QUARKONIUM CORRELATORS ON THE LATTICE

Gert Aarts

Physics Department, Swansea University

Swansea University
Prifysgol Abertawe

IN-MEDIUM PROPERTIES OF QUARKONIUM

MOTIVATION

quark-gluon plasma and heavy ion collisions:

- fate of heavy quark bound states
- J/ψ suppression: signal for deconfinement?
- strong interactions in the QGP?
- spectral function studies in quenched lattice QCD have shown that J/ψ and η_c may survive up to $T \sim 2T_c$

Asakawa & Hatsuda (2004) Datta, Karsch, Petreczky & Wetzorke (2004)

Umeda, Nomura & Matsufuru (2005) Jakovac, Petreczky, Petrov & Velytsky (2007)

what happens in dynamical QCD?

OUTLINE

- spectral functions from the lattice
- Maximum Entropy Method
- mesons at zero ...
  ... and finite temperature
- dynamical QCD
- reconstructed correlators
- the future: nonzero momentum, quark diffusion
- summary

SPECTRAL FUNCTIONS

FROM THE LATTICE

- meson spectral functions

$$\rho(t, \mathbf{x}) = \langle [J(t, \mathbf{x}), J^\dagger(0, \mathbf{0})] \rangle$$

- euclidean correlators on the lattice

$$G_E(\tau, \mathbf{x}) = \langle J(\tau, \mathbf{x}) J^\dagger(0, \mathbf{0}) \rangle$$

with meson operators

$$J_H(\tau, \mathbf{x}) = \bar{q}(\tau, \mathbf{x}) \Gamma_H q(\tau, \mathbf{x}) \quad \text{with} \quad \Gamma_H = \{1\!\!1, \gamma_5, \gamma^\mu, \gamma^\mu \gamma_5\}$$

- all expectation values in equilibrium $\sim e^{-H/T}$

$G_E(\tau)$ is computed $\Rightarrow$ what is corresponding $\rho(\omega)$?

SPECTRAL FUNCTIONS

FROM THE LATTICE

- dispersion relation

$$G_E(i\omega_n) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\rho(\omega)}{\omega - i\omega_n} \qquad \omega_n = 2\pi n T \ (n \in \boldsymbol{Z})$$

- or

$$G_E(\tau) = \int_0^{\infty} \frac{d\omega}{2\pi} K(\tau,\omega)\rho(\omega)$$

with the kernel

$$K(\tau,\omega) = \frac{\cosh[\omega(\tau - 1/2T)]}{\sinh(\omega/2T)} = [1 + n_B(\omega)]\, e^{-\omega\tau} + n_B(\omega) e^{\omega\tau}$$

LATTICE QCD

AND SPECTRAL FUNCTIONS

program:

- compute lattice correlators $G_E(\tau)$ numerically
- finite amount of information at $0 \leq \tau/a < N_\tau$
- "invert" integral equation

$$G_E(\tau) = \int_0^\infty \frac{d\omega}{2\pi} K(\tau, \omega) \rho(\omega)$$

- e.g. using Maximum Entropy Method (MEM)
  Asakawa & Hatsuda (2000)

MAXIMUM ENTROPY METHOD

RECONSTRUCT $\rho(\omega)$ FROM $G(\tau)$

solve ill-posed inversion problem:

$$G(\tau) = \int_0^\infty \frac{d\omega}{2\pi} K(\tau,\omega)\rho(\omega)$$

$$K(\tau,\omega) = \frac{\cosh[\omega(\tau - 1/2T)]}{\sinh(\omega/2T)}$$

- $G(\tau)$ known at $\mathcal{O}(10)$ data points
- $\rho(\omega)$ needed at $\mathcal{O}(10^3)$ values, $0 < a\omega < a\omega_{\rm max} \sim 5$

  $\omega_{\rm max} \sim 35$ GeV

provide (minimal amount of) prior information:

- positivity
- asymptotic behaviour

ZERO TEMPERATURE SPECTRAL FUNCTIONS

warm-up problem:

meson correlators at zero temperature

- confinement: bound states
- ground state: $G(\tau) \sim \exp(-M\tau) \Leftrightarrow \rho(\omega) \sim \delta(\omega - M)$
- UKQCD and RBC: dynamical QCD with $2+1$ flavours of domain wall fermions, generated on QCDOC

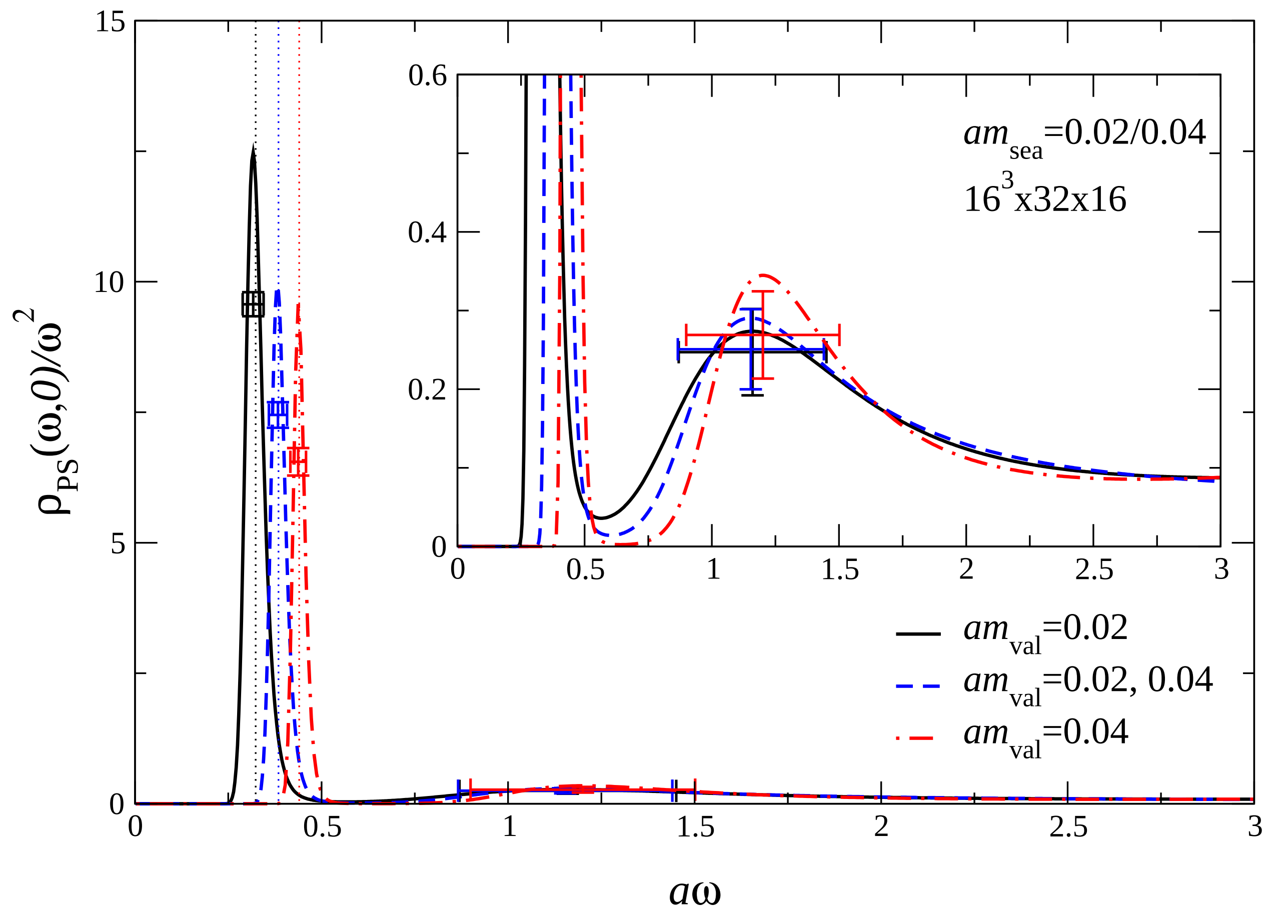
$16^3 \times 32$ with $N_s = 16$ in fifth dimension
$\beta = 2.13$ (Iwasaki action) $a^{-1} \sim 1.6$ GeV

ZERO TEMPERATURE SPECTRAL FUNCTIONS

DOMAIN WALL QCD WITH $2 + 1$ FLAVOURS

pseudoscalar channel

Aarts & Foley (2007)



