BI-TP 97/25

hep-ph/9707447

Generalized Vector Dominance and low x inelastic electron-proton scattering*

D. Schildknecht and H. Spiesberger

Fakultät für Physik, Universität Bielefeld, D-33501 Bielefeld

ABSTRACT

The HERA data on inelastic lepton proton scattering for values of the Bjorken scaling variable $x \lesssim 0.05$ are confronted with predictions based on Generalized Vector Dominance. Good agreement between theory and experiment is found over the full kinematic range of the squared four-momentum-transfer, Q^2, from $Q^2 = 0$ (photoproduction) to $Q^2 \lesssim 350$ GeV^2.

* Supported by the Bundesministerium für Bildung und Forschung, Bonn, Germany, Contract 05 7BI92P (9) and the EC-network contract CHRX-CT94-0579.

In the 1972 paper by Sakurai and one of the present authors [1] it was conjectured that the large cross section then observed [2] in deep inelastic electron scattering at large values of ω' (i.e., for small values of $x \simeq Q^2/W^2$ in present-day notation) was caused by contributions from vector states more massive than ρ^0, ω, ϕ to the imaginary part of the virtual Compton forward scattering amplitude (compare Fig. 1). The dominant role of ρ^0, ω, ϕ, which saturate the imaginary part of the forward Compton amplitude in photoproduction at the level of 78 % [1], with increasing spacelike Q^2 was expected to be rapidly taken over by more massive states coupled to the photon. It was shown, that the deep inelastic electron scattering data available at the time in the region of $x \lesssim 0.15$ were in accord with this hypothesis of Generalized Vector Dominance. If this hypothesis were viable, it was argued, that in generalization of the role of the low-lying vector mesons ρ^0, ω, ϕ, in (real) photoproduction, high-mass states should be diffractively produced in deep inelastic scattering at small x. Accordingly it was suggested to look for the production of such states and to compare their properties with the ones of the final state in e^+e^- annihilation experiments still in the state of planning in 1972. Moreover, as a further test of this picture for deep inelastic scattering at small x, shadowing in the scattering from complex nuclei, observed in photoproduction at the time, was predicted [3] to persist at small x in deep inelastic scattering.

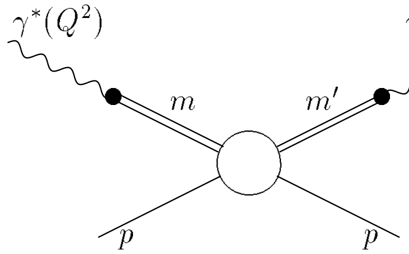

Figure 1: *The virtual Compton forward scattering amplitude in Generalized Vector Dominance. The spacelike photon ($Q^2 \geq 0$) virtually dissociates into $q\bar{q}$ states of masses m, m' which undergo forward scattering on the target proton.*

Shadowing in electron scattering, after many years of confusion, was established [4] in recent years in semi-quantitative agreement with expectation [5]. Moreover, large-rapidity gap events with typically diffractive features and masses much higher than the masses of ρ^0, ω, ϕ have been observed [6] at HERA. Encouraged by these experimental results, which are in qualitative accord with expectations based on Generalized Vector Dominance, in the present paper we examine the question in how far predictions based on Generalized Vector Dominance are in quantitative agreement with the results on the proton structure function at low x.

More specifically, it is the purpose of the present note to show that the simple 1972 ansatz [1], appropriately generalized to take care of the rise of (hadronic) cross sections with energy, not yet known in 1972, yields predictions for the low x proton structure function which are in good agreement with the experimental data from HERA [7].

The starting point of Generalized Vector Dominance for a theoretical description of the transverse photon absorption cross section $\sigma_T(W^2, Q^2)$ at small values of x is the mass dispersion relation [1]

$$\sigma_T(W^2, Q^2) = \int dm^2 \int dm'^2 \frac{\tilde{\rho}_T(W^2; m^2, m'^2) m^2 m'^2}{(m^2 + Q^2)(m'^2 + Q^2)}, \tag{1}$$

where the double spectral weight function $\tilde{\rho}_T(W^2; m^2, m'^2)$ is proportional to the imaginary part of the forward amplitude for $V + p \to V' + p$ where V and V' are states of masses m and m' coupled to the photon. In (1), we have adopted the usual conventions, in which the total center-of-mass energy of the virtual-photon-proton system, $\gamma^* p$, is denoted by W, the squared four-momentum transfer to the proton by Q^2 and the Bjorken scaling variable by $x = Q^2/(W^2 - M_p^2 + Q^2) \simeq Q^2/W^2$ for $W^2 \gg Q^2$. If the diagonal approximation

