Quarkonium correlators and potential models

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Summary

1. Introduction
2. Parameterization of lattice free energy
3. Heavy quarks bound states above T_c
4. Lattice correlators and spectral functions
5. Calculated spectral functions and correlators
6. Conclusions

Introduction

• Main focus: the properties of heavy quarkonia above critical temperatures: which ones and how long they survive dissociation.

• Interest in obtaining temperature dependent $Q\overline{Q}$ potentials, fitted on lattice free energy, to handle these properties within QM approach

• Main goal: consistently reproduce lattice results for spectral functions and mesonic correlators.

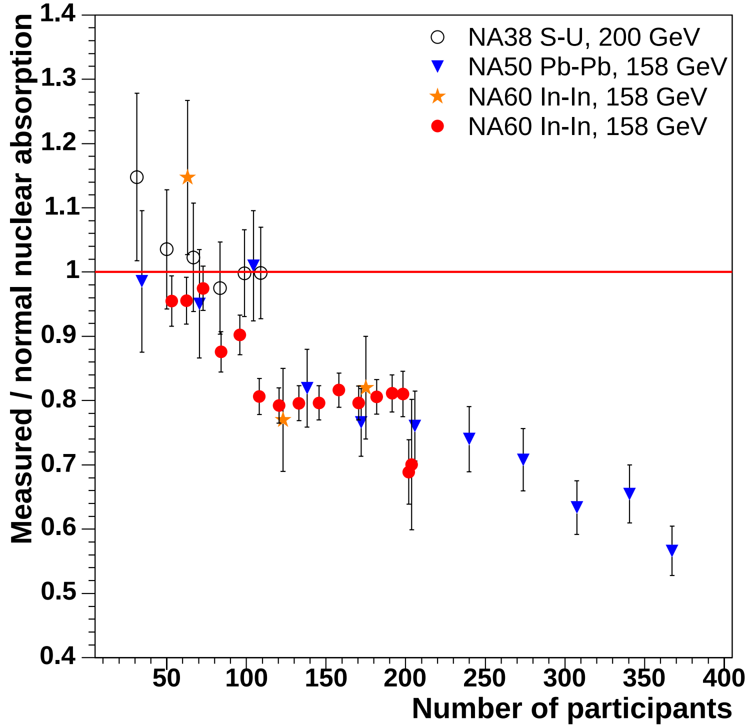
• Hints for sequential suppression pattern as observed at SPS (NA50, NA60, J/Ψ produced also from decays of excited charmonium states): different $c\bar{c}$ states dissociate at different temperatures.

$\Rightarrow$ But:

• Melting of $c\bar{c}$ states above T_c is not guaranteed: screened potential can still support bound states.

• Recombination can be not negligible at RHIC and LHC

• Hence the need for a precise study of heavy $Q\overline{Q}$ states



Sequential suppression pattern: Ψ' (10%), χ_c (30%), J/Ψ (60%).

NA60 and previous SPS data (NA38 and NA50) (R. Arnaldi talk at QM2005)

Heavy Quark Free Energy on the lattice

Color singlet free energy of a (infinitely) heavy $Q\overline{Q}$ pair at a distance $r = |\boldsymbol{x}|$ in a thermal bath of gluons and N_f light dynamical quarks

$\Rightarrow$ Limiting behaviour:

- Very short distance ($r \ll 1/T$): *perturbative one-gluon exchange.*

$$F_1(r,T) \underset{rT \ll 1}{\sim} -\frac{4}{3}\frac{\alpha(r)}{r}$$

Thermal effects are negligible, the running of the coupling is set by the $Q\overline{Q}$ distance.

- High temperature and ($T \gg T_c$) large distance ($rT \gg 1$): exchange of a *resummed electrostatic gluon*.

$$F_1(r,T) \underset{rT \gg 1}{\sim} -\frac{4}{3}\frac{\alpha(T)}{r} e^{-m_D(T)r} + F_1(r=\infty,T)$$

$\alpha = \alpha(T)$ runs with the temperature, $m_D(T)$ is the Debye screening mass (dressing of the electrostatic gluon).

$\Rightarrow$ In both limits the running of the coupling is given by a *Renormalization Group Equation* (RGE) $\rightarrow \alpha = \alpha(\mu)$ with

- $\mu \sim 1/r$ (short distance);
- $\mu \sim T$ (high temperature and large distance).

$\Rightarrow$ Functional form of the free energy (Ansatz I):

$$F_1(r,T) = -\frac{4}{3}\frac{\alpha(r,T)}{r}e^{-M(T)r} + C(T).$$

$\Rightarrow$ Running of the coupling (Ansatz II):

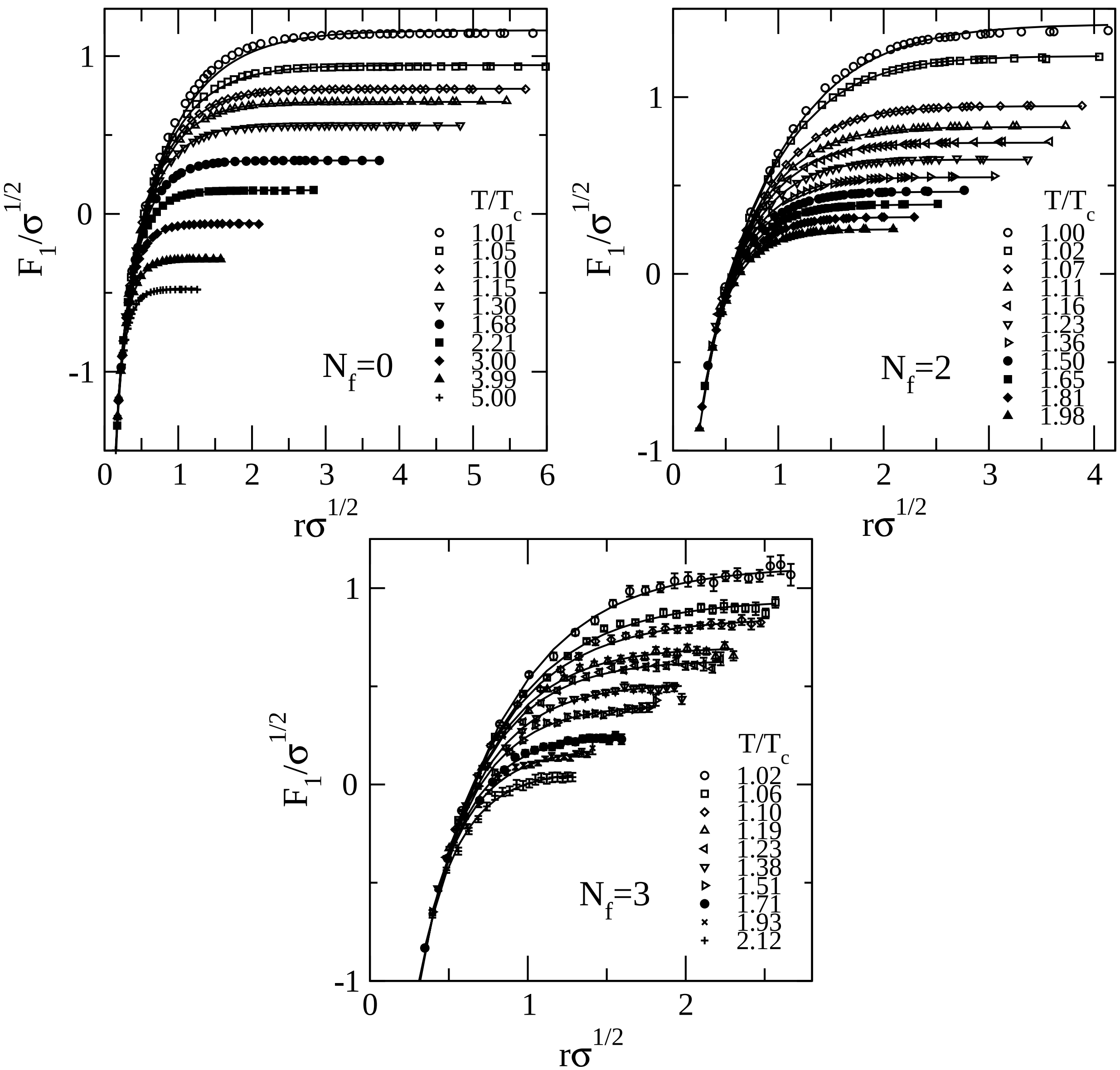
$$\alpha(r,T) = \alpha(\mu = c_r/r + c_t T)$$

$\Rightarrow$ $\boxed{\text{Our parameterization}}$

$$\frac{F_1}{\sqrt{\sigma}} = -\frac{4}{3}\frac{\alpha(\tilde{\mu})}{x}e^{-a_3 x} + a_0 \quad \text{with} \quad \tilde{\mu} = \frac{a_1}{x} + a_2 \ ,$$

with $x = r\sqrt{\sigma}$ and $\alpha(\tilde{\mu})$ solution of RGE with 2-loop beta function.

$\Rightarrow$ Lattice data from O. Kaczmarek et al.,; P. Petreczky and K. Petrov.


