Thermodynamics of a heavy $Q\bar{Q}$-pair near T_c

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MARIE CURIE ACTIONS

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$$L_{\rm mfp} \sim \frac{1}{n\sigma_t},$$

where $n \sim T^3$ is the particle-number density, σ_t is the Coulomb transport cross section:

$$\sigma_t = \int d\sigma(1-\cos\theta) \sim g^4 \int_{(gT)^2} \frac{d^2p_\perp}{p_\perp^4}\frac{p_\perp^2}{T^2} \sim \frac{g^4}{T^2}\ln\frac{1}{g}.$$

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$\Rightarrow$ a strong contradiction with the experiment:

$$\frac{L_{\rm mfp}}{\beta} \sim \frac{1}{g^4 \ln\frac{1}{g}} \gg 1.$$

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On the lattice, the two-point correlation function of Wilson lines (Polyakov loops) $L(\mathbf{R}) = \mathcal{P} \exp\left[ig \int_0^\beta dt A_4(\mathbf{R}, t)\right]$ in the singlet channel was measured (F. Karsch, O. Kaczmarek, P. Petreczky, F. Zantow, '05):

$$\frac{1}{3}\,\mathrm{Tr}\,\left\langle L(\mathbf{R}) L^\dagger(\mathbf{0})\right\rangle = \frac{\mathcal{Z}_{Q\bar{Q}}(\mathbf{R}, T)}{\mathcal{Z}(T)} =$$

$$= \frac{1}{\mathcal{Z}(T)} \int \mathcal{D}A_\mu^a \mathcal{D}\bar{\psi}\mathcal{D}\psi\, \frac{1}{3}\,\mathrm{Tr}\, L(\mathbf{R}) L^\dagger(\mathbf{0}) \exp\left[-\int_0^\beta dt \int d^3x \mathcal{L}_{\mathrm{QCD}}(\mathbf{x}, t)\right].$$

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The free energy of the static $Q\bar{Q}$-pair at a fixed large separation $|\mathbf{R}| \geq 1.5\,\mathrm{fm}$:

$$F(T) = -T\ln\frac{\mathcal{Z}_{Q\bar{Q}}(\mathbf{R},T)}{\mathcal{Z}(T)}\Big|_{\mathbf{R}\,\mathrm{fixed}}.$$

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The corresponding entropy $S(T) = -\frac{\partial F(T)}{\partial T}$ and the internal energy $U(T) = F(T) + TS(T)$ exhibit maxima at $T \to T_c$, which cannot be explained by perturbation theory alone.

Introduction

This talk is devoted to an attempt to explain these data theoretically.

Strategy and models:

• to determine an effective string tension $\sigma_{\rm eff}(T)$ in quenched SU(3) QCD.
Model: gluon chain = the $Q\bar{Q}$-string with multiple valence gluons.

• with the use of $\sigma_{\rm eff}(T)$ extrapolated to the unquenched case, to calculate $S(T)$ and $U(T)$ at $T < T_c$ for heavy-light mesons and heavy-light-light baryons, which are formed upon the string breaking and hadronization.
Model: the relativistic quark model.

Static $Q\bar{Q}$-pair with gluons at $T < T_c$

At low enough temperatures, the free energy of one string bit in the gluon chain $>$ thermal gluon mass, which grows linearly with T. This situation changes at a certain temperature T_0, which is smaller than T_c.

$T < T_0 \Rightarrow$ an elastic string, gluons move collectively with it;
$T > T_0 \Rightarrow$ a sequence of static nodes with adjoint charges, connected by independently fluctuating string bits.

To form the gluon chain, the string originating at Q performs a random walk to $\bar{Q}$ over the lattice of such nodes. The large entropy of such a random walk eventually leads to the deconfinement phase transition.

