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Direct Measurements of the Electromagnetic Coupling Constant at Large q^2

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Abstract

Recent direct measurements of the electromagnetic coupling constant $\alpha_{\rm em}$ at q=58 GeV and q=157 GeV are reviewed. Reinterpreting measurements of leptonic cross-sections and asymmetries at lower energies, a plot of the measured dependence of $\alpha_{\rm em}(q)$ between q=10 and 157 GeV is presented for the first time, and compared to the Standard Model prediction. The determination of $\alpha_{\rm em}$ at 157 GeV can be used to set improved limits on the QED cut-off parameter Λ, if the input data are refit without imposing the Standard Model relation between the weak and electromagnetic couplings. An estimate for the ultimate precision of direct $\alpha_{\rm em}$ determinations from LEP-2 is presented, which will reduce the correlation with the Higgs mass in unconstrained electroweak fits for $\alpha_{\rm em}(m_Z)$.

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1 Introduction

The running of the electromagnetic coupling constant $\alpha_{\rm em}$ as function of the momentum transfer q^2 is an immediate consequence of the renormalisation of the photon propagator in Quantum Electrodynamics. Phenomenologically, it can be understood from the screening of an infinite bare charge by fermion pairs due to the polarisation of the vacuum (i.e. fermion loops in the photon propagator). At larger momentum transfer (smaller distances) a probe starts to penetrate the cloud of fermion pairs and feels a larger charge. Theoretically, one has to chose a renormalisation convention to define the infinite bare charge, that cancels the infinities of the fermion loops, and a renormalisation scale μ^2, at which the value of the coupling is measured.

Usually, in QED, the "on-shell" renormalisation scheme is chosen for the photon propagator, i.e. divergencies are removed for a photon propagator of $q^2 = \mu^2 = 0$. Then, from the salient reference value of $\alpha_{\rm em}(0)$=127.0359979$\pm$0.0000032, precisely measured independent from higher order QED corrections via the Quantum Hall Effect [1], the running of $\alpha_{\rm em}(q^2)$ can be predicted from theory to be

$$\alpha_{\rm em}(q^2) = \frac{\alpha_{\rm em}(0)}{1 - \sum_f \Pi_f(q^2)} \qquad (1)$$

where $\Pi_f(q^2)$ is the renormalised vacuum polarisation contribution for a fermion f. Non-perturbative and resonant QCD effects and the unknown masses of light quarks would cause a considerable uncertainty in the contribution $\Pi_{\rm hadrons}(q^2)$ from quark loops. From the optical theorem, and the theorem of the residues, however, this can be calculated using a dispersion integral over the measured hadronic cross-section in $\rm e^+e^-$ annihilation. This procedure yields $\Pi_{\rm hadrons}(q^2)$ with a relative precision of 2.5% at $q^2 = m_{\rm Z}^2$. The combined effect of all fermions then results in a prediction of $\alpha_{\rm em}(m_{\rm Z}^2)^{-1} = 128.896 \pm 0.090$ [2], which is 5.9% different from $\alpha_{\rm em}(0)$. Despite of this rather accurate prediction, it's uncertainty of 0.07% constitutes the dominant error in the prediction of the Higgs Boson mass from electroweak fits [3]. A major part of the uncertainty originates from systematic errors of experimental hadronic cross-section measurements at low $\sqrt{s}$ between 1 and 5 GeV. To improve the accuracy, one possibility is a precise new measurement of these cross-sections. Another one, that will be dealt with below, is the direct measurement of $\alpha_{\rm em}$ at large q^2.

Due to its sizable effect, the running $\alpha_{\rm em}(q^2)$ had to be taken into account in many QED tests in the past, e.g. performed in $\rm e^+e^-$ annihilations at the PEP, PETRA, and TRISTAN accelerators [4]. It also has considerable impact on the weak sector of the Standard Model, where in improved Born approximation a Weinberg angle $\sin^2\overline{\theta}_{\rm w}$ can be e.g. defined by the relation

$$\sin^2\overline{\theta}_{\rm w}(m_{\rm Z}^2)\cos^2\overline{\theta}_{\rm w}(m_{\rm Z}^2) = \frac{\pi}{\sqrt{2}}\frac{\alpha_{\rm em}(m_{\rm Z}^2)}{m_{\rm Z}^2 G_F} \qquad (2)$$

based on the relation between the weak and electromagnetic coupling as predicted by the electroweak unification. Although many experiments would therefore have observed sizable deviations of their measurements from the predictions, if they would have ignored the running of $\alpha_{\rm em}$, there was until recently no attempt to directly measure it from the data and confront it to the prediction.

Attempts for a measurement of $\alpha_{\rm em}(m_{\rm Z})$ at the Z peak [3, 5, 6], though potentially the most precise ones, have important conceptual disadvantages. Due to the predominance of the Z propagator in this energy regime, they actually measure primarily the effective Weinberg angle $\sin^2\theta_{\rm w}^{\rm eff}(m_{\rm Z}^2)$ in the weak coupling constant

$$\alpha_{\rm weak}(m_{\rm Z}^2) = \alpha_{\rm em}(m_{\rm Z}^2)/\sin^2\theta_{\rm w}^{\rm eff}(m_{\rm Z}^2). \qquad (3)$$

Modulo higher order corrections relating the two Weinberg angles $\sin^2\overline{\theta}_{\rm w}$ and $\sin^2\theta_{\rm w}^{\rm eff}$, this weak coupling constant is predicted by Eq. (2), and a value of $\alpha_{\rm em}(q^2)$ can be extracted, since $m_{\rm Z}$ and G_F are very precisely known, The photon propagator itself has little influence in this procedure, with the exception of its contribution to the off-peak forward-backward asymmetries. Therefore here the running of the electromagnetic coupling is only indirectly measured, via the unification of the electroweak force.