$$\tilde{\rho}_T(W^2; m^2, m'^2) = \rho_T(W^2, m^2)\delta(m^2 - m'^2) \tag{2}$$

is adopted, (1) takes the particularly simple form

$$\sigma_T(W^2, Q^2) = \int_{m_0^2} dm^2 \frac{\rho_T(W^2, m^2) m^4}{(m^2 + Q^2)^2}, \tag{3}$$

where the threshold mass, m_0, is to be identified with the mass scale at which the cross section for the process $e^+e^- \to$ hadrons starts to become appreciable. As depicted in Fig. 1, the spectral weight function $\rho_T(W^2, m^2)$ in (3) is proportional to

i) the transition strength of a timelike photon to the hadronic state of mass m as observed in e^+e^- annihilation at the energy $\sqrt{s} \equiv m$, and

ii) the imaginary part of the forward scattering amplitude of this state of mass m on the nucleon.

For real photons, $Q^2 = 0$, the cross section $\sigma_T(W^2, Q^2 = 0) \equiv \sigma_{\gamma p}$ in the representation (3) is almost saturated (up to $\simeq 78\%$ [1]) by contributions from the discrete vector meson states, ρ^0, ω and ϕ. For spacelike Q^2, the contribution of ρ^0, ω, ϕ becomes rapidly unimportant, and their role, according to Generalized Vector Dominance, at small x is taken over by a sum of those more massive states which are produced in e^+e^- annihilation experiments beyond ρ^0, ω, ϕ at the energy of $\sqrt{s} \equiv m$.

Before elaborating on the ansatz (3) let us also quote its generalization [1] to the longitudinal photon absorption cross section, σ_L. A priori, the ratio of high-mass longitudinal-to-transverse forward scattering amplitudes for hadronic states of mass m evolving from $q\bar{q}$ vector states is unknown and has to be left open by introducing a parameter ξ for this ratio. Moreover, generalizing ρ^0 dominance, where the coupling of the ρ^0 to a conserved source implies a factor Q^2/m_ρ^2 for longitudinal photons [8], a factor Q^2/m^2 is introduced in the ansatz for σ_L. It assures the required vanishing of σ_L for $Q^2 \to 0$. Accordingly,

$$\sigma_L(W^2, Q^2) = \int_{m_0^2} dm^2 \frac{\xi \rho_T(W^2, m^2) m^4}{(m^2+Q^2)^2} \frac{Q^2}{m^2}. \tag{4}$$

Let us note that, first of all, the diagonal approximation, and secondly the equality of the spectral weight functions in (3) and (4) apart from the factors Q^2/m^2, and ξ, as well as the constancy of the factor ξ, constitute fairly drastic assumptions in view of the fact that with increasing Q^2 higher and higher masses, m, will dominate the integrals in (3) and (4).

The underlying assumptions can be empirically tested by analyzing the observed diffractive production (large rapidity gap events) as a function of Q^2 and m^2 at momentum transfer $t \to 0$. We note that ρ^0 production already, i.e. the contribution due to $m = m_\rho$ in (3) and (4), at large Q^2 does not exactly follow [9] the ρ^0-dominance form of the diagonal approximation. This in itself does not contradict the approximate validity of (3) and (4), as the ρ^0 contribution becomes rapidly unimportant as soon as $Q^2 \gg m_\rho^2$, but it may indicate that the ansatz (3) and (4) needs refinements in the future, e.g. by allowing for a mass dependence of ξ, or by modifying the factor Q^2/m^2 in the region of $Q^2 \gg m^2$, or by allowing for off-diagonal terms [10] according to (1). As for the time being, we will see that the gross features of the HERA data on the sum of σ_T and σ_L, i.e. on F_2, will be adequately represented by the very simple forms (3) and (4), once ρ_T will have been specified.

