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A Natural Solution to the Neutrino Mixing Problem.

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Abstract

The combined requirements, of (i) a natural solution to the fermion mass hierarchy problem and (ii) an explanation of both the atmospheric and solar neutrino problems, lead to an essentially unique picture of neutrino masses and mixing angles. The electron and muon neutrinos are quasi-degenerate in mass with maximal mixing, giving $\nu_e - \nu_\mu$ vacuum oscillations. The overall neutrino mass scale is set by the atmospheric neutrino requirement $\Delta m^2 \sim 10^{-2}$ eV^2, implying a mass for ν_e and ν_μ of order 1 eV in models with a natural mass hierarchy, whilst the tau neutrino is expected to be much lighter than this and only weakly mixed. We present an explicit example based on the anti-grand unification model of fermion masses.

1 Introduction

The observed hierarchy of quark and lepton masses and quark mixing angles strongly suggests the existence of an approximately conserved chiral flavour symmetry [1] beyond the Standard Model (SM). For theories in which this chiral symmetry group forms part of the extended gauge group, the values of the chiral flavour charges are strongly constrained by anomaly cancellation conditions. Several models of this type have been constructed [2, 3, 4, 5, 6, 7] which give a realistic quark and charged lepton mass spectrum, without any fine-tuning. In this letter we consider the structure of the neutrino mass matrix in such models with a natural mass hierarchy. We show that consistency with atmospheric and solar neutrino data can then only be obtained if they are both due to $\nu_e - \nu_\mu$ vacuum oscillations.

As we pointed out some time ago [8] the effective three generation light neutrino mass matrix M_ν in models with approximately conserved chiral flavour charges, generated for example by the usual see-saw mechanism, can have two qualitatively different types of eigenstate. This is a consequence of the hierarchical structure and symmetry $M_\nu = M_\nu^T$ of the mass matrix. In the first case, a neutrino can combine with its own antiparticle to form a Majorana particle and has small mixing angles with the other neutrinos. We shall be interested in the case where the tau neutrino combines with the tau antineutrino. The second type of eigenstate corresponds to a neutrino combining with an antineutrino, which is not the CP conjugate state, to form a 2-component massive neutrino. Such states naturally occur in pairs with quasi-degenerate masses and maximal mixing ($\sin^2 2\theta \simeq 1$). We shall be interested in the case where the electron neutrino combines with the muon antineutrino; the other member of the quasi-degenerate pair is formed by combining the muon neutrino with the electron antineutrino. The fractional mass difference between the two eigenstates is suppressed by the appproximately conserved chiral charges ($\Delta m/m \ll 1$).

In the next section, we discuss the structure of the neutrino mass matrix in models with a natural fermion mass hierarchy. We then consider the phenomenology of neutrino oscillations in such models in section 3. It is shown that the only way of obtaining a simultaneous solution of the atmospheric and solar neutrino problems without fine-tuning is via $\nu_e - \nu_\mu$ vacuum oscillations.

An explicit example based on the anti-grand unification model (AGUT) is then presented in section 4. The chiral charges of the quarks and leptons in the AGUT model are essentially uniquely determined by the anomaly cancellation conditions. However the overall neutrino mass scale is not explained in the model, since the natural see-saw mass scale is set by the Planck mass, which gives a too low neutrino mass scale of $\frac{\langle\phi_{WS}\rangle^2}{M_{Planck}} \sim 3 \times 10^{-6}\ eV$. It is therefore necessary to introduce the overall neutrino mass scale by hand. This is done by introducing an effective Higgs field which is a triplet under the electroweak SU(2) gauge group and assigning it an ad hoc vacuum expectation value, determined phenomenologically by the atmospheric neutrino data.