In addition, via the higher order weak corrections, a dependence of the resulting $\alpha_{\rm em}(m_{\rm Z}^2)$ on $m_{\rm t}$ and especially $m_{\rm H}$ is introduced, which renders it impossible to considerably improve $m_{\rm H}$ fits on the Z-peak by fitting $\alpha_{\rm em}(m_{\rm Z})$ simultaneously, since their errors are highly correlated. A recent fit to preliminary LEP-1 data [3], without constraining $\alpha_{\rm em}(m_{\rm Z}^2)$ to its Standard Model prediction, yields [6]

$$\alpha_{\rm em}(m_{\rm Z}^2) = 129.13^{+0.24}_{-0.70} \tag{4}$$

with a correlation coefficient of 0.90 between its error and that of the fitted Higgs boson mass.

2 Direct measurements of $\alpha_{\rm em}$ at large q^2

The first measurement of the strengthening of the electromagnetic coupling with increasing momentum transfer, that was based on directly measuring the photon propagator at high q^2, has been recently performed by the TOPAZ collaboration [7]. It takes the ratio of the measured number of $\mathrm{e^+e^-} \to \mu^+\mu^-$ events produced at an average $q^2 = (57.77\ \mathrm{GeV})^2$ to the measured number of $\mathrm{e^+e^-} \to \mathrm{e^+e^-}\mu^+\mu^-$ events in untagged two-photon collisions, at an average median $q_1 q_2 = (0.01\ \mathrm{GeV})^2$ of the two photons. This ratio is therefore proportional to $\alpha_{\rm em}^2(57.77\ \mathrm{GeV})\ /\ \alpha_{\rm em}^4(0.01\ \mathrm{GeV})$, and yields [7]

$$\alpha_{\rm em}^{-1}(57.77\ \ \mathrm{GeV}) = 128.5 \pm 1.8 \pm 0.7. \tag{5}$$

This is consistent with the Standard Model prediction of 129.6 ± 0.1, and more than 4 standard deviations away from $\alpha_{\rm em}^{-1}(0)$. Assumptions on the running of $\alpha_{\rm em}$ at low $q^2 \sim (5\ \mathrm{GeV})^2$, that would enter in cross-sections via the luminosity measurement, cancel in the measured ratio of events.

At $\mathrm{e^+e^-}$ centre-of-mass energies above the $\mathrm{Z^0}$, the amplitude for s-channel photon exchange amplitude becomes comparable to – and for leptons even larger than – that for Z exchange. The measurements of fermion pair production at LEP-2 are therefore opening the possibility to study the photon propagator at momentum transfers larger than $m_{\rm Z}$. A first measurement of $\alpha_{\rm em}$ at these energies has been performed by the OPAL collaboration [8]. The cross-section and asymmetry measurements for $s'/s > 0.8$ have been used to investigate the running of $\alpha_{\rm em}$ within the framework of the Standard Model. At three centre-of-mass energies from 133 GeV to 172 GeV, $\alpha_{\rm em}(\sqrt{s})$ was varied so as to minimize the χ^2 between the measurements and the Standard Model predictions calculated using ZFITTER [9], keeping all other Standard Model parameters fixed. In addition to performing a fit for each centre-of-mass energy a combined fit to all energies, in which $\alpha_{\rm em}$ runs with energy with a slope that corresponds to its fitted value, was performed.

As inputs to the fits, the measured values of hadronic, muon pair and tau pair cross-sections, $\sigma_{\rm had}$, $\sigma_{\mu\mu}$, and $\sigma_{\tau\tau}$, the ratio $R_{\rm b}{=}\sigma_{\rm b\overline{b}}/\sigma_{\rm had}$, and the combined muon and tau asymmetry values, $A_{\rm FB}^{\ell}$, were used. These measurements were all obtained with the requirement of $s'/s > 0.8$, where s' is the effective centre-of-mass energy after initial state radiation, in order to assure that $q^2 \approx s$ for the s-channel propagator. They can be employed in two different ways.

- Using only cross-section ratios $\sigma_{\mathrm{b\bar{b}}}/\sigma_{\mathrm{had}}$, $\sigma_{\mu\mu}/\sigma_{\mathrm{had}}$, $\sigma_{\tau\tau}/\sigma_{\mathrm{had}}$, and leptonic forward-backward asymmetries A_{FB}^{ℓ}, giving 4 measurements at each energy:
  This leads, like above, to a determination of α_{em} that is independent of any assumptions on the running of α_{em} at low q^2. The sensitivity to α_{em} is retained in the cross-section ratios of leptons and hadrons, because above the Z^0 the relative contribution of the photon amplitude to the total cross-section is much smaller for hadrons than for leptons. In addition changes to $\alpha_{\mathrm{em}}(q^2)$ affect the hadronic vector coupling in the Z amplitude via $\sin^2\theta_{\mathrm{w}}(q^2)$ in opposite direction to the photon amplitude.