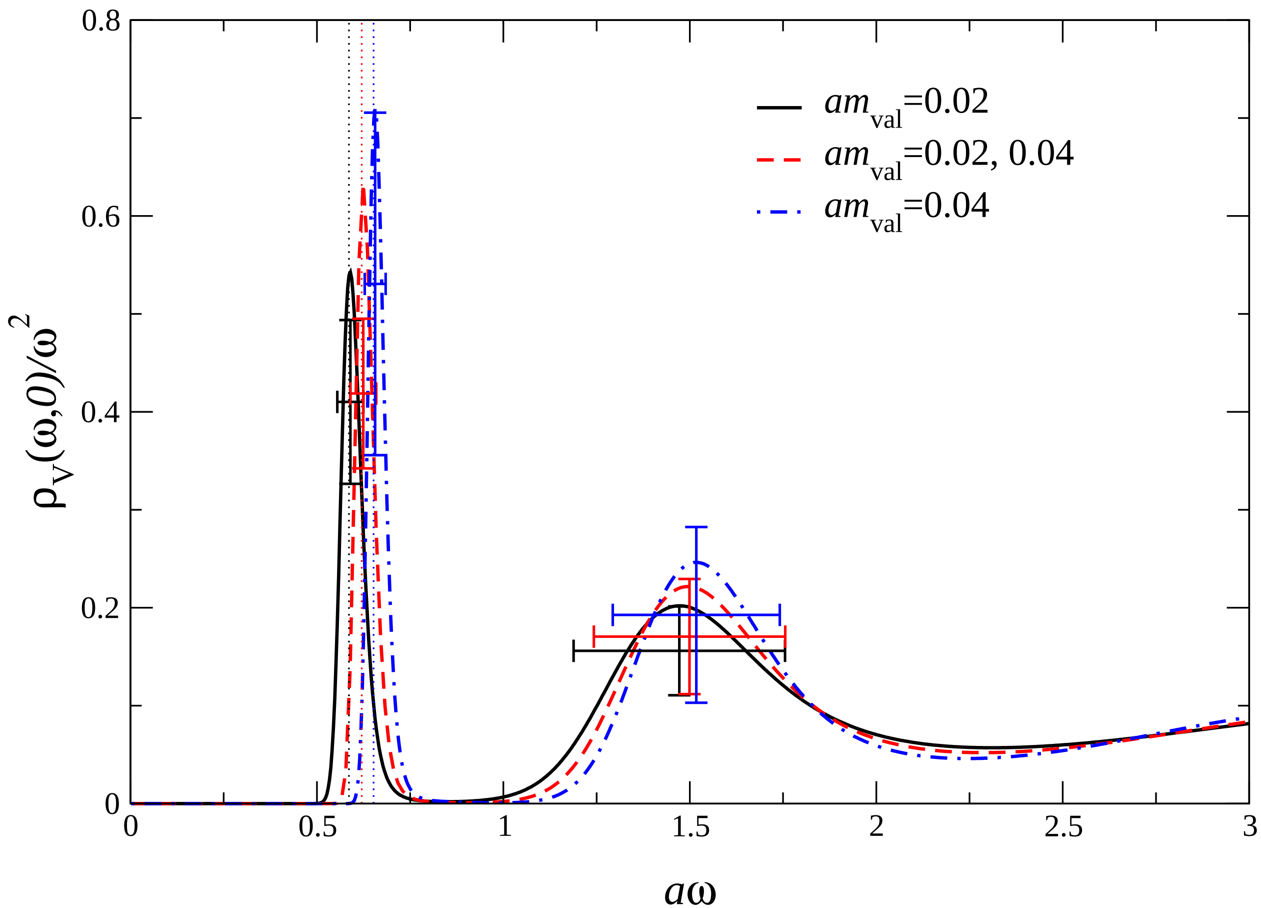
dotted lines: conventional cosh fits

ZERO TEMPERATURE SPECTRAL FUNCTIONS

DOMAIN WALL QCD WITH $2+1$ FLAVOURS

vector channel

Aarts & Foley (2007)

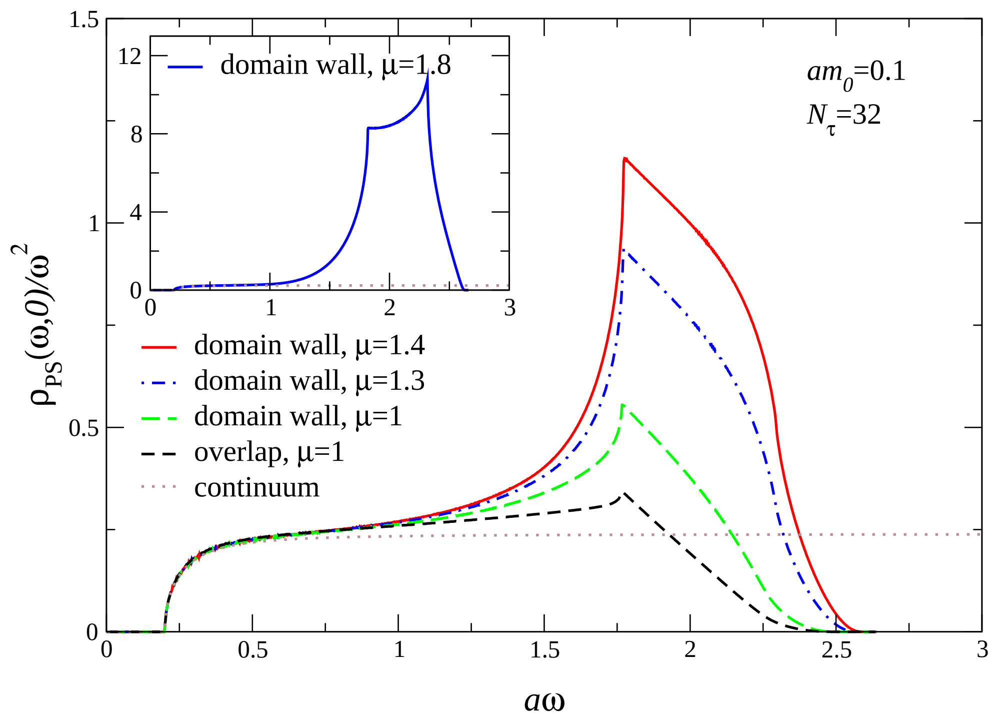


dotted lines: conventional cosh fits

LATTICE ARTEFACTS

FREE FERMION STUDY

- features when $1 < a\omega < 2$: presumably lattice artefacts



- high frequencies: edge of the Brillouin zone
- deviation from continuum spectral functions

Karsch et al. (2004), Aarts & Martínez Resco (2005) & Foley (2007)

CHARMONIUM

IN THE QUARK-GLUON PLASMA

quenched lattice QCD:

J/ψ and η_c may survive up to $T \sim 2T_c$

Asakawa & Hatsuda (2004) Datta, Karsch, Petreczky & Wetzorke (2004)

Umeda, Nomura & Matsufuru (2005)

recent, most complete study: different lattice spacings, anisotropies, charmonium and bottomonium Jakovac, Petreczky, Petrov & Velytsky (2007)

what happens in dynamical QCD?

a spectral function analysis

TrinLat $+$ SwanLat: Skullerud, Peardon, Oktay, Allton & Aarts (2007)

MAXIMUM ENTROPY METHOD

ALGORITHMIC IMPROVEMENTS

- reduction: from $\mathcal{O}(10^3)$ to $\mathcal{O}(10)$ degrees of freedom
- Bryan's algorithm: singular value decomposition of the kernel $K(\tau, \omega)$

finite temperature:

- unstable for small ω: $\lim_{\omega \to 0} K(\omega, \tau) = \frac{2T}{\omega} + \mathcal{O}\left(\frac{\omega}{T}\right)$
- amended algorithm required Aarts, Allton, Foley, Hands & Kim (2007)

(mostly) zero temperature:

- SVD problematic when N_τ is large
- different formulation Jakovac, Petreczky, Petrov & Velytsky (2007)

CHARMONIUM

IN THE QUARK-GLUON PLASMA

dynamical QCD:

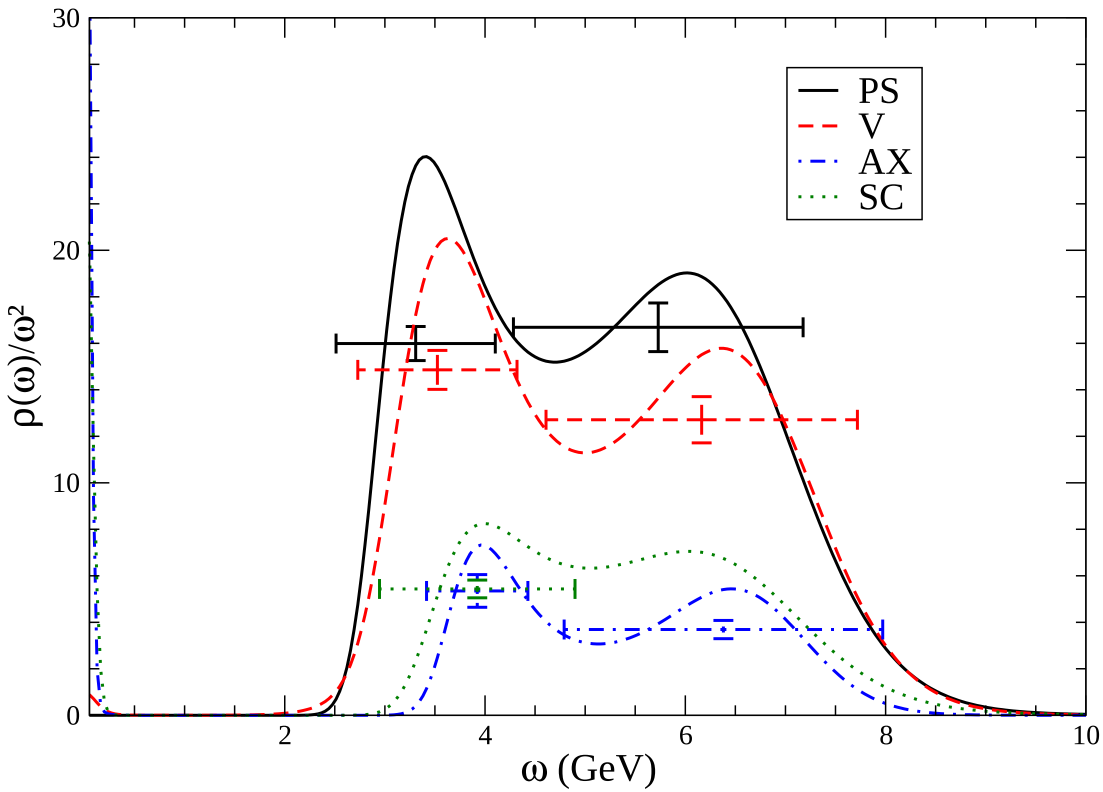
- TrinLat (Dublin) + SwanLat (Swansea): $N_f = 2$ QCD on highly anisotropic lattices
- $\xi = a_s/a_\tau \approx 6$, $1/a_\tau \approx 7$ GeV, $m_\pi/m_\rho \approx 0.54$
- $8^3 \times N_\tau$ and $12^3 \times N_\tau$
- temperature scan in steps of $\Delta T \approx 7$ MeV

