UB/FI 97-1
June 1997

Is the solar neutrino deficit energy-dependent? *

G. Conforto[a,b], A. Marchionni[b], F. Martelli[a,b] and F. Vetrano[a,b]

[a] Università degli Studi, I-61029 Urbino, Italy
[b] Istituto Nazionale di Fisica Nucleare, I-50125 Firenze, Italy

Abstract

All existing measurements of the solar neutrino flux are compared with the predictions of the most recent solar model by Bahcall and Pinsonneault, modified by introducing the hypothesis of neutrino oscillations with mass differences large enough to render energy-independent any quantity observable on earth. It is concluded that the data are consistent with this hypothesis and that, at least for the time being, any energy-dependence of the solar neutrino deficit must be regarded as just an attractive theoretical possibility, but not as a compelling reality.

*Contributed paper to the XVIII International Symposium on Lepton-Photon Interactions, Hamburg, Germany, 28 July - 1 August 1997

1 Introduction

It is by now well established that the measurements of the solar neutrino flux on earth fall short of the expectations based on solar models.

The question of whether this deficit can be proven to be energy-dependent is of primary importance in neutrino physics. In fact, if this were the case and ascribing the effect to oscillations, this would imply the existence of a δm^2 smaller than $\approx 10^{-5}$ eV^2.

A careful statistical analysis of all the available evidence was carried out in ref. [1] with the conclusion that the data are consistent with an energy-independent depletion of the solar neutrino flux due to vacuum oscillations. Other authors [2, 3] have also taken this viewpoint.

However, different results have been obtained in ref. [4] and more recently in ref. [5]. In particular, in this last paper, it is claimed that "an energy independent suppression of the solar neutrino spectrum as a result of neutrino oscillation/transitions into active neutrinos or antineutrinos is ruled out by the data from the four operating experiments at 99.96 % C.L."

Two opposite conclusions cannot be both right. To investigate the origin of this disagreement, we have retraced the steps of the analysis of ref. [5], starting from the same input data detailed in sect. 2.

We can reproduce all results up to the final value of the minimum χ^2, but we find that the confidence level for an energy-independent depletion of the solar neutrino flux is still acceptable, in agreement with our previous results [1]. A possible explanation for this discrepancy is discussed in sect. 3. We present our final conclusions in sect. 4.

2 Input data

In order to make our comparison as tight as possible, we have used exactly the same input data as those of ref. [5]. The experimental results originate from ref [6, 7, 8, 9], the predictions of the "reference" Standard Solar Model from ref. [10, 11]. They are reported in table 1. An update of the experimental information is presented in sect. 4.

The uncertainties on the theoretical predictions are obviously correlated. However, at least for the model of ref. [10], the information available allows [12] to calculate the relevant correlation matrix ρ reported in table 2.

Within the approach of ref. [5], the χ^2 for the hypothesis of an oscillation-induced energy-independent reduction of all predictions is then simply written as

$$\chi^2(F) = \sum_{i=1}^{4}\sum_{j=1}^{4} (e_i - t_i)\,(e_j - t_j)\,(S^{-1})_{ij}$$

where:

- the indices i and j run over the "four operating experiments";

- e_i are the experimental results;
- t_i are the theoretical expectations. For the Gallium (Ga), Chlorine (Cl) and Kamiokande (Ka) experiments they are obtained from the results T_i of the model of ref. [10, 11] through the relations

$$\begin{aligned} t_{Ga,Cl} &= FT_{Ga,Cl} \\ t_{Ka} &= F(1-f)T_{Ka} + fT_{Ka} \end{aligned}$$

  where F is the energy-independent depletion factor and $f = 0.155$ is the fraction of the Kamiokande detection efficiency due to flavour-blind Neutral Currents;
- S is the covariance matrix. With exactly the same procedure as above, the errors on the theoretical expectations δ_i are obtained from the estimates of the solar model uncertainties Δ_i through the relations

$$\begin{aligned} \delta_{Ga,Cl} &= F\Delta_{Ga,Cl} \\ \delta_{Ka} &= F(1-f)\Delta_{Ka} + f\Delta_{Ka} \end{aligned}$$

