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Dilatonic Dark Matter and Unified Cosmology
– A New Paradigm –

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Abstract

We study the possibility that the dilaton – the fundamental scalar field which exists in all the existing unified field theories including the superstring– plays the role of the dark matter of the universe. We find that the condition for the dilaton to be the dark matter strongly restricts its mass to be around 0.5 keV or 171 MeV. For the other mass ranges, the dilaton either undercloses or else overcloses the universe. The 0.5 keV dilaton has the free-streaming distance of about 1.4 Mpc and becomes an excellent candidate of a warm dark matter, while the 171 MeV one has the free-streaming distance of about 12 pc and becomes a cold dark matter. We discuss the possible ways to detect the dilaton experimentally.

I. INTRODUCTION

The standard big bang cosmology has been very successful in many ways. For example it naturally explains the Hubble expansion, the cosmic microwave background, and the primordial nuclear synthesis of the universe. But at a deeper level the model also raises more challenges. Is the inflation really necessary, and if so, what drives the inflation? What kind of dark matter, and how much of it, does the universe contain? How was the observed structure of the universe formed, and how did the dark matter affect this? There may be two attitudes that one can adopt in dealing with these challenges. One is a phenomenological approach. Here one introduces a minimum number of new parameters (a cosmological constant, for example) to the standard model, and try to obtain a best model which can fit as many data as possible. This is indeed the popular approach at present. The other one is a theoretical approach. Here one tries to construct a model which is most appealing from the logical point of view, based on the fundamental principles. For example, if one believes in the unification of all interactions, one may ask what is the best cosmological model that one can obtain from the unified theory. In this talk we will take the second attitude and discuss how the unification of all interactions could modify the big bang cosmology. The reason for this is partly because few people in cosmology takes this attitude, and partly because the popular approach has been thoroughly discussed by many authors. We believe that our approach could add a new insight to the cosmology.

The standard big bang cosmology may need a generalization from the observational point of view. But it must be emphasized that there is also a strong motivation to generalize it from the theoretical point of view. This is because the standard model is based on the Einstein's theory of gravitation, which itself may need a generalization. Of course the Einstein's theory has been a most beautiful and successful theory of gravitation. But from the logical point of view there are many indications that something is missing in the Einstein's theory. We mention just a few:

(1) The unification of all interactions may require a generalization of Einstein's theory. All the

known interactions are mediated by the spin-one or spin-two fields. However, the unification of all interactions inevitably requires the existence of a fundamental spin-zero field. In fact all modern unified theories—the Kaluza-Klein theory, the supergravity, and the superstring—contain a fundamental scalar field. What makes this scalar field unique is that, unlike others like the Higgs field, it couples directly to the (trace of) energy-momentum tensor of the matter field. As such it should generate a new force which will modify the Einstein's gravitation in a fundamental way.

(2) The Newton's constant G, which is supposed to be one of the fundamental constants of Nature, plays a crucial role in Einstein's theory. But the ratio between the electromagnetic fine structure constant α_e and the gravitational fine structure constant α_g of the hydrogen atom is too small to be considered natural,

$$\alpha_g/\alpha_e \simeq 10^{-40}. \tag{1}$$

This implies that not both of them may be regarded as the fundamental constants of Nature. From this Dirac [1] conjectured that G may not be a fundamental constant, but in fact a time-dependent parameter. If so, one must treat it as a fundamental scalar field which couples to all matter fields. Obviously the Dirac's conjecture requires a drastic generalization of Einstein's theory.

(3) The conformal transformation which changes the scale of the space-time metric at each space-time point,

$$g_{\mu\nu} \longrightarrow e^{\sigma(x)} g_{\mu\nu}, \tag{2}$$

is not a fundamental symmetry of Nature. At a deeper level (in the high energy limit), however, there is a real possibility that the conformal invariance and its scale factor may play an important role in physics. But in Einstein's theory there is no place where the conformal invariance and its scale factor could play a role.

(4) Finally, in cosmology the inflation at the early stage of evolution may be unavoidable. But for a successful inflation we need a dynamical mechanism which can (not only initiate

but also) stop the inflation smoothly. Unfortunately the Einstein's gravitation alone can not provide enough attraction to stop the inflation. Obviously one need an extra attractive force to end the exponential expansion, as shown in Fig.1. Again we may have to generalize the Einstein's theory, if we are to accommodate this extra attraction.

All these arguments, although mutually independent, suggest the existence of a fundamental scalar field which we call the dilaton which could affect the gravitation (and consequently the cosmology) in a fundamental way. In the following we discuss how the dilaton comes about in the unified field theory, and how it could affect the gravitation and cosmology.

II. GRAVITATION AND UNIFIED FIELD THEORY

To see how the dilaton appears in the unified theory, let us first consider the $(4+n)$–dimensional Kaluza-Klein theory whose fundamental ingredient is the $(4+n)$–dimensional metric g_{AB} $(A, B = 1, 2, \cdots, 4+n)$

$$g_{AB} = \begin{pmatrix} \tilde{g}_{\mu\nu} + e_0\kappa_0\phi_{ab}A^a_\mu A^b_\nu & e_0\kappa_0 A^a_\mu \phi_{ab} \\ e_0\kappa_0\phi_{ab}A^b_\nu & \phi_{ab} \end{pmatrix}, \tag{3}$$

where e_0 is a coupling constant, and κ_0 is a scale parameter which sets the scale of the n-dimensional internal space. When the metric has an n-dimensional isometry G, one can reduce the $(4+n)$-dimensional Einstein's theory to a 4-dimensional unified theory [2,3]. Indeed with $e_0^2\kappa_0^2 = 16\pi G$ and with

$$\begin{aligned} &\tilde{g} = |\det\ \tilde{g}_{\mu\nu}|, \qquad \phi = |\det\ \phi_{ab}|, \\ &\rho_{ab} = \phi^{-\frac{1}{n}}\phi_{ab}, \qquad (|\det\ \rho_{ab}| = 1), \end{aligned} \tag{4}$$

the $(4+n)$-dimensional Einstein's theory is reduced to the following 4-dimensional Einstein-Yang-Mills theory,

