The Higgs boson mass bound in MSSM broken at high scale

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Abstract

We study the dependence of the Higgs boson mass on the supersymmetry breaking scale in the minimal supersymmetric extension of the standard Weinberg-Salam model. In particular, we find that for supersymmetry breaking scale 10^8 GeV $\leq M_s \leq 10^{16}$ GeV and for $m_{top}^{pole} = 175 \pm 5$ GeV the Higgs boson mass is 120 GeV $\leq m_h \leq$ 160 GeV.

At present one of the most urgent problems in high energy physics is the search for the Higgs boson. The lower LEP1 bound on the Higgs boson mass is [1]

$$m_h > 66\, Gev\,. \tag{1}$$

In standard Weinberg-Salam model there are several theoretical bounds on the Higgs boson mass :

(i) Tree level unitarity requirement leads to $m_h \leq 1$ Tev [2].

(ii) The requirement of the absence of the Landau pole singularity for the effective Higgs self-coupling constant for energies up to 10^{14} GeV gives $m_h \leq 200$ GeV for $m_{top}^{pole} \leq 200$ GeV [3].

(iii) The vacuum stability requirement leads to the lower bound on the Higgs boson mass which depends on the top quark mass [4].

The minimal supersymmetric extension of the standard model (MSSM) [5] predicts at tree level that the lightest Higgs boson has to be lighter than the Z-boson [5]. Radiative corrections slightly increase the value of the lightest Higgs boson mass [6].

In our previous paper [7] we studied the dependence of the Higgs boson mass on the scale of supersymmetry breaking in MSSM using one loop renormalization group equations for the effective coupling constants. In this note we reanalize this problem using two loop renormalization group equations for the effective coupling constant and recent more accurate world average value of the strong coupling constant $\alpha_s(M_z) = 0.118 \pm 0.003$ [8] and the value of the top quark mass $m_{top}^{pole} = 175 \pm 6$ Gev [9].

Our main assumption is that the standard Weinberg-Salam model originates from its minimal supersymmetric extension which is explicitly broken due to soft supersymmetry breaking terms at scale M_s. The tree level Higgs potential in the

MSSM model with general soft supersymmetry breaking terms is given by [5]

$$V = V_0 + V_{soft}, \tag{2}$$

$$V_0 = (g_1^2 + g_2^2)(H_1^+ H_1 - H_2^+ H_2)^2/8 + g_2^2 |H_1^+ H_2|^2/2, \tag{3}$$

$$V_{soft} = m_1^2 H_1^+ H_1 + m_2^2 H_2^+ H_2 + m_3^2 (H_1^T i\tau_2 H_2 + h.c.). \tag{4}$$

Here g_1 and g_2 are the $U(1)$ and $SU(2)$ gauge coupling constants and the Higgs doublets H_1 and H_2 couple with $q = -1/3$ and $q = 2/3$ quarks respectively. We assume that one of the combinations of the H_1 and H_2

$$H_{light} = H_2 \cos(\phi) + i\tau_2 H_1^+ \sin(\phi) \tag{5}$$

is relatively light, $m_{light} \approx O(M_z)$, whereas the other orthogonal combination

$$H_{heavy} = -H_2 \sin(\phi) + i\tau_2 H_1^+ / cos(\phi) \tag{6}$$

acquires a mass $m_{heavy} \approx O(M_s)$. We also assume that the masses of the superpartners of ordinary particles are of the order of $O(M_s)$. It is clear that for such scenario for $M_s \geq O(1)$ TeV it is necessary to have fine tuning among the soft supersymmetry breaking terms.

