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The Scale Dependence of the Multiplicity in Quark and Gluon Jets and a Determination of C_A/C_F

Preliminary

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Abstract

The charged multiplicity of quark and gluon jets is investigated for various three-jet topologies observed in Z decays and compared for different energy and transverse-momentum-like scales. Gluon jets are identified in heavy quark events using impact parameter information. Quark jet properties are obtained from events with 2 jets and a hard photon and from light-quark-dominated three-jet events.

The increase of the gluon multiplicities with scale is about twice as strong as the corresponding increase for quarks. The observed quark and gluon multiplicities can be described simultaneously by an ansatz based on NLO QCD calculations. This implies that the observed hadronic multiplicity can be understood as resulting from a gluon radiation process which is proportional to the colour charges of the initial quarks and gluons. The colour factor ratio:

$$\frac{C_A}{C_F} = 2.34 \pm 0.04(stat.) \pm 0.11(scale)$$

determined from this measurement coincides with the QCD expectation.

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1 Introduction

High energy inelastic reactions involving final state quarks or gluons give rise to a large number of final state hadrons that can be clustered into jets, the remnants of the elementary fields of the reaction. The total hadron multiplicity shows an approximately logarithmic rise with the relevant scale of the process. This behaviour is successfully described by perturbative QCD calculations which assume the hadrons to originate from coherent gluon bremsstrahlung off the outgoing partons of the elementary reaction. As bremsstrahlung is proportional to the charge of the radiating objects, the hadron multiplicity in gluon and quark jets is expected to reflect the colour charge of these objects, i.e. the ratio of the *radiated gluon multiplicities* is expected to be C_A/C_F. The increase in multiplicity, which is concentrated at small hadron momentum, is directly complementary to the scaling violation of the fragmentation function at large hadron momenta [1].

It was however noted already in the first paper on the gluon to quark multiplicity [2] that this prediction does not immediately apply to the *observed hadron multiplicity* at finite energy, as this is also influenced by differences of the fragmentation of the *primary* quark or gluon. In fact these differences must be present, due to the fact that quarks are valence particles of the hadrons whereas gluons are not. This is most clearly evident from the behaviour of the gluon fragmentation function at large hadron momentum, which is suppressed by about one order of magnitude compared to the quark fragmentation function. This also causes an initially higher multiplicity to be expected from very low energy quarks compared to gluons. In a previous publication [1], it has been shown that this effect is even visible in the primary splitting probabilities (splitting kernels) of quarks and gluons and that it is indeed responsible for the observed small multiplicity ratio. If heavy quark jets are also included in the comparison, a further reduction of the multiplicity ratio is evident, due to the high number of particles from the decays of the primary heavy particles.

The definition of quark and gluon jets in three-jet events in e^+e^- annihilation requires the use of jet algorithms which combine the hadrons into jets. Low energy particles at large angles with respect to the original parton direction are likely to be assigned to a different jet. As gluon jets are initially wider than quark jets, this presumably leads to a loss of multiplicity for gluon jets and a corresponding gain for quark jets.

The effects discussed above lead to an observed charged multiplicity ratio which is smaller than the radiated gluon multiplicity ratio. So far these effects have mainly been neglected, both in experimental and in more elaborate theoretical investigations. However, as we will show in this paper, at current energies these non-perturbative effects are still important and need to be considered in a proper test of the prediction [2].

In this paper we attempt to account for the non perturbative effects directly, by studying the scale dependence of the multiplicity in quark and gluon jets. A scale dependence of quark and gluon properties was first demonstrated in [3] with the jet energy as an intuitive scale. This result was later confirmed by other measurements [4, 5] and has recently been extended to a transverse momentum-like scale [6].

The stronger radiation in gluon jets is expected to become directly evident from a stronger increase of the multiplicity with scale in gluon jets than in quark jets. In this approach, the size of the non-perturbative terms can also be estimated directly from the quark and gluon multiplicities at very small scales.

This paper is based on a data analysis which is similar to that presented in previous papers [1, 3]. We have therefore chosen to restrict our experimental discussion in section 2 to the relevant differences with respect to these papers. More detailed information can also be found in [7, 8]. In section 3, we then compare quark and gluon jet multiplicities obtained from symmetric and non-symmetric event topologies and justify the selection of scales for further analysis. Then the slopes of the multiplicities with scale are presented and fitted by a (N)NLO QCD prediction extended by an ansatz for the non-perturbative terms, yielding a direct measurement of C_A/C_F. Finally we summarize and conclude.

2 Data Analysis

Cuts applied to charged and neutral particles and to events in order to select hadronic Z decays are identical to those given in [1] for the $q\bar{q}g$ analysis and to those in [3] for the $q\bar{q}\gamma$ analysis. Three-jet events are clustered using the Durham-algorithm [9] with a jet resolution parameter $y_{cut} = 0.015$. The angles $\theta_{2,3}$ of the low energy jets with respect to the leading jet must be in the range 100^o to 170^o (see Fig. 1a). The leading jet is not considered in this analysis. From the overall set of events defined by these cuts, which is dominated by non-symmetric topologies, five symmetric reference samples of Y type events are extracted requiring $\theta_2 \approx \theta_3 = 120° \pm 5°,\ 130° \pm 5°,\ 140° \pm 5°,\ 150° \pm 5°$, and $160° \pm 5°$ (see Fig.1b).