$$\begin{aligned} \mathcal{L}_0 = -\frac{1}{16\pi G}\sqrt{\tilde{g}}\sqrt{\phi}\ \Big[\tilde{R} + \tilde{S} \\ &-\frac{n-1}{4n}\frac{(\partial_\mu\phi)^2}{\phi^2} + \frac{1}{4}\rho^{ab}\rho^{cd}(D_\mu\rho_{ac})(D_\mu\rho_{bd}) + \Lambda + \lambda(|\det\ \rho_{ab}| - 1) \\ &+4\pi G\phi^{\frac{1}{n}}\rho_{ab}{F_{\mu\nu}}^a{F_{\mu\nu}}^b + \cdots\Big], \end{aligned} \tag{5}$$

where $\tilde{R}$ and $\tilde{S}$ are the scalar curvature of $\tilde{g}_{\mu\nu}$ and ϕ_{ab}, Λ is the $(4+n)$-dimensional cosmological constant, λ is a Lagrange multiplier. But notice that the above Lagrangian has a crucial defect. First the metric $\tilde{g}_{\mu\nu}$ does not represent the Einstein metric because $\tilde{g}$ does not describe the proper 4-dimensional volume element. But more seriously the ϕ-field appears with a negative kinetic energy, and thus can not be treated as a physical field [2]. To cure this defect one must perform the following conformal transformation, and introduce the physical metric $g_{\mu\nu}$ and the dilaton field σ by

$$\begin{aligned} g_{\mu\nu} &= \sqrt{\phi}\ \tilde{g}_{\mu\nu}, \\ \sigma &= \frac{1}{2}\sqrt{\frac{n+2}{n}}\ln\phi. \end{aligned} \tag{6}$$

With this the Lagrangian (5) is written as

$$\begin{aligned} \mathcal{L}_0 = -\frac{\sqrt{g}}{16\pi G}\Big[R &+ \frac{1}{2}(\partial_\mu\sigma)^2 \\ &+ Se^{-\alpha\sigma} + \Lambda e^{-\alpha'\sigma} - \frac{1}{4}(D_\mu\rho^{ab})(D_\mu\rho_{ab}) + \lambda(|\det\,\rho_{ab}|-1) \\ &+ 4\pi G e^{\alpha\sigma}\rho_{ab}F_{\mu\nu}{}^a F_{\mu\nu}{}^b + \cdots\Big], \end{aligned} \tag{7}$$

where R and S are the scalar curvature of $g_{\mu\nu}$ and ρ_{ab}, α and α' are the coupling constants given by $\alpha = \sqrt{(n+2)/n}$ and $\alpha' = \sqrt{n/(n+2)}$. This shows that in the Kaluza-Klein theory the dilaton appears as the volume element of the internal metric which, as a component of the metric g_{AB}, must couple to all matter fields.

In superstring theory the dilaton appears as the massless scalar field that the mass spectrum of the closed string must contain. After the full string loop expansion, the 4-dimensional effective Lagrangian of the massless modes has the following form in the string frame [4,5]

$$\mathcal{L}_S = -\frac{\sqrt{\tilde{g}}}{\kappa}\Big[\tilde{C}_g(\varphi)\tilde{R} + \tilde{C}_\varphi(\varphi)(\partial_\mu\varphi)^2 - \frac{\kappa}{4}\tilde{C}_1(\varphi)(F_{\mu\nu}{}^a)^2 + \cdots\Big], \tag{8}$$

where κ is the string slope parameter, $\tilde{g}_{\mu\nu}$ is the string frame metric, φ is the string dilaton, and $\tilde{C}_i(\varphi)$ $(i = g, \varphi, 1, 2, 3, \cdots)$ are the dilaton coupling functions to various fields. At present their exact forms are not known beyond the fact that in the limit φ goes to $-\infty$ they should admit the following loop expansion

$$\tilde{C}_i(\varphi) = e^{-2\varphi} + a_i + b_i e^{2\varphi} + c_i e^{4\varphi} + \cdots. \tag{9}$$

Notice that the effective Lagrangian (8) looks quite similar to the Lagrangian (5) which we obtained from the Kaluza-Klein theory. Indeed, introducing a new metric $g_{\mu\nu}$ with the conformal transformation and replacing the original dilaton field φ with a new one σ by

$$\begin{aligned} g_{\mu\nu} &= \tilde{C}_g(\varphi)\tilde{g}_{\mu\nu}, \\ \sigma &= \int \Big[\, \frac{3}{4}\frac{1}{\tilde{C}_g}\frac{d\tilde{C}_g}{d\varphi} + 2\frac{1}{\tilde{C}_g}\frac{d\tilde{C}_\varphi}{d\varphi} + 2\frac{\tilde{C}_\varphi}{\tilde{C}_g}\,\Big]^{1/2} d\varphi, \end{aligned} \tag{10}$$

one may put the Lagrangian (8) into the following standard form

$$\mathcal{L}_S = \frac{\sqrt{g}}{\kappa}\Big[R + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{\kappa}{4}C_1(\sigma)F_{\mu\nu}{}^a F_{\mu\nu}{}^a + \cdots\Big]. \tag{11}$$

Notice that in the standard form the dilaton coupling function to gravity $\tilde{C}_g(\varphi)$ and the self coupling function $\tilde{C}_\varphi(\varphi)$ disappear completely with the redefinition of the fields. Only the coupling functions to the other matter fields remain.

Now the Lagrangian (11) looks almost identical to the Lagrangian (7) of the Kaluza-Klein theory. In both cases the dilaton appears as a fundamental scalar field. Of course there are some differences. One of them is the form of the dilatonic coupling functions to various matter fields. In the Kaluza-Klein theory they have simple exponential forms, whereas in the string theory their explicit forms are not known. Another is the mass of the dilaton. In the Kaluza-Klein theory the dilaton can easily acquire a mass, but in the superstring theory it remains masslss to all orders of perturbation [4,5]. But these differences is may not be so serious as it appears. To understand this one has to keep in mind two things. First, the Lagrangian (7) in the Kaluza-Klein theory is valid only at the tree level, because it did not take into account the full renormalization effect. Moreover (9) shows that, at the tree level in the string loop expansion, the string coupling functions also have the exponential forms. So with the quantum correction in the Kaluza-Klein theory, the difference in the dilaton coupling functions between the two theories becomes insignificant. As for the mass of the dilaton, there is no fundamental principle which can keep it massless, even enough the perturbative expansion leaves it massless in the string theory. So it could acquire a mass though some unknown non-pertubative or topological mechanism. From this one may conclude that *as far as the dilaton is concerned the string theory and the Kaluza-Klein theory give us practically the same effective Lagrangian, at least in the low energy approximation.* In both cases the dilaton comes into play an important ingredient as the spin-zero partner of the spin-two graviton which is responsible for the Einstein's gravitation. So in the unified theory one must take the dilatonic modification of the Einstein's theory seriously, whether one likes it or not.