N_τ	32	31	30	29	28	24	20	18	16
T/T_c	1.05	1.08	1.12	1.16	1.2	1.4	1.68	1.86	2.1
T(MeV)	221	228	235	243	252	294	353	392	441

CHARMONIUM

IN THE QUARK-GLUON PLASMA

$T/T_c = 1.05$, $T \approx 221$MeV ($N_\tau = 32$)

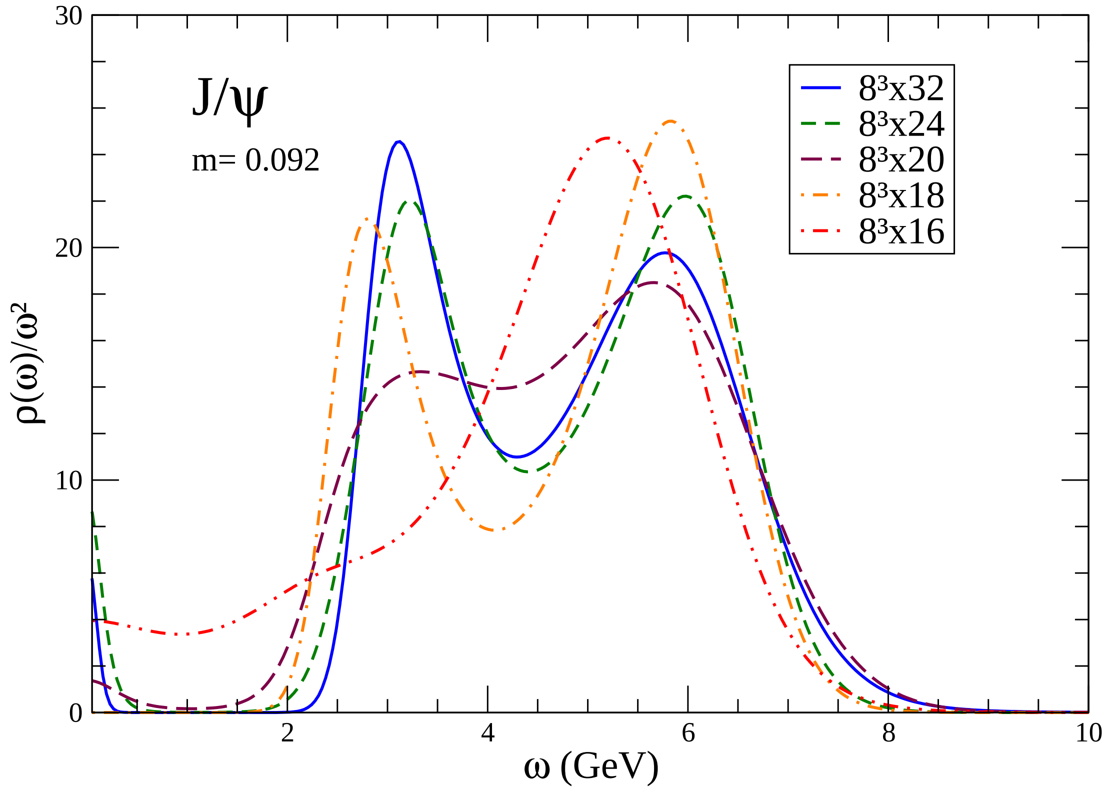


spectral functions in pseudoscalar (PS), vector (V), axial-vector (AV) and scalar (SC) channels

CHARMONIUM

IN THE QUARK-GLUON PLASMA

increasing the temperature
vector:



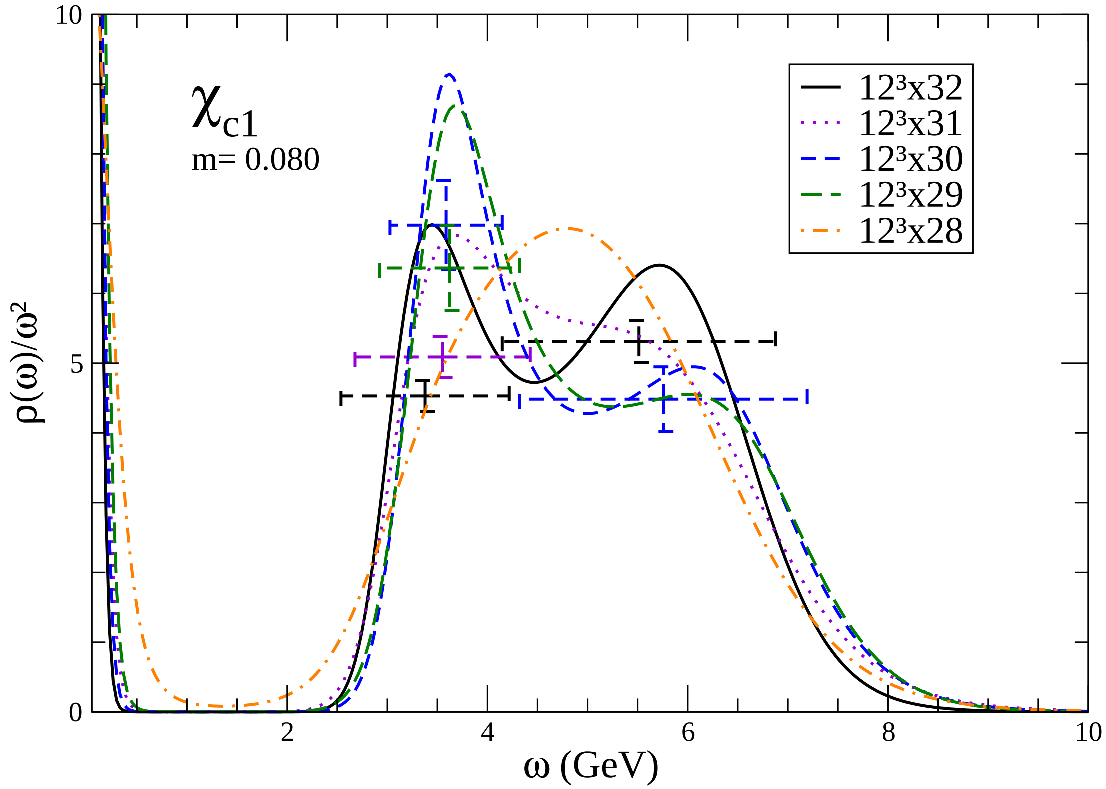
T/T_c
1.05
1.40
1.68
1.86
2.09

groundstate peak seems to disappear at $N_\tau \sim 20 - 16$
or $T \sim 1.7 - 2T_c$

CHARMONIUM

IN THE QUARK-GLUON PLASMA

increasing the temperature
axial
vector:



T/T_c
1.05
1.08
1.12
1.16
1.20

groundstate peak seems to disappear at $N_\tau \sim 28$
or $T \sim 1.2 T_c$

CHARMONIUM

RECONSTRUCTED CORRELATORS

MEM difficult:

- compare correlators at different temperatures

$$G(\tau;T) = \int_0^\infty \frac{d\omega}{2\pi} K(\tau,\omega;T)\rho(\omega;T),$$

$$K(\tau,\omega;T) = \frac{\cosh[\omega(\tau - 1/2T)]}{\sinh(\omega/2T)}$$

- temperature dependence of correlators: two sources

  1. kinematical: kernel $K(\tau,\omega;T)$ (boring)
  2. dynamical: spectral function $\rho(\omega;T)$ (interesting)

- eliminate effect 1

  Datta, Karsch, Petreczky & Wetzorke (2004)

CHARMONIUM

RECONSTRUCTED CORRELATORS

eliminate effect 1:

- determine spectral function at low reference temperature $\rho(\omega; T_0)$
- reconstruct "fake" correlator at higher T

$$G_{\rm rec}(\tau; T, T_0) = \int_0^\infty \frac{d\omega}{2\pi} K(\tau, \omega; T)\rho(\omega; T_0)$$