In the present paper, we concentrate on deep inelastic scattering in the high-energy region of $W \gtrsim 30$ GeV with Q^2 between zero and an upper limit determined by the restriction to sufficiently low values of $x \lesssim 0.05$, where the dynamical assumptions of the present work are expected to hold. In the energy range of $W \gtrsim 30$ GeV hadronic cross sections as well as photoproduction ($Q^2 = 0$) rise with increasing energy. Accordingly, we adopt a logarithmic rise[1] of the imaginary part of the forward scattering amplitude of the state of mass m on the proton in addition to a $1/m^2$ fall from dimensional analysis[2], and an additional $1/m^2$ decrease from e^+e^- annihilation due to the couplings of the photon to the initial and final states in Fig. 1, i.e.

$$\rho_T(W^2, m^2) = N \frac{\ln(W^2/am^2)}{m^4}. \tag{5}$$

The parameter N contains the normalization of the cross section for $e^+e^- \to$ hadrons and determines the overall normalization of the cross section for the scattering of the state of mass m on the proton. The parameter a sets the scale for the logarithmic W dependence of

[1] A power behavior [11], $\rho_T \propto (W^2)^\alpha$, would be possible as well. However, it turns out that when fitting the resulting expression for F_2 to the HERA data, only data in a restricted range of x and Q^2 values can be described.

[2] Bjorken [12] conjectures that this $1/m^2$ fall may be the consequence of the fact that only those $q\bar{q}$ configurations interact with the proton which are properly aligned in the direction of the incident virtual photon in the photon-proton rest frame. Alternatively, off-diagonal transitions implying an effective $1/m^2$ behavior have been suggested [10].

the forward scattering amplitude for the scattering of the state of mass m. Obviously, the scale of the logarithmic W dependence need not coincide with the mass m of the hadronic vector state being scattered. We will assume that a is constant, independent of m as well as independent of the quark flavor in the state of mass m.

The transverse photon absorption cross section, σ_T, then becomes

$$\sigma_T(W^2, Q^2) = N \int_{m_0^2}^{\infty} dm^2 \frac{\ln(W^2/am^2)}{(m^2+Q^2)^2}. \tag{6}$$

The ansatz (6) again contains a fairly crude approximation insofar as we do not separate the contributions due to charm, i.e. $c\bar{c}$ states with a higher threshold of $\overline{m}_0^2 \simeq (3\,\mathrm{GeV})^2$, from the contributions due to $q\bar{q}$ configurations of light quarks, "dual" to ρ^0, ω, ϕ, with thresholds m_0^2 of the order of m_ρ^2 and m_ϕ^2. From (6) we obtain

$$\sigma_T(W^2, Q^2) = N \left[\frac{1}{Q^2+m_0^2} \ln \frac{W^2}{am_0^2} - \frac{1}{Q^2} \ln \left(1 + \frac{Q^2}{m_0^2} \right) \right]. \tag{7}$$

We also note the limits of photoproduction, $Q^2 \to 0$, in (7)

$$\sigma_T(W^2, Q^2 \to 0) = \sigma_{\gamma p} = \frac{N}{m_0^2} \left(\ln \frac{W^2}{am_0^2} - 1 \right), \tag{8}$$

and of deep inelastic scattering, $Q^2 \gg m_0^2$,

$$\sigma_T(W^2, Q^2 \gg m_0^2) \simeq \frac{N}{Q^2} \ln \frac{W^2}{aQ^2}. \tag{9}$$

According to (8), the normalization of the experimental photoproduction cross section determines the ratio N/m_0^2, while $a \cdot m_0^2$ is determined by the scale of its energy dependence. The threshold mass m_0 being essentially fixed by e^+e^- annihilation into hadrons, the parameter a describes the "hadronlike" energy dependence of photoproduction.

The longitudinal photon absorption cross section, $\sigma_L(W^2, Q^2)$, according to (4) and (5) becomes

$$\begin{aligned} \sigma_L(W^2, Q^2) \;=\; & N\xi \Bigg[\left(\frac{1}{Q^2} \ln \left(1 + \frac{Q^2}{m_0^2} \right) - \frac{1}{Q^2+m_0^2} \right) \ln \frac{W^2}{am_0^2} \\ & + \frac{1}{Q^2} \left(\ln \left(1 + \frac{Q^2}{m_0^2} \right) + \mathrm{Li}_2 \left(- \frac{Q^2}{m_0^2} \right) \right) \Bigg]. \end{aligned} \tag{10}$$

Li_2 denotes the dilogarithm defined by $\mathrm{Li}_2(z) = -\int_0^z dt \ln(1-t)/t$. The longitudinal cross section (10) vanishes as $Q^2 \ln Q^2$ for $Q^2 \to 0$, while for $Q^2 \gg m_0^2$,

$$\begin{aligned} \sigma_L(W^2, Q^2 \gg m_0^2) \;=\; & \frac{N\xi}{Q^2} \Bigg\{ \left(\ln \frac{Q^2}{m_0^2} - 1 \right) \ln \frac{W^2}{aQ^2} \\ & - \frac{1}{2} \ln^2 \frac{Q^2}{m_0^2} - \frac{\pi^2}{6} \Bigg\}, \end{aligned} \tag{11}$$

where the asymptotic formula $\lim_{z\to\infty} \mathrm{Li}_2(-z) = -\frac{1}{2}\ln^2 z - \frac{\pi^2}{6} + O(z)$ was used.