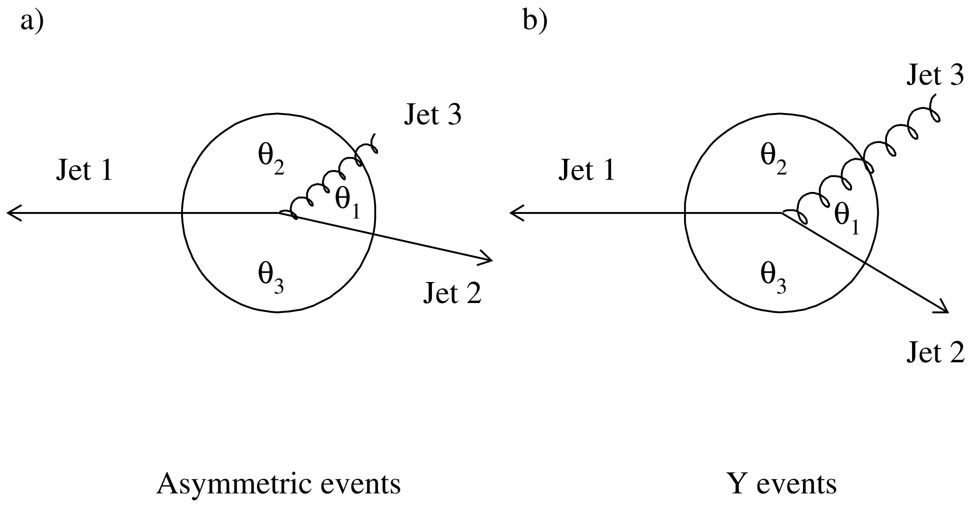

Figure 1: Definition of event topologies and angles used throughout this analysis. The lenght of the jet lines indicates the energies.

The identification of gluon jets by anti-tagging of heavy quark jets is identical to that described in [1, 3]. Quark jets are taken from $q\bar{q}\gamma$ events or $q\bar{q}g$ events which have been depleted in b quark events using an impact parameter technique. In order to extract multiplicities in pure quark and gluon jets, the data have been corrected using simulated events generated with JETSET 7.3. This is justified by the good agreement between data and simulation. Furthermore, the model independent techniques described in [1] for symmetric events give results compatible with those obtained with the simulation correction. The correction for b quark events remaining in the $q\bar{q}g$ sample does not influence the slope of the measured multiplicity with scale, but only leads to a shift of the absolute value. Moreover the effects of finite resolution and acceptance of the detector are corrected using a full simulation of the DELPHI detector and the same analysis chain.

In the simulation, quark and gluon jets are identified at "parton level". Partons are clustered into three jets using the Durham algorithm. Then, for each jet, the quarks and antiquarks with the flavour of the initial $q\bar{q}$ pair are summed over, where quarks contribute with weight +1 and antiquarks with weight −1. These sums are expected to yield +1 for quarks jets, −1 for anti-quark jets, and 0 for gluon jets. The small amount (2%) of events not showing this expected pattern were discarded. Finally the parton jets were mapped onto the jets found at hadron level by requiring the sum of the angles between the parton and hadron jets to be minimal.

In the Y type events, the gluon jet purities vary between about 60% and 80%. In the asymmetric events, they are in the range 95% for low energy gluons and 46% for the highest energy gluons. A few bins with lower purities have been excluded from the analysis. The quark purities are in the range 47% - 59% (43% - 81%) in the symmetric (asymmetric) samples respectively.

3 Results

In order to determine a scale dependence, the scale underlying the physics process needs to be specified. If an outer scale like the centre of mass energy is varied this is less important, because the actual physical scale is necessarily proportional to this outer scale. As only the relative change in scale usually matters, this outer scale can directly replace the physical scale. For this analysis the situation is different. The jets entering the analysis stem from Z decays, thus from a fixed centre of mass energy. So the relevant scales have to be determined from the properties of the jets and the event topology. From the above discussion, the scale has to be proportional to the jet energy as, for similar events, this quantity scales with the energy in the centre of mass system. Studies of hadron production in processes with non-trivial topology have shown that many characteristics of the parton cascade prove to depend on the hardness of the process producing the jet [10]:

$$\kappa_{DDT} = E_{jet} \sin \frac{\theta}{2} \tag{1}$$

where E_{jet} is the energy of the jet and θ the angle with respect to the closest jet. The scale κ_{DDT} [1] is similar to the transverse momentum of the jet and is also related to $\sqrt{y_{cut}}$ as used by the jet algorithms.

In the dipole model of gluon radiation, the properties of the gluon jet in a three jet $q\bar{q}g$ event arise due to an incoherent superposition of the radiation of the dipoles spanning between the quark and the gluon and between the anti-quark and the gluon. In a fully symmetric (mercedes star type) event, a gluon radiates twice as much as a quark or anti-quark (correspondingly, $C_A/C_F = 2$ in the dipole model). For quantities which are logarithmically scale dependent, as is usually the case in QCD, the gluon scale can thus be taken as the harmonic mean of the dipole scales:

$$\kappa_{dipole} = \sqrt{\kappa_{qg} \cdot \kappa_{\bar{q}g}} \tag{2}$$

[1] named after Dokshitzer, Diakonov and Trojan

Where the scales κ_{qg} and $\kappa_{\bar{q}g}$ are assumed to be given through Eqn. 1 by the gluon jet energy and the angles θ_1, θ_2 of the gluon with respect to the two (anti-)quark jets. These scales have recently been applied in a study of quark and gluon jet multiplicities [6]. Furthermore, for comparison, the multiplicity as a function of the jet energy and as a function of the transverse momentum of a jet:

$$\kappa_T = E_{jet} \sin \Theta \tag{3}$$

are also presented below. Here Θ is the angle with respect to the leading jet.