- Using the leptonic and hadronic cross-sections separately together with $R_{\mathrm{b}}=\sigma_{\mathrm{b\bar{b}}}/\sigma_{\mathrm{had}}$, and A_{FB}^{ℓ}, giving 5 measurements at each energy:
  Here the Standard Model running of α_{em} from $q^2 = 0$ to $q^2 \sim (5\ \mathrm{GeV})^2$ has to be assumed for the luminosity measurement. This is the region where most of the uncertainty of the the hadronic vacuum polarisation comes from. This measurement therefore measures the running from $q^2 \sim (5\ \mathrm{GeV})^2$ onwards an will not contribute to reduce the uncertainty in α_{em} originating from low q^2 values.

Error correlations between all the measurements were taken into account in the fit. They are rather small, the most significant correlations occurring between measurements of the same quantity at different energies, about 0.12 for the hadronic cross-sections, 0.07 for R_{b}, and 0.02 for the leptonic cross-sections. Since fewer measurements contribute to the first set of 3×4 inputs, and additional correlations of about 0.12 arise between the ratios of leptonic and hadronic cross-sections, the first fit gives somewhat larger errors. The results for both fits are listed in Table 1.

Table 1: *Results of fits for α_{em}. The first three rows show the fits to data at each centre-of-mass energy, the last row the combined fit to all energies. The Standard Model values of $1/\alpha_{\mathrm{em}}$, and the χ^2 between the measurements and the Standard Model (SM) predictions are also given for comparison.*

	Fit to 3×4 inputs		Fit to 3×5 inputs		SM for 3×5 inputs	
E_{cm} (GeV)	$1/\alpha_{\mathrm{em}}$	$\chi^2/d.o.f$	$1/\alpha_{\mathrm{em}}$	$\chi^2/d.o.f$	$1/\alpha_{\mathrm{em}}$	$\chi^2/d.o.f$
133.17	$114.9^{+9.9}_{-7.0}$	2.8/3	$116.9^{+7.6}_{-5.6}$	4.2/4	128.3	6.2/5
161.34	$119.6^{+10.7}_{-8.0}$	3.1/3	$115.5^{+8.2}_{-5.6}$	3.4/4	128.1	5.3/5
172.12	$127.0^{+14.9}_{-10.5}$	0.7/3	$128.7^{+13.5}_{-8.9}$	0.9/4	128.0	0.9/5
157.42	$119.9^{+6.6}_{-5.4}$	7.5/11	$119.9^{+5.1}_{-4.1}$	10.6/14	128.1	12.9/15

The combined result of the fit to 3×4 cross-section ratios is $1/\alpha_{\mathrm{em}}(157.42\ GeV) = 119.9^{+6.6}_{-5.4} \pm 0.1$, where the first error comes from the experimental measurements, and the second error from varying $\alpha_{\mathrm{s}}(m_{\mathrm{Z}})$ in the range 0.112 to 0.124. This fitted value is 2.6 standard deviations below the low energy limit of $1/\alpha_{\mathrm{em}}(0)$, and thus represents the second direct measurement demonstrating the running of α_{em} at large q^2. Assuming Standard Model running between m_{Z} and 157 GeV, with a slope corresponding to the fitted value, it can be translated in a measurement of

$$1/\alpha_{\mathrm{em}}(m_{\mathrm{Z}}) = 121.4^{+6.0}_{-4.9} \pm 0.1. \tag{6}$$

This measurement is completely statistics limited. At the end of LEP-2, there will be about 20 times more integrated luminosity available, and it is not unreasonable to assume that the large statistics will help the systematic measurement errors stay below the statistical ones. Then, combining all four LEP-experiments, one can hope to reach a precision of

$$\Delta\alpha_{\mathrm{em}}^{-1}(m_{\mathrm{Z}})\Big|_{LEP-2} = 0.6, \tag{7}$$

corresponding to a relative error of 0.5%, essentially uncorrelated with the Higgs mass. Including the complete set of LEP-2 data in electroweak fits, not constraining $\alpha_{\rm em}$ to its theoretical prediction, would then considerably reduce the overall error on $\alpha_{\rm em}$ in Eq. (4) and its correlation with the error of $m_{\rm H}$.

As shown above, the $\alpha_{\rm em}$ measurements from LEP-2 can be performed without assuming the Standard Model running of $\alpha_{\rm em}$ at low q^2, whose uncertainty dominates the error of the present theoretical prediction for $1/\alpha_{\rm em}(m_{\rm Z})$. The combined precision of a fit to LEP-1 and LEP-2 data will, however, not quite reach the current theoretical precision of 0.09 [2], but may still lead to a small improvement of its error.

3 A textbook plot for running $\alpha_{\rm em}$

As mentioned before, many QED tests at PEP, PETRA, and TRISTAN were not successful, if the running of $\alpha_{\rm em}$ would have been ignored in these analyses. Measurements of lepton cross-sections and asymmetries performed at these storage rings were used to set limits on new physics, fixing all Standard Model parameters, including $\alpha_{\rm em}(q^2)$, to their accepted values [4].