Static $Q\bar{Q}$-pair with gluons at $T < T_c$

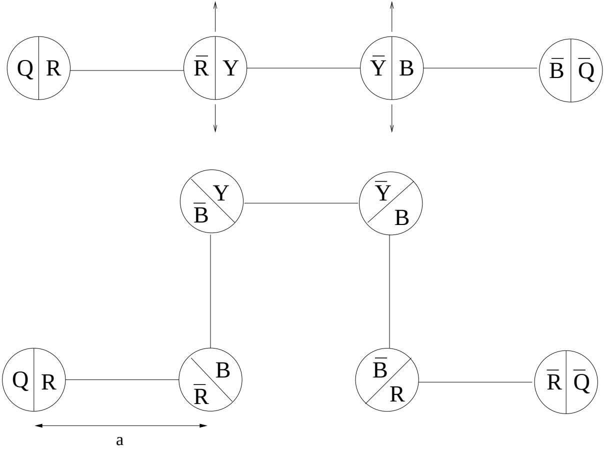

Figure: Gluon chain at $T < T_0$ and $T > T_0$. Below T_0, valence gluons move together with the string, while at $T > T_0$ they become static. Color may change from one string bit to another.

Static $Q\bar{Q}$-pair with gluons at $T < T_c$

Every string bit may transport each of the N_c colors $\Rightarrow$ the total number of states of the gluon chain is $N_c^{L/a}$, where L is the length of the chain and a is the length of one bit.

The partition function of the random walk ($R \equiv |\mathbf{R}|$):

$$\mathcal{Z}(R,T) = \sum_{n=-\infty}^{+\infty} \int_0^\infty \frac{ds}{(4\pi s)^2} \exp\left[-\frac{R^2 + (\beta n)^2}{4s} - \frac{s}{a}\left(\frac{\sigma}{T} - \frac{\ln N_c}{a}\right)\right].$$

The effective string tension and the critical temperature:

$$\sigma(T) = \sigma - \frac{T}{R} \ln \frac{\mathcal{Z}(R,T)}{\mathcal{Z}(R,T_0)}\bigg|_{R\to\infty} =$$

$$= \sigma + \frac{T}{\sqrt{a}}\left[\sqrt{\frac{\sigma}{T} - \frac{\ln N_c}{a}} - \sqrt{\frac{\sigma}{T_0} - \frac{\ln N_c}{a}}\right]$$

$$\Rightarrow \quad T_c\Big|_{N_c>1} = \frac{\sigma a}{\ln N_c}.$$

Static $Q\bar{Q}$-pair with gluons at $T < T_c$

High-energy scattering data yield $a = 0.302\,\mathrm{fm}$ (H.-J. Pirner *et al.*, '02) $\Rightarrow$ $T_c = 270\,\mathrm{MeV}$ in a perfect agreement with the modern lattice value (Bielefeld-Brookhaven group).

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$\sigma(T_c) = 0 \Rightarrow$ the temperature below which the lattice of valence gluons does not exist:

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An important result is the critical behavior

$$\sigma(T) \sim \sqrt{T_c - T} \quad \mathrm{at} \quad T \to T_c,$$

which is the same as in the Nambu-Goto model for the two-point correlation function of Polyakov loops (R.D. Pisarski and O. Alvarez, '82).

Static $Q\bar{Q}$-pair with gluons at $T < T_c$

Comparing to the limiting case when string bits cannot alter color:

$$\sigma(T) = \sigma + \sqrt{\frac{\sigma T}{a}}\left(1 - \sqrt{\frac{T}{T_0}}\right) \sim (T_c - T) \ \text{ at } T \to T_c \ \Rightarrow$$

the universality class of the 2d (!) Ising model, defined by the critical exponent $\nu = 1$, cannot be the right one for the 4d Yang-Mills theory.

The same linear fall-off of $\sigma(T)$ with $(T_c - T)$ one finds also in the

- Hagedorn phase transition: $S = \sigma R/T_H$, $F = \sigma R - TS$;
- deconfinement scenario based on the condensation of long closed strings: $S = \ln N$, $N = (2d-1)^{L/a}$ is the number of possibilities to realize on a hypercubic lattice a closed trajectory of length L, $F = \sigma L - TS \Rightarrow$ $T_c = \frac{\sigma a}{\ln(2d-1)}$, which yields 270 MeV only at $a = 0.54\,\text{fm} \simeq R/2$ (!).

Static $Q\bar{Q}$-pair with light quarks at $T < T_c$

In the unquenched case, the $Q\bar{Q}$-string breaks due to the production of a light $q\bar{q}$-pair. Hadronization $\Rightarrow$ formation of heavy-light mesons ($Q\bar{q}$), heavy-light-light baryons (Qqq), and their antiparticles. Considering the ($N_f = 2$)-case, with light u- and d-quarks, and using the value $T_c = 200\,\mathrm{MeV}$.