In order to test and compare these scales, Fig. 2 compares the multiplicities measured for quark and gluon jets from symmetric (Y-type) events and from the overall asymmetric three jet sample using identical scales. It is observed that the multiplicities obtained for symmetric and asymmetric event topologies disagree when using the energy and the transverse momentum scales. They also disagree if the κ_{Dipole} scale is applied to quark jets (not shown in the plot). However the agreement is satisfactory for the κ_{DDT} scale and, for gluon jets, best for κ_{Dipole}. The latter result was also obtained in [6]. Therefore κ_{DDT} and κ_{Dipole} are employed for further studies.

In Fig. 3, the multiplicity for quarks as obtained from $q\bar{q}g$ and $q\bar{q}\gamma$ events and for gluons from $q\bar{q}g$ events is shown as a function of κ_{Dipole} and is compared with the predictions of the JETSET [11] and HERWIG [12] fragmentation models. The errors for the $q\bar{q}\gamma$ sample are larger, due to the much smaller number of hard radiated γ's compared with gluons. The results for quarks from $q\bar{q}g$ and $q\bar{q}\gamma$ events are perfectly consistent [2]. This is a strong indication for the validity of our analysis and especially for the choice of the scale of the quark jets. For all multiplicities, an approximately logarithmic increase with scale is observed which is significantly stronger for gluon jets than for quark jets. This was already noted in a previous paper [3], where the jet energy was chosen as scale. Meanwhile this observation has been confirmed also by other measurements [4] and has been extended to other scales [6]. Both fragmentation models predict an comparable increase of the multiplicities which is in good agreement with the data. The agreement in absolute normalization is better for JETSET. At the smallest values of κ, where non-perturbative effects are expected to set in, the models indicate a slight deviation from the standard behaviour for the quark jets.

It is in order to compare this measurement to previous data. In fact the measurement [6] is in reasonable agreement with these data for gluon jets, but shows a smaller slope and a higher overall multiplicity for quark jets. A study with a fragmentation model indicates that a large part of the difference in normalization (0.4 units of multiplicity) is due to the inclusion of heavy quark events in [6].

As explained in the introduction, we want to obtain information on the relative colour charges of quarks and gluons from the slopes of the dependence of their multiplicities on the scale. Assuming the validity of the perturbative QCD prediction, the ratio of the gluon and quark multiplicities has to approach a constant value (approximately the colour factor ratio) at large scale. This trivially implies that also the ratio of the slopes of quark and gluon multiplicities must approach the same limit. This fact is a direct consequence of de l'Hôpitals rule, but is also directly evident from the linearity of the derivative:

$$\text{at large scale:} \quad n_{gluon}(\kappa) = C \cdot n_{quark}(\kappa) \quad \rightarrow \quad \frac{dn_{gluon}/d\kappa}{dn_{quark}/d\kappa} = C \tag{4}$$

[2]A slight difference in normalization is expected due to the different quark content of the two samples.