$\Rightarrow$ reconstructed correlator

- compare real and fake correlators at temperature T

$$G(\tau; T)/G_{\rm rec}(\tau; T, T_0)$$

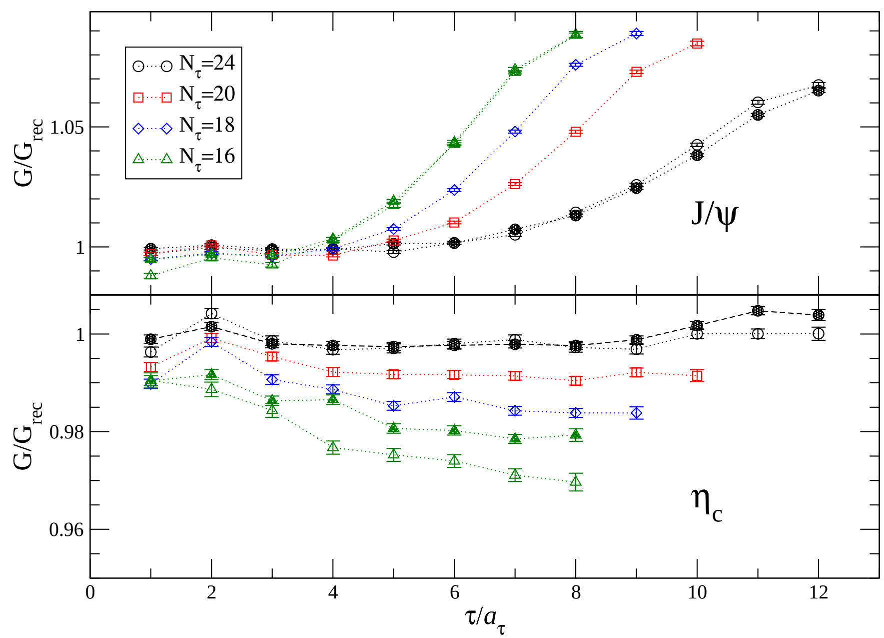
any deviation from 1 is due to changes in spectral function

CHARMONIUM

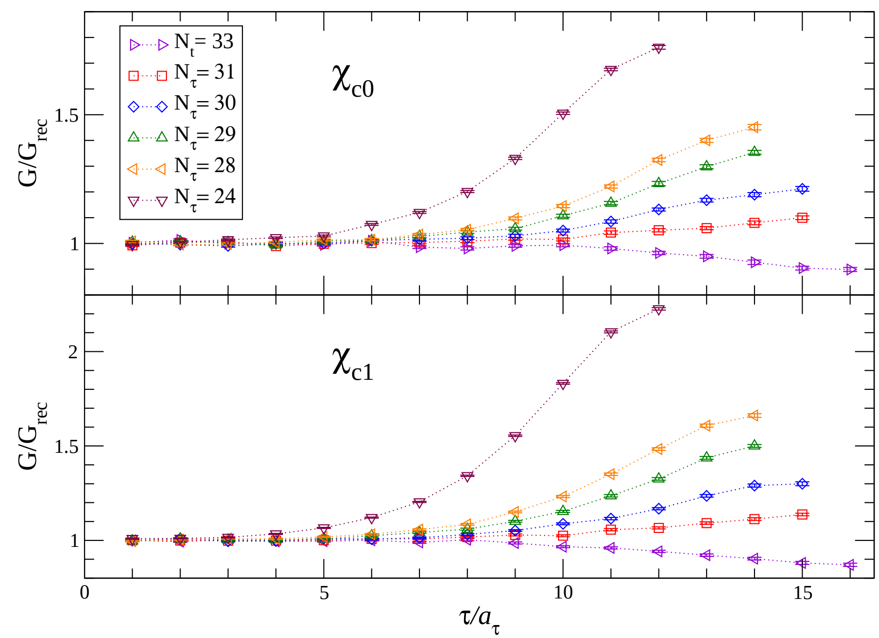
RECONSTRUCTED CORRELATORS

temperature dependence of correlators ($N_f = 2$ QCD)

S-waves:
vector, pseudoscalar



P-waves:
scalar, axial vector

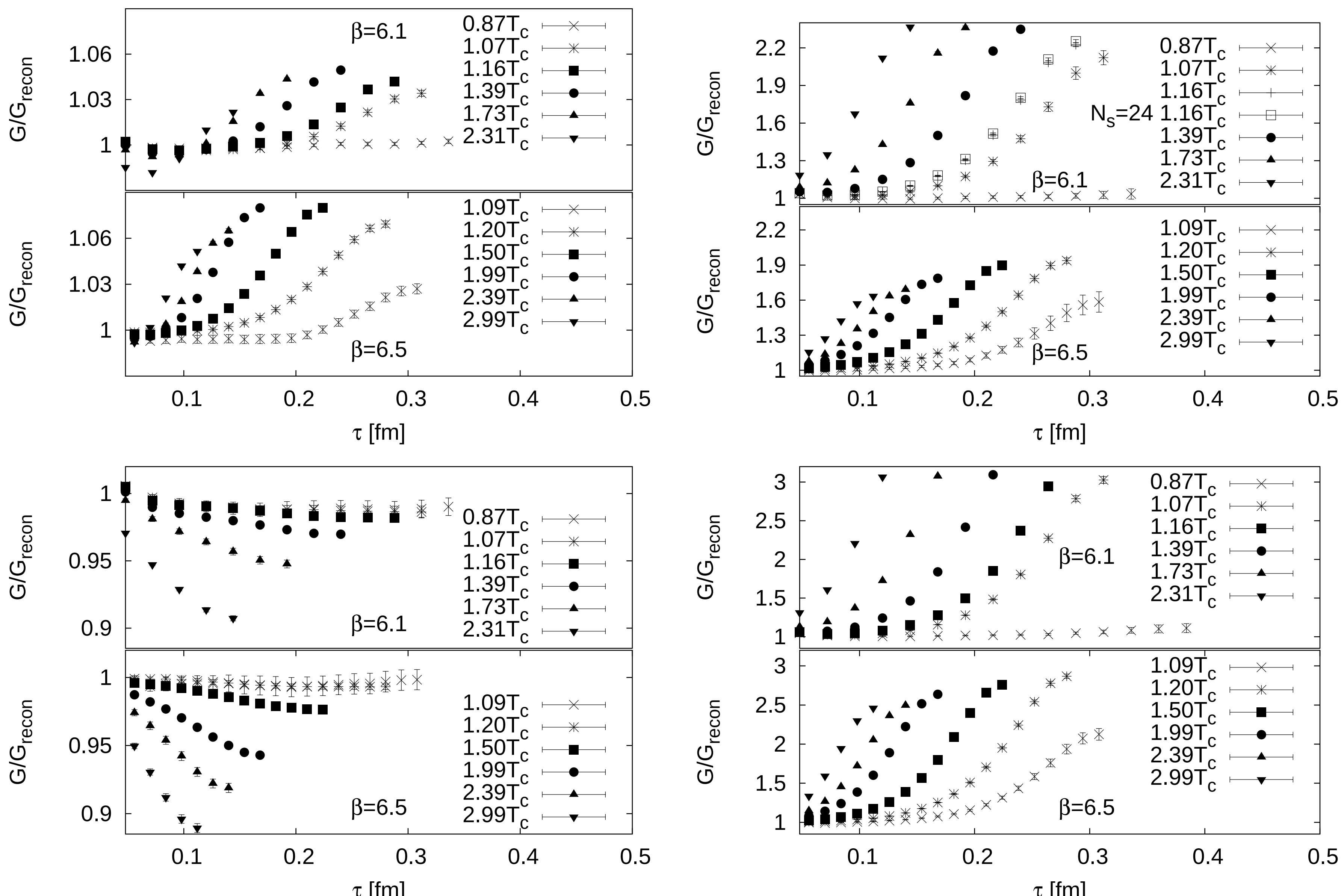


S-waves: a few percent

P-waves:
significant T dependence

CHARMONIUM

JAKOVAC, PETROV, PETRECZKY & VELYTSKY (2007) QUENCHED



CHARMONIUM

RECONSTRUCTED CORRELATORS

after removal of trivial temperature dependence:

- ratio of correlators/reconstructed correlators remarkably similar in quenched and dynamical simulations
- S waves: at most a few percent up to $2T_c$
- P waves: strong temperature dependence (up to 100%) at all $T > T_c$
- MEM is only used at lowest $T = T_0$: robust result

note: a small change in $G(\tau)$ does not exclude a large change in $\rho(\omega)$!

