
The X Files
The Truth Is Out There

Line Shapes of the $X(3872)$

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Line Shapes of the $X(3872)$

- What is the $X(3872)$?
- Line shapes of the $X(3872)$
- ... within ~ 1 MeV of $D^{*0}\bar{D}^0$ threshold
- ... within ~ 10 MeV of $D^*\bar{D}$ threshold

<u>References</u>

Braaten and Lu, arXiv:0709.2697 [hep-ph]
Braaten and Lu, to appear soon on arXiv [hep-ph]

What is the $X(3872)$?

Two crucial experimental facts

- mass is extremely close to $D^{*0}\bar{D}^0$ threshold

$$M_X - (M_{D^{*0}} + M_{D^0}) = -0.6 \pm 0.6 \text{ MeV}$$

measured in $J/\psi\,\pi^+\pi^-$ decay mode by Belle, CDF, Babar, D0

precise determination of D^0 mass by CLEO

- quantum numbers $J^{PC} = 1^{++}$ strongly preferred

observation of $X \to J/\psi\,\gamma$ by Belle

analyses of $X \to J/\psi\,\pi^+\pi^-$ by Belle, CDF

observation of $X \to D^0\bar{D}^0\pi^0$ by Belle, Babar

What is the X? (cont.)

Two crucial experimental facts

- $J^{PC} = 1^{++}$

$\Longrightarrow$ S-wave coupling to $D^{*0}\bar{D}^0$ (and $D^0\bar{D}^{*0}$)

- $M_X - (M_{D^{*0}} + M_{D^0}) = -0.6 \pm 0.6$ MeV

$\Longrightarrow$ resonant interaction with $D^{*0}\bar{D}^0$ (and $D^0\bar{D}^{*0}$)

Conclusion: $X(3872)$ is either/or

- weakly-bound charm meson molecule

$$X = \frac{1}{\sqrt{2}}\left(D^{*0}\bar{D}^0 + D^0\bar{D}^{*0}\right)$$

- virtual state of charm mesons

What is the X? (cont.)

Nonrelativistic Quantum Mechanics

2-body system with short-range interactions
and S-wave resonance sufficiently close to threshold
has universal properties that

- depend only on the large scattering length a

- are insensitive to details of interactions at shorter distances
  structure of constituents
  mechanism for resonance
  fine-tuning of potential
  tuning of energy of molecule
  etc.

"Universality of Few-Body Systems with Large Scattering Length"
Braaten and Hammer, arXiv:cond-mat/0410417 (Physics Reports)

What is the X? (cont.)

Universal features

- large scattering length a

- cross section at low energy E

$$\sigma(E) = \frac{4\pi a^2}{1 + 2M_{D^*\bar{D}}\, a^2 E}$$

- shallow S-wave bound states

  $a < 0$: none

  $a > 0$: one
  binding energy: $E_X = 1/(2M_{D^*\bar{D}}\, a^2)$
  mean separation: $\langle r \rangle_X = a/2$

What is the X? (cont.)

$X(3872)$ has universal properties

determined by large scattering length a in $D^{*0}\bar{D}^0 + D^0\bar{D}^{*0}$ channel
insensitive to all shorter length scales of QCD

Universal results for $a > 0$:

$$\begin{aligned} E_X &= 1/(2M_{D^*\bar{D}}\, a^2) \\ \langle r \rangle_X &= a/2 \end{aligned}$$

measured binding energy: $E_X = 0.6 \pm 0.6$ MeV

predicted mean separation: $\langle r \rangle_X = 2.9^{+\infty}_{-0.9}$ fm

What is the X? (cont.)

Beauty and the Beast

$$B \longrightarrow K + X(3872)$$

$$\langle r \rangle_X = 2.9^{+\infty}_{-0.9} \text{ fm}$$

LeFou: Tell us again, old man, just how big was the Beast?
Maurice: It was enormous, I'd say at least 8,
no, more like 10 fermis!
LeFou: Well, you don't get much crazier than that!
Belle: My father's not crazy and I can prove it!

