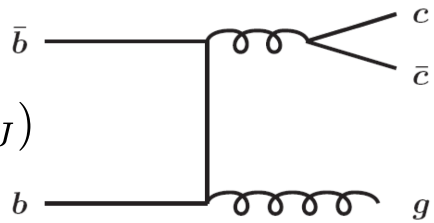
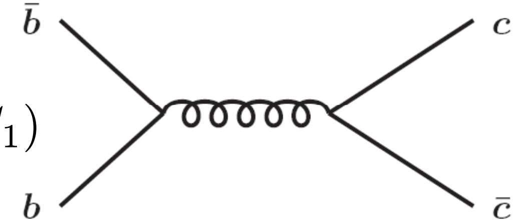
Momentum distributions for D^+



Bodwin, Braaten, Kang, Lee, PRD76('07)

Resummation of logarithmic corrections to all orders will cure unphysical negative rates near at the end point.

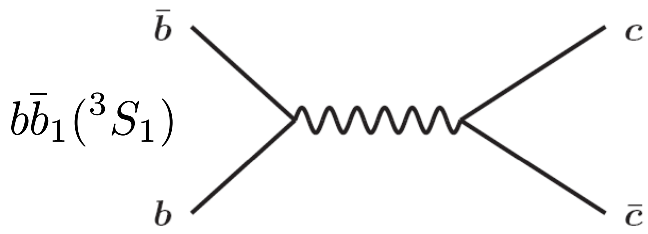
Color-octet contributions in Υ decays

	4-fermion operator	Fock state	Suppression factor
$b\bar{b}_8(^1S_0)$ 	$\sim \mathcal{O}(1)$	$\sim \mathcal{O}(v^3)$	$\sim \mathcal{O}(v^3/\alpha_s)$
$b\bar{b}_8(^3P_J)$ 	$\sim \mathcal{O}(v^2)$	$\sim \mathcal{O}(v^2)$	$\sim \mathcal{O}(v^4/\alpha_s)$
$b\bar{b}_8(^3S_1)$ 	$\sim \mathcal{O}(1)$	$\sim \mathcal{O}(v^4)$	$\sim \mathcal{O}(v^4/\alpha_s^2)$

$$\Gamma[b\bar{b}_8(^3P_J)] : \Gamma[b\bar{b}_8(^1S_0)] : \Gamma[b\bar{b}_8(^3S_1)] \simeq 1 : 3 : 4.7$$

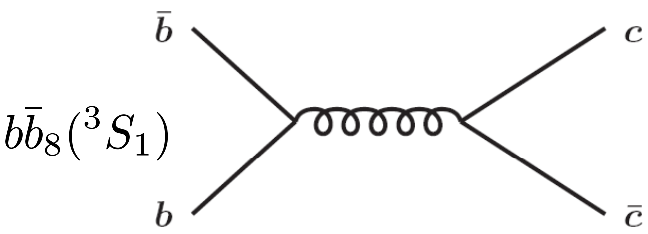
Color-octet contribution

Color−singlet



$$\Gamma_1^{(c/\gamma)} \approx \frac{1}{2} e_b^2 e_c^2 \alpha^2 N_c \langle \mathcal{O}_1 \rangle$$