At scales lower than the supersymmetry breaking scale M_s we have the standard Weinberg-Salam model with the single Higgs isodoublet $H = H_{light}$. The crusial point is that the self-interaction effective coupling constant $\bar{\lambda}(M_s)$ at scale M_s is

$$0 \leq \bar{\lambda}(M_s) = (\bar{g}_1^2(M_s) + \bar{g}_2^2(M_s))(\cos(2\phi))^2/4 \leq (\bar{g}_1^2(M_s) + \bar{g}_2^2(M_s))/4 \tag{7}$$

To preserve the supersymmetry the gauge couplings $\bar{g}_1(M_s)$ and $\bar{g}_2(M_s)$ have to be calculated within the $\bar{DR}$-scheme [10]. The relation between the gauge coupling constants in the $\bar{MS}$-scheme and $\bar{DR}$-scheme has the form [10]

$$\frac{1}{\alpha i_{\bar{MS}}} = \frac{1}{\alpha_{i_{\bar{DR}}}} + \frac{C_2(G)}{12\pi}, \tag{8}$$

where $C_2(G)$ is the quadratic Casimir operator for the adjoint representation ($C_2(SU(N)) = N$.

So the assumption that standard Weinberg-Salam model originates from its supersymmetric extension with the supersymmetry broken at scale M_s allows us to obtain non-trivial information about the low energy effective Higgs self-coupling constant and hence to obtain nontrivial information about the Higgs boson mass. To relate the high energy value of the Higgs self-interaction effective coupling constant $\bar{\lambda}(M_s)$ with the low energy value of $\bar{\lambda}(M_z)$ it is necessary to use the renormalization group equations. The renormalization group equations for the effective coupling constants in neglection of all Yukawa coupling constants except top-quark Yukawa coupling constant in one-loop approximation read

$$\frac{d\bar{g}_3}{dt} = -7\bar{g}_3^3\,, \tag{9}$$

$$\frac{d\bar{g}_2}{dt} = -(\frac{19}{6})\bar{g}_2^3\,, \tag{10}$$

$$\frac{d\bar{g}_1}{dt} = (\frac{41}{6})\bar{g}_1^3\,, \tag{11}$$

$$\frac{d\bar{h}_t}{dt} = (\frac{9\bar{h}_t^2}{2} - 8\bar{g}_3^2 - \frac{9\bar{g}_2^2}{4} - \frac{17\bar{g}_1^2}{12})\bar{h}_t\,, \tag{12}$$

$$\frac{d\bar{\lambda}}{dt} = 12(\bar{\lambda}^2 + (\bar{h}_t^2 - \frac{\bar{g}2_1}{4} - \frac{3\bar{g}_2^2}{4})\lambda - \bar{h}_t^4 + \frac{\bar{g}_1^4}{16} + \frac{\bar{g}_1^2\bar{g}_2^2}{8} + \frac{3\bar{g}_2^4}{16}\,, \tag{13}$$

$$t = (\frac{1}{16\pi^2})\ln(\mu/m_Z)\,. \tag{14}$$

Here $\bar{g}_3$, $\bar{g}_2$ and $\bar{g}_1$ are the $SU(3)$, $SU(2)$ and $U(1)$ gauge coupling, respectively, and $\bar{h}_t$ is the top quark Yukawa coupling constant. In our numerical analysis we studied the renormalization group equations for the effective coupling constants in two loop approximation [11] in $\bar{M}S$-scheme. We used the following central values for the initial effective coupling constants at M_z-scale [8],[12],[13]:

$$\alpha_3(M_z)_{\bar{M}S} = 0.118 \pm 0.003, \tag{15}$$

$$\sin^2(\theta_W)(M_z) = 0.2320 \pm 0.0005, \tag{16}$$

$$(\alpha_{em,\bar{M}S}(M_z))^{-1} = 127.79 \pm 0.13 \tag{17}$$

For boundary condition (7) for the Higgs self-coupling constant $\bar{\lambda}(M_s)$ we have integrated numerically the renormalization group equations in two loop approximation. Also we took into account one loop correction to the Higgs boson mass [14] (running Higgs boson mass $\bar{m}_h(\mu) = \sqrt{\bar{\lambda}(\mu)}v$) does not coincide with pole Higgs boson mass. We used two loop formulae of ref.15 which relate the running top quark mass with pole top quark mass. Our results for the Higgs boson mass for different values of M_s and m_{top}^{pole} are presented in Table 1. Here $k = 0$ corresponds to the boundary condition $\bar{\lambda}(M_s) = 0$ and $k = 1$ corresponds to the boundary condition $\bar{\lambda}(M_s) = \frac{1}{4}(\bar{g}_1^2(M_s) + \bar{g}_2^2(M_s))$. We have found that our numerical results practically do not depend on the uncertainties in the determination of the electroweak couplings at M_z-scale and also on the use of the boundary condition (6) for electroweak coupling constants in the $\overline{DR}$-scheme instead of the $\overline{MS}$-scheme. The uncertainty in the determination of the strong coupling constant at M_z-scale leads to the uncertainty in the determination of the Higgs boson mass less than 2 GeV. The dependence of the Higgs boson mass on the scale of supersymmetry breaking M_s is very weak in the interval 10^8 GeV $\leq M_s \leq 10^{16}$ GeV.