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Calculating the partition function of two noninteracting heavy-light mesons and two heavy-light-light baryons within the relativistic quark model, e.g.

$$H_{\bar{Q}q} = m_{\bar{Q}} + \sqrt{\mathbf{p}^2 + m_q^2} + V(r),$$

where $V(r) = \sigma(T)r - C\sqrt{\sigma(T)}$, m_q is the constituent mass of a light quark, $m_q \simeq 300\,\mathrm{MeV}$, and $C \simeq 1.65$ is fixed by the limit $T \to 0$.

Calculating further the entropies and the internal energies of these mesons and baryons together $\Rightarrow$

Static $Q\bar{Q}$-pair with light quarks at $T < T_c$

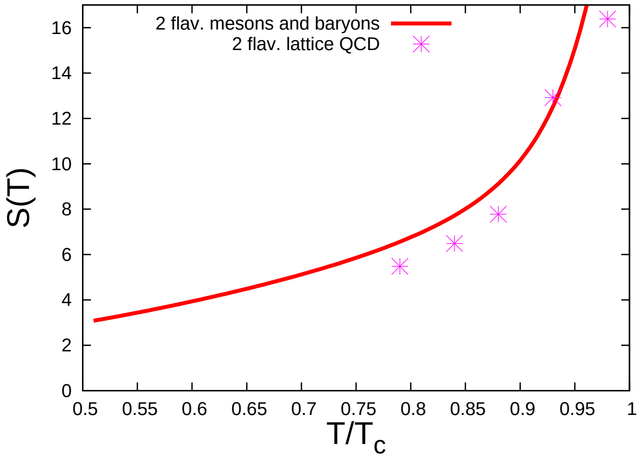

Figure: The calculated entropy $S(T)$ (full drawn curve) of two mesons and two baryons as a function of T/T_c with $T_c = 200\,\mathrm{MeV}$. The stars show the lattice data (O. Kaczmarek and F. Zantow, '05).

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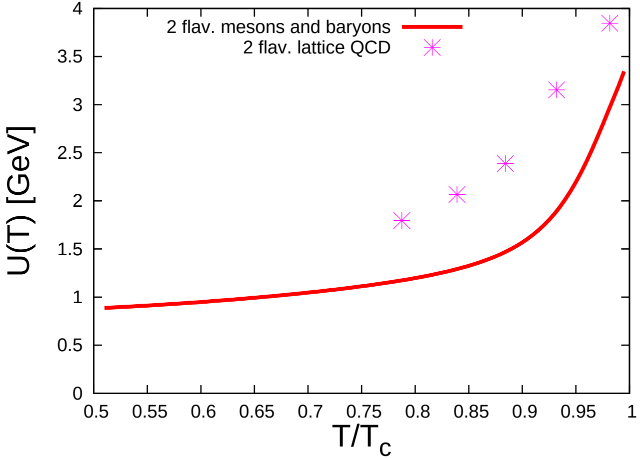

Figure: The calculated internal energy $U(T)$ (full drawn curve) of two mesons and two baryons as a function of T/T_c with $T_c = 200\,\mathrm{MeV}$. The stars show the lattice data (O. Kaczmarek and F. Zantow, '05).

Conclusions

• Gluon-chain model in quenched SU(N_c) QCD below T_c $\Rightarrow$ $\sigma(T) \sim \sqrt{T_c - T}$ at $T \to T_c$; a correct estimate for T_c at $N_c = 3$.

• Unquenched case below T_c: the canonical partition function of two heavy-light mesons and two heavy-light-light baryons, which are formed after the string breaking, with $\sigma(T)$ for $N_f = 2 \Rightarrow$ entropy and internal energy reproduce well the corresponding lattice data for the static $Q\bar{Q}$-pair.

Outlook

To calculate $S(T)$ and $U(T)$ at $T > T_c$.