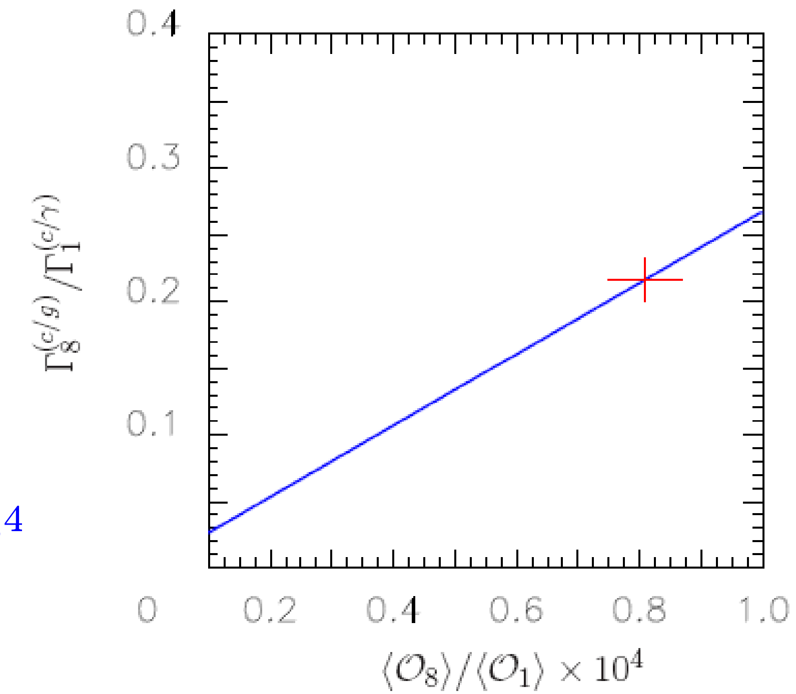
Color−octet



$$\Gamma_8^{(c/g)} \approx \frac{1}{4} \alpha_s^2 \langle \mathcal{O}_8 \rangle \qquad \frac{\langle \mathcal{O}_8 \rangle}{\langle \mathcal{O}_1 \rangle} \sim v^4$$

Ratio of bottomonium color−octet matrix element to color−singlet matrix element was calculated in lattice simulation. Bowdin,Lee,Sinclair,PRD72('05)

$$\frac{\langle \mathcal{O}_8({}^3S_1) \rangle}{\langle \mathcal{O}_1({}^3S_1) \rangle} = (8.1 \pm 0.6) \times 10^{-5}\ \mathrm{GeV}^2$$



Conclusions

- We have provided the predictions for the branching fractions and charm-quark momentum distributions for inclusive charm production in bottomonium decays.

- In Υ(nS) decays, the virtual-photon contributions are about 1.5 times larger than the QCD contributions.

- The infrared divergences in χ_{bJ} decays disappears by inclusion of the color-octet contribution.

- We have also provided the momentum distributions of charmed hadrons.

- The negative decay rate at the end point in χ_{bJ} decays may be cured by resumming logarithmic corrections to all orders.

Conclusions

- The inclusive charm production rate in bottomonium decays may serve as a probe of the color-octet matrix elements phenomenologically.

- It will be interesting to check our leading-order predictions by comparing with the CLEO-III data.

Thank you!

Backup

Fragmentation function

Belle,PRD73,032002(2006)

The Belle Collaboration has measured the charm quark fragmentation at 10.6 GeV, based on a data sample of 103 fb^{-1}.

A	B	Ratio
$D^{*0} + D^{*+}$	$D^+ + D^0$	$0.527 \pm 0.013 \pm 0.024$
D_s^+	$D_s^+ + D^+ + D^0$	$0.099 \pm 0.003 \pm 0.002$
Λ_c^+	$D_s^+ + D^+ + D^0$	$0.081 \pm 0.002 \pm 0.003$

where the ratios are defined by $\sigma(e^+e^- \to AX)/\sigma(e^+e^- \to BY)$ for the continuum sample.