III. BRANS-DICKE THEORY

One may wonder why the string theory and the Kaluza-Klein theory give us almost identical effective Lagrangian. There is a good reason for this. To understand this it is instructive to discuss the Brans-Dicke theory first. In an attempt to generalize the Einstein's gravitation with a fundamental scalar field, Brans and Dicke arrived at the following Lagrangian [6]

$$\mathcal{L}_{BD} = \mathcal{L}_g + \mathcal{L}_m, \tag{12}$$

$$\mathcal{L}_g = -\sqrt{\tilde{g}}\ \Big[\phi\tilde{R} + \frac{\omega}{\phi}(\partial_\mu\phi)^2\Big], \tag{13}$$

$$\mathcal{L}_m = -\sqrt{\tilde{g}}\ \Big[\frac{1}{4}(F_{\mu\nu})^2 + (D_\mu\Psi)^*(D_\mu\Psi) - m^2\Psi^*\Psi + \overline{\Psi}i\gamma^\mu D_\mu\Psi - m\overline{\Psi}\Psi\Big], \tag{14}$$

where $\tilde{g}_{\mu\nu}$ is the Jordan metric, ϕ is the Brans-Dicke scalar field, and ω is the coupling constant. Notice that here we have kept the electromagnetic field $F_{\mu\nu}$ and a charged scalar field Ψ as the matter fields for simplicity. But the important point here is that in the Jordan frame the dilaton coupling functions to the matter fields (other than the graviton and the dilaton) are all chosen to be trivial, so that the dilaton does not couple directly to the matter fields. In other words the ordinary matter is allowed to couple to the gravitation only "minimally" through the Jordan metric, in spite of the fact that the Brans-Dicke scalar field is an important element of gravitation. What is remarkable is that, if we change the Jordan frame to another with a conformal transformation, this minimal coupling no longer holds. To see this let us introduce the Pauli metric $g_{\mu\nu}$ and the Brans-Dicke dilaton σ by [7]

$$\begin{aligned} g_{\mu\nu} &= e^{\alpha\sigma}\tilde{g}_{\mu\nu} \ , \\ \phi &= \frac{1}{16\pi G}e^{\alpha\sigma}, \end{aligned} \tag{15}$$

where

$$\alpha = \frac{1}{\sqrt{2\omega+3}}. \tag{16}$$

Now in the Pauli frame one can easily show that the Brans-Dicke Lagrangian acquires the following standard form,

$$\begin{aligned} \mathcal{L}_{BD} = &-\frac{\sqrt{g}}{16\pi G}\ \Big[R + \frac{1}{2}(\partial_\mu\sigma)^2\Big] \\ &-\sqrt{g}\ \Big[\frac{1}{4}(F_{\mu\nu})^2 + e^{-\alpha\sigma}(D_\mu\Psi)^*(D_\mu\Psi) + e^{-2\alpha\sigma}m^2\Psi^*\Psi\Big]. \end{aligned} \tag{17}$$

So the dilaton coupling function acquires a non-trivial form in the Pauli frame (except for the $U(1)$ gauge field), so that now the dilaton has a direct coupling to the matter field. The reason why the coupling function to the gauge field remains trivial is simply because the gravitational interaction of the gauge field is conformally invariant under (15).

Brans and Dicke has argued that the above theory is the only acceptable theory of gravitation containing a massless scalar graviton which respects the weak equivalence principle. But it is clear that Lagrangian (12) is a special case of (8), in which all the dilaton coupling functions are uniquely determined. The reason why they have chosen the above coupling functions was very simple. The equivalence principle requires a test particle made of the matter fields to follow geodesics determined by the physical metric. To guarantee this the physical metric must couple *minimally* to the matter fields, which means that the dilaton should not couple directly to the matter field. This requires the dilaton coupling functions to the matter fields to be trivial. Of course Brans and Dicke identified the Jordan metric as physical, and thus arrived at the Lagrangian (12).

This tells us the followings. First, there is practically one way to generalize the Einstein's theory with a (massless) scalar graviton. The only freedom one can have is the dilaton coupling functions to the matter fields. This is why all the above theories—the Brans-Dicke theory, the Kaluza-Klein theory, and the superstring theory—give us practically the same effective Lagrangian. The other point is that the form of dilaton coupling functions depends on the conformal frame that one chooses. For example, in the Brans-Dicke theory the dilaton coupling functions are trivial in the Jordan frame, but acquire a non-trivial form in the Pauli frame. This means that a test particle will follow a geodesic in the Jordan frame, but not in the Pauli frame. So it is crucial for us to decide what is the physical frame before we discuss the physics. Considering the fact that the conformal invariance is clearly broken in the real world, this is perhaps what one should have expected. Nevertheless we find that this point is not so well appreciated in the literature.

As we have emphasized, the equivalence principle was the underlying principle for the Brans-Dicke theory. But can one really maintain the equivalence principle in the Bran-Dicke theory? Brans and Dicke argued that one can do so, if (and only if) one treats the Jordan metric as physical. To guarantee this they have forbidden a direct coupling of Brans-Dicke scalar field to the ordinary matter in the Jordan frame, even though the scalar field was an essential ingredient of gravitation. Unfortunately this does not guarantee the equivalence principle in the Brans-Dicke theory [8]. This is because the minimal coupling of the Jordan metric to the ordinary matter becomes unstable under the quantum fluctuation. Indeed, when the quantum correction takes place, the ordinary matter must couple to the Brans-Dicke scalar field through the Jordan metric, as shown in Fig.2. So the quantum fluctuation inevitably induces a direct coupling of the Brans-Dicke scalar field to the ordinary matter, even in the Jordan frame. This tells that, when the quantum correction takes place, there is no way to enforce the equivalence principle in the Brans-Dicke theory.