Model: thermodynamic perturbation theory in the Debye-screened color Coulomb potential of the $Q\bar{Q}$-pair + the constraint that the particle density should vanish at $T = T_c$.
The non-Abelian interaction energy of a plasma constituent a with the $Q\bar{Q}$ pair:

$$\mathcal{V}(\mathbf{r}) = C^{aQ}\mathcal{U}(|\mathbf{r}-\mathbf{x}_Q|) + C^{a\bar{Q}}\mathcal{U}(|\mathbf{r}-\mathbf{x}_{\bar{Q}}|),$$

where C^{aQ} and $C^{a\bar{Q}}$ are the corresponding products of SU(3) generators, and the interaction potential $\mathcal{U}(\mathbf{r})$ is a screened gluon exchange between the colored sources:

$$\mathcal{U}(\mathbf{r}) = 4\pi \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{\alpha_s(k,T)}{k^2+m_D^2} \mathrm{e}^{i\mathbf{kr}}.$$

Outlook

The Debye mass is

$$m_D = \sqrt{\frac{N_c}{3} + \frac{N_f}{6}}\, gT,$$

where $g \simeq 2.5$ according to the selfconsistency equation $g = \sqrt{4\pi\alpha_s(m_D, T)}$ (J. Braun and H.-J. Pirner, '06). The running coupling $\alpha_s(k, T)$ increases for small momenta k, has a maximum at $k \simeq T$ and finally decreases at $k \to \infty$.

The perturbative expansion of the free energy

$$F = F_0 + F_{Q\bar{Q}} + \langle \mathcal{V}(\mathbf{r}_1, \ldots, \mathbf{r}_N) \rangle - \frac{1}{2T} \left(\langle \mathcal{V}^2(\mathbf{r}_1, \ldots, \mathbf{r}_N) \rangle - \langle \mathcal{V}(\mathbf{r}_1, \ldots, \mathbf{r}_N) \rangle^2 \right),$$

where

$$\langle \mathcal{O}(\mathbf{r}_1, \ldots, \mathbf{r}_N) \rangle = \mathrm{Tr}_{\mathrm{singlet}} \left(\prod_{i=1}^{N} \int \frac{d^3\mathbf{p}_i d^3\mathbf{r}_i}{(2\pi)^3} \right) \mathcal{O}(\mathbf{r}_1, \ldots, \mathbf{r}_N) \mathrm{e}^{\beta(F_0 - E_0)},$$

$E_0 = \sum_{i=1}^{N} \sqrt{\mathbf{p}_i^2 + m_i^2}$, $N = \sum_a N_a$.

Outlook

- The masses: $m_g = m_D/\sqrt{2}$,

$$m_q^2 = m_{\bar{q}}^2 = m_0^2 + 2gT\sqrt{\frac{N_c^2-1}{16N_c}}\left(m_0 + gT\sqrt{\frac{N_c^2-1}{16N_c}}\right)$$

(A. Peshier, B. Kämpfer, and G. Soff, 2000), where $m_0 = 30\,\mathrm{MeV}$ is the current quark mass.

- To compare with the lattice results, we leave out the contribution F_0 of the quark-gluon plasma without the static $Q\bar{Q}$-pair.

Estimating the influence on $S(T)$ and $U(T)$ of color-octet $q\bar{q}$ bound states, interacting with the $Q\bar{Q}$-pair, by assuming that all quarks and antiquarks are bound in such states with the mass $2m_q$.

Outlook

- The $Q\bar{Q}$-interaction,

$$F_{Q\bar{Q}}(R,T) = -\frac{4}{3}\alpha_s \frac{\mathrm{e}^{-m_D R}}{R}, \quad \mathbf{R} \equiv \mathbf{x}_Q - \mathbf{x}_{\bar{Q}},$$

can be neglected because it is of the order of a few MeV at $R \simeq 1.5\,\mathrm{fm}$.

- The term $\langle \mathcal{V}(\mathbf{r}_1, \ldots, \mathbf{r}_N) \rangle$ vanishes due to color neutrality of the plasma.

- The term

$$F_2 = -\frac{1}{2T} \langle \mathcal{V}^2(\mathbf{r}_1, \ldots, \mathbf{r}_N) \rangle$$

is due to two interactions of $q\bar{q}$ and g with either Q or $\bar{Q}$, or with both of them:

$$F_2 = -\frac{n_g^{\mathrm{eff}} + n_{q\bar{q}}^{\mathrm{eff}}}{\pi T} \int d^3\mathbf{q} \frac{\alpha_s(q,T)^2}{(q^2 + m_D^2)^2} \left(1 - \mathrm{e}^{i\mathbf{qR}}\right).$$

Outlook

In order to get the overall coefficient in the last formula, one should:

- collect the color structure of a diagram where either g or $q\bar{q}$ exchanges by two gluons with Q or/and $\bar{Q}$;

- project this structure onto the $Q\bar{Q}$-singlet state by the projection operator $(P^{Q\bar{Q}}_{\rm singlet})_{ij,kl} = \frac{1}{9}\delta_{ij}\delta_{kl} + \frac{2}{3}t^a_{ij}(t^a_{kl})^T$.