2 The Fermion Mass Hierarchy

The masses of the charged fermions range over five orders of magnitude from the electron to the top quark. It is only the top quark which has a mass of order the electroweak scale $\langle\phi_{WS}\rangle = 174$ GeV and has a SM Yukawa coupling of order unity. All of the other quark and lepton masses are suppressed relative to this scale. It is natural to interpret the different orders of magnitude of the suppression factors as due to different products of small symmetry breaking parameters, arising from some approximate chiral gauge symmetry beyond that of the Standard Model Group (SMG). The SMG is then the low energy remnant of some larger gauge group G and the SM Yukawa couplings are effective coupling constants which, in general, are forbidden by gauge invariance under G. The gauge group G is supposed to be spontaneously broken to the SMG at some high energy scale and the effective SM Yukawa couplings are thereby generated. These suppressed effective couplings of left-handed to right-handed quarks and leptons are mediated by vector-like super-heavy intermediate states. If all the appropriate superheavy states exist, with masses of order the fundamental mass scale M_F of the extended theory, the suppression factors are determined by the gauge quantum numbers of the fermions and the Higgs fields. In this way charged fermion mass matrices are generated, for which the different matrix elements can naturally be of different orders of magnitude and give a realistic mass and mixing hierarchy.

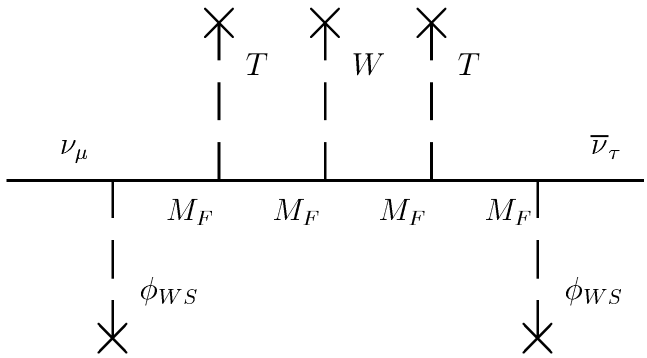

Figure 1: Example Feynman diagram for a neutrino mass matrix element generated by the see-saw mechanism and suppressed by the approximately conserved chiral gauge quantum numbers of the AGUT model. The crosses indicate the couplings of the Higgs fields to the vacuum.

This scheme is readily extended [8] to generate a non-zero light neutrino mass matrix M_ν:

$$\mathcal{L}_m = (M_\nu)_{ij} \nu_{L_i} C \nu_{L_j} + \text{h.c.} \tag{1}$$

It is then necessary to exchange either the Weinberg-Salam Higgs field ϕ_{WS} twice—the see-saw mechanism [8, 9]—as illustrated in fig. 1, or a weak isotriplet Higgs field Δ [10]. As an example, the Higgs field exchanges required to generate the mass matrix element connecting the left-handed muon neutrino to the right-handed tau antineutrino, via the see-saw mechanism, are shown in fig. 1 for an AGUT model of the fermion masses [7, 11]. Assuming all the fundamental Yukawa couplings are of order unity, this diagram gives the order of magnitude expression:

$$(M_\nu)_{\mu\tau} = \frac{\langle \phi_{WS} \rangle^2}{M_F} \frac{\langle W \rangle}{M_F} \frac{\langle T \rangle^2}{M_F^2} \tag{2}$$

for the matrix element. The first factor $\frac{\langle \phi_{WS} \rangle^2}{M_F}$ is the see-saw neutrino mass scale, while $\frac{\langle W \rangle}{M_F}$ and $\frac{\langle T \rangle^2}{M_F^2}$ are suppression factors arising from the exchanges of the Higgs fields, W and T, needed to match the chiral gauge quantum number differences between ν_μ and $\overline{\nu}_\tau$ in the AGUT model. A similar structure is obtained in other models with approximately conserved $U(1)$ charges [12, 13].

As is the case for the charged fermion mass matrices, the neutrino mass matrix M_ν is determined up to factors of order unity by the quantum numbers of the neutrinos and Higgs fields, provided we assume the existence of all the necessary intermediate states at the fundamental mass scale M_F. In some models this is not true and the superheavy fermion spectrum is constrained, often by specifying a heavy Majorana (right-handed neutrino) mass matrix. The quantum numbers of the SM neutrino states are of course the same as those of the charged leptons in the corresponding weak isodoublets.