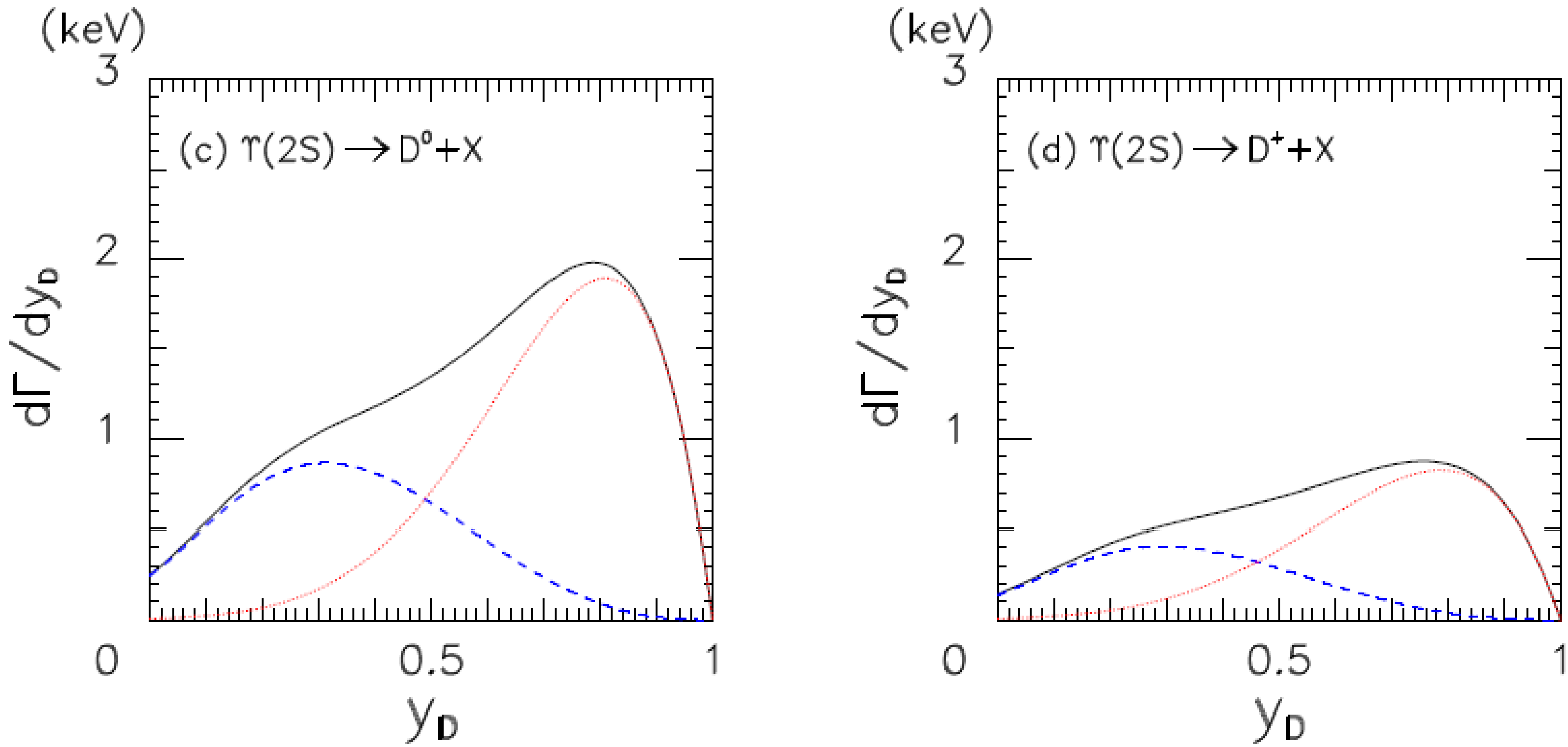
These ratios imply that the direct production rate of D^+ from the charm quark is about 0.197, while that of D^{*+} is about 0.220.
This escapes a naïve prediction for the ratio of the two rates.