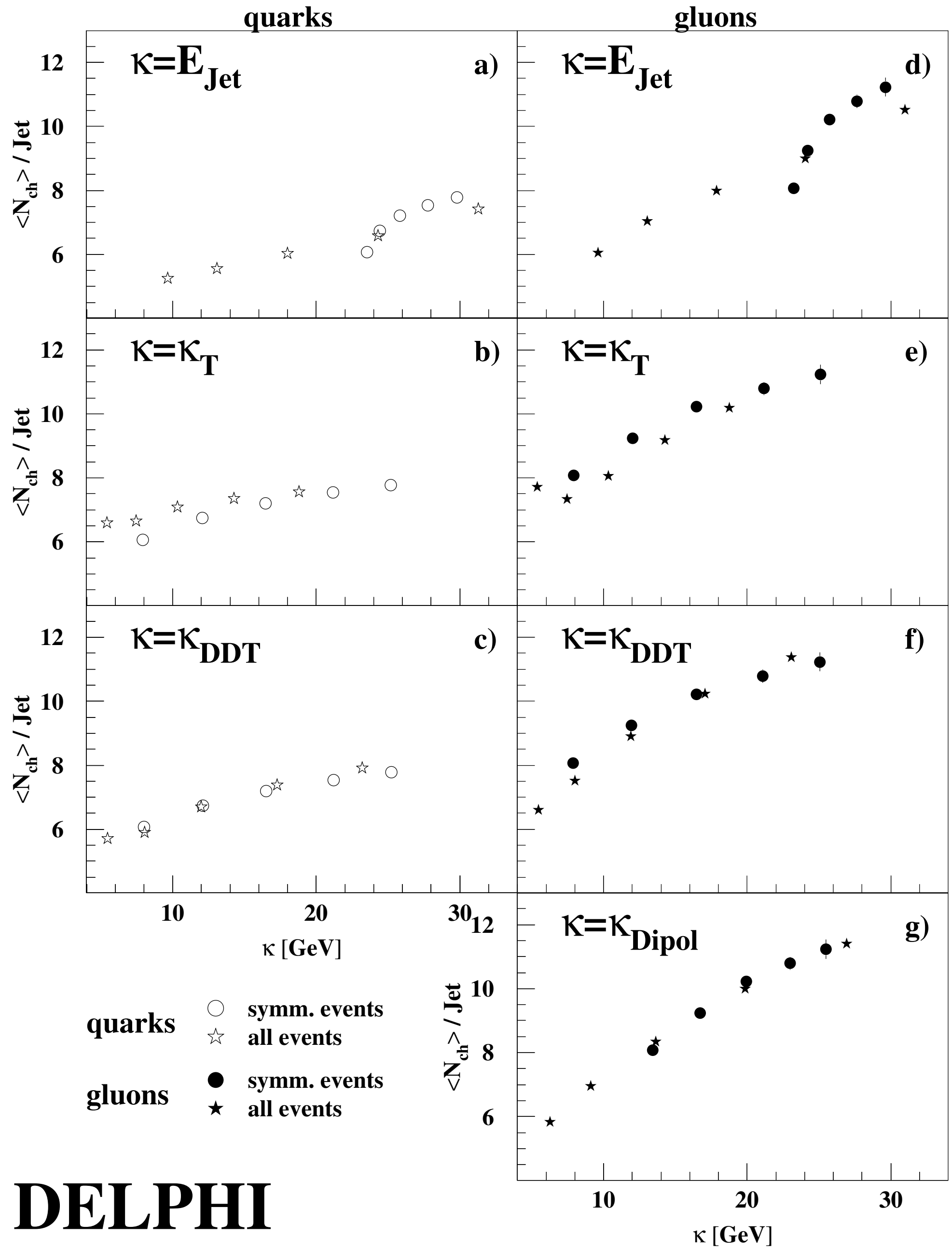

Figure 2: Average charged particle multiplicity for quark and gluon jets as determined from symmetric and non-symmetric event topologies as a function of different choices of scales as defined in the text.

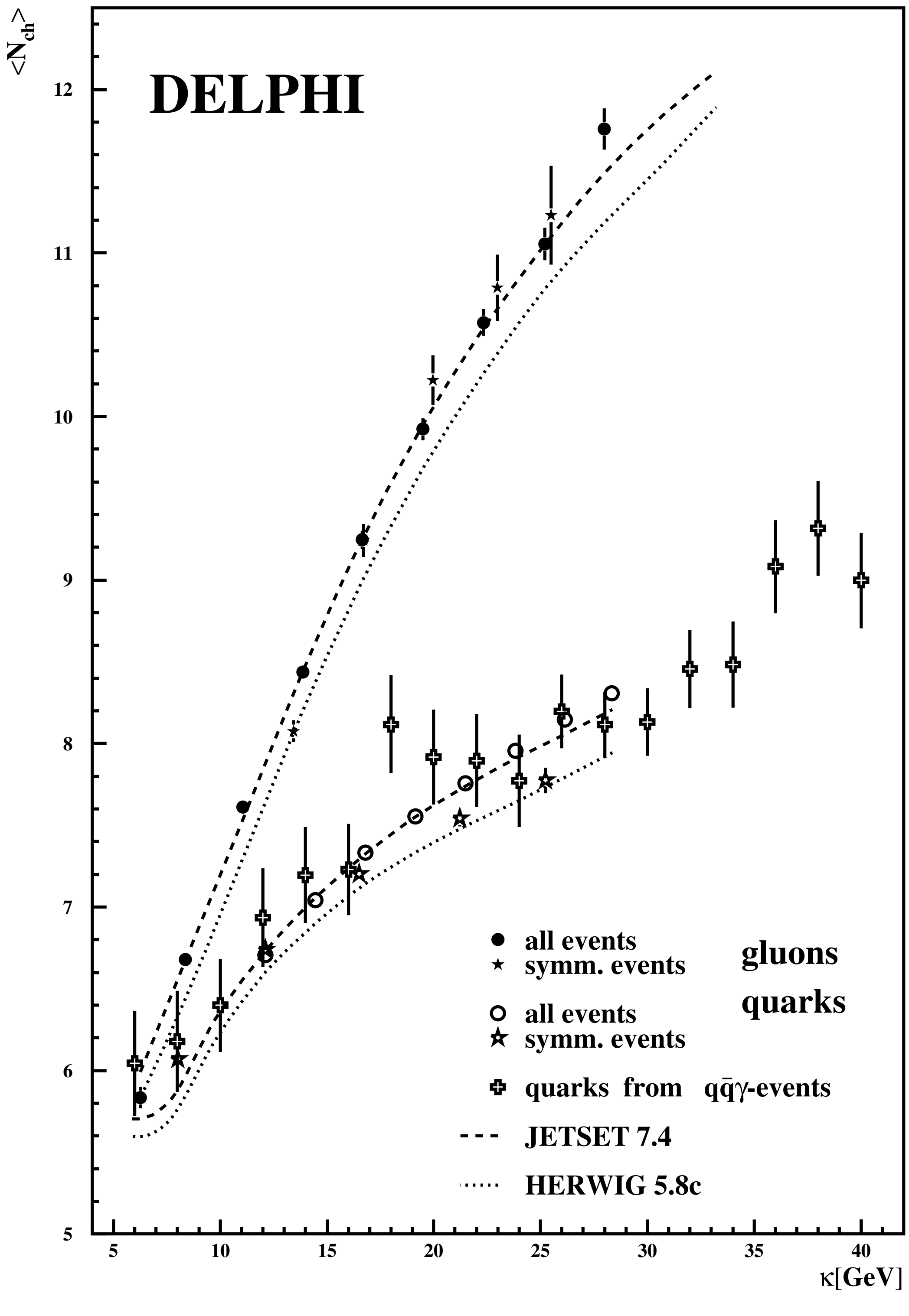

Figure 3: Average charged particle multiplicity for quark and gluon jets as determined from symmetric and non-symmetric event topologies as function of $\kappa = \kappa_{Dipole}$ compared with the predictions of JETSET and HERWIG. Parameter settings of the models are as in [13].

i.e. the QCD predictions for the ratio of multiplicities applies equally well to the ratio of the slopes of the multiplicities. In fact it is to be expected that the slope ratio is closer to the QCD prediction than the multiplicity ratio, as it should be less affected by non-perturbative effects. Beyond this, it is clear that any transformation applied to both the quark scale and the gluon scale leaves the slope ratio unaffected.