Line Shapes of the $X(3872)$

Line shape of X in decay mode C
= invariant mass distribution of C: $M_{D^{*0}} + M_{D^0} + E$

$$\frac{d\Gamma}{dE}[B \to K + C]$$

Mass measurements of $X(3872)$ ...

... in $J/\psi\,\pi^+\pi^-$

$$M_X - (M_{D^{*0}} + M_{D^0}) = -0.6 \pm 0.6 \text{ MeV}$$

... in $D^0 \bar{D}^0 \pi^0$

$$\begin{aligned} M - (M_{D^{*0}} + M_{D^0}) &= +4.1 \pm 0.7^{+0.3}_{-1.6} \text{ MeV} \quad \text{(Belle)} \\ &= +4.3 \pm 1.1 \pm 0.5 \text{ MeV} \quad \text{(Babar)} \end{aligned}$$

Line Shapes of X (cont.)

short distances: $\ll |a| \sim 6$ fm

large momenta: $\gg 1/|a| \sim 30$ MeV

Qualitative difference between decay modes

$X \longrightarrow J/\psi\, \pi^+\pi^-$

decay products have large momenta

$\Longrightarrow$ constituents must come within short distance

"short-distance decay mode"

$X \longrightarrow D^0 \bar{D}^0 \pi^0$

involves decay of constituent $D^{*0} \to D^0\pi^0$

$\bar{D}^{*0} \to \bar{D}^0\pi^0$

decay products have small momenta $\sim 1/|a|$

Line Shapes of X (cont.)

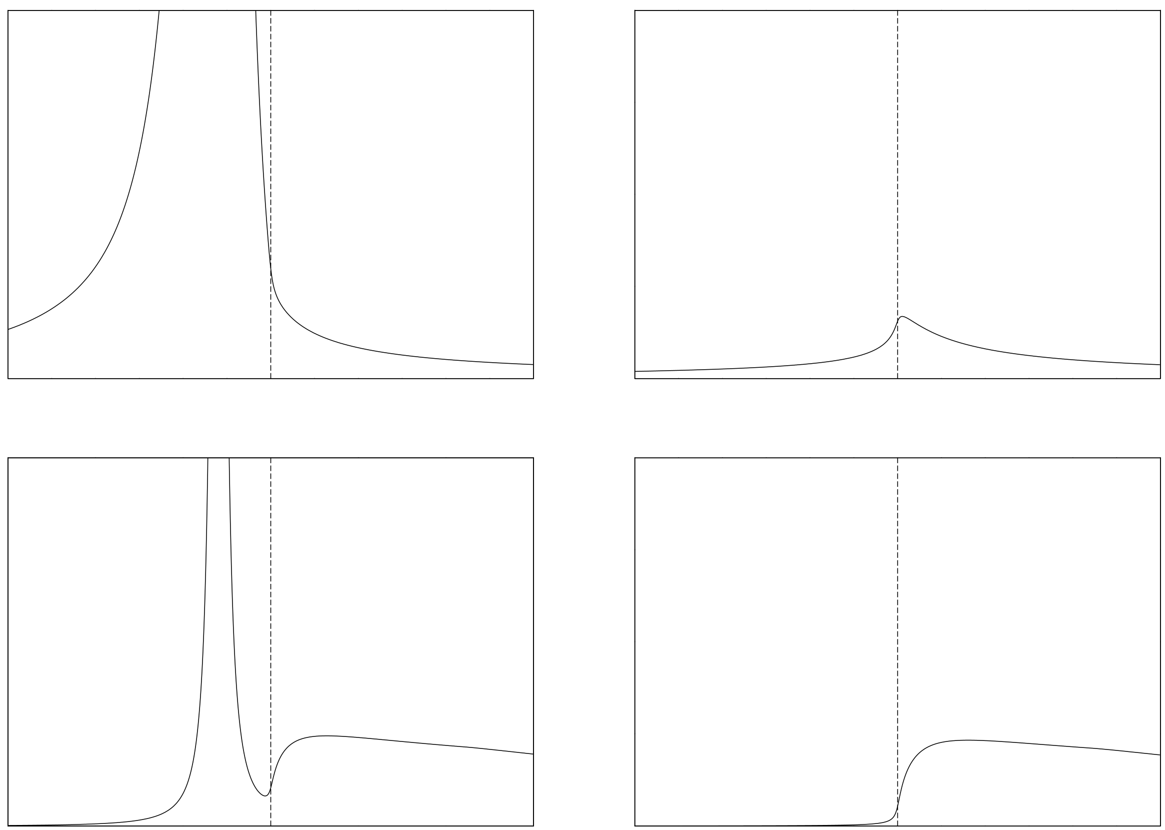
Qualitative difference between bound state and virtual state