F1/σ^1/2
rσ^1/2
T/Tc
1.01
1.05
1.10
1.15
1.30
1.68
2.21
3.00
3.99
5.00
Nf=0
1.00
1.02
1.07
1.11
1.16
1.23
1.36
1.50
1.65
1.81
1.98
Nf=2
1.02
1.06
1.10
1.19
1.23
1.38
1.51
1.71
1.93
2.12
Nf=3

$Q\overline{Q}$ Bound states in the QGP

• Extract the internal energy (=potential energy for infinitely heavy sources). From $F = U - TS$:

$$U_1 = -T^2 \frac{\partial(F_1/T)}{\partial T} \ .$$

• $Q\overline{Q}$ Effective potential

• Necessary to eliminate, in U_1, the contribution of gluons and dynamical light quarks (C.Y. Wong, Phys.Rev. C **72**, 034906 (2005)).

$$\begin{aligned} V_1(r,T) &\equiv U_1^{Q\bar{Q}}(r,T) - U_1^{Q\bar{Q}}(r \to \infty, T) \\ &= f_F(T)\left[F_1(r,T) - F_1(r\to\infty,T)\right] + f_U(T)\left[U_1(r,T) - U_1(r\to\infty,T)\right], \end{aligned}$$

where

$$f_F(T) = \frac{3}{3+a(T)}, \qquad f_U(T) = \frac{a(T)}{3+a(T)}$$

and $a(T) = 3p/\epsilon$ (pressure and energy density of quarks and gluons).

• Resulting potential more similar to free energy and less attractive. Higher (than in PDG) quark masses are employed, to get agreement with lattice data.

• Schrödinger equation

$$\left[-\frac{\boldsymbol{\nabla}^2}{2\mu} + V_1(r,T)\right]\psi(\boldsymbol{r},T) = \varepsilon(T)\psi(\boldsymbol{r},T)\ ,$$

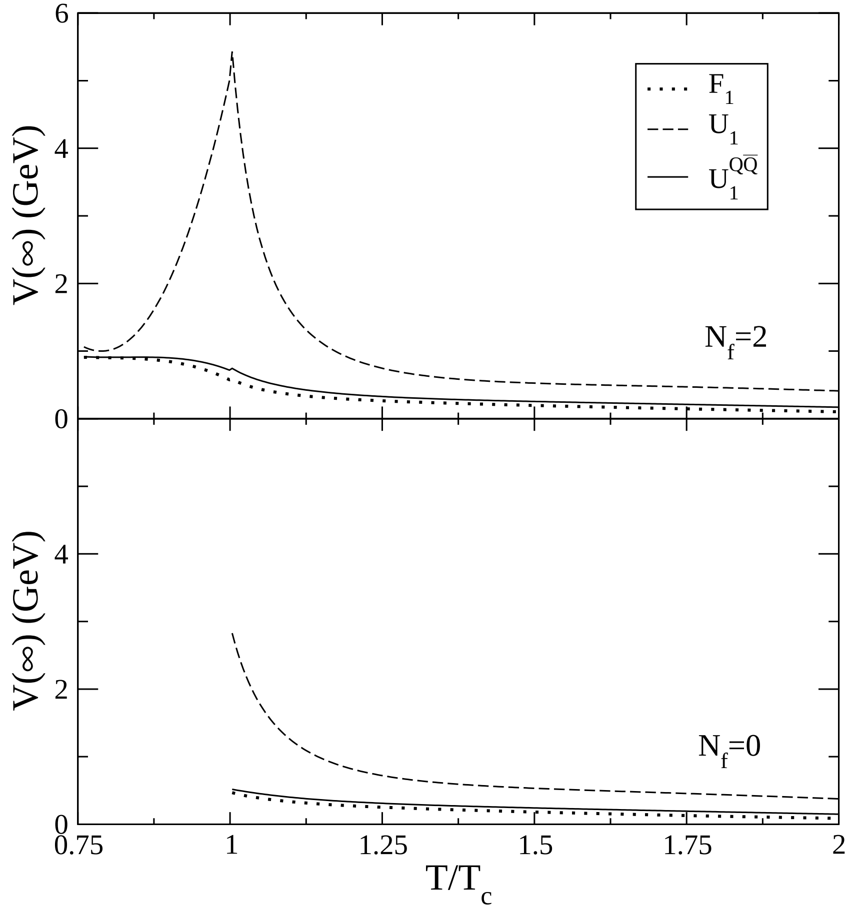
(μ is the *reduced mass*);

• Quarkonium mass

$$M(T) = 2m_{c(b)} + \varepsilon(T) + U_1(r \to \infty, T)\ ;$$

$\Rightarrow$ Quark masses from the PDG: $1.15 < m_c < 1.35$, $4.1 < m_b < 4.4$ GeV. Here used $1.4 < m_c < 1.6$, $4.3 < m_b < 4.7$ GeV.

asymptotic values of the lattice free energy and QQbar potential



Dissociation temperatures

	$N_f = 0$		$N_f = 2$	
	$m_c = 1.4$ GeV	$m_c = 1.6$ GeV	$m_c = 1.4$ GeV	$m_c = 1.6$ GeV
$J/\psi, \eta_c$	1.40	1.52	1.45	1.59
χ_c	< 1	< 1	1.00	1.00
ψ'	< 1	< 1	0.98	0.99
	$m_b = 4.3$ GeV	$m_b = 4.7$ GeV	$m_b = 4.3$ GeV	$m_b = 4.7$ GeV
Υ, η_b	2.96 [4.5]	3.18	3.9(*) [6.7(*)]	4.4(*)
χ_b	1.13 [1.55]	1.15	1.15 [1.63]	1.17
Υ'	1.12 [1.40]	1.14	1.13 [1.43]	1.15

Thermal meson correlation functions

Thermal meson propagator along the (imaginary) temporal direction:

$$G_M(-i\tau, \boldsymbol{p}=0, T) = \int_0^{+\infty} d\omega\ \sigma_M(\omega, \boldsymbol{p}=0) K(\tau, \omega, T)$$

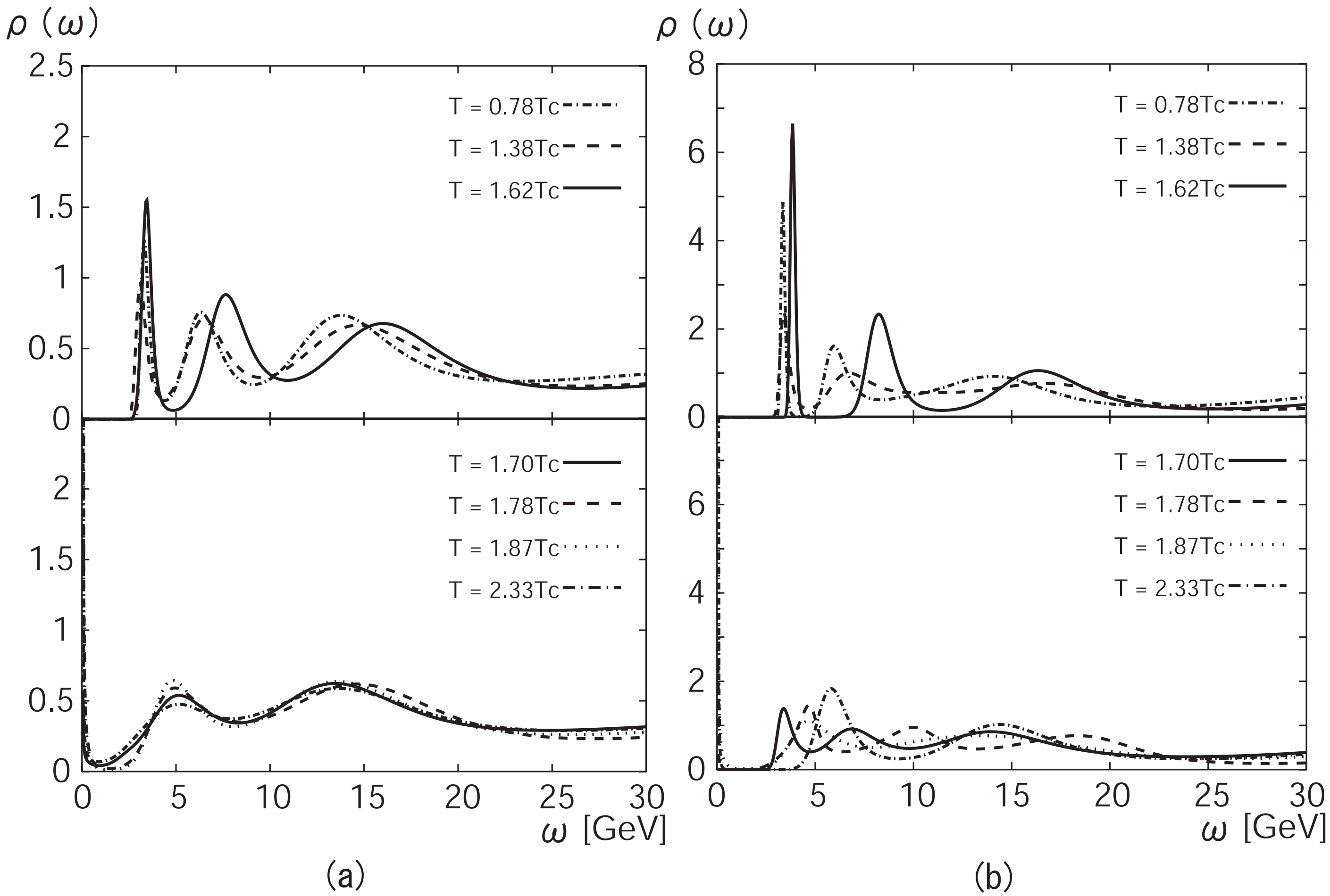
where

$$K(\tau, \omega, T) = \frac{\cosh(\omega(\tau - \beta/2))}{\sinh(\omega\beta/2)}$$

$$\sigma_M(\omega, \boldsymbol{p}) = \frac{1}{\pi} \mathrm{Im}\, \chi_M(\omega + i\eta, \boldsymbol{p})$$