CHARMONIUM

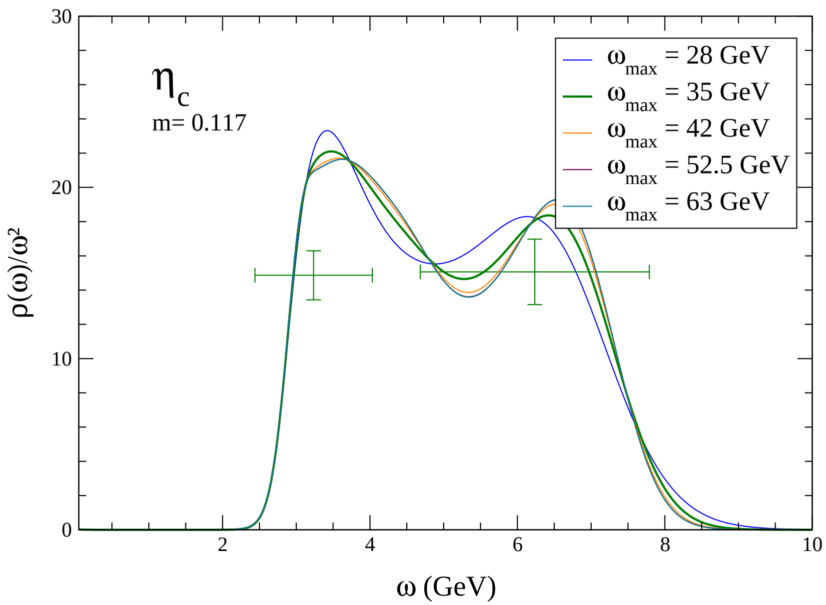
SYSTEMATICS

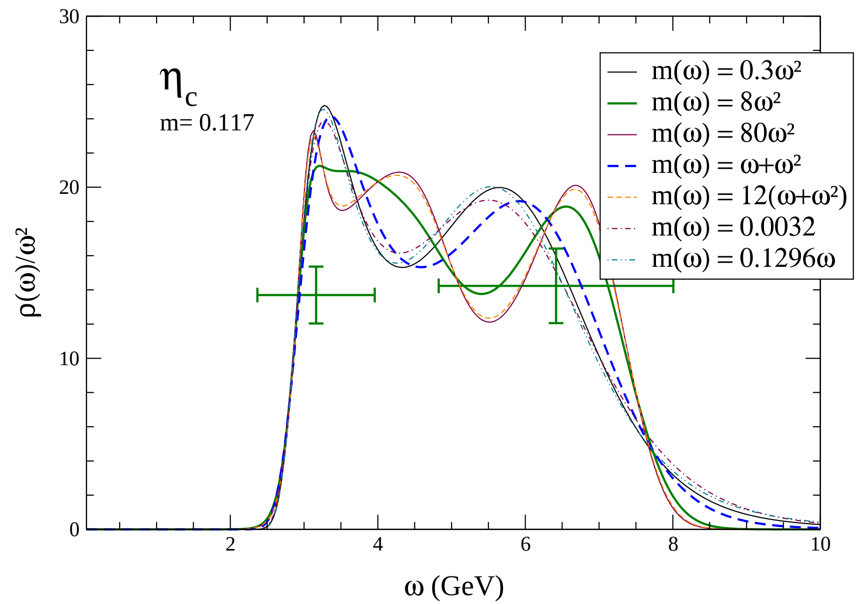
systematic checks in MEM:

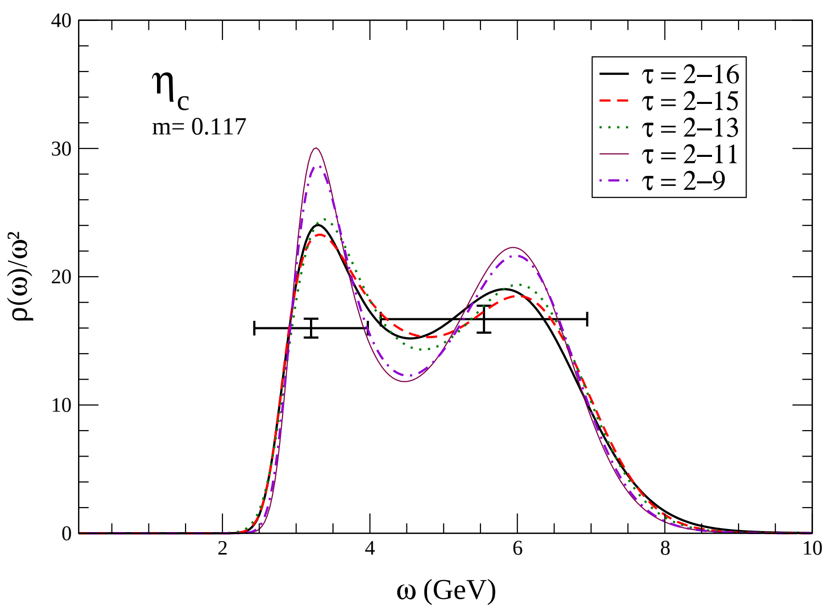
- energy range $0 < \omega < \omega_{\max}$
- time range included in analysis $\tau_{\min} \leq \tau \leq \tau_{\max}$
- default model dependence
- statistical uncertainties

CHARMONIUM

SYSTEMATICS







dependence on:

- energy range
- default model
- time range

towards the future:

- spectral functions at nonzero momentum
- transport and quark diffusion

MOMENTUM DEPENDENT SPECTRAL FUNCTIONS

MOVING MESONS

$$G(\tau,\mathbf{p}) = \int d^3x\, e^{i\mathbf{p}\cdot\mathbf{x}} \langle J(\tau,\mathbf{x}) J^\dagger(0,\mathbf{0})\rangle$$

- mesons moving with respect to the heatbath

Datta, Karsch, Petreczky & Wetzorke (2004)

Jakovac, Petreczky, Petrov & Velytsky (2007)

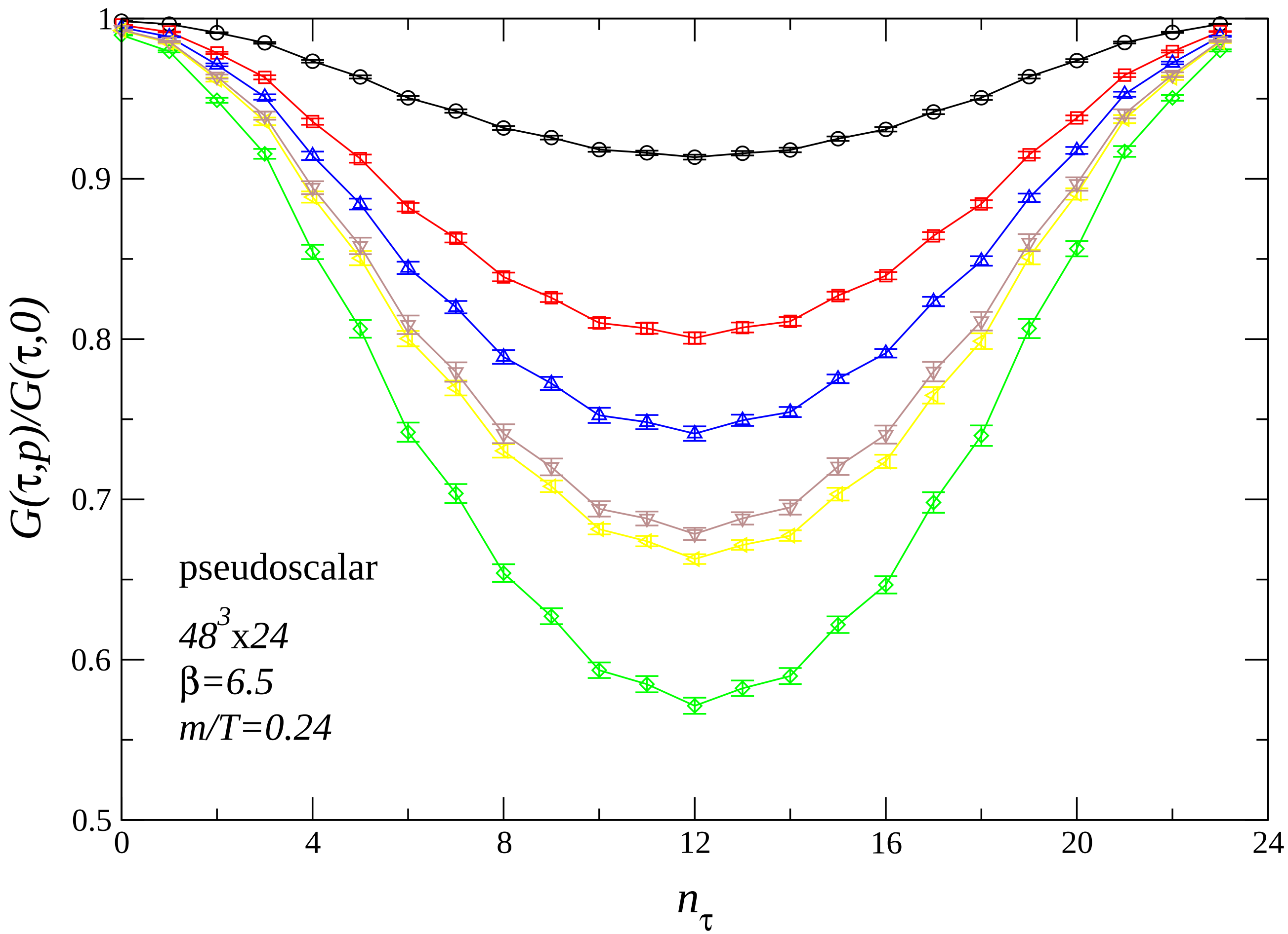
- use twisted (nonperiodic) boundary conditions
- quark field: $\psi(x_i + L) = e^{i\theta_i}\psi(x_i)$
- meson momentum: $\mathbf{p} = \frac{2\pi}{L}\mathbf{n} - \frac{\boldsymbol{\theta}_1 - \boldsymbol{\theta}_2}{L}$

Aarts, Allton, Foley, Hands & Kim (2007)