The neutrino mass matrix M_ν is, by its very definition eq. (1), symmetric. Also, in models with approximately conserved chiral $U(1)$ charges, the matrix elements are generally of different orders of magnitude due to the presence of various suppression factors similar to those in eq.(2). Thus the generic structure for M_ν is a matrix in which the various elements typically each have their own order of magnitude, except in as far as they are forced to be equal by the symmetry $M_\nu = M_\nu^T$. The largest neutrino mass eigenvalue is then given by the largest matrix element of M_ν. If it happens to be one of a pair of equal off-diagonal elements, we get two very closely degenerate states as the heaviest neutrinos and the third neutrino will be much lighter and, in first approximation, will not mix with the other two. If the largest element happens to be a diagonal element, it will mean that the heaviest neutrino is a Majorana neutrino, the mass of which is given by this matrix element, and it will not be even order of magnitude-wise degenerate with the other, lighter neutrinos. These light neutrinos may or may not get their masses from off-diagonal elements and thus, in first approximation, be degenerate.

In models with approximately conserved chiral charges, there is a tendency for a pair of quasi-degenerate neutrinos to form; these are typically the heaviest neutrinos [8]. These neutrino states may couple dominantly to any pair of charged leptons. Thus the strongly mixed quasi-degenerate pair are essentially just as likely to be electron and muon neutrinos as muon and tau neutrinos or electron and tau neutrinos.

The lepton mixing matrix U is defined analogously to the usual CKM quark mixing matrix, in terms of the unitary transformations U_ν and U_E, on the left-handed lepton fields, which diagonalise the squared neutrino mass matrix $M_\nu M_\nu^\dagger$ and the squared charged lepton mass matrix $M_E M_E^\dagger$ respectively:

$$U = U_\nu^\dagger U_E \tag{3}$$

The charged lepton unitary transformation U_E is expected to be quasi-diagonal, with small off-diagonal elements due to the charged lepton mass hierarchy. On the other hand when there is a quasi-degenerate pair of neutrinos, because off-diagonal elements dominate their masses, the mixing angle contribution from U_ν will be very close to $\pi/4$. This is because then, in first approximation, U_ν has to diagonalise the σ_x Pauli matrix, leading to eigenstates which are 50% probability mixtures of two of the original neutrino states. The lepton mixing matrix U will have a similar structure, since U_E is quasi-diagonal. If there is no pair of quasi-degenerate neutrinos, U_ν and U are expected to be quasi-diagonal like U_E

So we conclude that there are two generic forms for the neutrino masses and mixing angles. In the first case there are a pair of quasi-degenerate neutrinos with essentially maximal mixing and a third essentially unmixed Majorana neutrino. In the second case the neutrino spectrum is similar to those of the charged fermion families, being hierarchical and having small mixing angles.

3 Neutrino Phenomenology

From the above discussion we see that models of this type could generate a neutrino spectrum which has small mixing between all three neutrinos. However, in order to explain the atmospheric neutrino problem it is necessary to have large mixing (for two neutrino mixing $\sin^2(2\theta) \gtrsim 0.7$). So we need only consider the case where we have two nearly degenerate neutrinos with almost maximal mixing.

There are three possibilities for neutrino mass matrices of this form; we may have the large mixing between the electron and mu neutrinos, the electron and tau neutrinos, or the mu and tau neutrinos. The atmospheric neutrino problem cannot be explained by $\nu_\tau - \nu_e$ mixing so we can immediately discount that scenario. Hence we are left with the cases of nearly maximal mixing between ν_e and ν_μ or ν_μ and ν_τ with the remaining neutrino mixing only slightly.

In the following we shall refer to the mass eigenstate neutrinos as ν_1, ν_2 and ν_3, with corresponding masses m_1, m_2 and m_3 (defined as the moduli of the mass eigenvalues). When ν_e and ν_μ are strongly mixed these mass eigenstates will be approximately given in terms of the flavour eigenstates by:

$$|\nu_1\rangle \simeq \frac{1}{\sqrt{2}}\left(|\nu_e\rangle + |\nu_\mu\rangle\right) \tag{4}$$

$$|\nu_2\rangle \simeq \frac{1}{\sqrt{2}}\left(|\nu_e\rangle - |\nu_\mu\rangle\right) \tag{5}$$

$$|\nu_3\rangle \simeq |\nu_\tau\rangle \tag{6}$$

In the case of large $\nu_\mu - \nu_\tau$ mixing we obtain similar relations between flavour and mass eigenstates by making the replacements $e \leftrightarrow \tau$ and $1 \leftrightarrow 3$ in the above equations. We also define the mass squared differences by $\Delta m^2_{ij} = |m_i^2 - m_j^2|$.