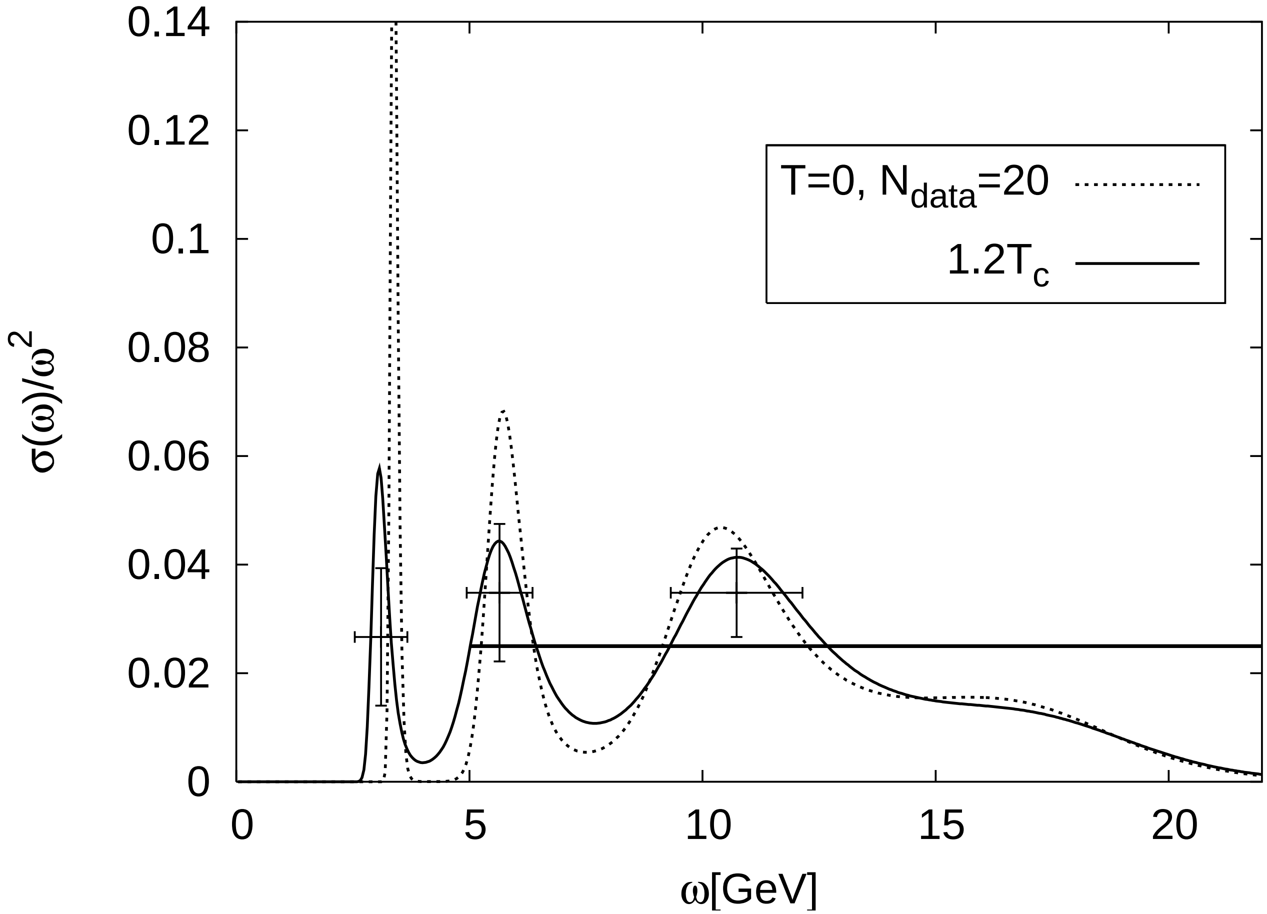
- σ_M is the thermal meson spectral function.
- $G_M(-i\tau, \boldsymbol{p}=\mathbf{0})$ is measured on the lattice for a finite set of values of τ (~ 20).
- $\sigma_M(\omega, \mathbf{0})$ has to be reconstructed (Maximum Entropy Method usually employed).

Lattice Meson Spectral Functions (T. Hatsuda, hep-lat/0509306)

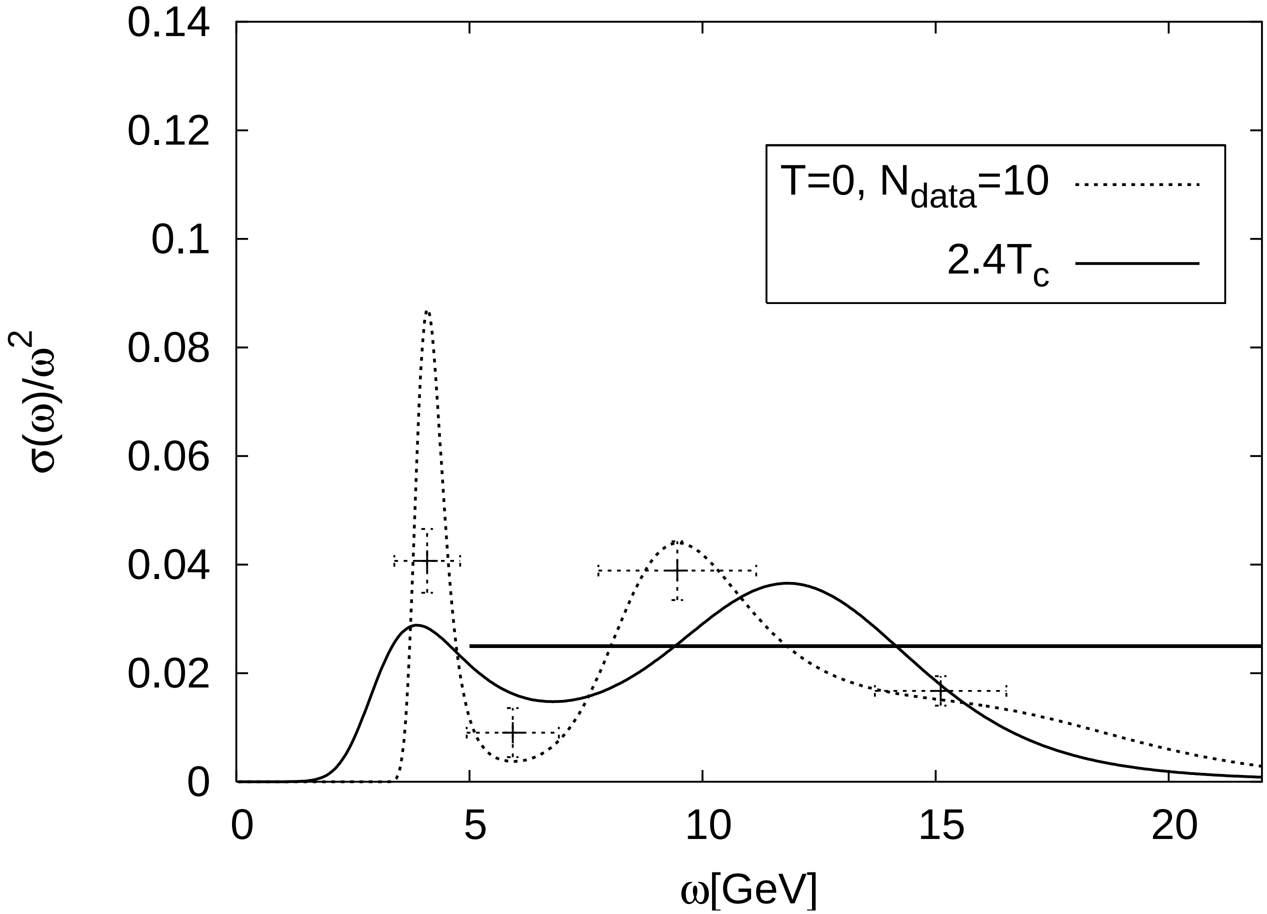


- J/Ψ (a) and η_c (b) dimensionless spectral functions.

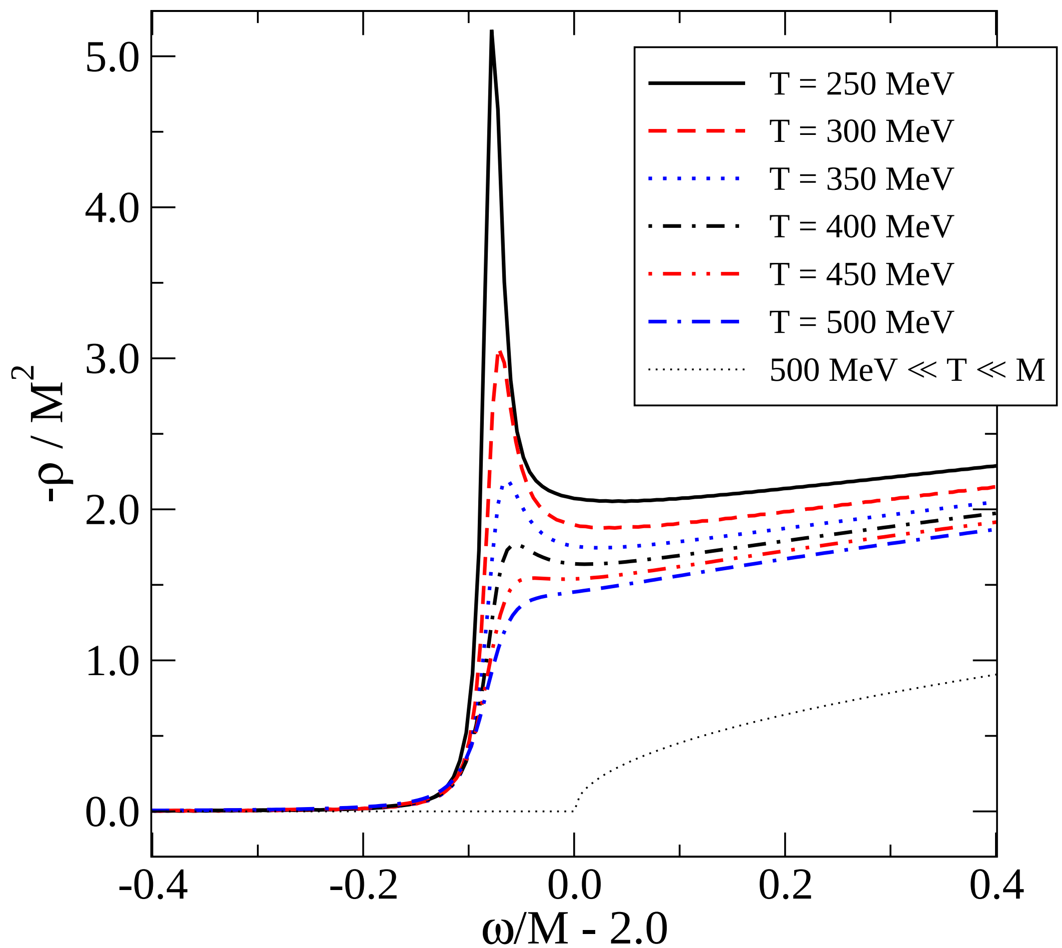
Vector Meson Spectral Functions at $T = 1.2T_c$ (A. Jacovak et al., PRD75, 014506 (2007))