This contiguity has been cross checked using the HERWIG model, which allows the number of colours, and thus by SU(n) group relations the colour factor ratio C_A/C_F, to be changed. The predictions of HERWIG are found to follow the expectation of the right hand side of Eqn. 4.

In Fig. 4, the data of Fig. 3 are shown again, but with a logarithmic x-coordinate. As expected, in this graph the increase of the multiplicity with scale appears to be approximately linear. It becomes immediately evident that the slope for the gluon multiplicity is about twice as big as for quarks, thus already strikingly confirming the QCD prediction.

In order to obtain quantitative information, several functions (see Tab. 1) are fitted to the data, requiring however that the multiplicities increase monotonically with energy. In the lower inset of Fig. 4, the ratio of the gluon to quark multiplicity is shown as well as the ratio of the corresponding slopes as function of the scale. The ratio of the multiplicities in the data is between 1 at small scale and 1.5 at the highest scales measured. However, the fits indicate that the quark multiplicity is bigger than the gluon multiplicity for very small scale. The ratio of the slopes for the different types of fit (indicated by the grey band in Fig. 4) for the whole range of the data, maybe with the exception of very small scales where non-perturbative effects are expected to set in, is in the range 2.0 to 2.5, well within the range of the naive QCD expectation.

The size of the non-perturbative constant terms attached to the multiplicity due to the primary gluon or quark fragmentation are larger for quarks (see Tab. 1) with the exception of the first fit for κ_{DDT}. The difference of these terms is up to 1.66 for κ_{DDT} and up to 3 in case of κ_{Dipole}. The larger non-perturbative constant term for quarks compared to gluons directly explains the different behaviour of the ratio of multiplicities and the slope ratio in the lower part of Fig. 4.

The comparison of the data to the fragmentation models and the fits suggest that the non-perturbative influence is concentrated at very small scale. Therefore it seems justified to simultaneously fit the general NLO prediction for the multiplicity in jets [14] extended by additive terms to account for the non-perturbative influence of the quark and gluon data. The ratio of the slopes of the multiplicity (using Eqn. 4) is taken directly from the NNLO QCD prediction [15]. Thus the full ansatz reads:

$$
\begin{aligned}
< n_q > &= N_{pert} + N_0^q \\
< n_g > &= N_{pert} \cdot r(\kappa) + N_0^g \\
N_{pert} &= K \cdot \alpha_s^b \cdot \exp \frac{c}{\sqrt{\alpha_s}} \, [1 + O(\sqrt{\alpha_s})] \\
r(\kappa) &= \frac{9}{4}(1 - r_1\gamma_0 - r_2\gamma_0^2)
\end{aligned}
\tag{5}
$$

with:

$$
b = \frac{1}{4} + \frac{2}{3}\frac{n_F}{\beta_0}\left(1 - \frac{C_F}{C_A}\right) \quad ; \quad c = \frac{\sqrt{32 C_A \pi}}{\beta_0} \quad ; \quad \beta_0 = 11 - \frac{2}{3} n_F \tag{6}
$$