MOMENTUM DEPENDENT SPECTRAL FUNCTIONS

MOVING MESONS

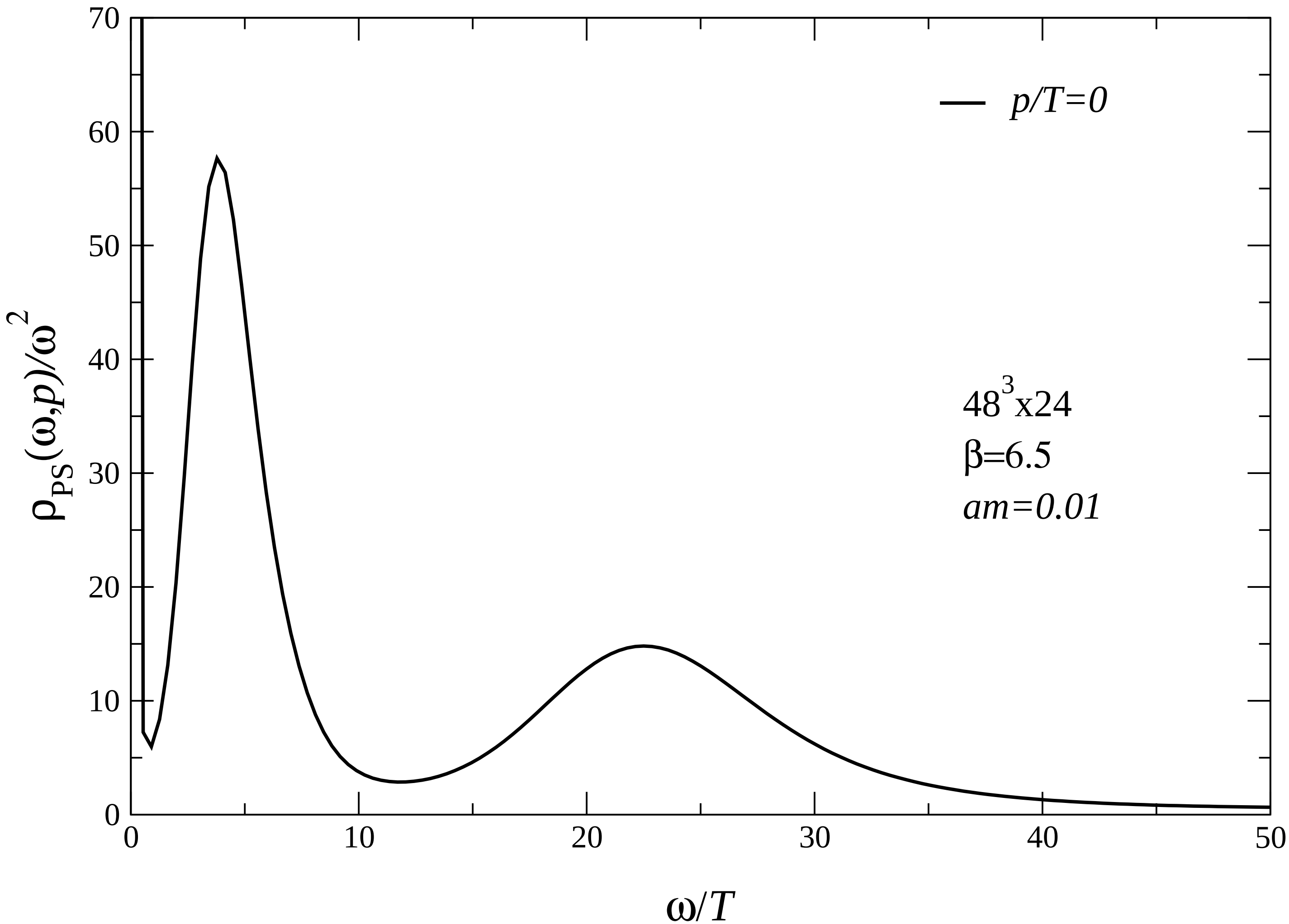
ratio of correlators: $G(\tau, \mathbf{p})/G(\tau, \mathbf{0})$ $\quad T \sim 0.6 T_c$



$p/T = 1,\ 1.6,\ 1.9,\ 2.1,\ 2.2,\ 2.6$

MOMENTUM DEPENDENT SPECTRAL FUNCTIONS

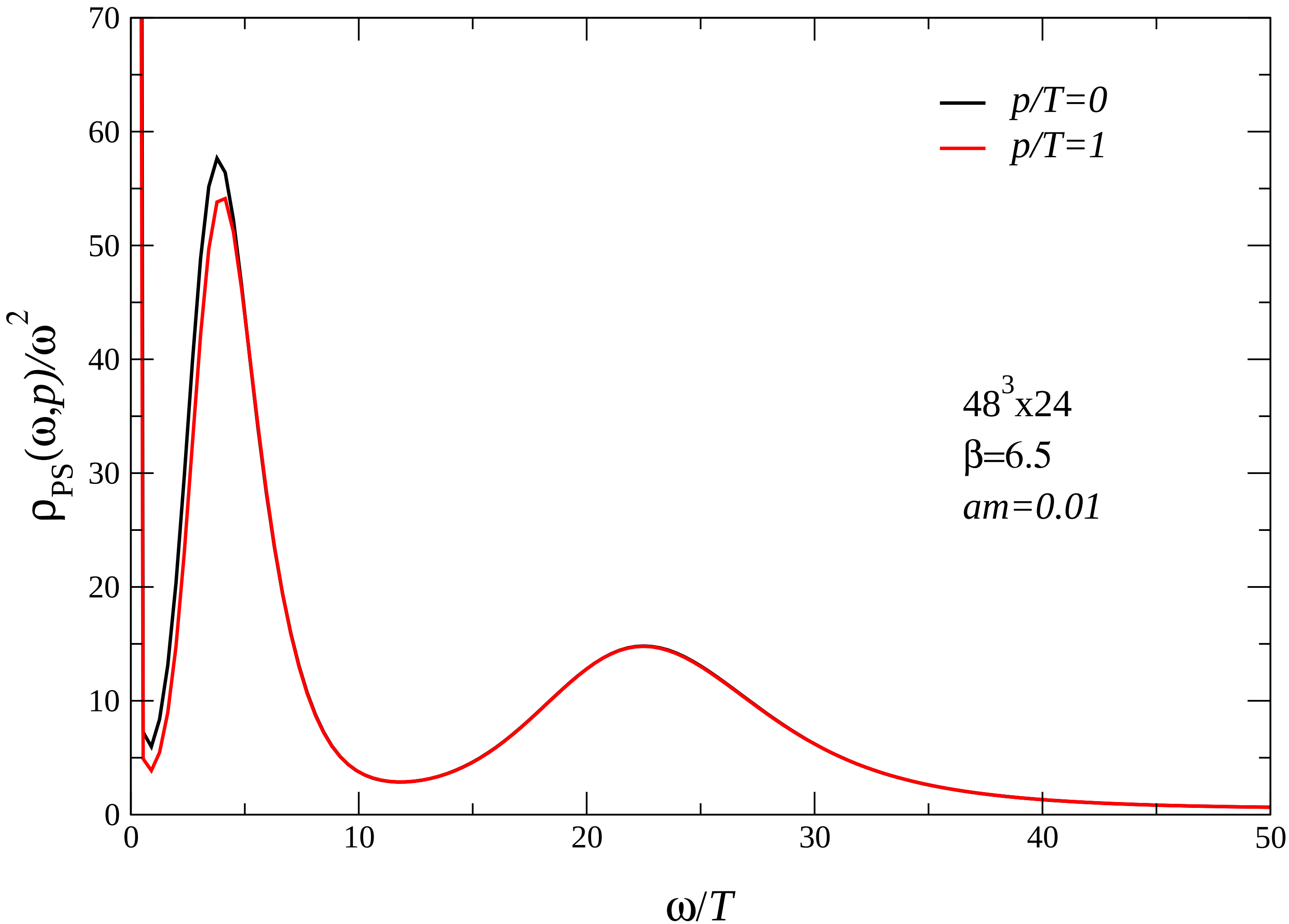
PSEUDOSCALAR, $T \sim 0.6T_c$, PRELIMINARY