Before turning to the comparison with experiment, we recall the connection of σ_T and σ_L with the proton structure function, $F_2(W^2, Q^2)$. For $x \simeq Q^2/W^2 \ll 1$ it reads

$$F_2(W^2, Q^2) \simeq \frac{Q^2}{4\pi^2\alpha}(\sigma_T + \sigma_L) \equiv \frac{Q^2}{4\pi^2\alpha}\sigma_{\gamma^* p}. \tag{12}$$

Following common practice, in (12) we have introduced the virtual photoproduction cross section,

$$\sigma_{\gamma^* p} \equiv \sigma_T + \sigma_L. \tag{13}$$

The explicit form of $F_2(W^2, Q^2)$ is easily obtained by substituting σ_T and σ_L from (7) and (10), respectively, into (12). Its asymptotic form for $Q^2 \gg m_0^2$, inserting (9) and (11) into (12), reads

$$F_2(W^2, Q^2 \gg m_0^2) = \frac{N}{4\pi^2\alpha}\left(\ln\frac{1}{ax}\right)\left[1 + \xi\left(\ln\frac{Q^2}{m_0^2} - 1 - \frac{\frac{1}{2}\ln^2\frac{Q^2}{m_0^2} + \frac{\pi^2}{6}}{\ln\frac{1}{ax}}\right)\right], \tag{14}$$

where the scaling variable $x \simeq Q^2/W^2$ was substituted. The transverse part of F_2, obtained by putting $\sigma_L = 0$ in (12) and $\xi = 0$ in (14), shows scaling behavior for Q^2 sufficiently large, $Q^2 \gg m_0^2$. The approach to scaling according to (7) depends on the scale m_0^2 and will be less fast, if our treatment will be refined by allowing for quark-flavor dependent threshold masses replacing the single scale, m_0. We note that the strict absence of scaling violation in the transverse part of F_2 for asymptotic values of Q^2 is related to the simplified input assumption of a $1/m^2$ decrease of e^+e^- annihilation as a function of the e^+e^- energy, $\sqrt{s} \equiv m^2$, as adopted in (5). If this assumption is dropped by allowing for realistic scaling violations in $R \equiv \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$, logarithmic scaling violations will also be induced in the transverse part of F_2. For the time being, we keep our simplified form (3), (4) with (5), also in view of the fact that experimental data separating σ_T and σ_L are not available so far in the region of small x.

For the comparison with the data from H1 and ZEUS from HERA, we have chosen for the free parameters the values of

$$\begin{aligned} N &= 5.13 \cdot 4\pi^2\alpha = 1.48 \\ m_0^2 &= 0.89 \text{ GeV}^2 \\ \xi &= 0.171 \\ a &= 15.1 \end{aligned} \tag{15}$$

which are obtained from fitting the H1 and ZEUS data for F_2 [7] in the range of $Q^2 < 200$ GeV^2. The value of m_0, obtained from the fit does not deviate much from the ρ^0 mass, as expected from its meaning as an effective threshold of e^+e^- annihilation. The fact that m_0^2 is nevertheless slightly larger than $m_\rho^2 \simeq 0.59$ GeV^2 is presumably due to the simplification of not having separated the charm contribution to σ_T and σ_L in (3) and (4) with its substantially higher threshold mass from the contribution of the light quarks. The value of the longitudinal to transverse ratio, ξ, in (15) seems reasonable.

Fig. 2 shows remarkably good agreement of $\sigma_{\gamma^* p}$ from (7), (10), (13), and (15) with the HERA data from H1 and ZEUS over the full Q^2 range from photoproduction to $Q^2 \simeq 350$ GeV^2, and energies W from $W \simeq 60$ GeV to $W \simeq 245$ GeV, corresponding to values of the scaling variable of $x \lesssim 0.05$. In Fig. 3, we show a comparison between Generalized Vector Dominance and the HERA data as a function of W for a series of values of Q^2. Fig. 3 obviously illustrates what happens if Fig. 2 is cut in a direction perpendicular to the abscissa at selected values of Q^2. Comparing the slope of the theoretical predictions at fixed W for different values of Q^2, one observes a rising slope in this log-log plot. The rising slope originates from the change in scale of the W dependence, $a \cdot m_0^2$ from photoproduction effectively being replaced by $a \cdot Q^2$ for $Q^2 \gg m_0^2$, in combination with the increasing importance of σ_L. The increase towards low x becomes more dramatic when plotting the structure function F_2 on a linear scale against log x, compare Fig. 4. This Figure explicitly also shows that our theoretical prediction for the transverse part of the structure function, $F_{2,T}(W^2, Q^2)$, defined by ignoring σ_L in (12), has reached its scaling limit for $Q^2 \gtrsim 12$ GeV^2. The rise of $F_2(W^2, Q^2)$ with increasing Q^2 for $Q^2 \gtrsim 12$ GeV^2, is due to the influence of σ_L, as discussed in connection with (14).