Bound state ($a > 0$)

Virtual state ($a < 0$)

$J/\psi\,\pi^+\pi^-$

$D^0\bar{D}^0\pi^0$



Line Shapes of X (cont.)

Quantitative behavior of line shapes

may depend on

- D^* widths

  $\Gamma[D^{*0}] = 65.5 \pm 15.4$ keV

- inelastic scattering channels of charm mesons

  $J/\psi\,\pi^+\pi^-$, $J/\psi\,\pi^+\pi^-\pi^0$, ...

- charged charm mesons

  $D^{*+}D^-$ threshold: $+8.1$ MeV

- 3-body channels: $D\bar{D}\pi$

  $D^0\bar{D}^0\pi^0$ threshold: -7.1 MeV

  $D^+\bar{D}^0\pi^-$, $D^0D^-\pi^+$ threshold: $+2.3$ MeV

  see talk by Tom Mehen

Line Shapes of X (cont.)

Recent analyses of data from Belle and Babar
on $B^+ \longrightarrow K^+ + X(3872)$
in decay channels $J/\psi\,\pi^+\pi^-$, $D^0\bar{D}^0\pi^0$

- Hanhart, Kalashnikova, Kudryavtsev, Nefediev [arXiv:0704.0605]
  included effects of $D^{*\pm}D^{\mp}$ channel
  included effects of inelastic channels $J/\psi\,\pi^+\pi^-$, $J/\psi\,\pi^+\pi^-\pi^0$
  ignored effects of D^* widths
  assumed bound state cannot decay into $D^0\bar{D}^0\pi^0$

  Conclusion: $X(3872)$ must be a virtual state

  See talk by Hanhart

- Braaten and Lu [arXiv:0709.2697]
  neglected effects of $D^{*\pm}D^{\mp}$ channel
  included effects of inelastic channels $J/\psi\,\pi^+\pi^-$, $J/\psi\,\pi^+\pi^-\pi^0$
  included effects of D^{*0} width

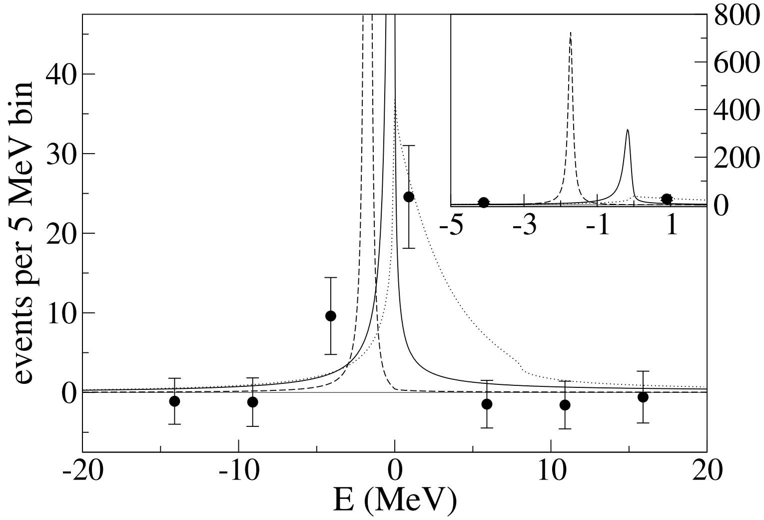
  Conclusion: data prefers $X(3872)$ to be a bound state
  but virtual state not excluded

Line Shapes of X (cont.)