Vector Meson Spectral Functions at $T = 2.4T_c$ (A. Jacovak et al., PRD75, 014506 (2007))



Vector Meson Spectral Functions at several T (Laine)



Spectral functions from potential model

With eigenfunctions and eigenvalues of Schroedinger equation we evaluate the spectral function as the imaginary part of the $Q\bar{Q}$ propagator:

$$\begin{aligned}\sigma_M(\omega, T) &= \frac{1}{\pi}\mathrm{Im}G_M(\omega) = \sum_n |\langle 0|j_M|n\rangle|^2\,\delta(\omega - E_n) \\ &= \sum_n F^2_{M,n}\delta(\omega - M_n) + \theta(\omega - s_0)F^2_{M,\omega - s_0}\end{aligned}$$

where $E_n = s_0 + \epsilon_n$, s_0 is the continuum threshold:

$$s_0(T) = 2m + U_1^{Q\overline{Q}}(r \to \infty, T)$$

NB Continuum is taken in Two ways:

- from corresponding solution of Schroedinger equation [consistent with present approach]
- from QCD perturbative calculations

The couplings $F_{M,n} \equiv \langle 0|j_M|n\rangle$ and $F_{M,\epsilon} \equiv \langle 0|j_M|\epsilon\rangle$ are related to the wavefunction in the origin:

- Pseudoscalar and vector channels:

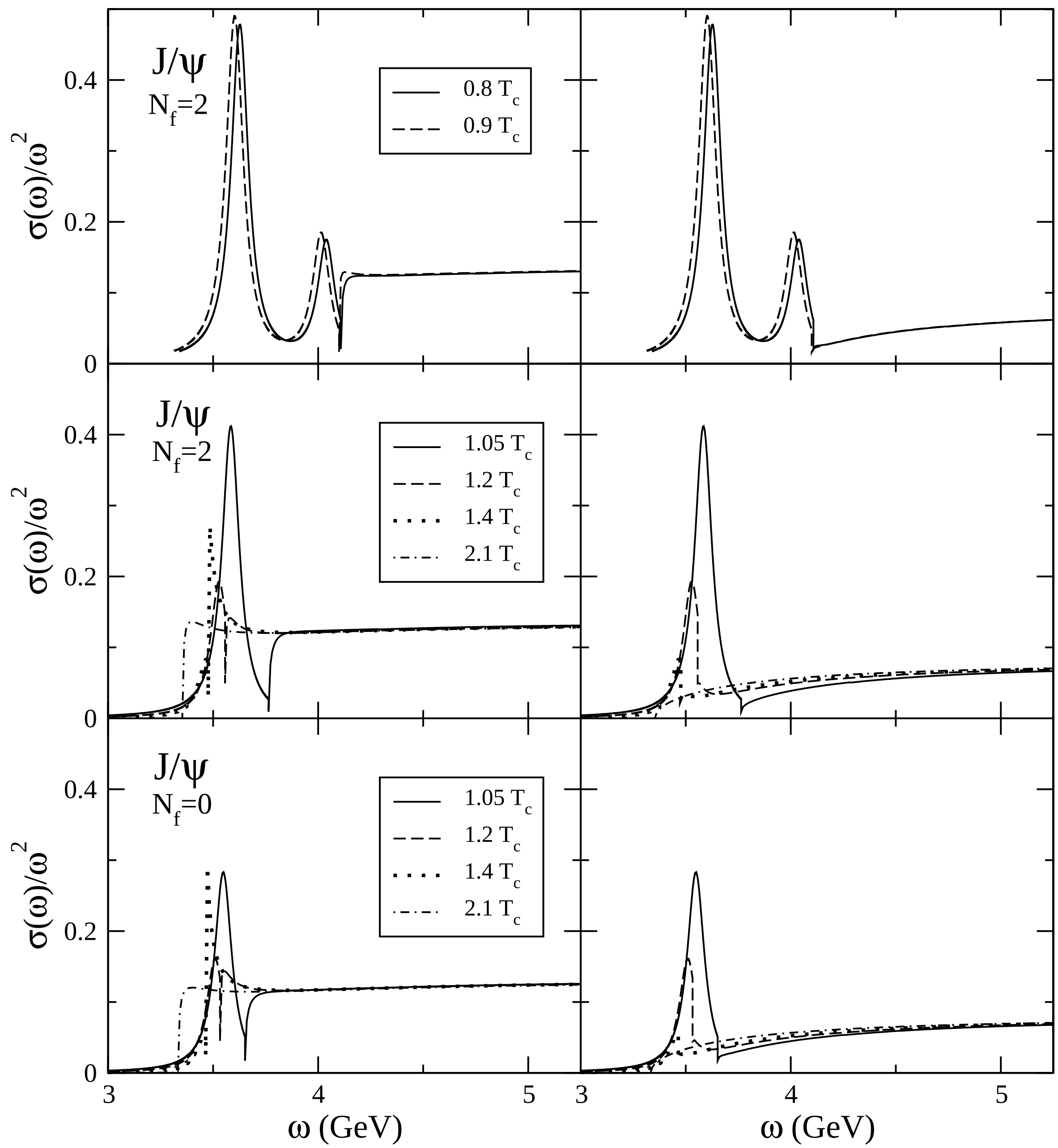
$$F_{PS}^2 = \frac{N_c}{2\pi}|R(0)|^2 \quad \text{and} \quad F_V^2 = \frac{3N_c}{2\pi}|R(0)|^2,$$

- scalar and axial-vector channels:

$$F_S^2 = \frac{9N_c}{2\pi m^2}|R'(0)|^2 \quad \text{and} \quad F_A^2 = \frac{9N_c}{\pi m^2}|R'(0)|^2,$$

For a better comparison with lattice spectral function we include a width Γ_n into the bound state part:

$$\sigma_M(\omega, T) = \sum_n F_{M,n}^2 \frac{1}{\pi} \frac{\Gamma_n/2}{(\omega - M_n)^2 + \Gamma_n^2/4} + \theta(\omega - s_0) F_{M,\omega - s_0}^2,$$



J/ψ
N_f=2
0.8 T_c
0.9 T_c
J/ψ
N_f=2
1.05 T_c
1.2 T_c
1.4 T_c
2.1 T_c
J/ψ
N_f=0
1.05 T_c
1.2 T_c
1.4 T_c
2.1 T_c
σ(ω)/ω^2
ω (GeV)

Correlators from potential model

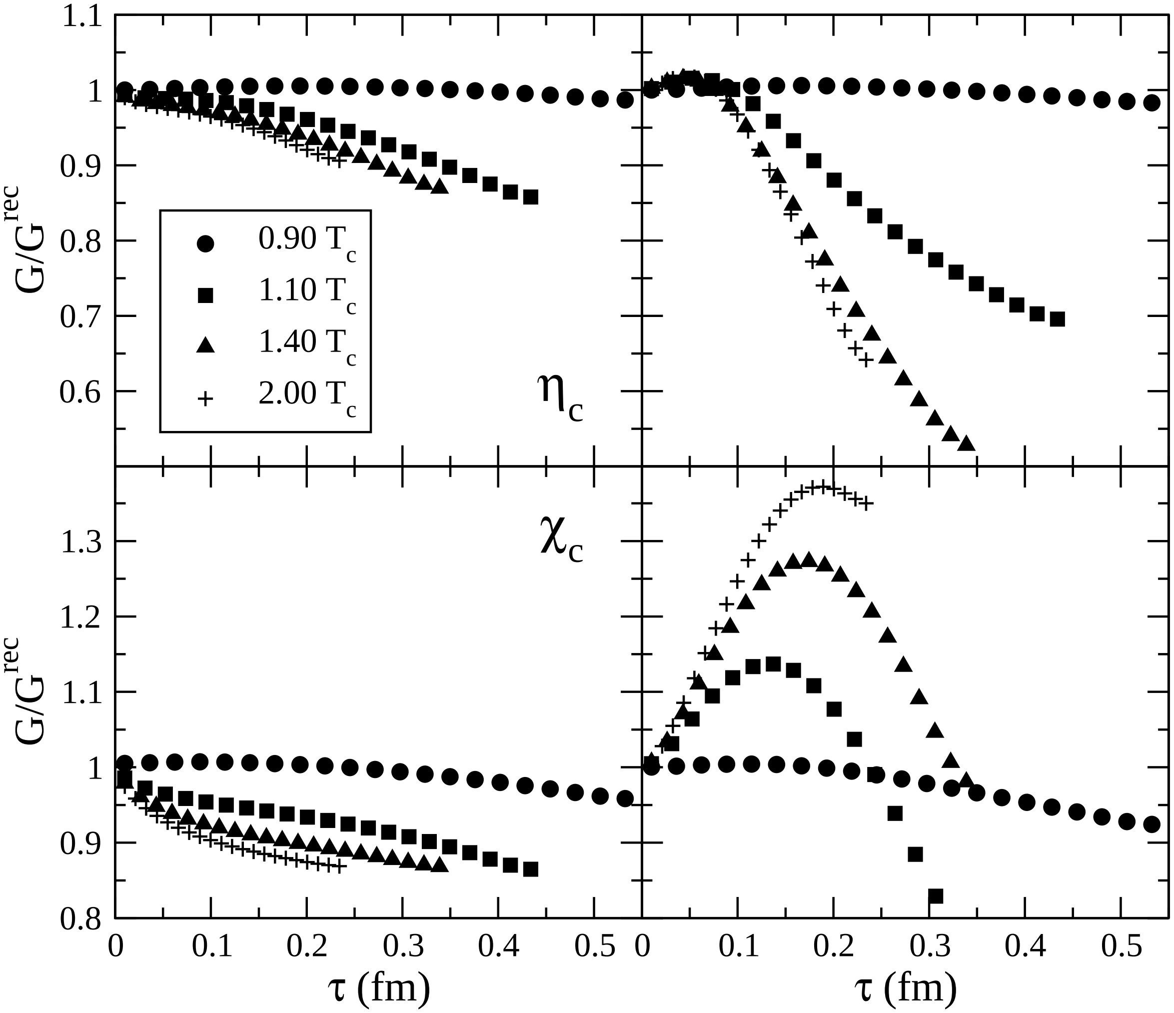
The Euclidean correlators follows from convolution of spectral functions and thermal Kernel:

$$G_M(\tau,T)=\sum_n F^2_{M,n}K(\tau,M_n,T)+\int_0^\infty d\epsilon\, F^2_{M,\epsilon}K(\tau,\epsilon+s_0,T).$$

On the lattice the following ratio is evaluated:

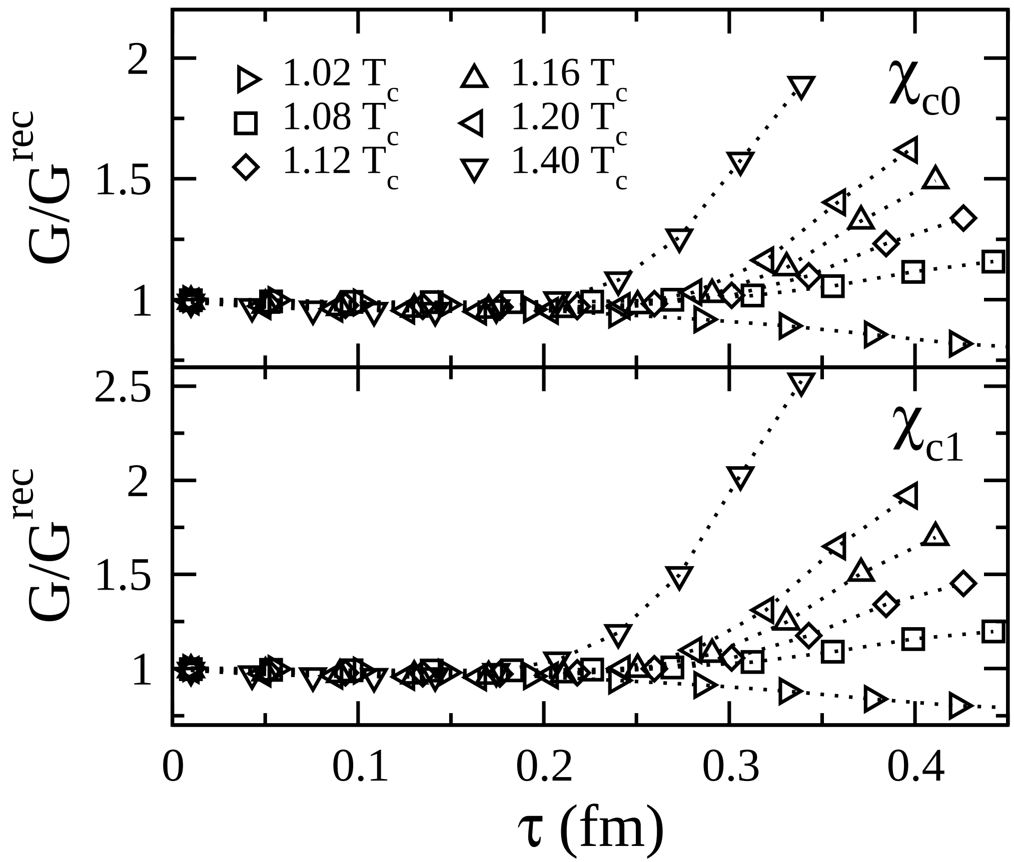
$$R=\frac{G_M(\tau,T)}{G_M^{\mathrm{rec}}(\tau,T,T_<)}=\frac{\int_0^\infty d\omega\,\sigma_M(\omega,T)K(\tau,\omega,T)}{\int_0^\infty d\omega\,\sigma_M(\omega,T_<)K(\tau,\omega,T)}$$

The denominator is calculated using the kernel at $T>T_c$ and the MEM spectral function at some reference temperature $T_<$ (reconstructed correlator).



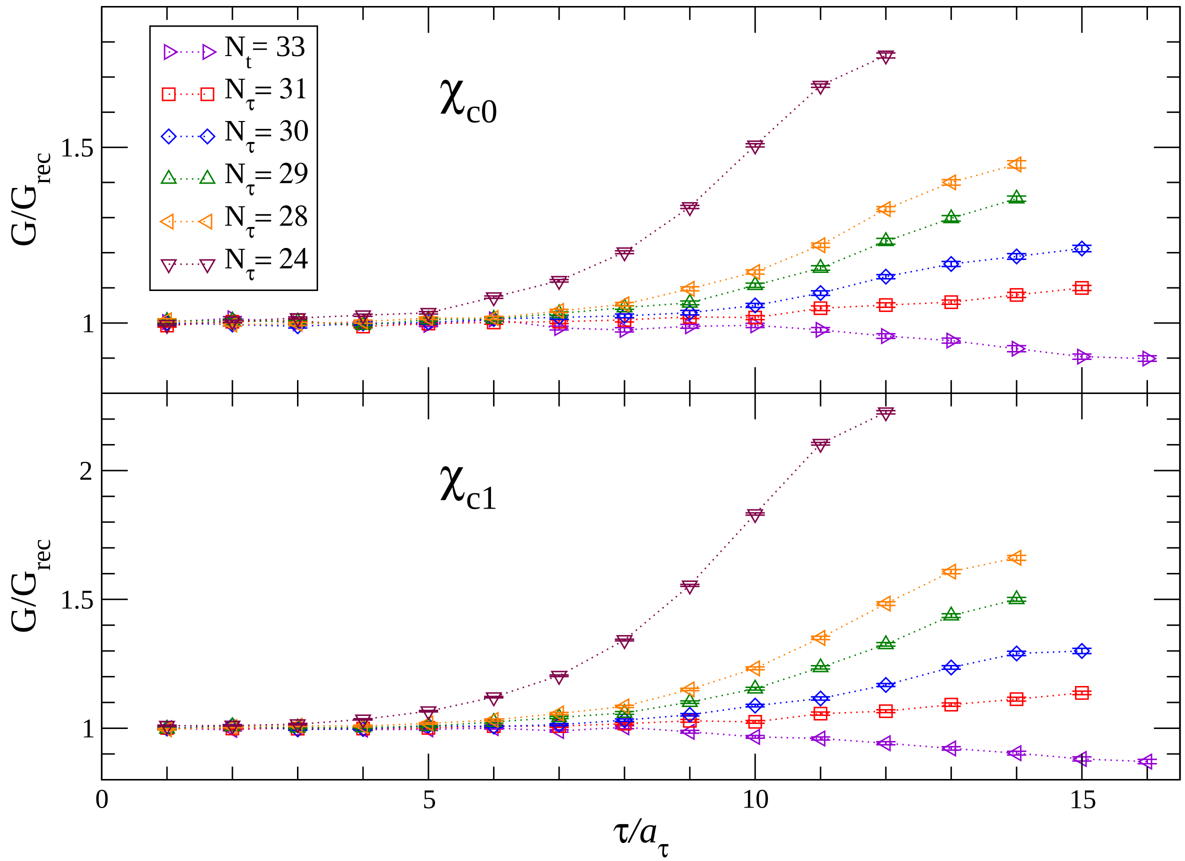
1.1
1
0.9
0.8
0.7
0.6
1.3
1.2
1.1
1
0.9
0.8
G/G^rec
G/G^rec
0.90 T_c
1.10 T_c
1.40 T_c
2.00 T_c
η_c
χ_c
0
0.1
0.2
0.3
0.4
0.5
τ (fm)
τ (fm)