Refinements of the present work immediately suggest themselves. The charm quark contribution should be treated separately in the basic ansatz, and the low energy behavior of photoproduction and electron scattering has to be incorporated. Moreover, a detailed analysis of the relation between $\sigma_{\gamma^* p}$ and diffractive production at $t = 0$ is to be carried out, improving and elaborating upon previous suggestions [13, 14].

Various parametrizations of the experimental data on low x deep inelastic scattering, including photoproduction, exist in the literature, either based on [15, 16] modifications of Regge theory or on a combination [17] of ρ^0, ω, ϕ dominance with the parton-model approach. The fit of the data presented in [18] is of interest in the context of the present paper, as logarithmic Q^2 and x dependences only are employed in the fit.

In the present work we have shown that the data on low x deep inelastic scattering are consistent with a picture in which the role of ρ^0, ω, ϕ in photoproduction is extended to more massive vector states in deep inelastic scattering, coupled to the photon with a strength known from e^+e^- annihilation. In this sense there is continuity in the underlying dynamics. With increasing Q^2, the scale in the logarithmic W dependence of photoproduction becomes gradually replaced by Q^2 as soon as Q^2 becomes large compared with the thresholds of light quark and charm quark production in e^+e^- annihilation. In addition, a scaling-violating longitudinal contribution to F_2 is turned on. While details of this picture may have to be refined, the principal dynamical ansatz, resting on the connection between the virtual photon absorption cross section, $\sigma_{\gamma^* p}$, and e^+e^- annihilation into hadronic $q\bar{q}$ states with subsequent diffractive forward scattering on the proton, as suggested twenty-five years ago, will be likely to stand the test of time.

Acknowledgment
We thank Dieter Haidt, Peter Landshoff and Günter Wolf for stimulating discussions on the HERA results. We also thank the H1 and ZEUS collaborations for providing us with their most recent F_2 data. One of us (D.S.) started to work on the subject matter of the present paper while visiting the Max Planck Institut für Physik in Munich in summer 1995. It is a pleasure to thank Wolfhart Zimmermann for hospitality and Leo Stodolsky for useful discussions on the subject of this paper.

References

[1] J.J. Sakurai and D. Schildknecht, Phys. Lett. 40B (1972) 121;
B. Gorczyca and D. Schildknecht, Phys. Lett. 47B (1973) 71.

[2] E.D. Bloom et al., Phys. Rev. Lett. 23 (1969) 930.

[3] D. Schildknecht, Nucl. Phys. B66 (1973) 398.

[4] EM Collaboration, J. Achman et al., Phys. Lett. B202 (1988) 603.

[5] C.D. Bilchak, D. Schildknecht and J.D. Stroughair, Phys. Lett. B214 (1988) 441; Phys. Lett. B233 (1989) 461.

[6] H1 Collaboration, T. Ahmed et al., Nucl. Phys. B429 (1994) 477;
ZEUS Collaboration, M. Derrick et al., Phys. Lett. B315 (1993) 481.

[7] H1 Collaboration, S. Aid et al., Nucl. Phys. B470 (1996) 3;
H1 Collaboration, C. Adloff et al., DESY 97-042, subm. to Nucl. Phys. B;
ZEUS Collaboration, M. Derrick et al., Z. Phys. C72 (1996) 399;
ZEUS Collaboration, J. Breitweg et al., DESY 97-135, subm. to Phys. Lett. B.

[8] H. Fraas and D. Schildknecht, Nucl. Phys. B14 (1969) 543;
J.J. Sakurai, Phys. Rev. Lett. 22 (1969) 981.

[9] H1 Collaboration, S. Aid et al., Nucl. Phys. B468 (1996) 3;
ZEUS Collaboration, M. Derrick et al., Phys. Lett. B356 (1995) 601.