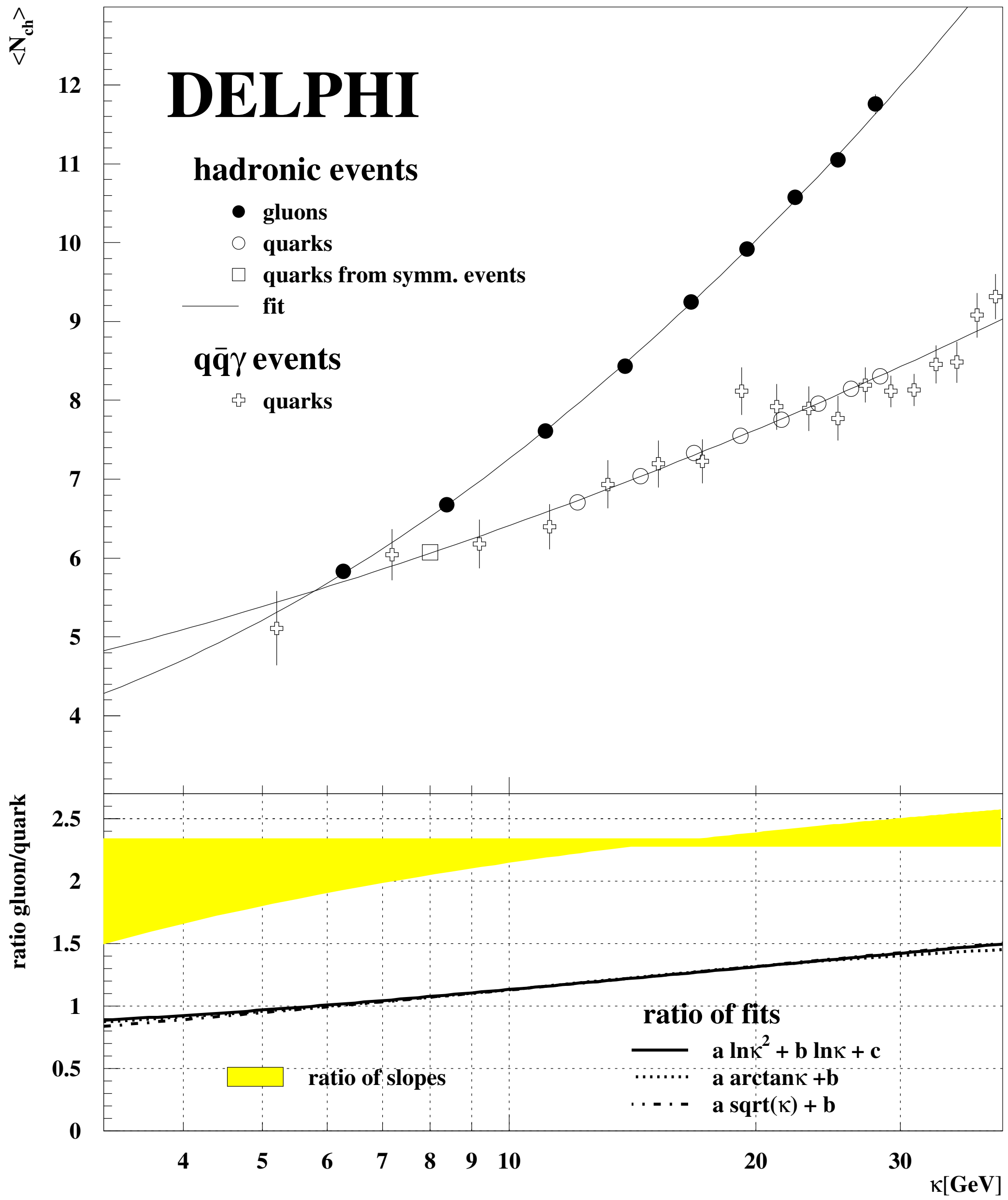

Figure 4: Average charged particle multiplicity for quark and gluon jets as a function of $\kappa = \kappa_{Dipole}$ fitted with different functions. The $q\bar{q}\gamma$ data are not included in the fit due to their slightly different quark content. The lower inset shows the ratio of the gluon to quark multiplicity, the range of the fitted ratio of the slopes of the two functions is shown by the shaded band. The corresponding parameters and their errors are given in Tab. 1.

fitted function		DDT	Dipole
$n_q = a(\ln\kappa)^2 + b^2\ln\kappa + N_0^q$ $n_g = r\cdot a(\ln\kappa)^2 + c^2\ln\kappa + N_0^g$	χ^2/N_{df}	1.56	0.72
	r	3.12 ± 0.05	3.86 ± 0.1
	N_0^q	3.79 ± 0.05	3.72 ± 0.04
	N_0^g	4.2 ± 0.1	3.26 ± 0.06
	a	0.205 ± 0.008	0.196 ± 0.005
	b	0.82 ± 0.02	0.85 ± 0.02
	c	0.55 ± 0.02	-0.008 ± 0.1
$n_q = a\arctan(b\cdot\kappa) + N_0^q$ $n_g = r\cdot a\arctan(b\cdot\kappa) + N_0^g$	χ^2/N_{df}	1.03	0.88
	r	2.08 ± 0.04	2.28 ± 0.04
	N_0^q	4.48 ± 0.07	4.62 ± 0.06
	N_0^g	4.27 ± 0.09	3.19 ± 0.1
	a	3.78 ± 0.05	3.86 ± 0.06
	b	$(5.6 \pm 0.3)\cdot 10^{-2}$	$(5.0 \pm 0.2)\cdot 10^{-2}$
$n_q = a\sqrt{\kappa} + N_0^q$ $n_g = r\cdot a\sqrt{\kappa} + N_0^g$	χ^2/N_{df}	4.19	2.98
	r	2.23 ± 0.04	2.34 ± 0.07
	N_0^q	3.56 ± 0.07	3.56 ± 0.07
	N_0^g	1.9 ± 0.2	0.55 ± 0.2
	a	0.90 ± 0.02	0.905 ± 0.02

Table 1: Functions and parameters fitted to the scale dependence of the quark and gluon multiplicities.

$$r_1 \quad = \quad 1 + \frac{n_f}{C_A} - \frac{2n_fC_F}{C_A^2} \quad ; \quad r_2 = r_1\left(\frac{25}{8} - \frac{3}{4}\frac{n_f}{C_A} - \frac{n_fC_F}{C_A^2}\right) \quad ; \quad \gamma_0 = \sqrt{\frac{\alpha(\kappa)C_A}{18\pi}}$$

The terms proportional to r_1 (r_2) correspond to the NLO and NNLO prediction respectively. $N_0^{q,g}$ and K are non-perturbative constant terms determined by the fit. For this fit, Λ was set to 50 MeV, which leads to a successful description of the overall multiplicity. Note that, due to the incompleteness of the calculation, this value of Λ is process dependent [14]. The data at the lowest scales, where non-perturbative effects seem to set in, are excluded from the fits.

This ansatz describes the data perfectly (see Fig. 5). Tab. 2 gives the parameters of the fit and their errors for both choices of scale. The χ^2/Ndf values for the fits are reasonable even without accounting for systematic errors.

The inhomogeneous terms given in Tab. 2 are estimates of the size of the non-perturbative terms discussed in the introduction. A rather large value is found for the quarks (2.3) whereas the value for gluons is even negative. The latter is expected, due to the presumed loss of particles from gluon jets into quark jets by the jet algorithms.