All these measurements can be reinterpreted by assuming the pure Standard Model without any contributions from new physics, and interpreting each cross-section or asymmetry as a measurement of the running $\alpha_{\rm em}$. In the following we employ about 60 measurements [10, 11] of cross-sections for $\rm e^+e^- \to \mu^+\mu^-$, $\rm e^+e^- \to \tau^+\tau^-$, $A_{\rm FB}^{\mu\mu}$, and $A_{\rm FB}^{\tau\tau}$ to determine $\alpha_{\rm em}(s)$ at 5 different centre-of-mass energies $\sqrt{s}=$ 10.4 GeV (DORIS), 29.0 GeV (PEP), 34.7 and 43.7 GeV (PETRA), and 57.8 GeV (TRISTAN). At each $\sqrt{s}$ the deviation of the measured cross-sections and asymmetries from their Standard Model prediction is translated in a measurement of $\alpha_{\rm em}(s)$, fixing all other parameters, like the axial coupling of the $\rm Z^0$ responsible for the asymmetries, to their Standard Model values. The difference between today's predictions [2] for the vacuum polarisation and the ones used at the time of the measurements [12] is taken into account.

As an example, the ratio C of the total observed $\rm e^+e^- \to \mu^+\mu^-$ continuum cross-section to its Monte Carlo expectation, has been measured in [10], averaged over six different $\sqrt{s}$ in the region of the Υ resonances. The result, $C = 0.999 \pm 0.006$, does not include the luminosity and efficiency errors, common to all six energies. Correcting for about 25% background, mainly from $\rm e^+e^- \to e^+e^-\mu^+\mu^-$, and including the common errors as given in [10], leads to

$$R_{\mu\mu}(10.4\ {\rm GeV}) \equiv \frac{\sigma(\rm e^+e^- \to \mu^+\mu^-)_{\rm measured}}{\sigma(\rm e^+e^- \to \mu^+\mu^-)_{\rm QED}} = 0.999 \pm 0.008 \pm 0.045. \tag{8}$$

for an average centre-of-mass energy of $\sqrt{s}$=10.4 GeV. Taking into account the prediction [12] of $\alpha_{\rm em}(q^2)$ used in [10] to derive the ratio C, this can be translated in a measured value of

$$\alpha_{\rm em}(10.4\ GeV) = 131.9 \pm 3.0. \tag{9}$$

The dominant systematic errors of all cross-section measurements are due to uncertainties in the acceptance and luminosity calculation, while the systematic error on the forward-backward asymmetry is usually due to uncertainties of background contributions with different angular distribution, especially Bhabha scattering. Combining many measurements of different experiments care has to be taken to account for correlated systematic errors between the experimental measurements, that originate from the common use of Monte Carlo software for the luminosity, acceptance, and background determination. In the following, fits to $\alpha_{\rm em}$ have been performed at each $\sqrt{s}$ separately, where for each pair of leptonic cross-section measurements the smaller of the two relative systematic errors was taken to be 100% correlated between the two measurements. An analogous procedure was applied for each pair