[10] H. Fraas, B. Read and D. Schildknecht, Nucl. Phys. B86 (1975) 346;
R. Devenish and D. Schildknecht, Phys. Rev. D14 (1976) 93.

[11] A. Donnachie and P.V. Landshoff, Nucl. Phys. B244 (1984) 322.

[12] J.D. Bjorken, SLAC-PUB-95-7096, presented at Conference on Fundamental Interactions of Elementary Particles, Moscow, Russia, 23-26 October 1995, hep-ph/9601363.

[13] L. Stodolsky, Phys. Lett. B325 (1994) 505.

[14] D. Schildknecht, BI-TP 94/38, Contribution to the XXVII International Conference on High Energy Physics, Glasgow, Scotland, U.K., 20-27 July 1994, unpublished.

[15] H. Abramowicz, E.M. Levin, A. Levy and U. Maor, Phys. Lett. B269 (1991) 465.

[16] A. Donnachie and P.V. Landshoff, Z. Phys. C61 (1994) 139.

[17] B. Badełek and J. Kwieciński, Phys. Lett. B295 (1992) 263.

[18] W. Buchmüller and D. Haidt, DESY 96-061, hep-ph/9605428;
D. Haidt, Contribution to the 5th International Workshop on Deep Inelastic Scattering and QCD, Chicago, April 14-18, 1997.