The result for the colour factor ratio is obtained directly from the fit, and is in the range 2.08 to 2.27 for the leading order prediction and 2.23 to 2.45 for the NNLO prediction. The results obtained with quark jets from $q\bar{q}\gamma$ events are fully consistent. They are assumed to be independent from the previous results as their statistical error is dominated by the large error on the quark jet multiplicities. The lower result in both cases corresponds to κ_{Dipole}. The quality of the fits is better for this scale than for κ_{DDT}. Together with the better agreement of the symmetric and non-symmetric topologies (see Fig. 2), this supports κ_{Dipole} as the relevant choice of scale. However the final decision on the scale awaits better theoretical justification.

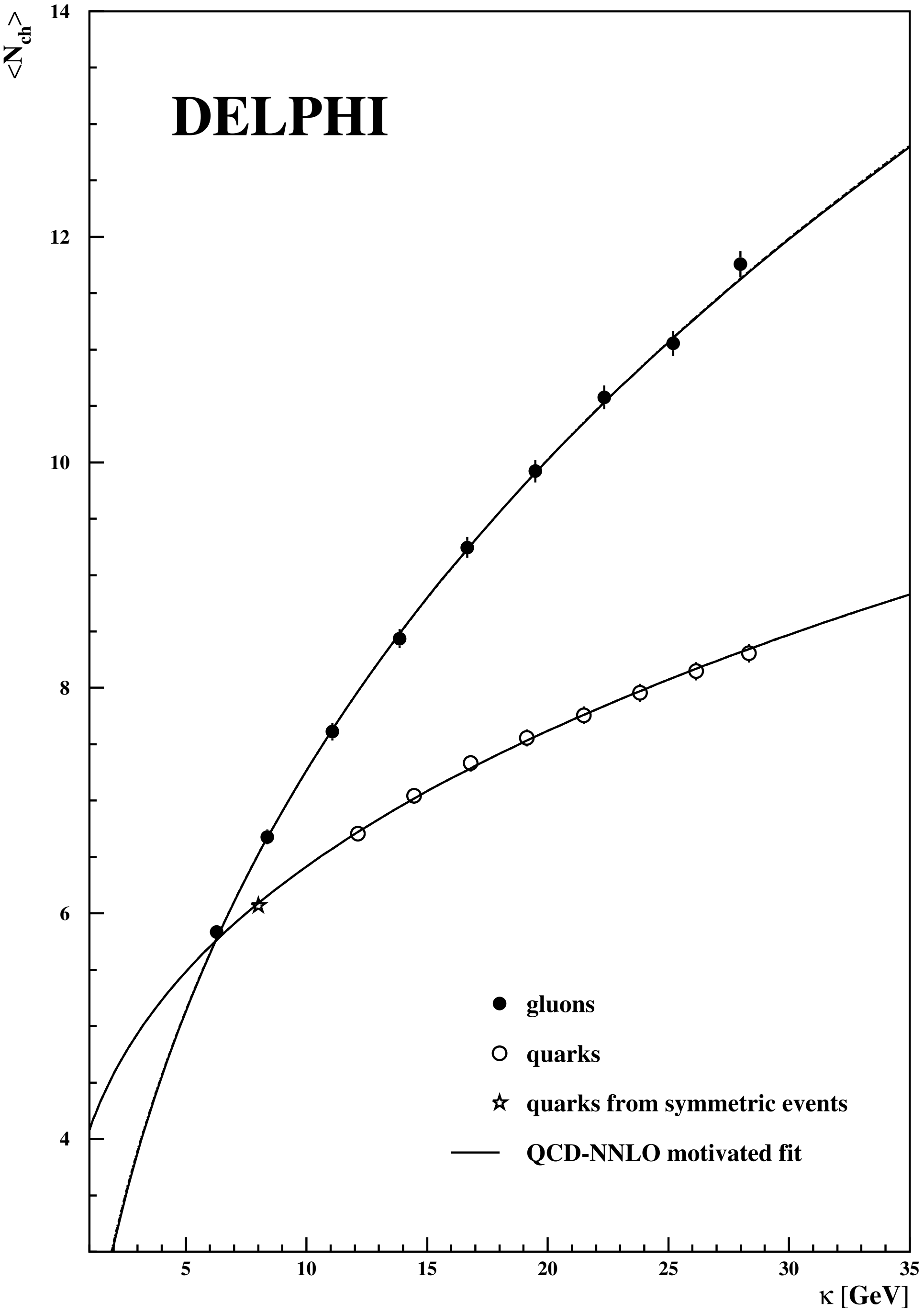

Figure 5: Average charged particle multiplicity for quark and gluon jets as a function of $\kappa = \kappa_{Dipole}$ fitted with the (N)NLO QCD result extended by non-perturbative terms. The corresponding parameters and their errors are given in Tab. 2.

$\Lambda = 50$MeV $N_f = 3$		DDT			Dipole		
		LO	NLO	(N)NLO	LO	NLO	(N)NLO
χ^2/N_{df}		1.79	1.78	1.78	1.23	1.21	1.21
non	$K \cdot 10^3$	6.82 ± 0.07	6.94 ± 0.07	6.96 ± 0.07	7.00 ± 0.06	7.13 ± 0.06	7.14 ± 0.06
pertur-	N_0^q	2.30 ± 0.07					
bative	N_0^g	-0.2 ± 0.2	-0.1 ± 0.2	-0.06 ± 0.1	-2.07 ± 0.1	-1.94 ± 0.1	-1.91 ± 0.1
$\frac{C_A}{C_F}$		2.08 ± 0.04	2.21 ± 0.06	2.23 ± 0.05	2.27 ± 0.04	2.43 ± 0.05	2.45 ± 0.04
χ^2/N_{df}					1.31	1.30	1.30
non	$K \cdot 10^3$				5.98 ± 0.3	6.09 ± 0.4	6.10 ± 0.3
pertur-	N_0^q				3.10 ± 0.3		
bative	N_0^g				-0.69 ± 0.2	-0.59 ± 0.2	-0.56 ± 0.2
$\frac{C_A}{C_F}$					2.25 ± 0.2	2.40 ± 0.2	2.42 ± 0.2

Table 2: Parameters of the QCD ansatz fitted to the scale dependence of the quark and gluon multiplicities. The upper part of the table shows the results as obtained from $q\bar{q}g$ events, the lower part the results as obtained from $q\bar{q}g$ events for the gluon jets and $q\bar{q}\gamma$ events for the quark jets.