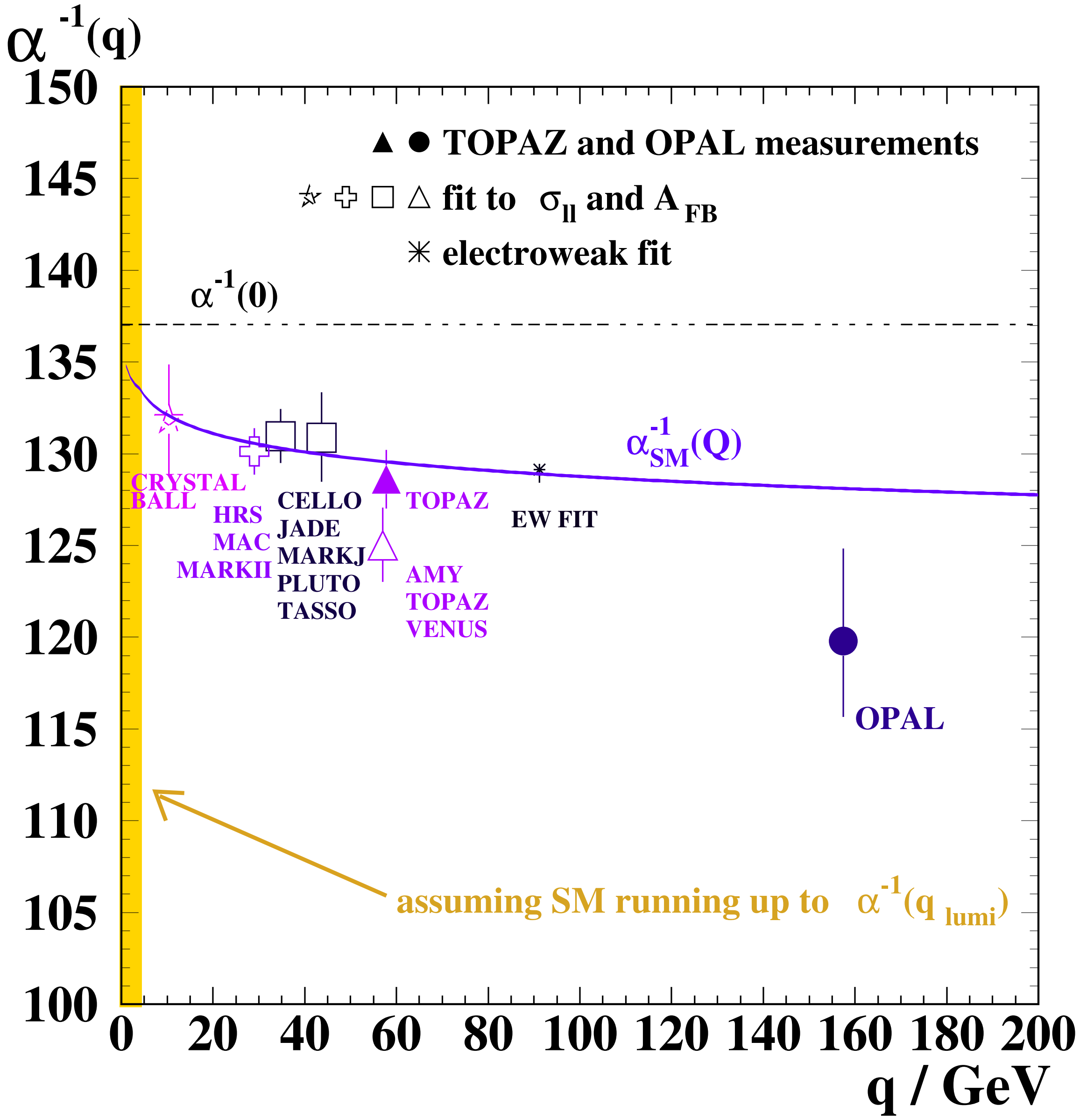

Figure 1: *Direct determination of $\alpha_{em}(s)$ at five centre-of-mass energies from a fit to 63 measurements of leptonic cross-sections and asymmetries at the DORIS, PEP, PETRA, and TRISTAN e^+e^- storage rings. All measurements rely on assuming the Standard Model running of α_{em} below 5 GeV. The results of the TOPAZ [7] and OPAL [8] collaborations are also shown. The star indicates a preliminary indirect determination of $\alpha_{em}(m_Z)$ from an electroweak fit [6] to LEP-1 data, assuming the Standard Model relation between the weak and electromagnetic couplings.*

of asymmetry measurements. Such a procedure leads to a conservative estimate of the error correlations, if the relative influence of systematic errors from the common usage of simulation packages is the same in all experiments. The result of the fit is summarized in Table 2, where the resulting values of $\chi^2/d.o.f.$ for each $\sqrt{s}$ do not indicate any gross underestimate of error correlations.

Table 2: *Results of fits for $1/\alpha_{\rm em}(s)$. The rows show the fits to data at each centre-of-mass energy and the χ^2 of the fit. The Standard Model values of $1/\alpha_{\rm em}(s)$, and the χ^2 between the measurements and the Standard Model predictions are also given for comparison.*

$\sqrt{s}$	storage	Fit		Standard Model	
(GeV)	ring	$1/\alpha_{\rm em}$	$\chi^2/d.o.f.$	$1/\alpha_{\rm em}$	$\chi^2/d.o.f.$
10.4	DORIS	131.9±3.0	0/0	132.1	0.1/1
29.0	PEP	130.1±1.3	11.7/9	130.6	11.8/10
34.7	PETRA	130.9±1.5	21.7/24	130.3	22.0/25
43.7	PETRA	$130.9^{+2.5}_{-2.4}$	6.0/14	129.9	6.2/15
57.0	TRISTAN	$125.0^{+2.1}_{-2.0}$	4.6/11	129.6	9.4/12

The results are shown in Fig. 1 together with the result of the electroweak fit [6], and the TOPAZ [7] and OPAL [8] measurements. For the latter, the combined fit using 3×5 inputs is plotted. The TOPAZ value is averaged with a second TOPAZ measurement [13] from the hadronic cross-sections. All measurements in this figure rely on the measurement of luminosity (and therefore assume the Standard Model running of $\alpha_{\rm em}$ at low q^2), but demonstrate consistency with the predicted running of $\alpha_{\rm em}$ for momentum transfers between (5 GeV)^2 and (160 GeV)^2.

While for the strong coupling constant, similar comparisons between the predicted and measured q^2 dependence have been successfully performed for many years, this is the first time, that such a plot has been made for the electromagnetic coupling.

4 Limits for new physics from the running $\alpha_{\rm em}$

As indicated above, the results of Fig. 1 are not well suited to reduce the uncertainty in $\alpha_{\rm em}(m_{\rm Z})$, since they assume Standard Model running up to momentum transfers of about 5 GeV. There are, however, models of new physics that only modify the effective running of $\alpha_{\rm em}$ for larger q^2. Such a model has been proposed by Drell [14] long time before the invention of the electroweak theory. In this scheme the photon propagator is modified according to

$$\frac{1}{q^2} \rightarrow \frac{1}{q^2} - \frac{1}{q^2-\Lambda^2} = \frac{1}{q^2}\left(1 - \frac{q^2}{q^2-\Lambda^2}\right)$$

where the cut-off parameter Lambda can be interpreted as the mass of an additional heavy photon with coupling strength α. Alternatively $1/\Lambda$ can be regarded as the scale where a substructure of the fermions becomes apparent.