If we consider the large $\nu_\mu - \nu_\tau$ mixing scenario then we must have $\Delta m^2_{23} \sim 10^{-2}$ eV^2 for the atmospheric neutrino problem. We also want to explain the solar neutrino problem, and this requires mixing with the electron neutrino. The only small mixing solution to this problem is the MSW solution [14] which has $\Delta m^2_{e2(3)} \sim 10^{-5}$ eV^2. Hence we would need:

$$\Delta m^2_{e2(3)} = \left|m^2_{\nu_e} - m^2_{2(3)}\right| \sim 10^{-5} \text{ eV}^2 \tag{7}$$

$$\Delta m^2_{23} = \left|m^2_2 - m^2_3\right| \sim 10^{-2} \text{ eV}^2 \ll m^2_2 \tag{8}$$

where m_2^2 is much greater than Δm^2_{23} because we have nearly degenerate ν_2 and ν_3.

Clearly the only way we can satisfy these equations is if the ν_e is nearly degenerate to ν_2 and ν_3 (indeed the degree of degeneracy would need to be much greater to one of them than that between ν_2 and ν_3). This would require extreme fine tuning since there is no reason to expect the slightly mixed neutrino to be nearly degenerate with the other two.

So, we are left with large mixing between ν_e and ν_μ. In this case we could solve the atmospheric neutrino problem by $\nu_e - \nu_\mu$ mixing, with $\Delta m^2_{12} \sim 10^{-2}$ eV^2, [15]. The MSW solution with $\nu_e - \nu_\tau$ mixing is again prevented since the fine tuning involved would again be unnatural. 'Just so' vacuum oscillations, where about one $\nu_e - \nu_\mu$ oscillation length lies between the sun and the earth, require $\Delta m^2_{12} \sim 10^{-10}$ eV^2, which is clearly incompatible with the atmospheric neutrino solution. It would seem that we have eliminated all the possible natural solutions with this type of model.

However, as pointed out in [16], recent standard solar model calculations allow greater freedom in the solutions to the solar neutrino problem. These calculations vary in their predictions of the ^8B flux by more than a factor of two. If this flux is treated as a free parameter within this range then it is possible to get acceptable 'energy-independent' solutions. By 'energy-independent' vacuum oscillation solutions we mean that Δm^2 is sufficiently large that many oscillation lengths lie within the sun-earth distance, so that the reduction in ν_e flux does not depend on the energy of the solar neutrinos. Whilst an acceptable solution can be found in this way it should be noted that changing the ^8B flux does not alter the disagreement between our prediction and the different flux suppressions measured at KAMIOKANDE and HOMESTAKE [15] and we would still require these experiments to measure the same flux suppression factor of $\frac{1}{2}$. Hence we can now solve the solar neutrino problem with large mixing and

$$10^{-10}\ \mathrm{eV}^2 \lesssim \Delta \mathrm{m}^2_{12} \lesssim 10^{-2}\ \mathrm{eV}^2. \tag{9}$$

This is clearly compatible with the atmospheric neutrino solution if we take $\Delta m^2_{12} \sim 10^{-2}$ eV^2. So we now have an essentially unique solution to the solar and atmospheric neutrino problems within models of this type.

There is also a controversial indication for neutrino masses from the LSND experiment, [17], a $\overline{\nu}_\mu \to \overline{\nu}_e$ appearance experiment. However, a large mixing angle ($\sin^2(2\theta) \sim 1$) fit to the LSND data gives $\Delta m^2_{12} \sim 6 \times 10^{-2}$ eV^2 and this is inconsistent with reactor data [18] which for large mixing require $\Delta m^2_{12} \lesssim 10^{-2}$ eV^2. So we would predict that the LSND result will prove to be unfounded.

It is interesting to note the range of masses we would expect to arise for ν_e and ν_μ in these models. Since we have a hierarchical structure (apart from the near degeneracies arising from large off-diagonal elements) it is reasonable to assume $\Delta m_{12} \lesssim 0.1 m_1$. We can also use the experimental limit on the electron neutrino mass $m_{\nu_e} < 15$ eV [1] [19] to obtain:

$$0.2 \text{ eV} \lesssim m_{1(2)} < 15 \text{ eV}. \tag{10}$$

It also follows from the limit on m_{ν_e} and $\Delta m_{12}^2 \sim 10^{-2}\text{eV}^2$ that we would require $\Delta m_{12} = |m_1 - m_2| \gtrsim 3 \times 10^{-4}$ eV.