The Lagrangian (17) tells that in the Pauli frame the dilaton couples directly to the charged scalar field, but not to the electromagnetic field. Naively this would imply that the charged particle will not follow the geodesic, but the photon will. This appearance, however, is misleading because the quantum fluctuation must necessarily induce a direct coupling of the dilaton to the gauge field. Indeed the quantum correction shown in Fig.3 should add an induced interaction [8]

$$\delta\mathcal{L} \simeq \alpha\alpha_e \sigma F_{\mu\nu}F_{\mu\nu}, \tag{18}$$

to the Lagrangian (17). Clearly this induced coupling is suppressed by the factor α_e, compared to the direct coupling which already exists at the classical level. This shows that the dilaton couples to different matters with different strengths. It is this "composition-dependent" coupling that violates the equivalence principle in the Brans-Dicke theory.

Since the Brans-Dicke theory is a prototype theory of gravitation that one finds in all existing unified theories, it is important to understand the theory in more detail. We emphasize a few characteristics of the Brans-Dicke theory :

1) It is the Pauli metric, not the Jordan metric, which describes the massless spin-two graviton and thus the Einstein's gravitation [7]. In fact the Jordan metric is a strange mixture of the spin-two graviton and spin-zero dilaton which does not even describe a mass eigenstate. This tells that, when one wants to compare the theory with the Einstein's gravitation, one must use the Pauli frame. Furthermore, when one tries to quantize the theory, obviously the Pauli frame comes in as the natural frame. So from the logical point of view the Pauli frame becomes the most natural frame to discuss the physics.

2) In the Pauli frame the Brans-Dicke dilaton describes a scalar component of gravitation which is absent in the Einstein's gravitation. Naturally this "new" gravitation could be interpreted as the dilatonic "fifth force" which modifies the Einstein's gravitation. In this view the huge Brans-Dicke coupling constant ω ($\omega > 600$) translates to a perfectly reasonable new constant α ($\alpha < 0.03$) through (16), which determines the coupling strength of the dilatonic fifth force to the ordinary matter. This tells that the Brans-Dicke theory is really a theory of fifth force.

The importance of the above discussion is that these characteristic features of Brans-Dicke theory, in particular the existence of the dilatonic fifth force and the violation of the equivalence principle, should also apply to the Kaluza-Klein theory and the superstring theory. The only difference is that the situation gets worse in the unified theories. For example, in these theories the violation of equivalence principle takes place already at the classical level. This is because here the dilaton couples to different matter with different strengths even without any quantum correction. To make the matter worse, in the string theory the dilatonic fifth force becomes intolerably large, because here the massless dilaton couples as strongly as the gravitation (with $\alpha \simeq 1$) to the matter field.

IV. DILATONIC FIFTH FORCE

Now we discuss the dilatonic fifth force [7,9] in a general setting. We have shown that the dilatonic coupling constant α may in principle depend on the type of matter field it couples, when there are more than one type of matter fields in the theory. But for simplicity one may assume that only one type of coupling, the dilatonic coupling to the baryonic matter, is important for the practical purpose. Given the fact that the baryonic matter is the only dominant matter of the universe verified so far, the assumption is well justified. In this case only one universal coupling constant α characterizes the fifth force. With this the important issue now becomes how strong is the fifth force, and how far does it act. The strength is determined by the coupling constant α, but the range is determined by the mass μ of the dilaton. Let F_g and F_5 be the gravitational and the fifth force between the two mass points m_1 and m_2 separated by r. From the dimensional argument one may express the total force F in the Newtonian approximation as

$$F = F_g + F_5 \simeq \frac{\alpha_g}{r^2} + \frac{\alpha_5}{r^2} e^{-\mu r} = \frac{\alpha_g}{r^2}(1 + \beta e^{-\mu r}), \qquad (19)$$

where α_g and α_5 are the fine structure constants of the gravitation and the fifth force, and β is the ratio between α_g and α_5. In terms of Feynman diagram the first term represents one graviton exchange but the second term represents one dilaton exchange in the zero momentum transfer limit. Notice that in the Brans-Dicke theory the Lagrangian (17) suggests

$$\beta \simeq \alpha^2, \tag{20}$$

from which one can easily estimate β. For instance $\omega > 600$ with (16) implies $\beta \simeq \alpha^2 < 10^{-3}$.

To proceed further one must know the mass of the dilaton. Of course the dilaton appears massless in the superstring and the Brans-Dicke theory. If the dilaton remains strictly massless, there is no way to differentiate the fifth force from the gravitation in this Newtonian approximation. This is because the net effect of the massless dilaton is simply to replace α_g with the effective gravitational constant $\overline{\alpha}_g = (1+\beta)\alpha_g$. In the absence of any simple mechanism which can keep the dilaton massless, however, it is reasonable to assume that the dilaton acquires a small mass through some unknown quantum correction. Unfortunately it is extremely difficult to estimate this quantum correction at present. Under this circumstance one may leave μ as a free parameter and consider the following possibilities :

a) $\mu \simeq 0$ (long range). From the existing experimental data on the long range fifth force we have $\beta < 10^{-9}$ [10]. Notice that in the Brans-Dicke theory $\beta < 10^{-9}$ amounts to $\omega > 10^8$, which gives us a much more stringent constraint on the Brans-Dicke coupling constant than the existing bound $\omega > 600$ [11]. Clearly the new bound is made possible by the interpretation that the Brans-Dicke theory is really a theory of a fifth force in which the new gravitational force is generated by the Brans-Dicke scalar field. Of course such a large ω (or such a small β) might be interpreted to imply that there is no such long range fifth force. At this point, however, it may be good to remember that the ratio between the gravitational and the electromagnetic coupling of the elementary particles is extremely small, $\alpha_g/\alpha_e \simeq 10^{-40}$.

b) $\mu \simeq 2 \times 10^{-10}$ eV (1 km range). In this medium range we have $\beta e^{-\mu r} \leq 10^{-4}$ experimentally. This is perfectly consistent with our estimate of β based on $\omega > 600$ in the Brans-Dicke theory. Of course β could still be much smaller than 10^{-4} here, in which case a best way to measure β is the laboratory (small size) experiments.

c) $\mu \simeq 1$ keV (2×10^{-8} cm range). In this atomic scale there is no experimental constraint yet, and the possibility of $\beta \simeq 1$ can not be ruled out. In fact all the unified theories predict β to be of the order one. This case is particularly interesting because the 1 keV dilaton could be an excellent candidate of the dark matter of the universe, as we will discuss in the following.

d) $\mu \simeq 10^{15}$ GeV (2×10^{-28} cm range). In this grand unification scale there is practically no way to detect the fifth force in the present universe, even though it may very well exist at this short distance. Notice, however, that in the early universe this fifth force could have played an important role to stop the inflation by providing an extra attractive force to curb the exponential expansion of the universe [8].