It is conservative to assume, that an extra heavy neutral boson does not mix with the electroweak gauge bosons, notably with the $\rm Z^0$, since in this case stringent bounds can be set on its mass and coupling from the high precision lineshape measurements at LEP-1 [15]. Then, the modification of the photon propagator can be absorbed as a formfactor in an effective running electromagnetic coupling constant

$$\alpha_{\rm eff}(q^2) = \alpha(0)\left(\frac{1}{1-\sum_f \Pi_f(q^2)} \mp \frac{q^2}{q^2-\Lambda_\pm^2}\right), \tag{10}$$

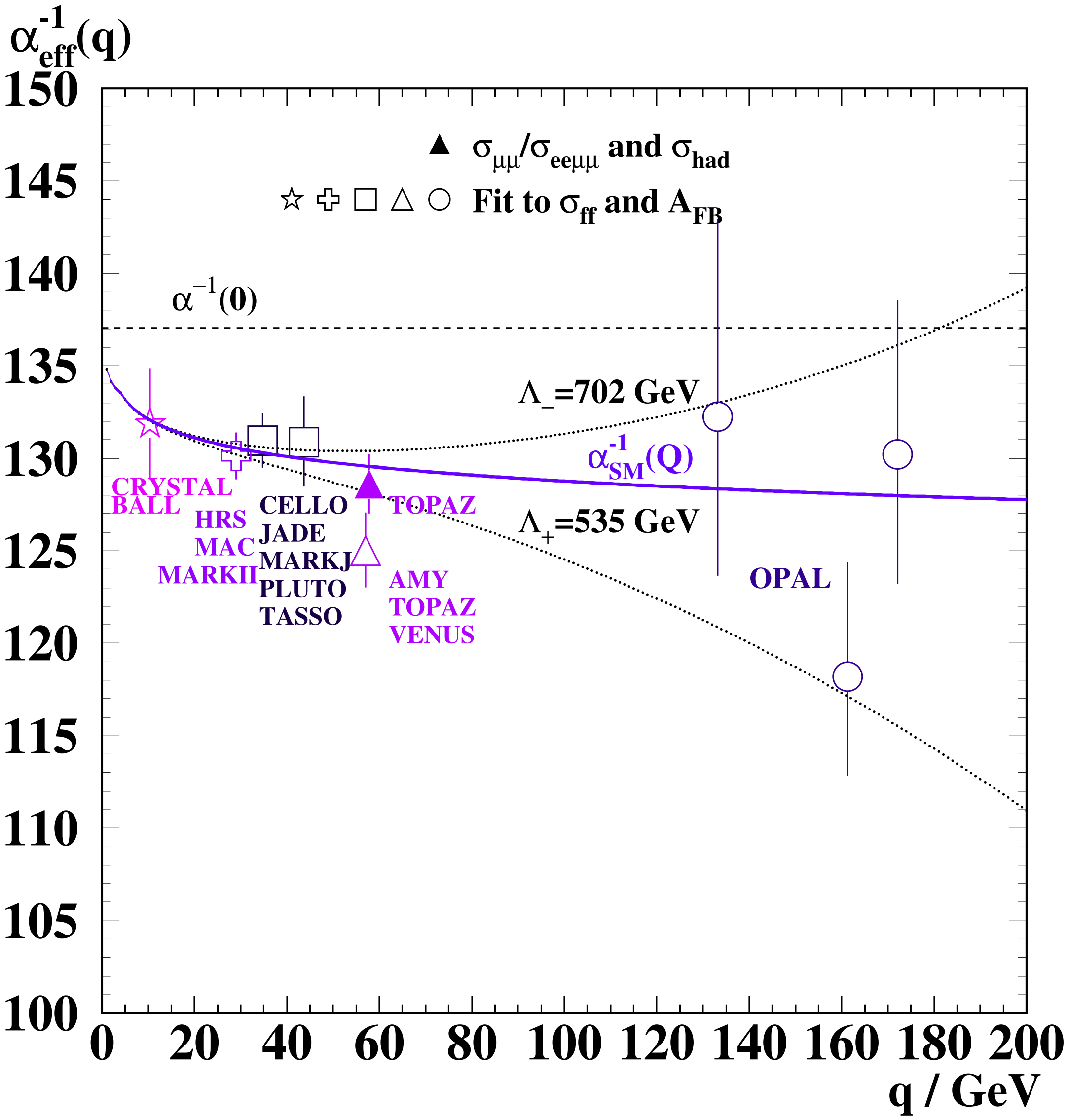

Figure 2: *Measurements of* $\alpha_{\rm eff}^{-1}(q^2)$ *for the photon propagator, as defined in the text. The values below the Z are taken to be the same as in Fig. 1, since effects of weak coupling constants are small. The OPAL data have been refit using Standard Model predictions for the weak coupling. The indicated limits on the cut-off parameter* Λ *are derived from the OPAL data, only.*

where the two cut-off parameters Λ_- and Λ_+ are for negative and positive interference with the 'normal' photon, respectively.

Fit results to $\alpha_{\rm em}(q^2)$ can then be interpreted in terms of limits on these Λ parameters. It has to be taken care of the fact, that the effective $\alpha_{\rm eff}^{-1}(q^2)$ should *not* enter in the determination of $\sin^2\theta_{\rm w}(q^2)$, where rather the undisturbed pure photon coupling $\alpha_{\rm em}(q^2)$ is relevant. This means that the electroweak fit to LEP-1 data [6] cannot be employed here, since it deduces most of its information on $\alpha_{\rm em}(q^2)$ from the measurement of $\sin^2\theta_{\rm w}(q^2)$. Also the above fits to OPAL data [8] at $\sqrt{s}$=130–172 GeV include the influence of $\alpha_{\rm em}(q^2)$ on $\sin^2\theta_{\rm w}(q^2)$, though above the Z^0 peak the influence of the weak coupling is reduced.

We have therefore repeated the fit to OPAL data at each $\sqrt{s}$, modifying ZFITTER so that $\alpha_{\rm eff}^{-1}(q^2)$, as defined above only enters in the photon coupling in the improved Born amplitude for ee$\rightarrow$ff . All weak couplings are calculated employing the Standard Model prediction for the running $\alpha_{\rm em}$. The result of these fits is given in Table 3. The results are similar to Table 1, with the exception of the value for 133 GeV, where the influence of the weak coupling is still large.