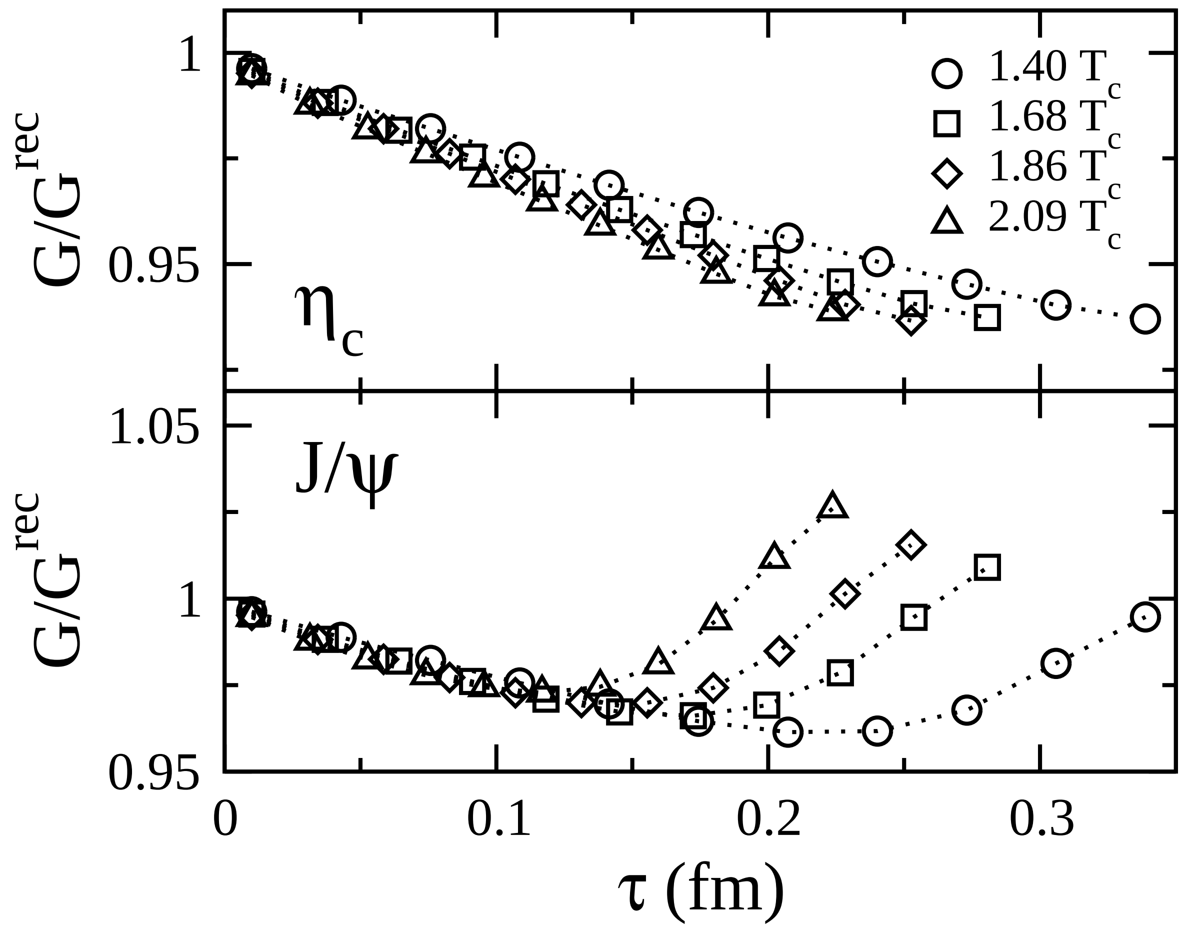
• BUT... by including the effect of zero modes in spectral functions



P-wave ratio of correlators

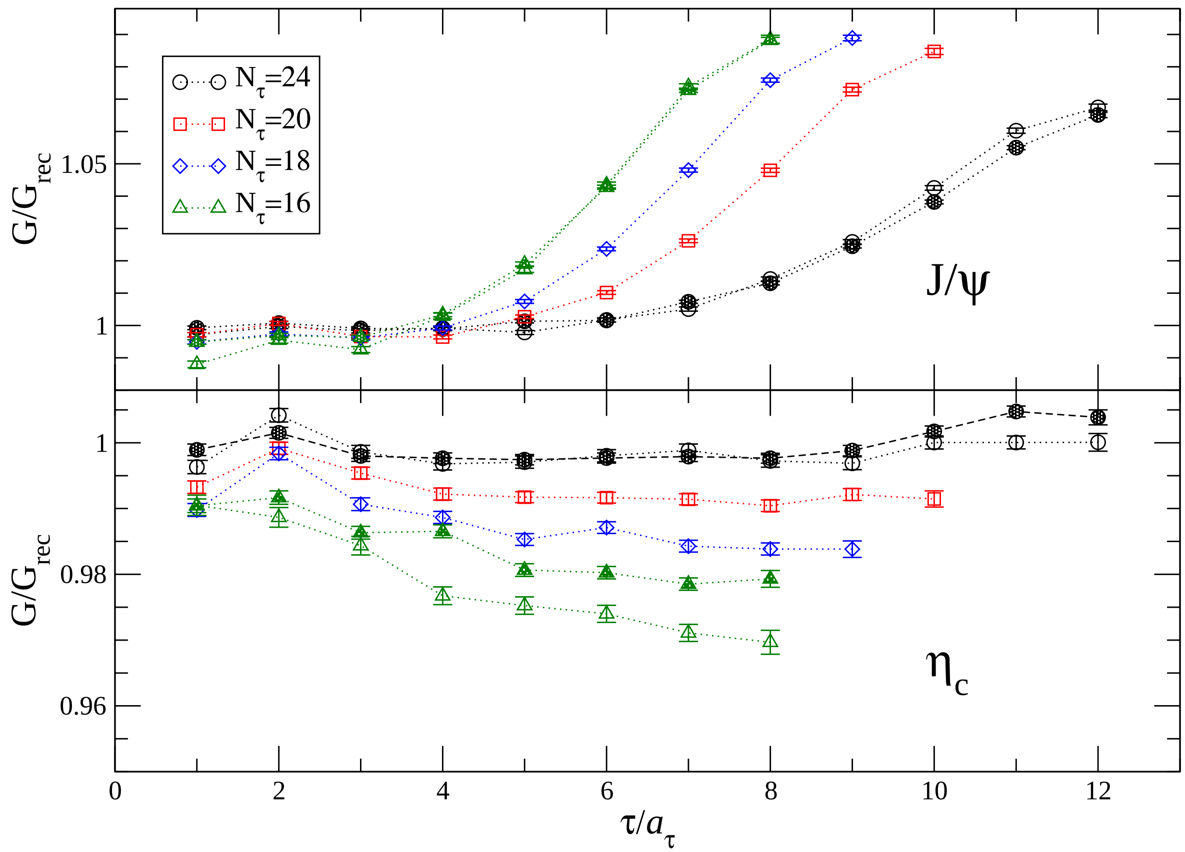
● To be compared with lattice (G.Aarts et al, arXiv: 0705.2198 [hep-lat])