meson at zero momentum

Momentum dependent spectral functions

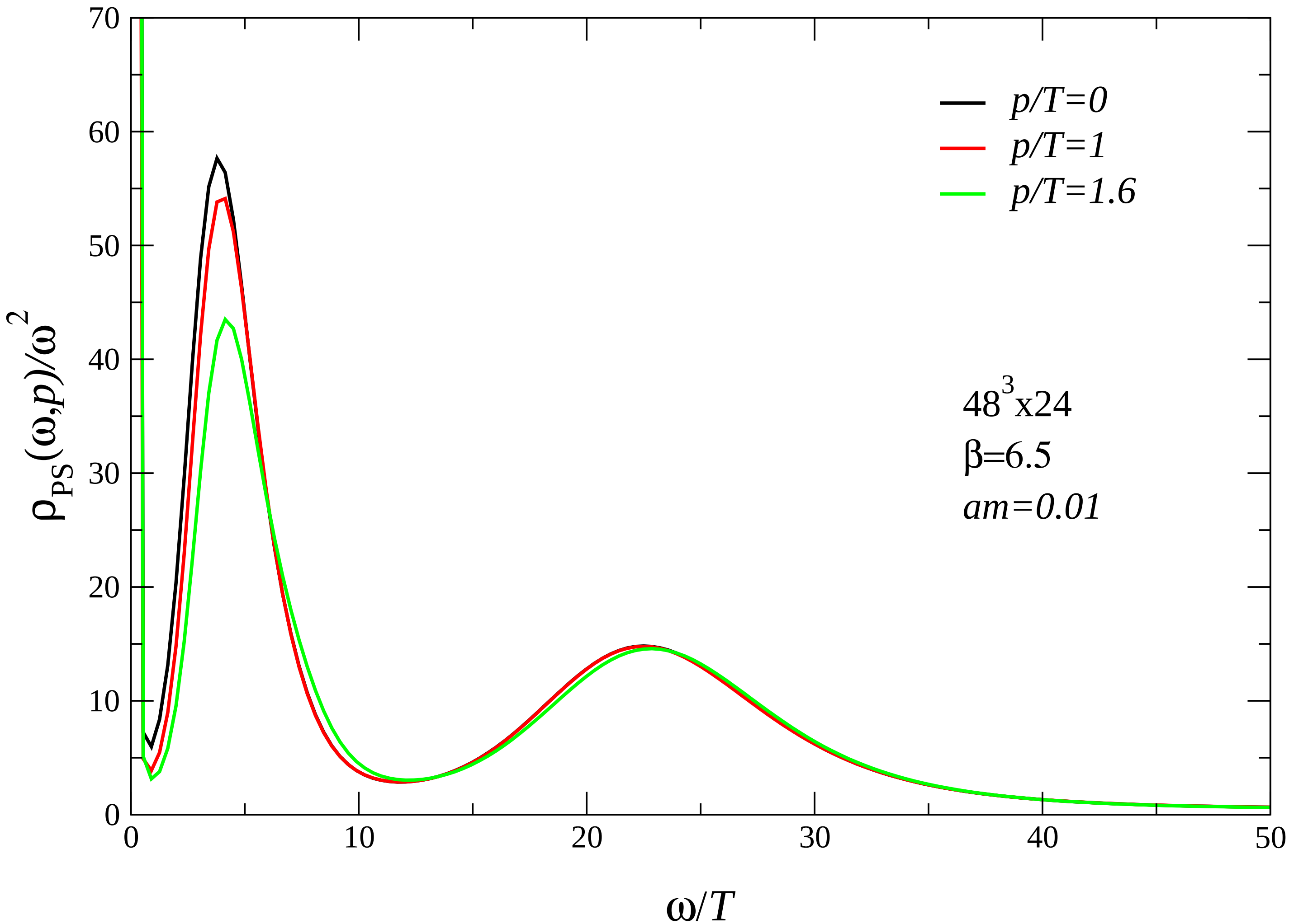
PSEUDOSCALAR, $T \sim 0.6T_c$, PRELIMINARY



increase momentum: moving mesons

MOMENTUM DEPENDENT SPECTRAL FUNCTIONS

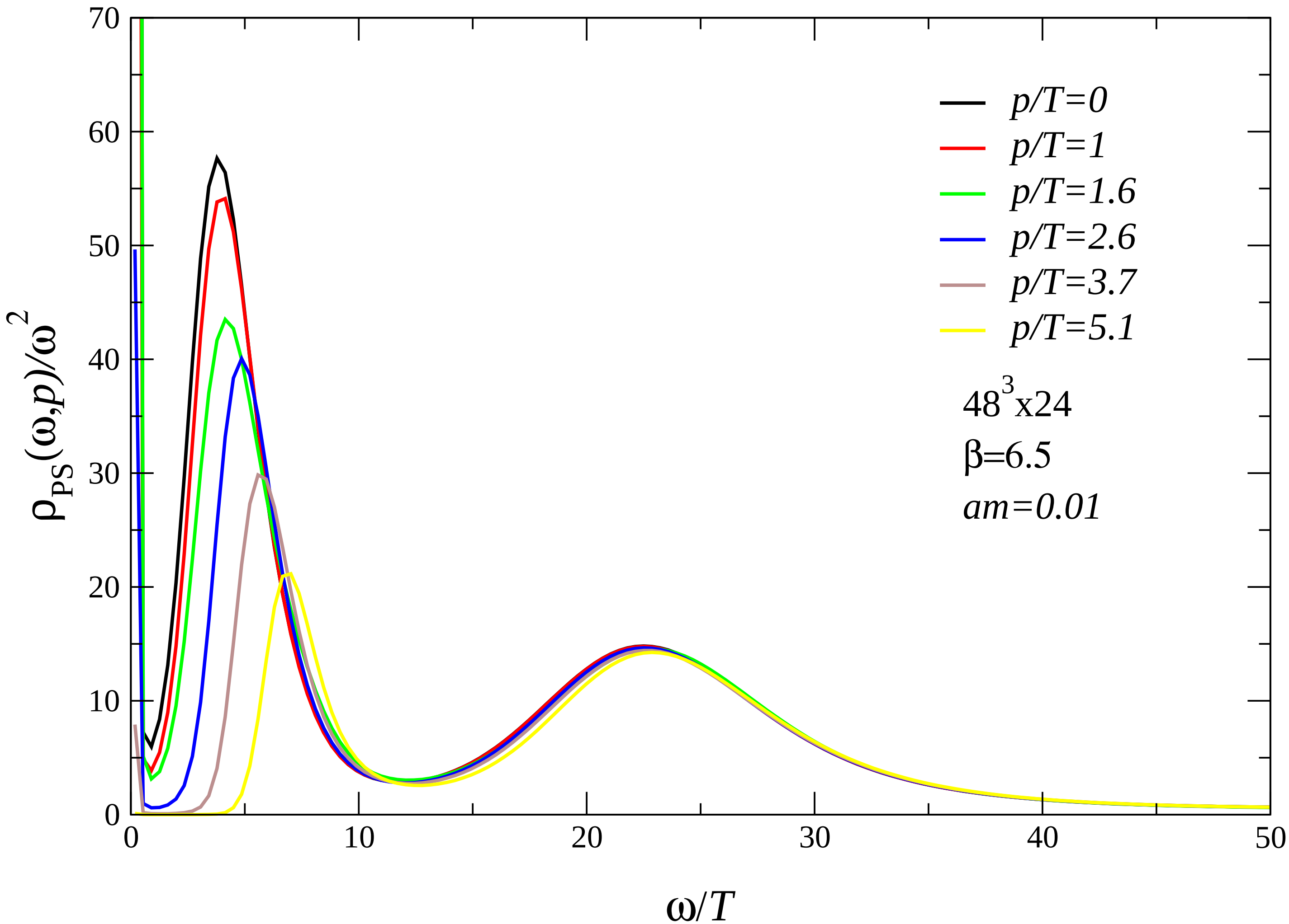
PSEUDOSCALAR, $T \sim 0.6T_c$, PRELIMINARY



increase momentum: moving mesons

MOMENTUM DEPENDENT SPECTRAL FUNCTIONS

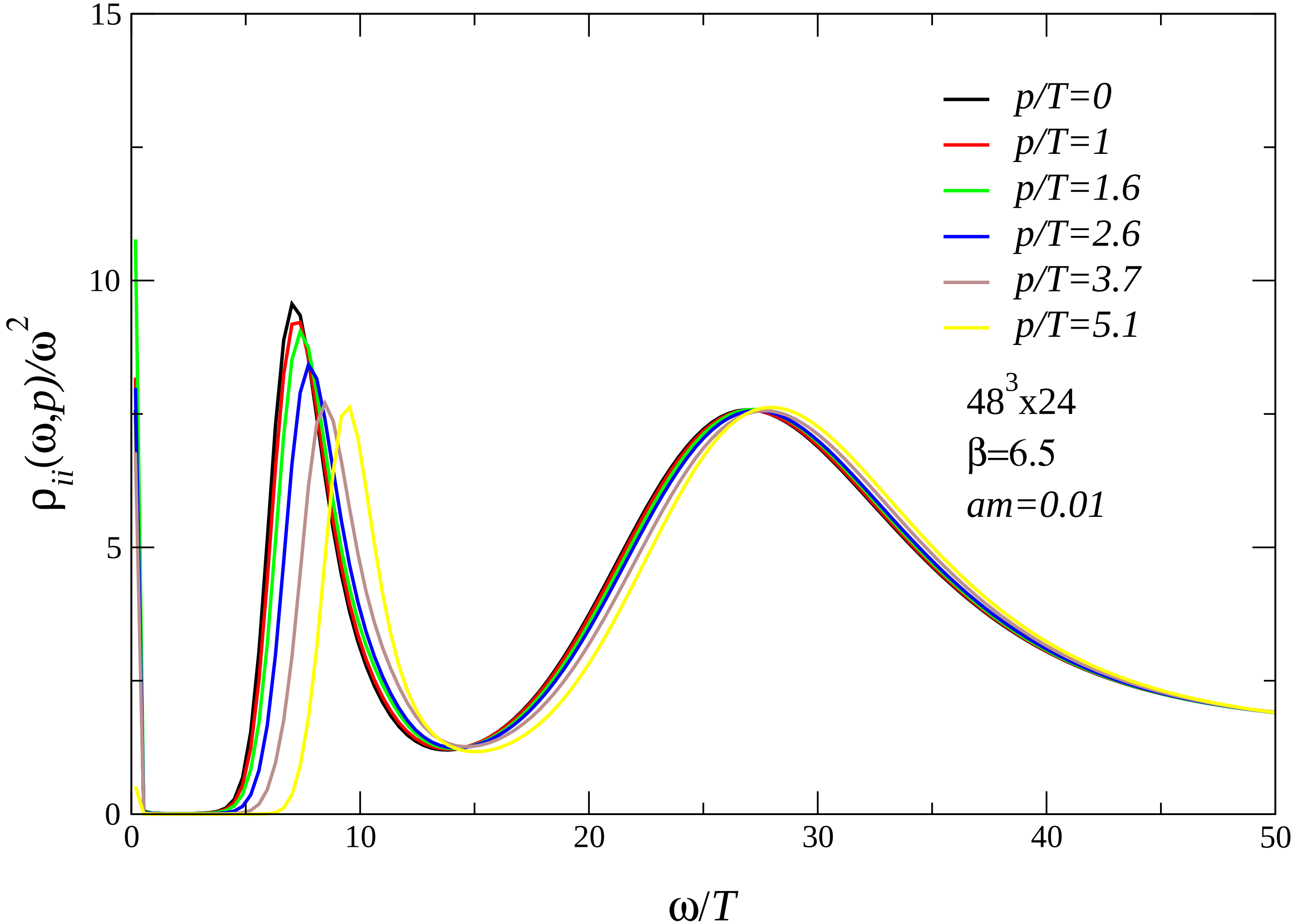
PSEUDOSCALAR, $T \sim 0.6T_c$, PRELIMINARY