From the discussion in the previous section we would expect it to be more usual for the tau neutrino to be lighter than the other neutrinos. Indeed, since the suppression factors are due to charge differences which are similar to those in the charged fermion sector, it seems likely that they will span a similar range (~ 5 orders of magnitude). If this is the case then the tau neutrino must be the lightest neutrino to avoid the cosmological bound for stable neutrinos $\sum m_\nu < 40$ eV.

Since masses with $\sum m_\nu \gtrsim 1$ eV will significantly contribute to dark matter we would expect it to be common (but not essential) for models of this nature to generate hot dark matter candidates. The tau neutrino would be much too light to contribute significantly, so we would expect any dark matter contribution to come from the electron and mu neutrinos.

So we have an essentially unique solution to the solar and atmospheric neutrino problems with mass matrices of the form:

$$M_\nu \sim \begin{pmatrix} B_1 & A & X_1 \\ A & B_2 & X_2 \\ X_1 & X_2 & C \end{pmatrix} \tag{11}$$

where A is the dominant element and we would have:

$$m_1 \sim m_2 \sim |A| \tag{12}$$

and would also expect $m_{\nu_\tau} \sim |C|$, although if $\left|\frac{2X_1X_2}{A}\right| \gtrsim |C|$ then X_1 and X_2 would also contribute to m_{ν_τ}. Since we expect m_{ν_τ} to be heavily suppressed we would also expect:

$$\Delta m_{12} \sim \max\{|B_1|, |B_2|\} > |C|. \tag{13}$$

[1]There are well known problems with the experimental limit on m_{ν_e}, and it should be noted that this is a fairly conservative bound; bounds as low as $m_{\nu_e} < 4.4$ eV are claimed by some experiments.

We also have a constraint on the amplitude of double beta decay expected in models of this type. This amplitude is proportional to:

$$\langle m \rangle \sim (M_\nu)_{11} = B_1 \tag{14}$$

It follows that $|\langle m \rangle| \lesssim \Delta m_{12}$ and we may use $\Delta m_{12} \lesssim 0.1 m_1$ together with $\Delta m_{12}^2 \sim 10^{-2}$ eV^2 to obtain:

$$|\langle m \rangle| \lesssim 0.02 \text{ eV} \tag{15}$$

which compares with the experimental bound [20, 21] of:

$$|\langle m \rangle| < (0.6 - 1.6) \text{ eV}. \tag{16}$$

Planned experiments [20, 22] are expected to reach a sensitivity of $|\langle m \rangle| \sim (0.1 - 0.3)$ eV.

4 An Explicit Model

The AGUT model provides an example of one model of the type discussed which can generate neutrino masses and mixings of the form required by the previous section, yielding a solution to the atmospheric and solar neutrino problems. This model has an extended gauge group

$$G = SMG_1 \otimes SMG_2 \otimes SMG_3 \otimes U(1)_f \tag{17}$$

where $SMG_i = SU_i(3) \otimes SU_i(2) \otimes U_i(1)$. This group breaks down at the Planck scale ($M_{Planck} \sim 10^{19}$ GeV) to the diagonal subgroup SMG of SMG^3, identified as the usual SM gauge group, with the $U(1)_f$ being totally broken. The fermions of the i'th generation are put into the same representations under SMG_i as their usual SM representation, and are trivial under the other two SMG_js. Their charges under $U(1)_f$ are then determined by anomaly cancellation requirements.

Four Higgs fields S, W, T and ξ (in addition to the Weinberg-Salam Higgs field, ϕ_{WS}) and their representations under G were chosen in [7, 11] to break G down to the usual SM group and generate a realistic charged fermion spectrum. The Higgs field S was chosen to have a VEV $\langle S\rangle = 1$, in units of M_{Planck}, and the other VEVs were determined by a fit to the quark-lepton masses and quark mixing angles:

$$\langle W\rangle = 0.179, \quad \langle T\rangle = 0.071, \quad \langle\xi\rangle = 0.099 \tag{18}$$