Table 3: *Results of fits for $\alpha_{\rm eff}^{-1}(q^2)$ to OPAL data between 130–172 GeV.*

	Fit		Standard Model	
$E_{\rm cm}$ (GeV)	$1/\alpha_{\rm em}$	$\chi^2/d.o.f.$	$1/\alpha_{\rm em}$	$\chi^2/d.o.f.$
133.17	$132.3^{+10.7}_{-8.6}$	6.0/4	128.3	6.2/5
161.34	$118.2^{+6.2}_{-5.4}$	3.0/4	128.1	5.3/5
172.12	$130.2^{+8.3}_{-7.0}$	0.8/4	128.0	0.9/5

A combined fit to all three centre-of-mass energies leads to 95% lower limits of

$$\begin{aligned}\Lambda_- &> 702\ \text{GeV}\\ \Lambda_+ &> 535\ \text{GeV}.\end{aligned}$$

They are consistent with limits obtained from the same OPAL data in a contact interaction approach [8], which parametrize a new effective interaction with an energy scale Λ at a coupling constant $\alpha = 1$. Scaling the OPAL limits for Vector-Vector contact interaction by the appropriate factor $1/\sqrt{\alpha_{\rm em}(0)} = 11.7$, leads to corresponding limits of $\Lambda_- > 660$ GeV and $\Lambda_+ > 530$ GeV, which are valid in the limit $\Lambda^2 \gg s$. The limits obtained are about a factor of two better than previous limits [4] from PEP, PETRA and TRISTAN data. These can be used to put constraints on new physics, contributing to loop corrections of the photon propagator, for instance effects due to an amomalous magnetic moment of the W boson [16], its possible SUSY partners, or other new heavy charged gauge bosons.

The running of $\alpha_{\rm eff}^{-1}(q^2)$ that would be seen for Λ values at their respective lower limit is indicated in Fig. 2, together with the fit results of $\alpha_{\rm eff}^{-1}(q^2)$ from the OPAL data. The values of $\alpha_{\rm em}(q^2)$ from measurements below the Z are also shown. Since the effect of weak interactions is negligible at low energies, they could still improve the limits in a combined fit, though the main constraint comes from the highest energy points. Such a combination, however, would need a study of correlated systematic errors between different storage rings.

A further improvement of the limits is obtained by combining the measurements of fermion-pair production above the Z^0 from all LEP collaborations. The results of the combination of published [8, 17, 18] and preliminary [19] cross-sections for hadron and lepton-pair production at four different centre-of-mass energies is listed in Table 4. All cross-sections have been corrected to a unified cut of $\sqrt{s'}/\sqrt{s} > 0.85$ before the average, and the effect of interference between intial- and final state radiation has been subtracted from the result, using correctiond determined in [8].