Decay rates

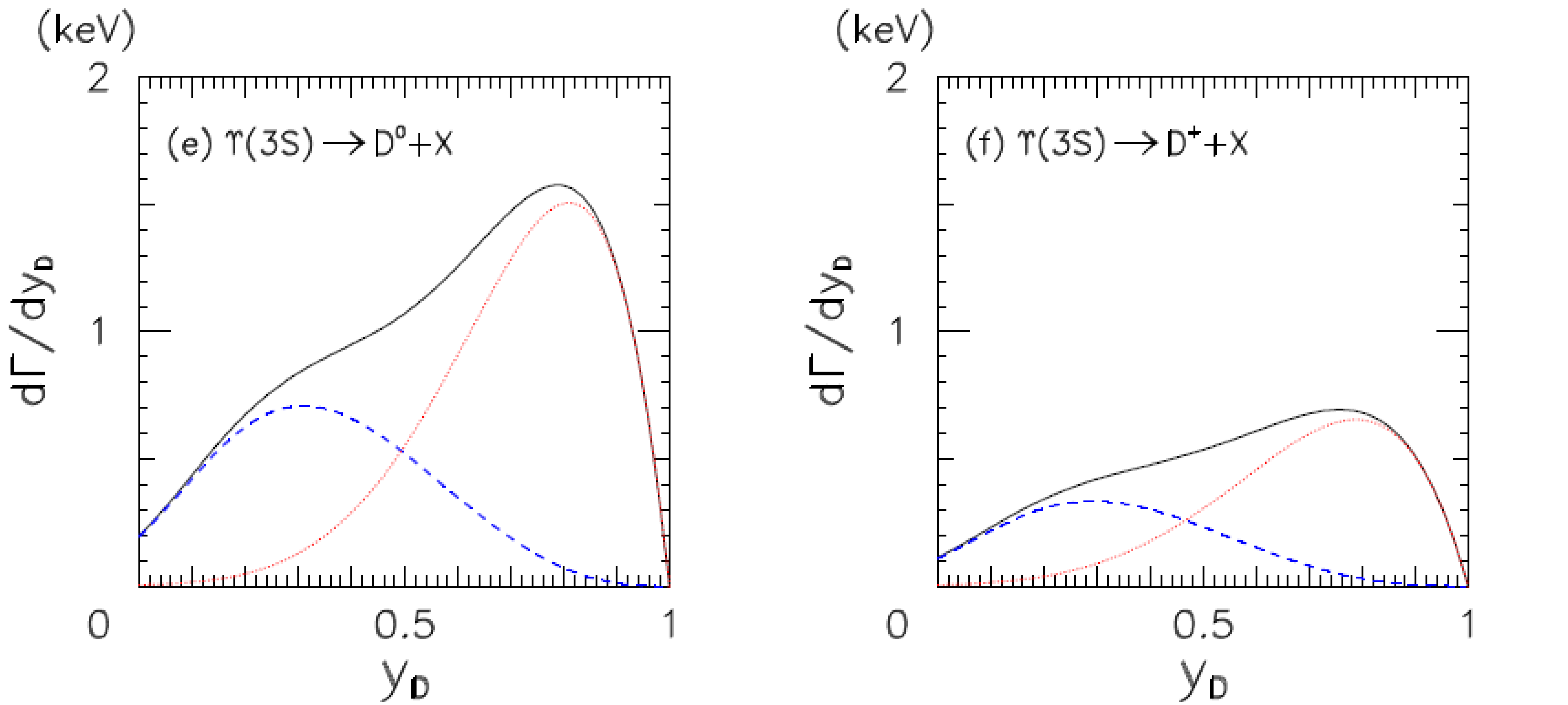
TABLE I: Inclusive charm production rate $\Gamma^{(c)}$ and partial widths $\Gamma^{(c/g^*)}$ and $\Gamma^{(c/\gamma^*)}$ in units of keV for $\alpha_s(m_b) = 0.215$, $m_b = 4.6 \pm 0.1$ GeV, and $\langle O_1 \rangle_\Upsilon$ in Eq. (23). Uncertainties are estimated as stated in the text. The partial widths $\Gamma^{(c\bar{c}gg)}$ and $\Gamma^{(c\bar{c}g\gamma)}$ can be obtained by multiplying $\Gamma^{(c/g^*)}$ by factors $F_\gamma^{-1} \approx 0.982$ and $1 - F_\gamma^{-1} \approx 0.0184$, respectively.

state \ Γ (keV)	$\Gamma^{(c/g^*)}$	$\Gamma^{(c/\gamma^*)}$	$\Gamma^{(c)}$
$\Upsilon(1S)$	1.47 ± 0.36	2.60 ± 0.65	4.07 ± 0.75
$\Upsilon(2S)$	0.83 ± 0.20	1.38 ± 0.34	2.21 ± 0.40
$\Upsilon(3S)$	0.68 ± 0.16	1.09 ± 0.27	1.77 ± 0.32