The above analysis teaches us the followings. First, the modification of the Einstein's gravitation by the massless dilaton (a long range fifth force) has to be extremely small, if there is any. Indeed the weak dilatonic coupling to the ordinary matter field restricts $\omega > 10^8$ in the Brans-Dicke theory, which sets the most stringent bound for the Brans-Dicke coupling constant. Furthermore this extreme weak dilatonic coupling must apply to all unified theories, as far as the dilaton remains massless. So in any theory with a massless dilaton one must find a theoretical justification why the dilatonic coupling is so weak. This (together with $\alpha \simeq 1$) creates a serious problem for the superstring theory, where the string dilaton remains massless to all orders of perturbation [5,12]. Secondly, a fifth force by a massive dilaton, even with a relatively small mass of 10^{-10} eV range, is very difficult to rule out experimentally. In fact there is practically no hope to detect a dilatonic fifth force at about the grand unification scale in the present universe. Nevertheless this provides a most interesting possibility from the cosmological point of view, because this type of fifth force may have played a crucial role in the evolution of the universe at the early stage. Furthermore the massive dilaton could provide an excellent candidate for a non-baryonic dark matter.

V. UNIFIED COSMOLOGY

It is generally believed that, to solve the major problems of the standard cosmology, one may need an inflation at the early stage of universe. A best way to implement the inflation is to introduce a scalar field as the inflaton field. But this inevitably leads us to a "generalized"

Brans-Dicke theory, which is exactly what we find in the unified field theories as we have discussed in the above. This implies that the unified field theories can naturally provide us an inflation [13]. In fact the possibility of an inflation in all the unified theories has been successfully argued by many authors [5,14]. This is because in these theories the dilaton could assume the role of the inflaton, so that by choosing a proper inflationary potential one could obtain a successful inflation.

There is, however, an important point that one must keep in mind when one tries to implement an inflation in these theories. To discuss the inflation one must first decide which conformal frame is the physical frame. This is because the actual expansion rate of inflation depends on the conformal frame that one chooses [13]. Unfortunately many of the authors in the literature have overlooked this important point, and have used unphysical frames without a proper justification.

Depending on the dilatonic potential the unified theory provides us with a wide range of cosmology. But there are a few characteristics of the unified cosmology [13]:
a) The matter in the unified cosmology consists of two parts, the dilatonic matter and the ordinary matter. Futhermore the density ρ_σ of the dilatonic matter is generically given by

$$\rho_\sigma \simeq \frac{1}{16\pi G}\frac{H^2}{\alpha^2} \simeq \rho_c \qquad (21)$$

so that Ω_σ could become of the order one. This suggests that the dilaton could easily become the dark matter of the universe.

b) In the unified cosmology the Dirac's conjecture is realized, but the time-dependence of the Newton's constant is given by

$$\frac{\dot{G}}{G} \simeq H \simeq 10^{-10}/year. \qquad (22)$$

So it could naturally accomodate the present experimental constraint on $\dot{G}/G$.

VI. DILATONIC DARK MATTER

The unified cosmology implies that the dilaton could be the dark matter of the universe. In this section we first show that the dilaton starts with the thermal equilibrium from the beginning and decouples from the other sources very early near the Planck scale. After the decoupling the fate of the dilaton crucially depends on the mass. We find that there are two mass ranges, $\mu \simeq 0.5$ keV and $\mu \simeq 171$ MeV, where the dilaton could be the dominant matter of the universe. The dilaton with mass larger than 171 MeV does not survive long enough to become the dominant matter of the present universe, and the dilaton with mass smaller than 0.5 keV survives but fails to be dominant due to the low mass. The dilaton with mass in between can not be seriously considered because it would overclose the universe. Remarkably the 0.5 keV dilaton has the free-streaming distance of about 1.4 Mpc and provide an excellent candidate of a warm dark matter, but the 171 MeV dilaton has much shorter free-streaming distance of about 12 pc so that it becomes a cold dark matter.

To show how the dilaton reaches the thermal equilibrium from the beginning notice that the dominant interaction modes of the dilaton with other matter fields are the Feynman diagrams shown in Fig.4. Normally the dilatonic coupling strength would be $\alpha\, m_q/m_p$, where α is the dimensionless coupling constant, m_p is the Planck mass, and m_q is the mass of the dominant matter (the quarks). But notice that at high temperature (at $T \gg m_q$), the coupling strength becomes $\alpha\, T/m_p$. With this one can easily estimate the dilaton creation (and annihilation) cross section shown in Fig.4 (a)

$$\sigma \simeq g^2 \alpha^2 \Big(\frac{T}{m_p}\Big)^2 \times \frac{1}{T^2}, \tag{23}$$

so that the creation rate Γ is given by

$$\Gamma \simeq n_q \sigma v \simeq g^2 \alpha^2 \Big(\frac{T}{m_p}\Big)^2 \times T. \tag{24}$$

Similarly the scattering cross section σ shown in Fig.4 (b) is given by

$$\sigma \simeq \alpha^4 \Big(\frac{T}{m_p}\Big)^4 \times \frac{1}{T^2} \tag{25}$$

with the following interaction rate Γ

$$\Gamma \simeq n_q \sigma v \simeq \alpha^4 \Big(\frac{T}{m_p}\Big)^4 \times T. \tag{26}$$

On the other hand the Hubble expansion rate H in the early universe is given by [15]

$$H \simeq \frac{T^2}{m_p}. \tag{27}$$

From this we conclude that *the dilaton is thermally produced from the beginning, and decouples with the other sources at around the Planck scale with the decoupling temperature T_d given by*

$$T_d \simeq \frac{m_p}{\alpha^4}. \tag{28}$$

Notice that the dilaton decouples with the other sources at around the same time as the graviton does. This is indeed what one would have expected, since the dilaton is nothing but the scalar counterpart of the Einstein's graviton.