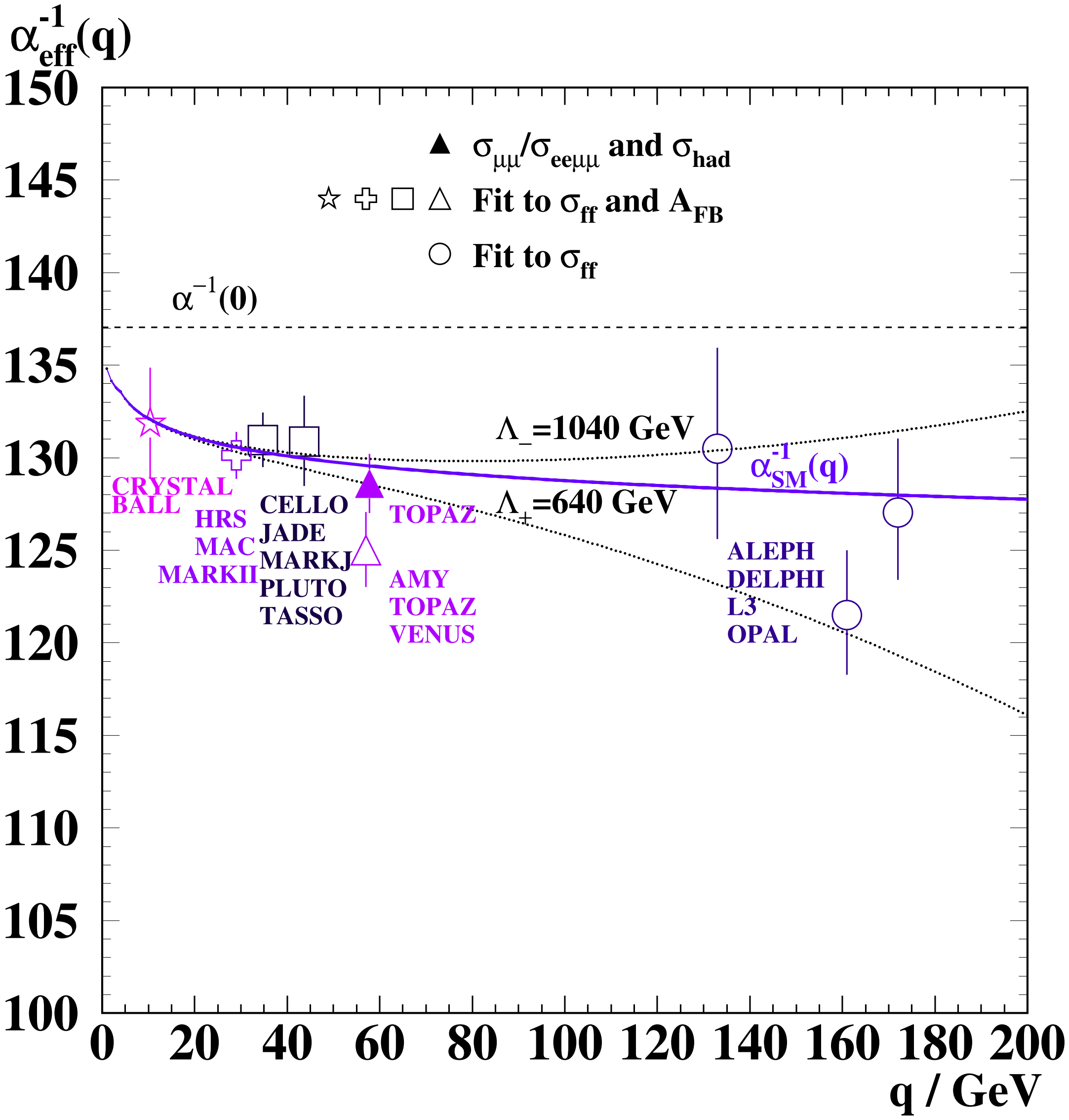

Figure 3: *Measurements of $\alpha^{-1}_{\rm eff}(q^2)$ for the photon propagator, as described in Fig. 2. The indicated limits on the cut-off parameter Λ are derived from a combined fit to fermion-pair cross-sections measured by the four LEP collaborations at centre-of-mass energies between 130 and 172 GeV.*

Table 4: *Average of LEP results [8, 17, 18, 19] for fermion-pair cross-sections between 130–172 GeV, for $\sqrt{s'}/\sqrt{s} > 0.85$.*

$E_{\rm cm}$ (GeV)	$\sigma_{\rm had}$ (pb) measured	SM	$\sigma_{\mu^+\mu^-}$ (pb) measured	SM	$\sigma_{\tau\tau}$ (pb) measured	SM
130.0	78.5±3.1	83.5	9.1±1.0	8.49	9.7±1.3	8.49
136.0	66.1±2.8	67.1	7.65±0.95	7.32	6.7±1.1	7.32
161.0	36.9±1.1	35.6	4.80±0.41	4.64	5.63±0.55	4.64
172.0	30.0±1.0	29.0	3.74±0.35	3.96	3.77±0.44	3.96

Table 5: *Results of fits for $\alpha_{\mathrm{eff}}^{-1}(q^2)$ to combined measurements of fermion-pair cross-sections at LEP between 130–172 GeV.*

	Fit		Standard Model	
E_{cm} (GeV)	$1/\alpha_{\mathrm{em}}$	$\chi^2/d.o.f.$	$1/\alpha_{\mathrm{em}}$	$\chi^2/d.o.f.$
133.0	$130.5^{+5.5}_{-4.9}$	4.2/2	128.3	4.3/3
161.0	$121.5^{+3.5}_{-3.2}$	1.5/2	128.1	4.8/3
172.0	$127.0^{+4.0}_{-3.6}$	1.6/2	128.0	1.7/3

The average cross-sections were subjected to the same fit procedure as described above, taking into account all correlations between them. They agree with the Standard Model prediction with a χ^2/d.o.f. = 11.7/12. From the χ^2 curve as a function of Λ, one obtains 95% lower limits of

$$\begin{aligned} \Lambda_- &> 1040 \ \ \mathrm{GeV} \\ \Lambda_+ &> 640 \ \ \mathrm{GeV}, \end{aligned}$$

which are indicated in Fig. 3. The fit results at the single centre-of-mass energies are given in Table 5 and also displayed in Fig. 3.

5 Summary

A comparison between the predicted and measured q^2 dependence of α_{em} over the wide range of q^2=(10 GeV)^2–(157 GeV)^2 has been performed for the first time. The measured values are consistent with the Standard Model expectation.

A refit to OPAL cross-sections and asymmetries at the highest centre-of-mass energies without imposing the Standard Model relation between the weak and electromagnetic couplings, leads to 95% lower limits for the QED cut-off parameter Λ of $\Lambda_- > 702$ GeV and $\Lambda_+ > 535$ GeV, about a factor of two better than previous limits. A fit to combined LEP data, using only cross-sections, allows to set limits of $\Lambda_- > 1040$ GeV and $\Lambda_+ > 640$ GeV, respectively.

At the end of LEP-2, and combining all four LEP-experiments, one can hope to reach a relative error of 0.5% on $\alpha_{\mathrm{em}}(m_{\mathrm{Z}})$ from LEP-2 alone, essentially uncorrelated with the Higgs mass. Including the complete set of LEP-2 data in electroweak fits, not constraining α_{em} to its theoretical prediction, a considerable reduction of the overall error on α_{em} and its correlation with the error of m_{H} can be anticipated.

The α_{em} measurements from LEP-2 can be performed without assuming the Standard Model running of α_{em} at low q^2, whose uncertainty dominates the error of the present theoretical prediction for $1/\alpha_{\mathrm{em}}(m_{\mathrm{Z}})$. The combined precision of a fit to LEP-1 and LEP-2 data will, however, not quite reach the current theoretical precision of 0.09, but may lead to a small improvement of its error.

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