  They give rise to the solar-model-related part of the covariance matrix through the relations $S_{ij} = \rho_{ij}\delta_i\delta_j$ in which ρ_{ij} are the elements of the correlation matrix reported in table 2. The complete S is then finally obtained by adding in quadrature the experimental errors σ_i to the diagonal elements: $S_{ii}^2 = \delta_i^2 + \sigma_i^2$. The σ_i are in turn obtained by adding in quadrature statistical and instrumental errors. When asymmetric errors are given, the average values have been used.

3 A trivial oversight ?

The minimum value of the function $\chi^2 = \chi^2(F)$ is $\chi^2_{\min} = 13.5$ corresponding to the result $F = 0.43 \pm 0.06$, in excellent agreement with the value $\chi^2_{\min} = 13.0$ quoted in ref. [5].

The corresponding Confidence Level (CL) for $n = 3$ (4 data points minus 1 unknown parameter) degrees of freedom is 0.36 %. Fig. 1 shows the plot of CL = CL(χ^2) for different values of n.

The value $\chi^2_{\min} = 13.0$ found in ref. [5] corresponds to a CL = 0.46 %. We find no justification for the low value (CL = 4×10^{-4}) invoked to reject the energy-independent-depletion hypothesis.

A possible explanation that we venture to put forward lies in the fact that in Fig. 1 a vertical line at $\chi^2 = 13.0$ intercepts the $n = 1$ curve in the range $3 \times 10^{-4} < \text{CL} < 4 \times 10^{-4}$. So it is perhaps plausible that the confidence level of ref. [5] was calculated for the wrong number of degrees of freedom ($n = 1$ instead of $n = 3$).

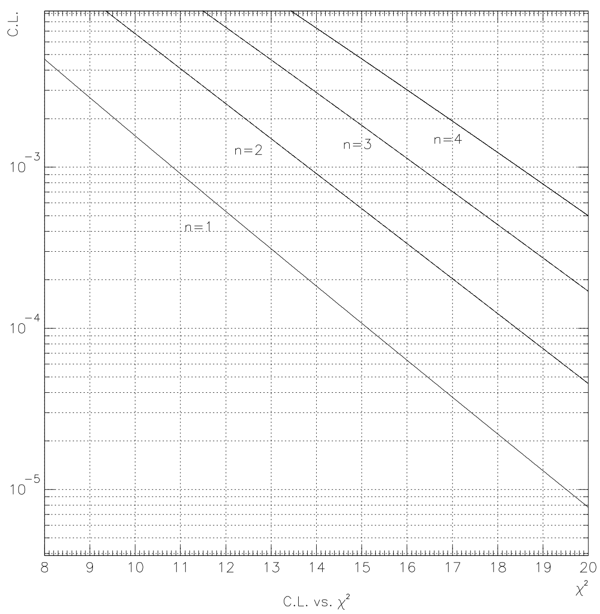

Figure 1: Confidence Levels (CL) vs. χ^2 for various degrees of freedom (n).

4 Conclusions

The validity of a χ^2 analysis relies on the two basic hypotheses that errors must be correctly calculated and Gaussian-distributed. Neither is really justified in the case of solar neutrinos.

On the experimental side, results are often affected by large systematics and asymmetric errors. Occasionally, doubts have been raised about the correctness of their evaluations.

On the theoretical side, different models of the same sun giving rise to different predictions cannot be all simultaneously right, implying that the quoted errors are probably underestimated. In some cases the assumption of a Gaussian probability density function is highly questionable and this in turn implies the likelihood of an underestimated variance.

For all these reasons, the χ^2 approach cannot be taken too seriously far away from the minimum where small deviations from a Gaussian behaviour in the tails of the probability density functions give rise to huge variations of the Confidence Level.