Once the dilaton acquires a mass, it becomes unstable and decays to the ordinary matter. A typical decay process is the two photon process and the fermion pair production process described by the following interaction Lagrangian

$$\begin{aligned} \mathcal{L}_{int} \simeq & -\frac{\alpha_1}{4}\sqrt{16\pi G}\,\phi F_{\mu\nu} F_{\mu\nu} \\ & -\alpha_2 \sqrt{16\pi G} m\,\phi\,\bar{\psi}\,\psi. \end{aligned} \tag{29}$$

For the two photon process we obtain the following life-time at the tree level [16]

$$\tau_1 \simeq \frac{1}{\alpha_1^2}\Big(\frac{m_p}{\mu}\Big)^2 \frac{1}{\mu} \quad \Big(\times \frac{T}{\mu}\Big), \tag{30}$$

where the last term in parenthesis is the time-dilatation effect which becomes important only at high temperature when the dilaton becomes relativistic. Similarly for the pair production we obtain

$$\begin{aligned} \tau_2 & \simeq \frac{1}{\alpha_2^2}\Big(1 - 4\frac{m^2}{\mu^2}\Big)^{-3/2} \Big(\frac{m_p}{\mu}\Big)^2 \frac{1}{\mu} \quad \Big(\times \frac{T}{\mu}\Big) \\ & \geq \frac{10.75}{\alpha_2^2}\Big(\frac{m_p}{\mu}\Big)^2 \frac{1}{\mu} \quad \Big(\times \frac{T}{\mu}\Big). \end{aligned} \tag{31}$$

A more detailed calculation which includes all possible decay channels allowed in the standard electroweak model gives us the graph of the dilaton life-time with respect to the dilaton mass shown in Fig.5 [16]. Notice that here we have assumed $\alpha_1 \simeq \alpha_2 \simeq 1$ for simplicity, but one should keep in mind that in reality the coupling constants could turn out to be much smaller.

To estimate how much the dilaton contributes to the matter density of the present universe one must estimate the number density of the dilaton at present time. From the entropy conservation of the universe one can easily estimate the present temperature T_ϕ of the dilaton. Based on the standard electroweak theory one finds

$$T_\phi \leq \Big(\frac{3.91}{106.75}\Big)^{1/3} T_0 \simeq 0.91^\circ K, \tag{32}$$

where T_0 is the present temperature of the background radiation. Notice that again this is the temperature of the graviton at present time. From this one can estimate the number density n_0 of the dilaton at present. Assuming that the dilaton is stable one has

$$n_0 = \frac{\zeta(3)}{\pi^2} T_\phi^3 \simeq 7.5\,/cm^3. \tag{33}$$

But as we have emphasized, the massive dilaton can not be stable, and the number density of the dilaton $n(\mu)$ must crucially depend on its mass. So for the dilaton to provide the critical mass of the universe one must have

$$\rho(\mu) = n(\mu) \times \mu = n_0\, e^{-t_0/\tau(\mu)} \times \mu \simeq 10.5 \;\; h^2 \;\; keV/cm^3. \tag{34}$$

where t_0 is the age of the universe, $\tau(\mu)$ is the life-time of the dilaton, and h is the Hubble constant in the unit 100Km/sec Mpc. A numerical calculation with $t_0 \simeq 1.5 \times 10^{10}$ years shows that *there are two mass ranges, $\mu \simeq 1.4 \;\; h^2 \;\; keV$ or $\mu \simeq 475 \;\; h^2 \;\; MeV$, which can make the dilaton a candidate of the dark matter in the universe.* Notice that with $h \simeq 0.6$ the mass becomes 0.5 keV or 171 MeV. Also notice that the $\rho(\mu)$ starts from zero when $\mu = 0$ and reaches the maximum value at 1.4 h^2 keV $< \mu <$ 475 h^2 MeV and again decreases to zero when $\mu = \infty$. This means that when $\mu <$ 1.4 h^2 keV or $\mu >$ 475 h^2 MeV the dilaton undercloses the universe, but when 1.4 h^2 keV $< \mu <$ 475 h^2 MeV it overcloses the universe. From this one may conclude that *the dilaton with 1.4 h^2 keV $< \mu <$ 475 h^2 MeV is not acceptable because this is incompatible with the cosmology.* In view of the fact that the dilaton must exist in all the unified field theories, the above constraint on the mass of the dilaton should provide us an important piece of information in search of the dilaton.

Now we discuss the possibility of the dilatonic dark matter in more detail:

a) $\mu \simeq 1.4\ h^2$ keV. In this case the available decay channels are the $\gamma\gamma$ process and the $\nu_e\bar{\nu}_e$ process. So the life-time is given by $\tau \simeq 5.0 \times 10^{25}$ years, which tells that it is almost stable. To determine whether this dilaton could serve as a hot or cold dark matter, one must estimate the free-streaming distance λ of the dilaton. The dilaton becomes non-relativistic around T$\simeq \mu/3 \simeq 0.47\ h^2$ keV, long before the matter-radiation equilibrium era. In terms of time this corresponds to [15]

$$t_{NR} \simeq 1.2 \times 10^7 \times \Big(\frac{keV}{\mu}\Big)^2 \Big(\frac{T_\phi}{T_0}\Big)^2 sec \simeq \frac{6.75 \times 10^5}{h^4} sec. \tag{35}$$

From this one obtains

$$\begin{aligned} \lambda &\simeq 0.16 \Big(\frac{keV}{\mu}\Big)\Big(\frac{T_\phi}{T_0}\Big)\Big[\ln\Big(\frac{t_{EQ}}{t_{NR}}\Big) + 2\Big] Mpc \\ &\simeq \frac{0.5}{h^2} Mpc. \end{aligned} \tag{36}$$

Certainly this is a very interesting number, which tells that the 1.4 h^2 keV dilaton becomes an excellent candidate of a warm dark matter.

b) $\mu \simeq 475\ h^2$ MeV. In this case the available decay channels are the $\gamma\gamma$, $\nu\bar{\nu}$ (3 types), and $e\bar{e}$ processes. The corresponding life-time is given by $\tau \simeq 1.2 \times 10^9$ years, so that only a fraction of the thermal dilaton survives now. For this dilaton one has

$$t_{NR} \simeq \frac{5.9 \times 10^{-6}}{h^4} sec, \tag{37}$$

and the corresponding free-streaming distance becomes

$$\lambda \simeq \frac{4.3}{h^2} pc. \tag{38}$$

Clearly this dilaton becomes a good candidate for a cold dark matter.