increase momentum: moving mesons

MOMENTUM DEPENDENT SPECTRAL FUNCTIONS

VECTOR, $T \sim 0.6T_c$, PRELIMINARY



heavier meson in the vector channel

TRANSPORT AND QUARK DIFFUSION

KUBO RELATIONS & LINEAR RESPONSE

quarks moving through a (quark-)gluon plasma $\Rightarrow$ diffusion

transport coefficients: diffusion constant D, conductivity σ

$$\sigma = D\Xi = \lim_{\omega\to 0} \frac{\rho_{xx}(\omega, \mathbf{0})}{2\omega}$$

current spectral density: $\rho_{\mu\nu}(\omega, \mathbf{p}) = \int d^4x\, e^{ipx} \langle [j_\mu(x), j_\nu(0)] \rangle$

$\Rightarrow$ hydrodynamic limit of spectral functions: $\mathbf{p} \to 0$, $\omega \to 0$

- light quarks
- heavy quarks

from the lattice?

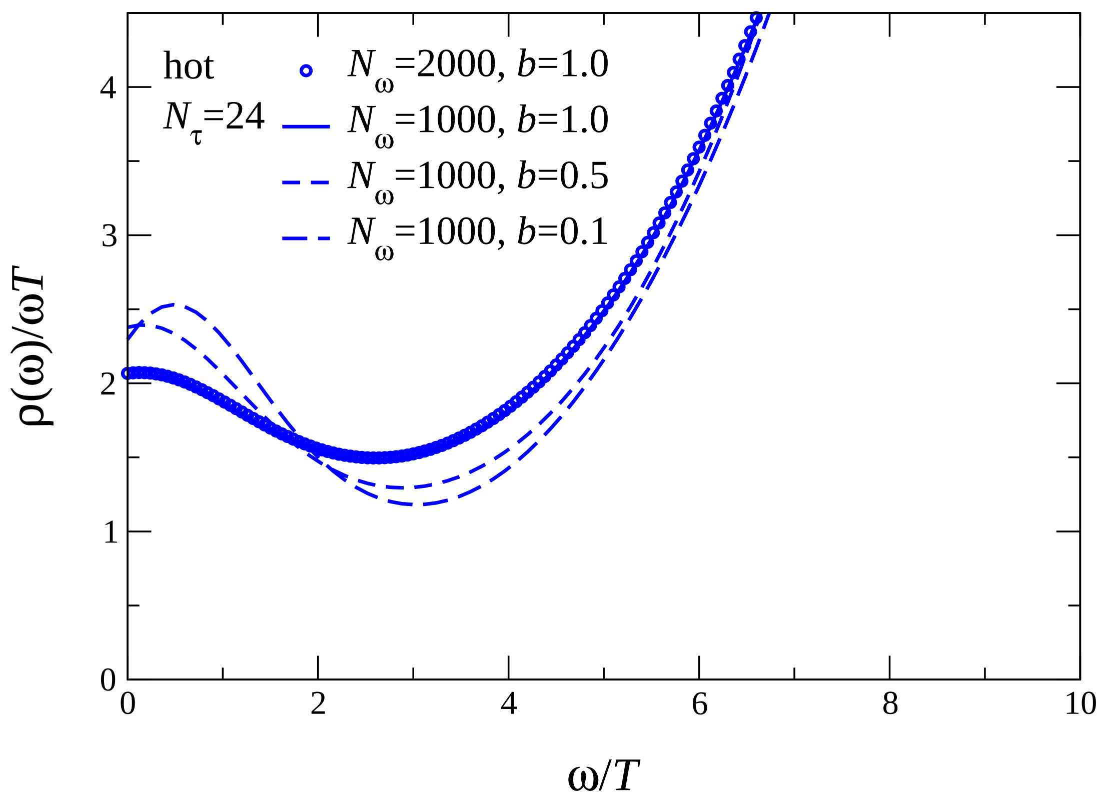
Aarts & Martínez Resco (2002), Petreczky & Teaney (2005)

LIGHT QUARKS: CONDUCTIVITY

LOW-ENERGY REGION OF SPECTRAL FUNCTIONS

hot: $T/T_c = 1.5$

quenched lattice QCD, $64^3 \times N_\tau$, $\beta = 7.192$



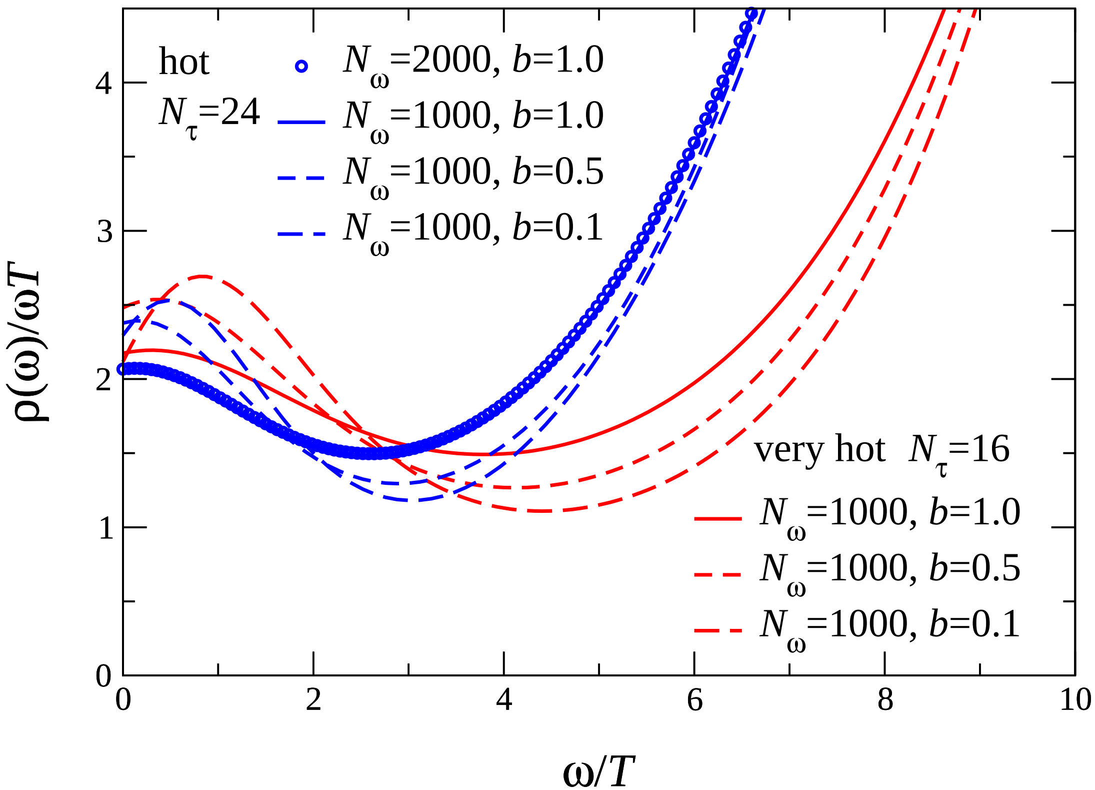
variations of default model: robust signal

Aarts, Allton, Foley, Hands & Kim (2007)

LIGHT QUARKS: CONDUCTIVITY

LOW-ENERGY REGION OF SPECTRAL FUNCTIONS

hot: $T/T_c = 1.5$ and 2.25



variations of default model: robust signal

Aarts, Allton, Foley, Hands & Kim (2007)

LIGHT QUARKS: CONDUCTIVITY

LOW-ENERGY REGION OF SPECTRAL FUNCTIONS

- conductivity $\sigma/T = 0.4 \pm 0.1$
- systematic error due to MEM reconstruction
- in line with the notion of sQGP

weakly coupled: $\sigma/T \sim 1/\alpha_s^2 \gg 1$

strongly coupled: $\sigma/T \sim 1$

- relevant for charge transport in heavy ion phenomenology

SUMMARY

lattice QCD and meson spectral functions:

- analytic continuation: imaginary to real time
- new complications at finite temperature
- charmonium in $N_f = 2$ lattice QCD
- first dynamical simulations are in agreement with (most) quenched studies
  - S-waves survive, P-waves don't
  - analysis using reconstructed correlators remarkably similar
- future: nonzero momentum, quark diffusion