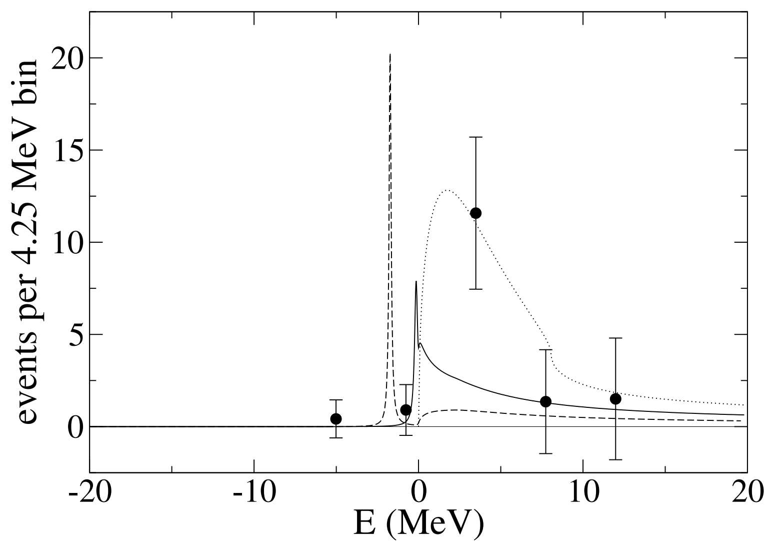
Analysis of Braaten and Lu [arXiv:0709.2697]

- Belle data on $B^+ \to K^+ + X$ with total experimental errors subtracted
- line shapes with 3 adjustable parameters
- two local minima of χ^2 with Re$\gamma = +17.3$ MeV and $+57.8$ MeV

$J/\psi\,\pi^+\pi^-$



$D^0\bar{D}^0\pi^0$



Conclusions

- data prefers $X(3872)$ to be a bound state, but virtual state not excluded
- can explain difference between measured mass in $J/\psi\,\pi^+\pi^-$ and in $D^0\bar{D}^0\pi^0$

Line Shapes of $X(3872)$ within ~ 1 MeV of $D^{*0}\bar{D}^0$ Threshold

S-wave resonance in 1^{++} channel: $D^{*0}\bar{D}^0 + D^0\bar{D}^{*0}$

1. begin with scattering amplitude $f(E)$ that satisfies unitarity exactly:

$$\mathrm{Im} f(E) = |f(E)|^2 \sqrt{2M_{D^*\bar{D}} E}$$

2. apply deformations that take into account
   D^{*0} width
   inelastic scattering channels

3. insert into factorization formulas for the line shapes

Line Shapes within ~ 1 MeV of Threshold (cont.)

1. scattering amplitude that satisfies unitarity exactly:

$$
\begin{aligned}
f(E) &= \frac{1}{-\gamma + \kappa(E)} \\
\gamma &= 1/a \\
\kappa(E) &= (-2M_{D^*\bar{D}}E - i\varepsilon)^{1/2}
\end{aligned}
$$

- $a > 0$: bound state

  pole at $E = -\gamma^2/(2M_{D^*\bar{D}})$

- $a < 0$: virtual state

  pole on second sheet of complex energy E

Line Shapes within ~ 1 MeV of Threshold (cont.)

2. Apply deformations to unitary scattering amplitude

$$f(E) = \frac{1}{-\gamma + \kappa(E)}$$

- take into account D^{*0} width: $M_{D^{*0}} \to M_{D^{*0}} - i\Gamma[D^{*0}]/2$

$$\kappa(E) = \left(-2M_{D^*\bar{D}}(E + i\Gamma[D^{*0}]/2)\right)^{1/2}$$

- take into account inelastic scattering channels

$$\gamma \longrightarrow \mathrm{Re}\,\gamma + i\,\mathrm{Im}\,\gamma, \qquad \mathrm{Im}\,\gamma > 0$$