Now the important question is how one could detect the dilaton. It seems very difficult to detect it through the dilatonic fifth force, because the range of the fifth force would be about 10^{-8} cm (for $\mu = 1.4\ h^2$ keV) or about 10^{-13} cm (for $\mu = 475\ h^2$ MeV). Perhaps a more promising way is to use the two photon decay process, which produces two mono-energetic X-rays of $E \simeq 0.7\ h^2$ keV or $E \simeq 237.5\ h^2$ MeV with the same polarization. With the local halo density of our galaxy $\rho_{HALO} \simeq 0.3$ GeV$/cm^3$ one can easily find the local dilaton number density to be $\bar{n} \simeq 2.1 \times 10^5/h^2/cm^3$ for $\mu = 1.4\ h^2$ keV and $\bar{n} \simeq 0.63/h^2/cm^3$ for $\mu = 475\ h^2$ MeV. In both cases the local velocity of the dilaton is about 10^{-3} c. So it is very important to look for the above X-ray signals from the sky (with the Doppler broadening of $\Delta E \simeq 10^{-3}E$) or to perform a Sikivie-type X-ray detection experiment with a strong electromagnetic field to enhance the dilaton conversion, although the long life-time (for $\mu = 1.4\ h^2$ keV) or the low local number density (for $\mu = 475\ h^2$ MeV) of the dilaton could make such experiments very difficult.

One might try to detect the dilaton from the accelerator experiments. The dilaton has a clear decay signal, but the production rate should be very small due to the extreme weak coupling. So one need a huge luminosity to produce the dilaton from the accelerators. There are, of course, other (indirect) ways to test the existence of the dilaton. For example, it may be worth to look for the impacts of the dilaton in the stellar evolution and the supernovae explosion. We will discuss these in a separate paper [16].

VII. DISCUSSION

In this talk we have discussed the possible impacts of the hypothetical dilaton in physics, in particular in cosmology. Remarkably the dilaton allows some definite predictions which could be tested by experiments. In particular, the dilaton with mass 0.5 keV or 171 MeV could make an exellent candidate of a dark matter. Of course the exact value of the mass may change later because the above results are based on the simplest assumptions. But the importance of our analysis is that the dilaton *must* be taken seriously because it exists in all the existing unified theories, including the superstring theory. In fact it is one of the very few predictions that the unified theories can provide. So by testing the dilaton experimentally one could test the unification scheme itself. Indeed any negative experimental result on the dilaton should make a serious damage to the credibility of the existing unified theories. For this reason it is very important to perform experiments which could test the existence of the dilaton.

From the theoretical point of view the importance of the above analysis is that one can test the equivalence principle, the fifth force, the inflation, and the dark matter problem within a single unified picture through the dilaton. The unified theories demand this. To be sure we still do not know whether the above unified picture is correct or not. Nevertheless it is nice to see that the present unified theories is able to provide such a unified picture of Nature.

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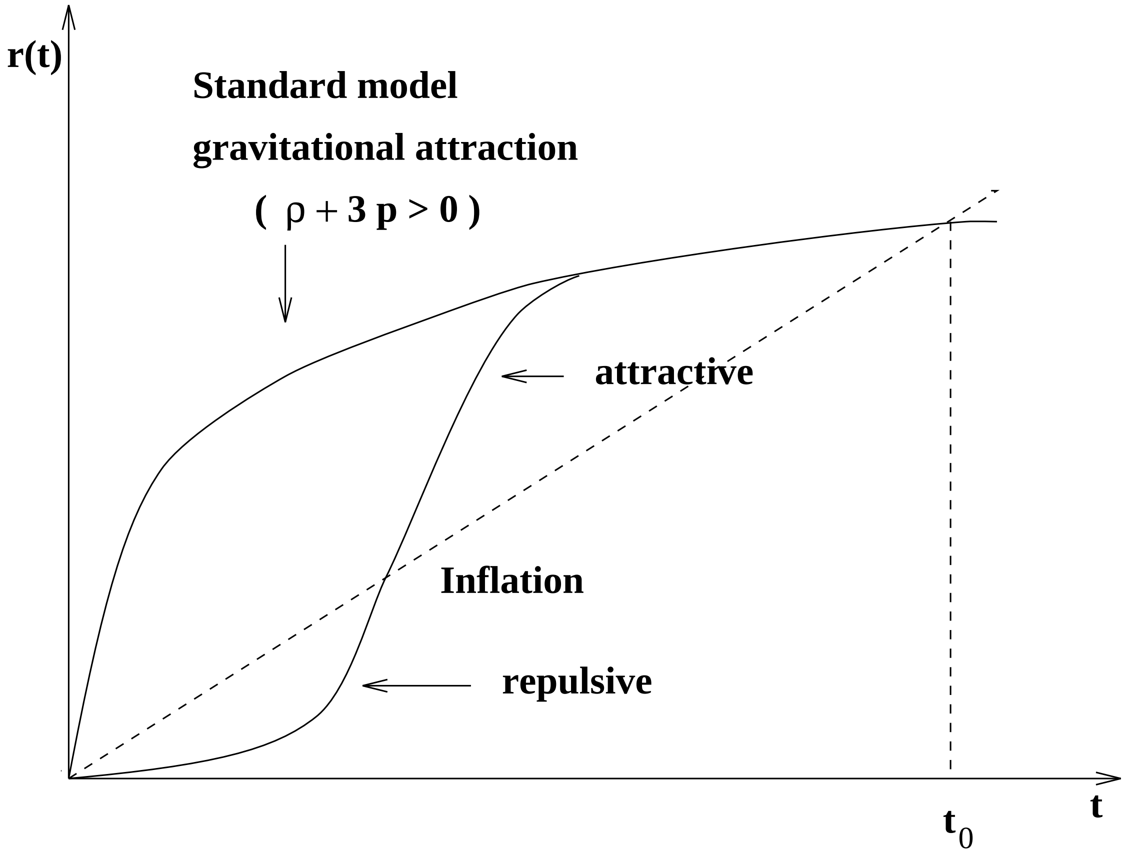
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Standard model
gravitational attraction
(ρ + 3 p > 0)
attractive
Inflation
repulsive
t
t0