It is observed that the leading order result for κ_{Dipole} agrees better with the QCD expectation than the (N)NLO result. As the choice of scales is explicitly model dependent, the average of the colour factors obtained with the two scales using the (N)NLO calculation is given as the final result :

$$\frac{C_A}{C_F} = 2.34 \pm 0.04(stat.) \pm 0.11(scale)$$

The first error is the average statistical error from the fit, the second error is half the difference between the results obtained with the different scale choices. Preliminary studies of further experimental uncertainties indicate that these are small compared with the scale error.

4 Summary

In summary we have measured the dependence of the charged particle multiplicities in quark and gluon jets for several choices of scales. From the comparison of symmetric and asymmetric events, two transverse momentum-like scales have been found to be suitable for quantitative studies of the scale dependence. The multiplicities of quark jets observed in $q\bar{q}g$ and $q\bar{q}\gamma$ events are found to agree well. The ratio of the slopes of the gluon and quark multiplicities is found to lie in the range 2.0 to 2.5. The slope ratio directly reflects the higher colour charge of gluons compared to quarks. This can also be interpreted as direct evidence for the triple gluon coupling, one of the basic ingredients of QCD. It is of special importance that this evidence is due to very soft radiated gluons and is therefore complementary to the measurement of the triple gluon coupling in four-jet events at large momentum transfer.

The simultaneous description of the quark and gluon multiplicities with scale also strongly supports the Local Parton Hadron Duality hypothesis.

Large non-perturbative terms mainly for the quark multiplicity are responsible for the observed relatively small gluon to quark multiplicity ratio. If these non-perturbative terms are considered as additive terms, the data can be described by the perturbative

NLO QCD expectation. From this fit, the colour factor ratio C_A/C_F is measured to be:

$$\frac{C_A}{C_F} = 2.34 \pm 0.04(stat.) \pm 0.11(scale)$$

This measurement is comparable in precision with the most precise measurements from four-jet events [16, 17].

A reduction of the dominating scale error of this measurement seems feasible, provided a better understanding of the physical scales determining jet evolution is reached.

Finally it should be remarked that it should be possible to verify this result at higher jet energies by comparing the multiplicities observed in e^+e^- annihilation as a function of the beam energy, which is $q\bar{q}$ production, and $p\bar{p}$ collisions at small x, which are dominated by gluon gluon scattering. However, multiplicities as function of the transverse energy, E_T, then have to be measured in $p\bar{p}$ collisions. So far, to our knowledge, these data are not available.

References

[1] DELPHI Collab., K. Hamacher, O. Klapp, P. Langefeld et al., contributed paper to the EPS-HEP Conf., Jerusalem 1997.

[2] S.J. Brodsky and J.F. Gunion, Phys. Rev. Lett. 37 (1976) 402.

[3] DELPHI Collab., P. Abreu et al., Z.Phys.C70 (1996) 179-196.

[4] CLEO Collab., M.S. Alam et al., CLNS-96-1452 (1997), hep-ex/9701006.
OPAL Collab., G. Alexander et al., Phys.Lett. B388 (1996) 659-672.
for a compilation see: J. Fuster et al., Proc. of QCD 96, S. Narison ed., Nucl. Phys. B (Proc. suppl.) 54A (1997).

[5] DELPHI Collab., R. Alemany et al., contributed paper to the EPS-HEP Conf., Jerusalem 1997.

[6] ALEPH Collab., R. Barate et al., CERN-PPE-97-003 (1997).

[7] S. Marti i Garcia, Tesi Doctoral, Univ. de València, 1995.

[8] M. Siebel, Diplomarbeit, Bergische Univ. - GH Wuppertal, 1997.

[9] S. Catani, Yu. L. Dokshitzer, M. Olsson, G. Turnock and B.R. Webber, Cavendish-HEP-91/5

[10] Yu. Dokshitzer, V.A. Khoze, A.H. Mueller ans S.I. Trojan, Basics of Perturbative QCD, Editions Frontières, 1991.

[11] T. Sjöstrand, Comp. Phys. Comm. 39 (1986) 347.

[12] G. Marchesini et al., Comp. Phys. Comm. 67 (1992) 465.

[13] DELPHI Collab., P. Abreu et al., Z. Phys. C 73 (1996) 11-59.

[14] B.R. Webber, Phys. Lett. 143B, (1984) 501.

[15] J.B. Gaffney and A.H. Mueller, Nucl. Phys. B250 (1985) 1109;
A.H. Mueller, Nucl. Phys. B241 (1984) 141.

[16] ALEPH Collab, R. Barate et al., CERN PPE 97-002 (1997).

[17] DELPHI Collab, A. Seitz et al., contributed paper to the EPS-HEP Conf., Jerusalem 1997.