Outlook

Calculating the *effective* densities $n^{\rm eff}_{g,q\bar{q}}$ through the free ones

$$n_g = 12T\frac{m_g^2}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n}K_2\left(\frac{m_g}{T}n\right), \; n_{q\bar{q}} = 32T\frac{N_f^2 m_q^2}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n}K_2\left(\frac{2m_q}{T}n\right).$$

Due to confinement, these densities vanish at $T = T_c \Rightarrow$

$$n_a^{\rm eff}(T) = h(T)n_a \;\; \text{with} \;\; h(T) = 1 - \exp\left(-\frac{T-T_c}{\lambda}\right).$$

Fixing the value of λ by comparing

$$P_{\rm eff}(T) \simeq h(T)^{4/3}\cdot\frac{T}{V}\ln\mathcal{Z}_{\rm grand}$$

with the corresponding lattice data (F. Karsch, E. Lärmann, A. Peikert, 2000) $\Rightarrow$

$$\lambda = 80\,{\rm MeV}.$$

Outlook

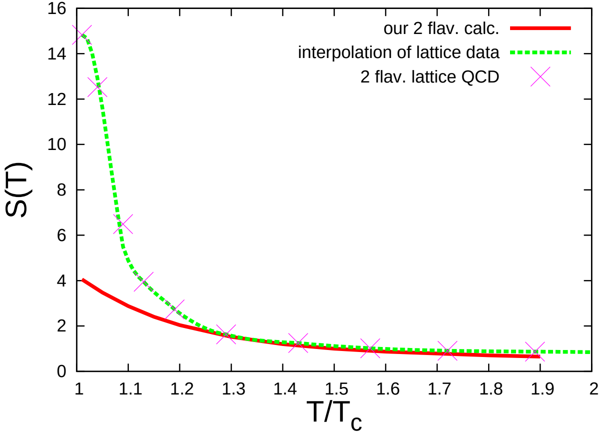

Figure: The calculated entropy $S(T)$ (full drawn curve) of a $Q\bar{Q}$ pair, interacting with $q\bar{q}$ bound states and gluons, as a function of T/T_c, with $T_c = 200$ MeV for two light flavors, at $R = 1.5$ fm. The dashed curve interpolates the lattice data (O. Kaczmarek, P. Petreczky, F. Zantow, '05).

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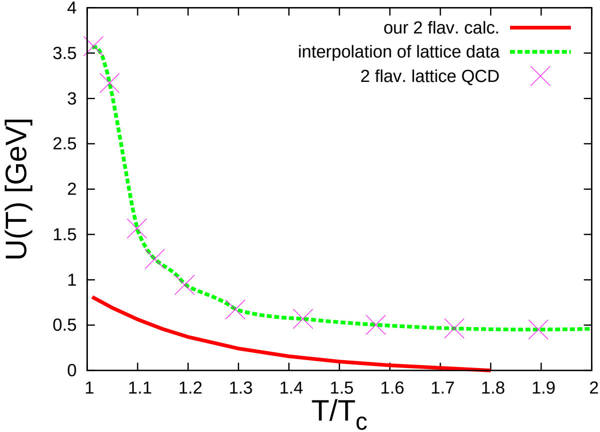

Figure: The calculated internal energy $U(T)$ in GeV (full drawn curve) of a $Q\bar{Q}$ pair, interacting with $q\bar{q}$ bound states and gluons, as a function of T/T_c, with $T_c = 200$ MeV for two light flavors, at $R = 1.5$ fm. The dashed curve interpolates the lattice data.

Outlook

$S(T)$ and $U(T)$ will hopefully become closer to the lattice results (especially at $T_c < T < 1.3T_c$) when the renormalization of Polyakov loops is taken into account.

REF: D.A., S. Domdey, H.-J. Pirner, Nucl. Phys. A 789 (2007) 357.