Optical theorem:

$$\mathrm{Im}\, f(E) = |f(E)|^2 \,(\mathrm{Im}\,\gamma - \mathrm{Im}\,\kappa(E))$$

consistent with multi-channel unitarity

3. Factorization formulas for the line shapes

factor rates into long-distance factor (depends on E, γ)
$\times$ short-distance factors (insensitive to E, γ)

$B^+ \to K^+ + C$, where $C = J/\psi\,\pi^+\pi^-$, $J/\psi\,\pi^+\pi^-\pi^0$, . . .

$$\frac{d\Gamma}{dE} = 2\,\Gamma_{B^+}^{K^+} \times |f(E)|^2 \times \Gamma^C$$

$B^+ \to K^+ + D^0\bar{D}^0\pi^0$

$$\frac{d\Gamma}{dE} = 2\,\Gamma_{B^+}^{K^+} \times |f(E)|^2 \left[M_{D^*\bar{D}}\left(\sqrt{E^2+\Gamma[D^{*0}]^2/4}+E\right)\right]^{1/2} \times \mathrm{Br}[D^{*0} \to D^0\pi^0]$$

Short-distance factors

- Γ_B^K different for $B^+ \to K^+$ and $B^0 \to K^0$
- Γ^C different for $J/\psi\,\pi^+\pi^-$ and $J/\psi\,\pi^+\pi^-\pi^0$

Line Shapes of $X(3872)$ within $\sim$ 10 MeV of $D^{*0}\bar{D}^0$ Threshold

S-wave resonance in two coupled 1^{++} channels:
$D^{*0}\bar{D}^0 + D^0\bar{D}^{*0}$, $D^{*+}D^- + D^+D^{*-}$

1. begin with scattering amplitudes $f_{00}(E)$, $f_{01}(E)$, $f_{11}(E)$
   that satisfy two-channel unitarity exactly
   with isospin symmetry at high energy
   $\Longrightarrow$ 2 scattering parameters: $\gamma_{I=0}$, $\gamma_{I=1}$

2. apply deformations that take into account
   D^{*0}, D^{*+} widths
   inelastic scattering channels

3. insert into factorization formulas for the line shapes,
   with short-distance factors constrained by isospin symmetry

Implications

1. Conceptual error by Braaten and Kusunoki [hep-ph/0412268]

   prediction: $B^0 \to K^0 + X$ is suppressed compared to $B^+ \to K^+ + X$

   Belle, Babar: no indication of strong suppression

   error: implicitly assumed $|\gamma_0|, |\gamma_1| \ll \sqrt{2M_{D^*\bar{D}}\nu} =$ 125 MeV

2. Conceptual error by Voloshin [arXiv:0704.3029]

   prediction: line shapes of X from $B^0 \to K^0$ same as from $B^+ \to K^+$

   errors: did not allow for resonant scattering

   between neutral and charged $D^*\bar{D}$ channels

   results inconsistent with isospin symmetry

   in short-distance factors for $B \to K$

Line Shapes within ~ 10 MeV of Threshold (cont.)

3. Interpretation of scattering amplitude of Hanhart et al.

   "generalization of Flatté parametrization for near-threshold resonance"
   limit $|\gamma_1| \gg |\gamma_0|, \sqrt{2M_{D^*\bar{D}}\nu}$ gives essentially same scattering amplitude

4. Line shapes depend on production process, decay channel

   determined by γ_0, γ_1
   different for $B^+ \to K^+ + X$ and $B^0 \to K^0 + X$
   different for $J/\psi\, \pi^+\pi^-$, $J/\psi\, \pi^+\pi^-\pi^0$, $D^0\bar{D}^0\pi^0$
   zeroes in line shapes of $J/\psi\, \pi^+\pi^-$
   $B^+ \to K^+ + X$: zero near $+6$ MeV
   $B^0 \to K^0 + X$: zero near -2 MeV
   no zeroes in line shapes of $J/\psi\, \pi^+\pi^-\pi^0$, $D^0\bar{D}^0\pi^0$

5. Ratios of production rates from $B^0 \to K^0$ and $B^+ \to K^+$

   determined by γ_0, γ_1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