Fig. 1

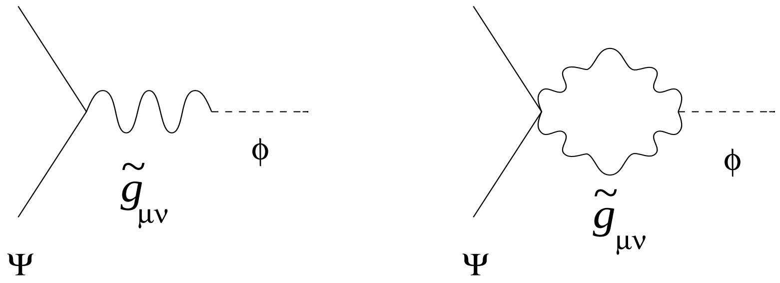

Fig. 2

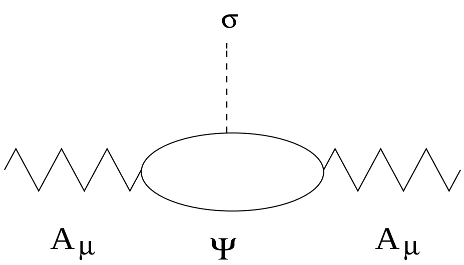

Fig. 3

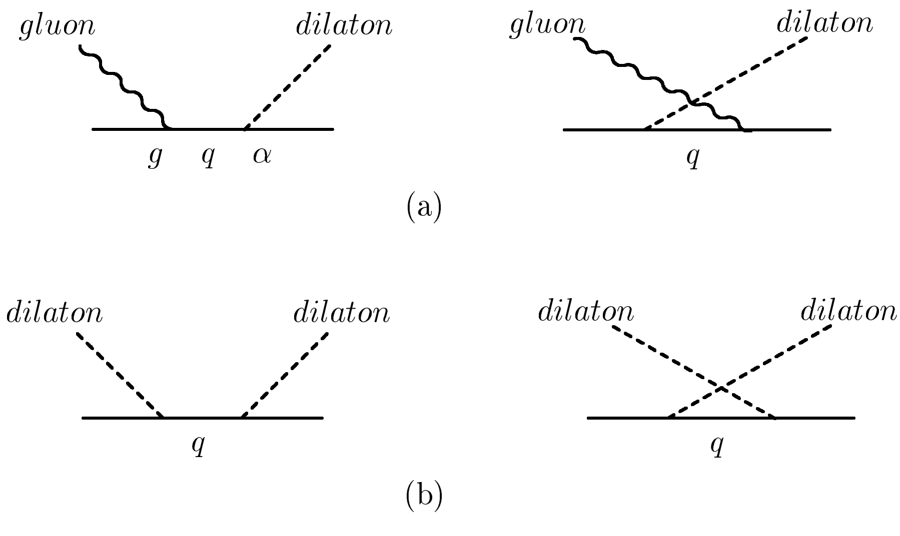

Fig. 4

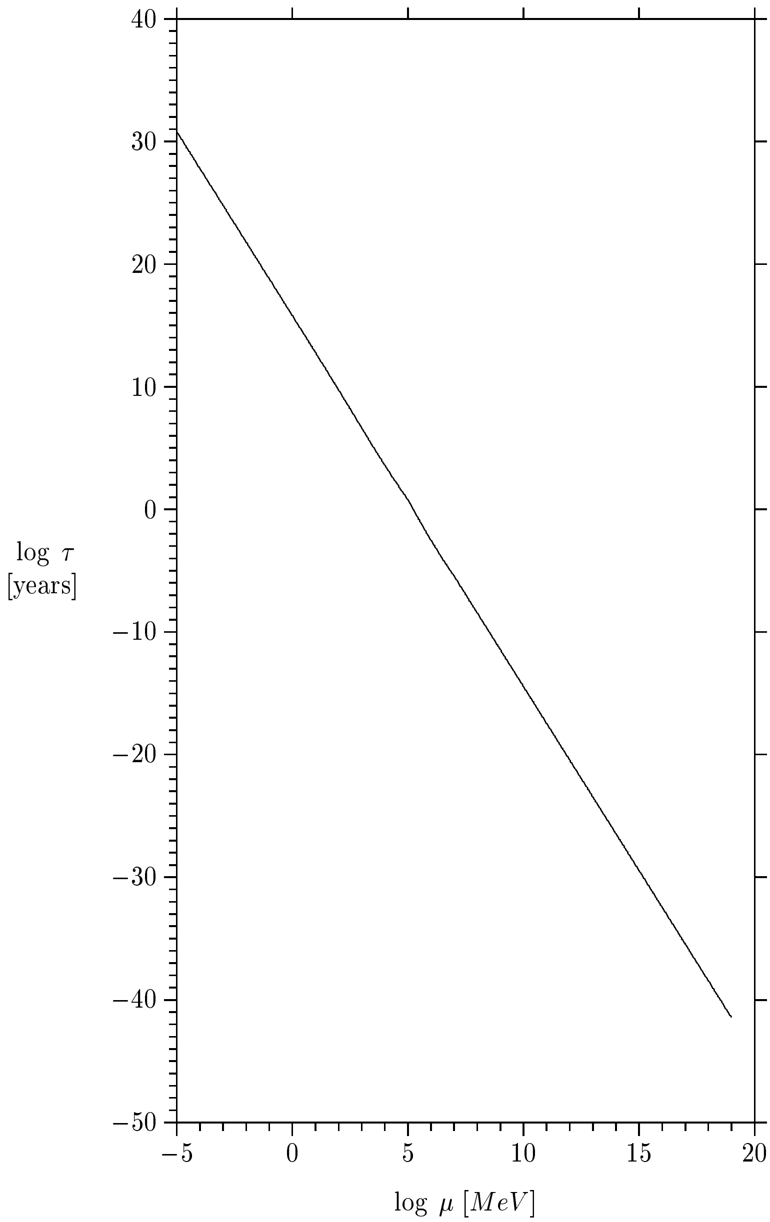

Fig. 5