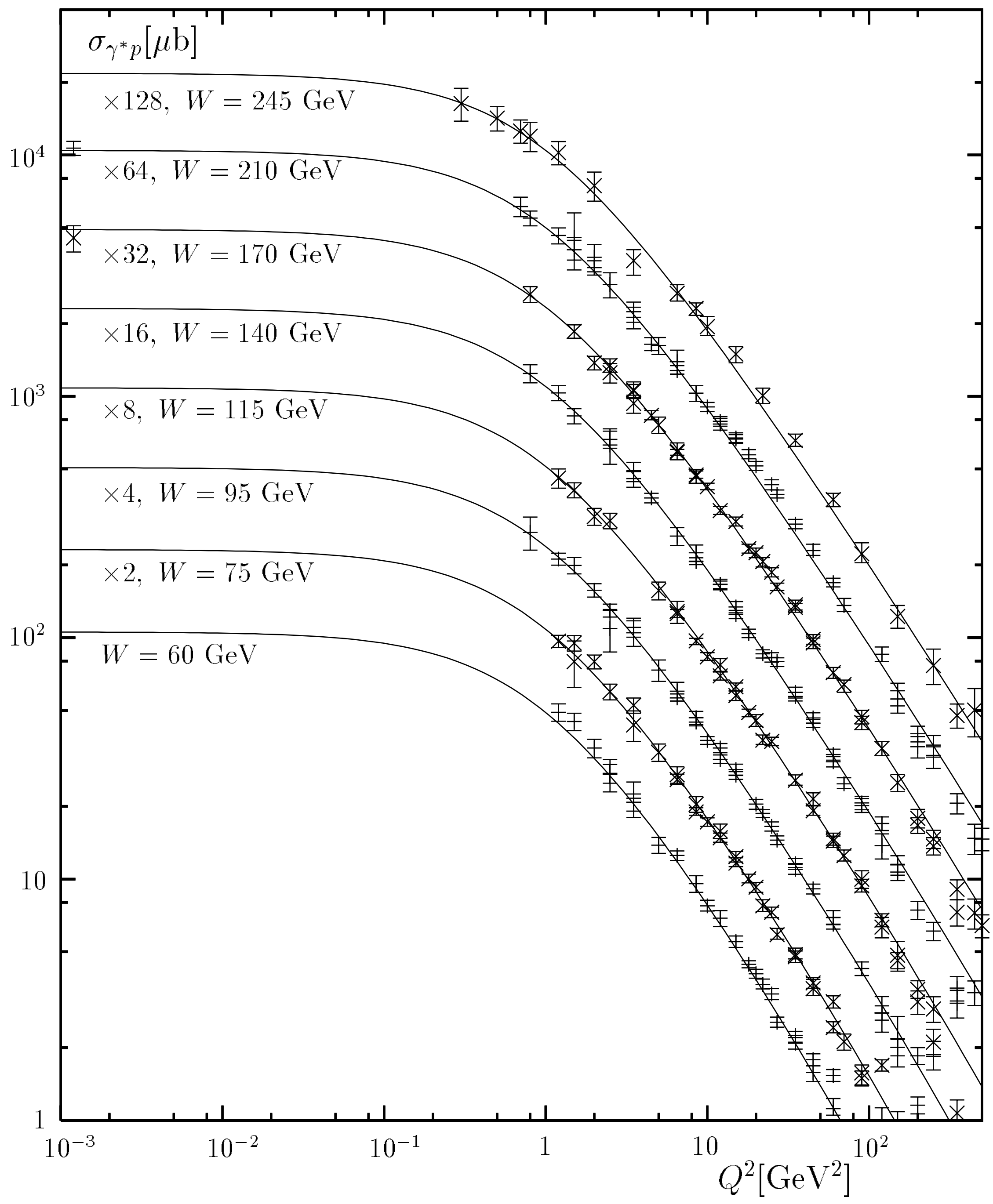

Figure 2: *Generalized Vector Dominance prediction for $\sigma_{\gamma^* p}$ based on (7), (10), (13) and (15) compared with the experimental data from the H1 and ZEUS collaborations at HERA.*