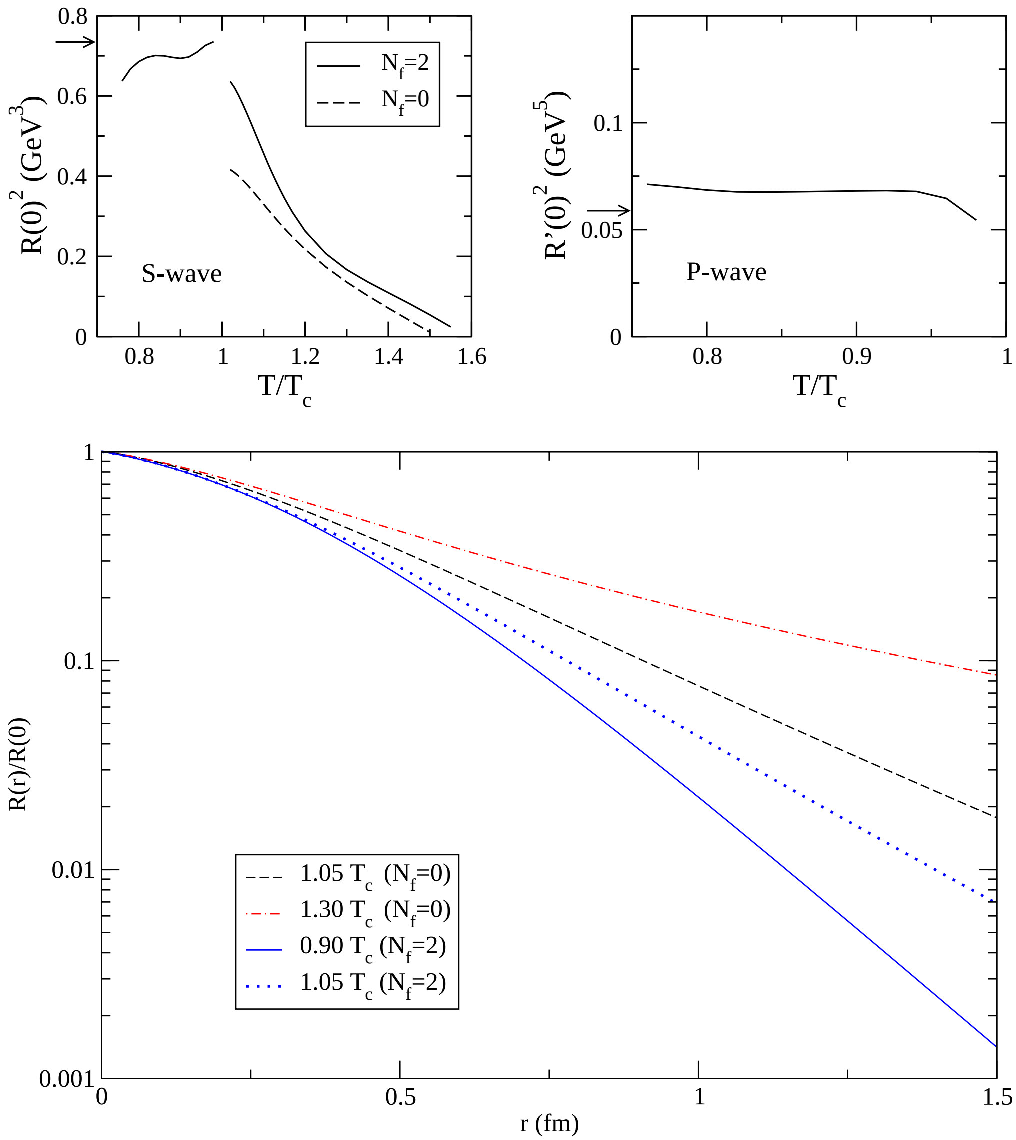




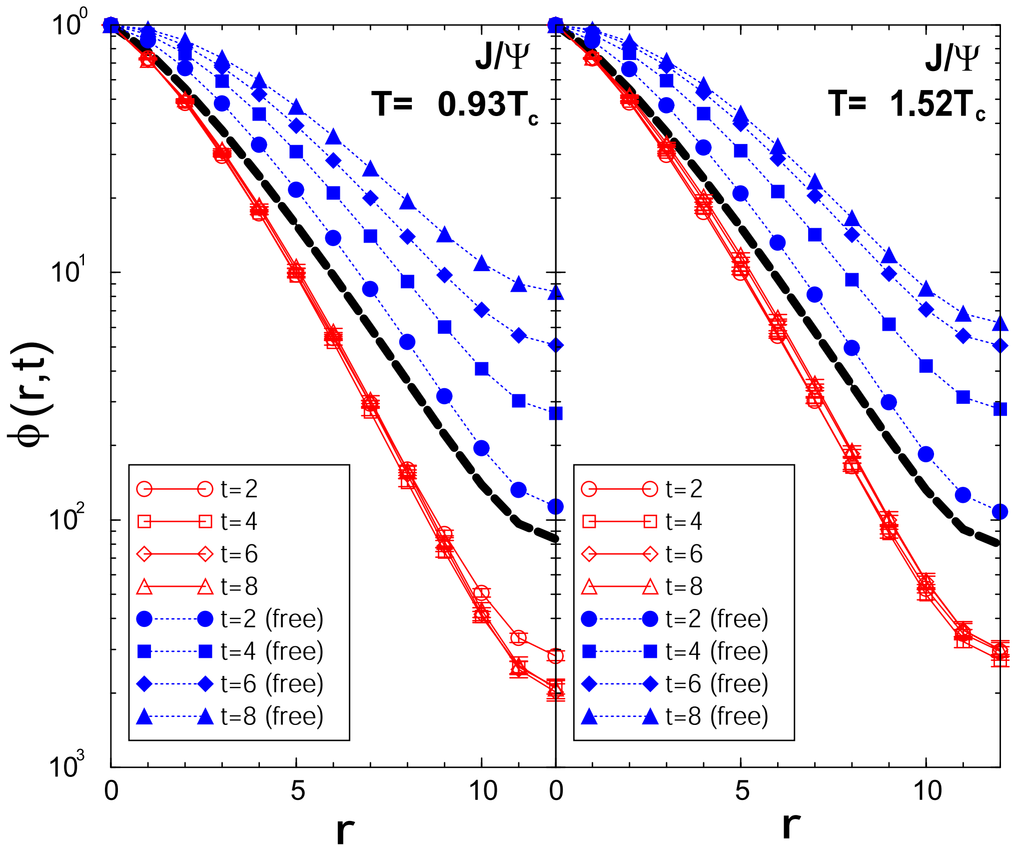
S-wave ratio of correlators

- To be compared with lattice (G.Aarts et al, arXiv: 0705.2198 [hep-lat])





R(0)^2 (GeV^3)
T/T_c
S-wave
N_f=2
N_f=0
R'(0)^2 (GeV^5)
P-wave
R(r)/R(0)
r (fm)
1.05 T_c (N_f=0)
1.30 T_c (N_f=0)
0.90 T_c (N_f=2)
1.05 T_c (N_f=2)



$\Phi = \Psi(r,t)/\Psi(0,t)$ (Umeda et al., hep-lat/0011085)

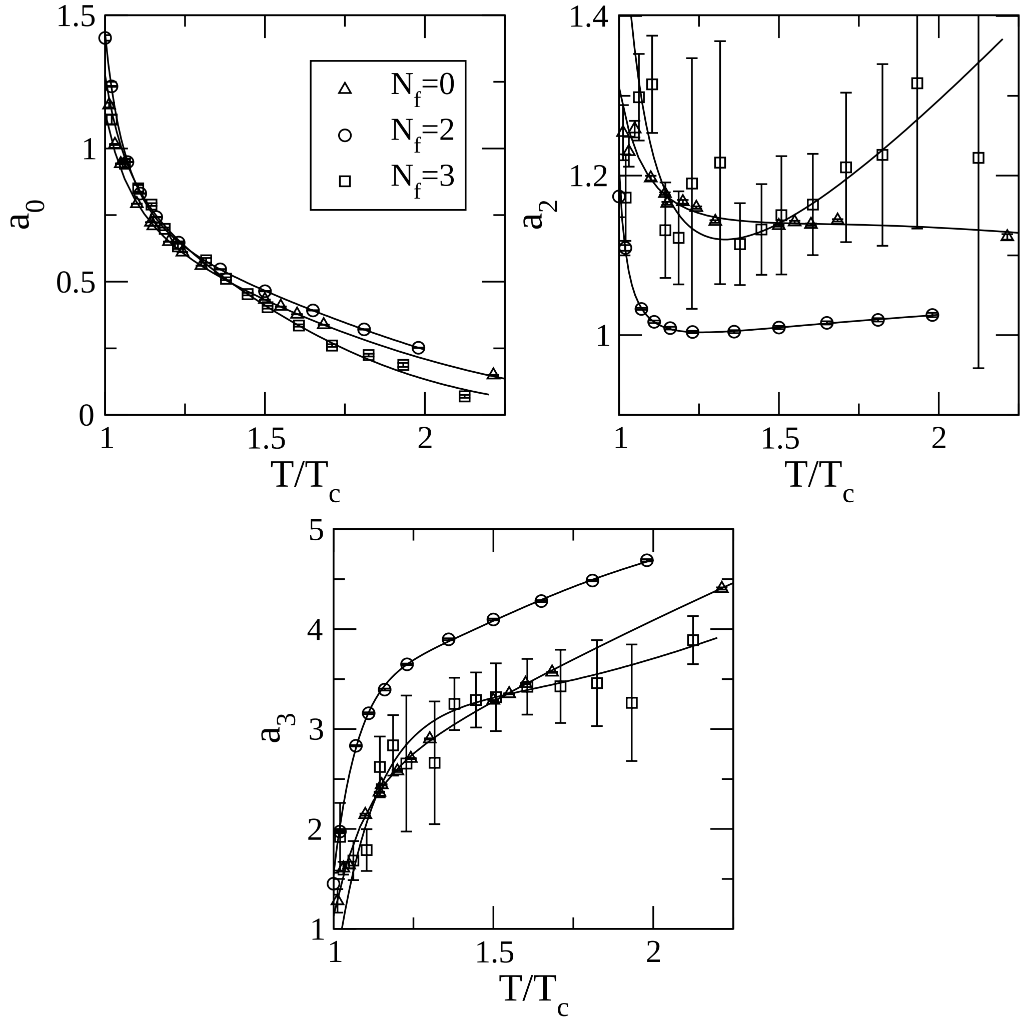
Conclusions

- We obtained a $Q\overline{Q}$ effective temperature dependent potential by fitting lattice data on free energy with $N_f = 0, 2, 3$
- Disentangling the potential energy from $F_1(r, T)$ quite important to get results in agreement with the lattice spectral function findings;
- From the solution of the Schrödinger equation we evaluated bound state energies of several quarkonium states and determinated the dissociation temperatures, in good agreement with the lattice
- Separation of gluon and light quark contributions to the internal energy improved the results attainable from effective potential
- Spectral functions and $Q\overline{Q}$ correlators, $\sigma(\omega) \to G(\tau)$ are in semi-quantitative agreement with corresponding lattice results.
- Open problem remains the behavior with temperature of our PS wave function: it decreases yet in presence of bound state, while the lattice ones are stable.