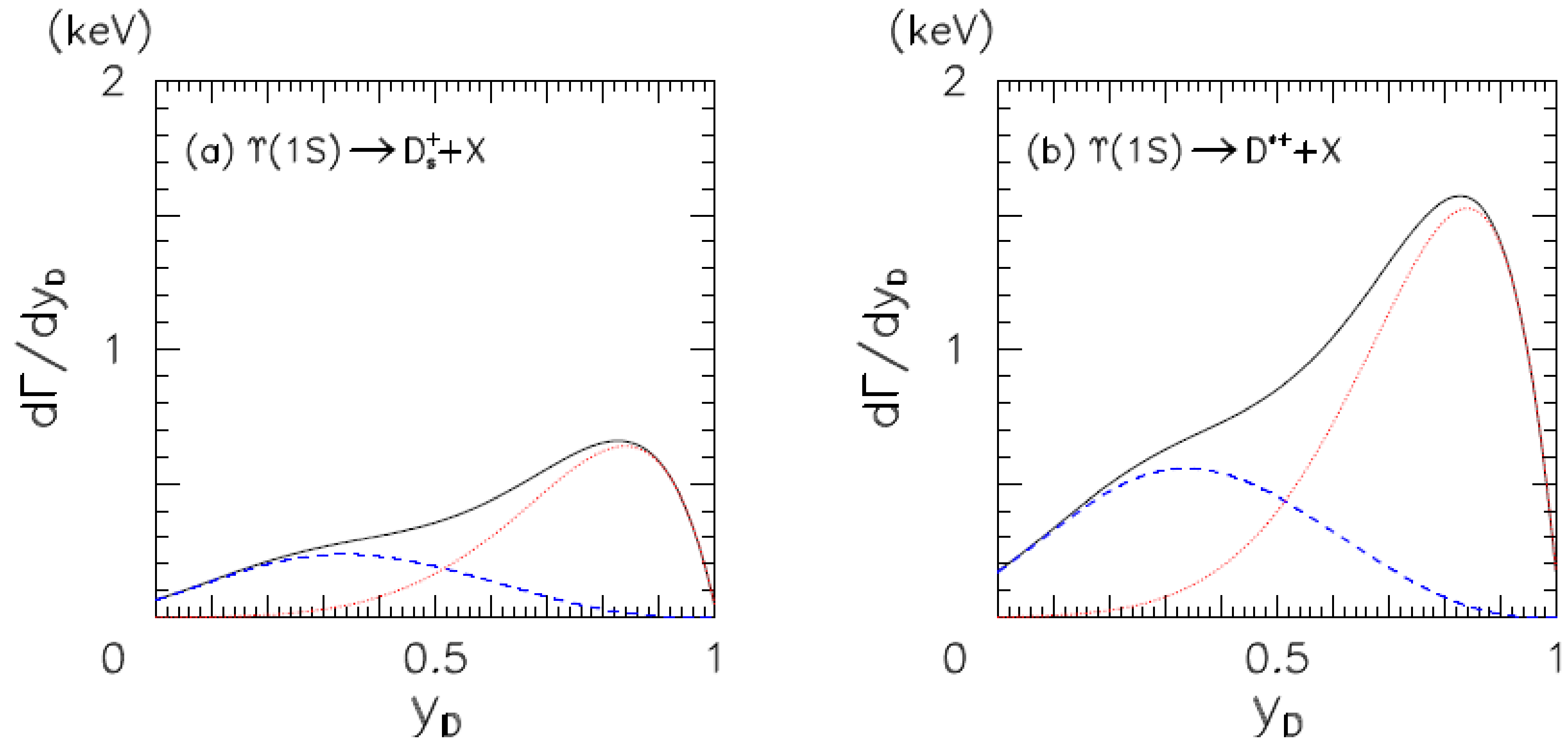
Momentum distributions for hadrons



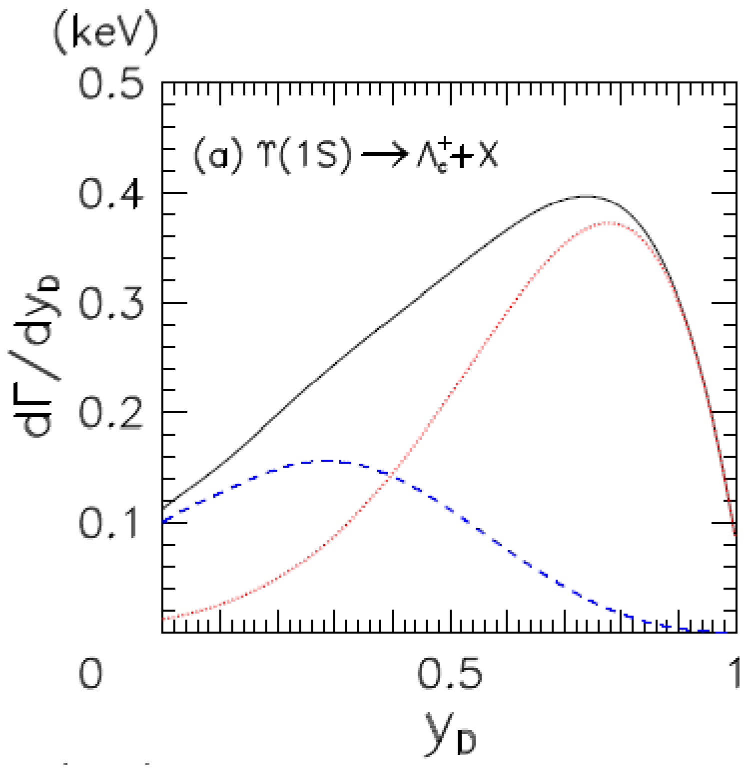
Momentum distributions for hadrons



Momentum distributions for hadrons



Momentum distributions for hadrons



Momentum distributions