However, even discarding all these caveats, no serious disagreement with the energy-independent-depletion hypothesis can be claimed to exist at present. Making use of the recent and more precise measurement of the solar neutrino flux by the SuperKamiokande experiment, $(2.65 \pm 0.15) \times 10^6$ cm^{-2} s^{-1} [13], yields $\chi^2_{\rm min}$

$= 14.2$, corresponding to $F = 0.41 \pm 0.05$ and CL $= 0.26$ %. The confidence levels are small, but not unacceptably small.

To gauge the acceptability of the CL obtained, it is perhaps worth remembering that "If this probability is larger than an agreed-upon value (0.001, 0.01, or 0.05 are common choices), the data are *consistent* with the assumptions; otherwise we may want to find improved assumptions. As for the converse, most people do not regard a model as truly *inconsistent* unless the probability is as low as that corresponding to four or five standard deviations for a Gaussian (6.3×10^{-5} or 5.7×10^{-7})" [14].

The onus of the proof is on the energy-dependence camp. As no definitive proof of the correctness of this hypothesis has been produced so far, the energy-dependence of the solar neutrino deficit must be regarded, at least for the time being, as just an attractive theoretical possibility, but not as a compelling reality.

References

[1] G. Conforto, Nucl. Phys. B (Proc. Suppl.) **38**, 308 (1995). Also, G. Conforto *et al.*, Astropart. Phys. **5**, 147 (1996) (hep-ph/9606226).

[2] P.F. Harrison, D.H. Perkins, W.G. Scott, Phys. Lett. **B349**, 137 (1995); Phys. Lett., **B396**, 186 (1997).

[3] A. Acker, S. Pakvasa, Phys. Lett. **B397**, 209 (1997).

[4] P.I. Krastev and S.T. Petcov, Phys. Rev. **D53**, 1665 (1996).

[5] P.I. Krastev and S.T. Petcov, Phys. Lett. **B395**, 69 (1997).

[6] K.Lande, to appear in Proceedings of Neutrino '96 Int. Conference, June 13-19 1996, Helsinki.

[7] T. Kirsten, to appear in Proceedings of Neutrino '96 Int. Conference, June 13-19 1996, Helsinki.

[8] V. Gavrin, to appear in Proceedings of Neutrino '96 Int. Conference, June 13-19 1996, Helsinki.

[9] Y. Fukuda *et al.*, Phys. Rev. Lett. **77**, 1683 (1996).

[10] J. Bahcall and M.H. Pinsonneault, Rev. Mod. Phys. **67**, 781 (1995).

[11] J. Bahcall *et al.*, Phys. Rev. **C54**, 411 (1996).

[12] G.L. Fogli and E. Lisi, Astropart. Phys. **3**, 185 (1995).

[13] E. Kearns, talk given at the "Workshop on fixed target physics at the Main Injector", May 1-4 1997, FNAL, Batavia, Illinois.

[14] Particle Data Group, R. M. Barnett *et al.*, Phys. Rev. **D54**, 1 (1996).

Table 1: Solar neutrino experimental results [6-9] and "reference" Standard Solar Model [10,11] theoretical predictions.

Experiment (Units)	Result	Prediction
Gallex (SNU)	$69.7 \pm 6.7^{+3.9}_{-4.5}$	136.8^{+8}_{-7}
Sage (SNU)	$72^{+12}_{-10}{}^{+5}_{-7}$	
Chlorine (SNU)	$2.56 \pm 0.16 \pm 0.14$	$9.5^{+1.2}_{-1.4}$
Kamiokande ($10^6 \mathrm{cm}^{-2}\mathrm{s}^{-1}$)	$2.80 \pm 0.19 \pm 0.33$	$6.62^{+0.93}_{-1.12}$

Table 2: Error correlation matrix of the theoretical predictions for the Gallium (Ga), Chlorine (Cl) and Kamiokande (Ka) experiments. The convention for the indices of the matrix elements ρ_{ij} is 1=Ga, 2=Cl, 3=Ka.

$$\rho = \begin{array}{ccc} 1 & 0.656 & 0.646 \\ 0.656 & 1 & 0.976 \\ 0.646 & 0.976 & 1 \end{array}$$