This spectrum leads to the following charged lepton mass matrix, where we ignore CP violating phases:

$$M_E \quad \sim \quad \langle\phi_{WS}\rangle \begin{pmatrix} \langle W\rangle\langle T\rangle^2\langle\xi\rangle^2 & \langle W\rangle T\rangle^2\langle\xi\rangle^3 & \langle W\rangle\langle T\rangle^4\langle\xi\rangle \\ \langle W\rangle\langle T\rangle^2\langle\xi\rangle^5 & \langle W\rangle\langle T\rangle^2 & \langle W\rangle\langle T\rangle^4\langle\xi\rangle^2 \\ \langle W\rangle\langle T\rangle^5\langle\xi\rangle^3 & \langle W\rangle^2\langle T\rangle^4 & \langle W\rangle\langle T\rangle \end{pmatrix} \tag{19}$$

However, as we noted in [11], in order to generate neutrino masses of the right size it is necessary to introduce a new scale, since the see-saw scale $\frac{\langle\phi_{WS}\rangle^2}{M_{Planck}} \sim 3 \times 10^{-6}$ eV is too small. Here we do this by introducing a triplet (under $SU(2)$ in the SM) Higgs field Δ to generate the neutrino masses and choose the Abelian charges of Δ so that $(M_\nu)_{12} = (M_\nu)_{21}$ is unsuppressed giving:

$$\left(\frac{y_1}{2}, \frac{y_2}{2}, \frac{y_3}{2}, y_f\right) = \left(-\frac{1}{2}, -\frac{1}{2}, 0, 0\right) \tag{20}$$

where $\frac{y_i}{2}$ and y_f are the charges under $U(1)_i$ and $U(1)_f$. This Higgs field Δ is a doublet under $SU(2)_1$ and $SU(2)_2$, but a singlet under all the other non-Abelian groups. It generates the neutrino mass matrix:

$$M_\nu \sim \langle\Delta^0\rangle \begin{pmatrix} \langle\xi\rangle^3 & 1 & \langle T\rangle^3\langle\xi\rangle^2 \\ 1 & \langle\xi\rangle^3 & \langle T\rangle^3\langle\xi\rangle \\ \langle T\rangle^3\langle\xi\rangle^2 & \langle T\rangle^3\langle\xi\rangle & \langle T\rangle^3\langle W\rangle^3\langle\xi\rangle \end{pmatrix} \tag{21}$$

by Feynman diagrams such as those in fig. 2. This matrix is clearly of the form required from the previous section, with $(M_\nu)_{12}$ dominating the matrix to give nearly degenerate electron and muon neutrino masses. The diagonal element $(M_\nu)_{33}$ corresponds to m_{ν_τ} and is heavily suppressed, so the tau neutrino is very light in this model and, in fact, the neutrino masses span 7 orders of magnitude.

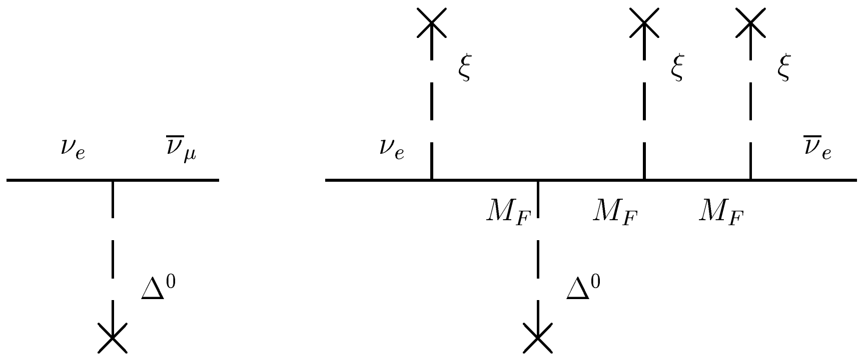

Figure 2: Example Feynman diagrams for neutrino mass. The crosses indicate the couplings of the Higgs fields to the vacuum, and $M_F \simeq M_{Planck}$

Diagonalisation of the lepton mass matrices M_E and M_ν then gives a lepton mixing matrix:

$$U \sim \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{\langle T\rangle^3\langle\xi\rangle}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{\langle T\rangle^3\langle\xi\rangle}{\sqrt{2}} \\ -\langle T\rangle^3\langle\xi\rangle & \langle T\rangle^3\langle\xi\rangle^2 & 1 \end{pmatrix} \sim \begin{pmatrix} 0.71 & -0.71 & 3\times10^{-5} \\ 0.71 & 0.71 & 3\times10^{-5} \\ -4\times10^{-5} & 4\times10^{-6} & 1 \end{pmatrix} \quad (22)$$