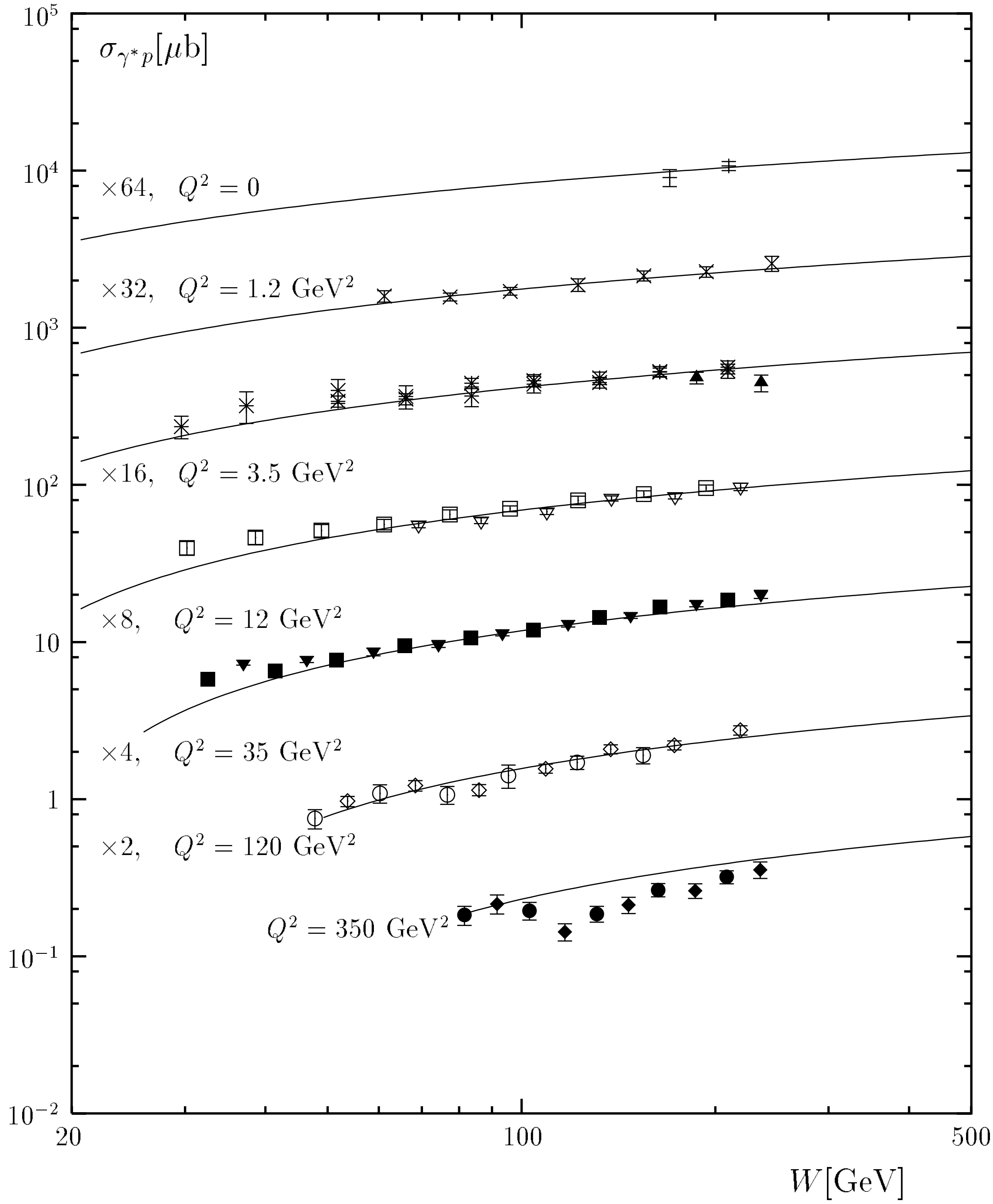

Figure 3: *Same as Fig. 2, but for various values of Q^2 as a function of W.*

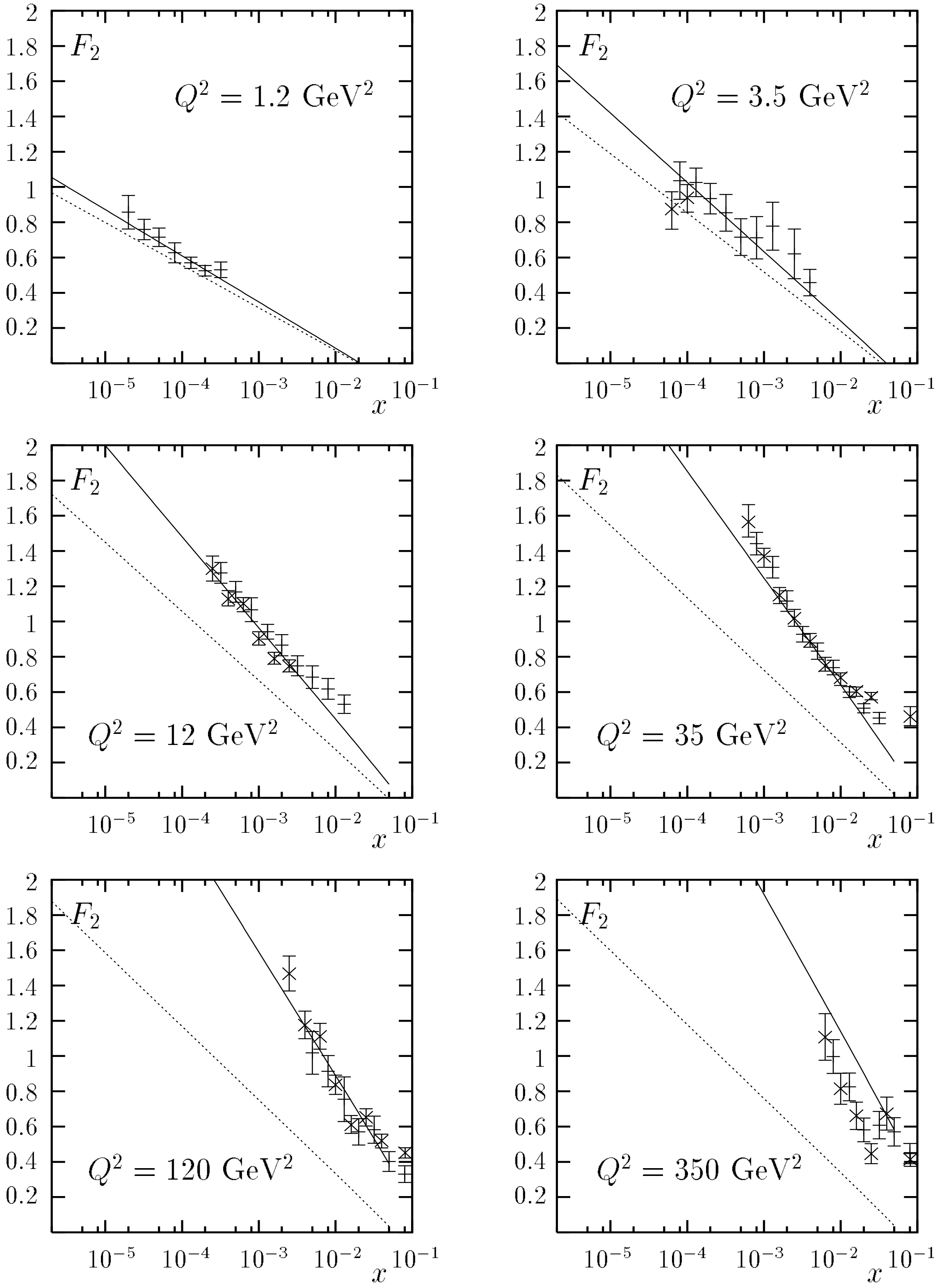

Figure 4: *Generalized Vector Dominance prediction for F_2 as a function of x compared with the HERA data for different values of Q^2. The dotted line shows the contribution to F_2 due to transverse virtual photons ($\xi = 0$).*