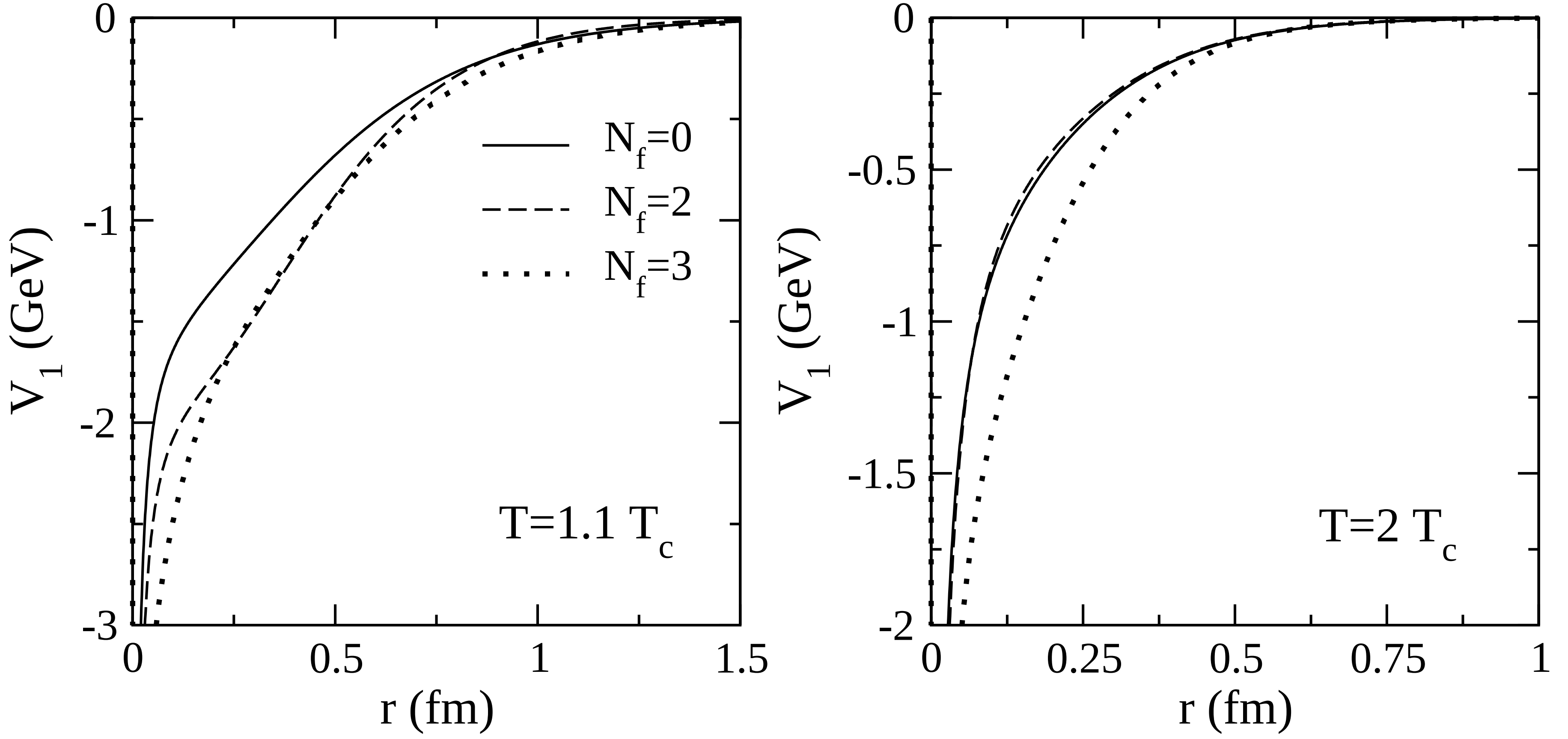
References

- W.M.A., A. Beraudo, A. De Pace, A. Molinari, Phys. Rev. **D72**, 114011 (2005)
- W.M.A., A. Beraudo, A. De Pace, A. Molinari, Phys. Rev. **D75**, 074009 (2007)
- W.M.A., A. Beraudo, A. De Pace, A. Molinari, arXive 0706.2846

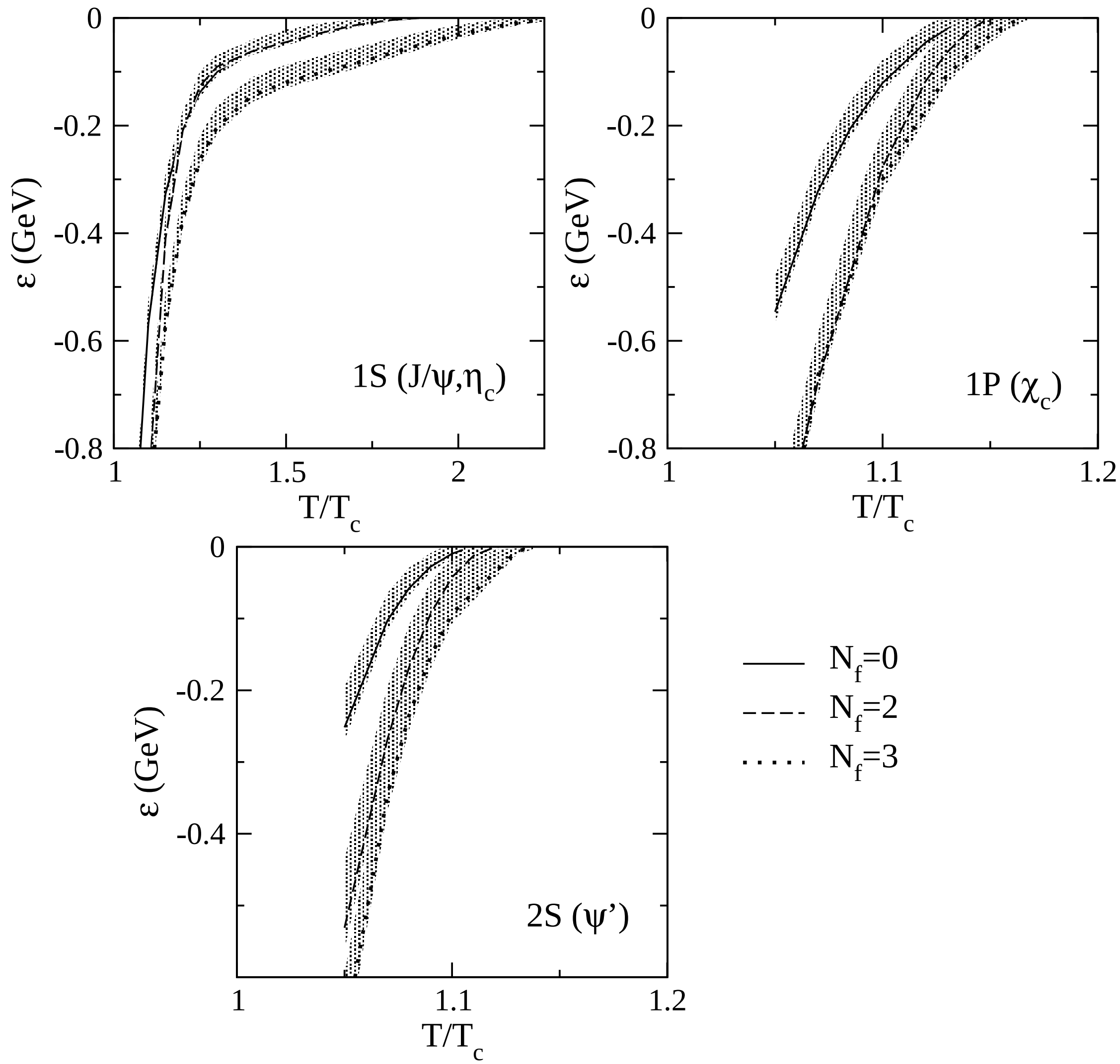
⇒ Parameters of the fit versus T/T_c:

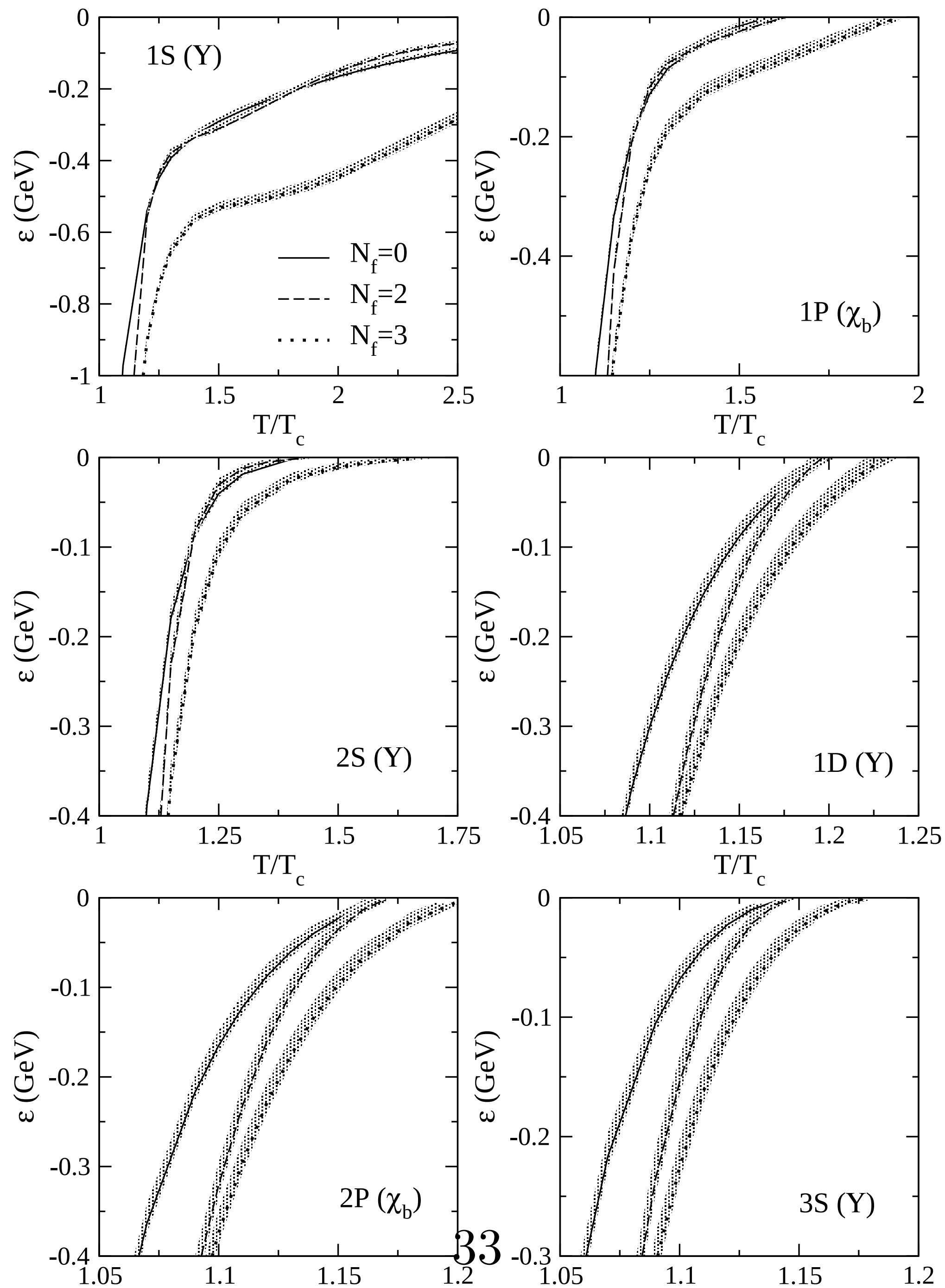


• QQbar potential:



• Binding energies (Charmonium):





1S (Υ)
1P (χ_b)
2S (Υ)
1D (Υ)
2P (χ_b)
3S (Υ)
N_f=0
N_f=2
N_f=3
ε (GeV)
T/T_c