As we can see from this mixing matrix the tau neutrino is virtually unmixed in this model, with the mixing being much less than the constraints given by CDHS [23] ($\sin^2 2\theta_{\mu\tau} \lesssim 0.1$), and the sensitivities of CHORUS [24] and NOMAD [25], which for $\Delta m^2_{\nu_\mu\nu_\tau} \sim 15$ eV^2 are sensitive to $\sin^2 2\theta_{\mu\tau} > 10^{-3}$. So we essentially have two neutrino mixing between the electron and muon neutrinos with $\sin^2(2\theta) \sim 1$.

We choose the mass scale $\langle\Delta^0\rangle \approx 2$ eV in order to get an appropriate value for Δm^2_{12} giving:

$$\Delta m^2_{12} \sim 2\langle\Delta^0\rangle^2\langle\xi\rangle^3 \approx 8\times10^{-3} \text{ eV}^2 \quad (23)$$

$$m_1 \approx m_2 \approx 2 \text{ eV}, \quad m_{\nu_\tau} \approx 2\times10^{-7} \text{ eV} \quad (24)$$

which, since we have almost maximal mixing between ν_e and ν_μ, is suitable for the solution to both the solar and atmospheric neutrino problems. As expected the model is incompatible with the LSND result, and it also gives masses suitable for hot dark matter, with $\sum m_\nu \sim 4$ eV. The model also makes a prediction for neutrinoless double beta decay of $|\langle m \rangle| \sim \langle \Delta^0 \rangle \xi^3 \sim 2 \times 10^{-3}$ eV, which as expected is much less than values accessible by current or planned experiments.

5 Conclusions

We have found that, in models which give a natural solution to the fermion mass hierarchy problem, the only way of naturally explaining both the solar and atmospheric neutrino problems is if both effects are due to nearly maximal $\nu_e - \nu_\mu$ mixing with $\Delta m_{12}^2 \sim 10^{-2}$ eV^2; this leads to our prediction of an electron neutrino flux suppression factor of $\frac{1}{2}$ in all solar neutrino experiments. The electron and muon neutrinos are nearly degenerate with masses of order 1 eV, and are therefore likely to be hot dark matter candidates, whilst the tau neutrino is much lighter and only slightly mixed.

The prospects for examining this scenario experimentally in the near future are very good; reactor experiments on $\overline{\nu}_e$ survival rates, such as CHOOZ [26] and PALO-VERDE [27], will be able to reach $\Delta m_{12}^2 \sim 10^{-3}$ eV^2, and we would expect to see a strong signal of neutrino oscillations there. The LSND result will also prove to be unfounded if our scenario is correct. It should be noted that a characteristic of this scenario is that we would not expect to see either seasonal or day/night effects from the solar neutrinos. It is harder to verify the tau neutrino mixing since it is much weaker; however if the muon neutrino is heavier than a few eV then in some models the tau neutrino mixing may be sufficiently large (e.g. the mixing could be of order $\frac{m_\mu}{m_\tau}$ coming from M_E) to give a signal at the CERN experiments CHORUS and NOMAD. We conclude that we must have essentially unique neutrino masses and mixing, with large $\nu_e - \nu_\mu$ mixing, which can be confirmed or excluded by experiment within the next few years.

From the theoretical point of view the above neutrino mass scale implies some new physics between the electroweak scale and the Planck scale. So the assumption of a total "desert" up to about one order of magnitude below M_{Planck}, as in the AGUT model, is not consistent with our interpretation of neutrino phenomenology. Either some intermediate mass see-saw fermions or a Higgs field like Δ is required. The latter could acquire a vacuum expectation value $\langle\Delta^0\rangle \sim 1$ eV, via its interaction with two Weinberg-Salam Higgs fields ϕ_{WS} and the other Higgs fields W, T, ξ and S; but only if, for some as yet unknown reason, it has a very small coefficient of Δ^2 in the Higgs potential compared to M^2_{Planck}.

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