From the requirement of the absence of Landau pole singularity for the Higgs self-coupling constant $\bar{\lambda}(\Lambda)$ for the scales up to $\Lambda = (10^3; 10^4; 10^6; 10^8; 10^{10}; 10^{12}; 10^{14})$ GeV (to be precise we require that at the scale Λ the Higgs self-coupling constant is $\frac{\bar{\lambda}^2(\Lambda)}{4\pi} \leq 1$) we have found the upper bound on the Higgs boson mass $m_h \leq$ 400; 300; 240; 200; 180; 170; 160) GeV, respectively.

It should be noted that in nonminimal supersymmetric electroweak models say in the model with additional gauge singlet σ we have due to the term $k\sigma H_1 i\tau_2 H_2$ term in the superpotential an additional term $k^2|H_1 i\tau_2 H_2|^2$ in the potential and as

a consequence our boundary condition for the Higgs self-coupling constant has to be modified, namely

$$\bar{\lambda}(M_s) = \frac{1}{4}(\bar{g}_1^2(M_s) + \bar{g}_2^2(M_s))\cos^2(2\varphi) + \frac{1}{2}\bar{k}^2(M_s)\sin^(2\varphi) \geq 0 \qquad (18)$$

The boundary condition (18) depends on unknown coupling constant $\bar{k}^2(M_s)$. However, it is very important to stress that for all nonminimal supersymmetric models broken to standard Weinberg-Salam model at scale M_s the effective Higgs self-coupling constant $\bar{\lambda}(M_s)$ is non-negative that is a direct consequence of the non-negativity of the effective potential in supersymmetric models. Therefore, the vacuum stability requirement results naturally if supersymmetry is broken at some high scale M_s and at lower scales standard Weinberg-Salam model is an effective theory.

As it follows from our results for 170 GeV $\leq m_{top}^{pole} \leq$ 180 GeV and for $M_s \leq 1$ TeV the Higgs boson mass is less than 120 GeV, while for the same values of the top quark mass and for the supersymmetry breaking scale $M_s \geq 10^8$ GeV the Higgs boson mass is larger than 120 GeV. Therefore, for such values of the top quark mass the measurement of the Higgs boson mass will discriminate standard scenario with low energy broken suppersymmetry and scenario with standard Weinberg-Salam model valid up to very high scale [16]. Moreover, for such values of the top quark mass the discovery at LEP2 the Higgs boson with the mass lighter than 85 GeV would mean that the scale of new physics is less than 5 TeV.

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Table 1. The dependence of the Higgs boson mass on the values of M_s, $m_{top}^{pole} \equiv m_t$ and $k = 0, 1$. Everything except k is in GeV.

m_t	165	165	170	170	175	175	180	180	185	185
M_S	k=0	k=1	k=0	k=1	k=0	k=1	k=0	k=1	k=0	k=1
10^3	69	111	74	114	78	117	83	120	88	123
$10^{3.5}$	81	117	86	120	92	124	98	128	104	132
10^4	89	121	95	125	101	130	108	134	114	139
10^6	105	129	113	135	121	141	129	147	137	153
10^8	112	132	120	138	129	147	138	152	146	159
10^{10}	115	133	124	140	133	147	142	154	151	161
10^{12}	117	134	126	141	136	147	145	154	154	161
10^{14}	118	134	127	141	132	148	147	156	156	164
10^{16}	118	134	128	141	138	148	148	156	158	164

References

[1] J.P.Martin, in Proceedings of the 28 th International Conference on High Energy Physics, Editors, Z.Ajduk and A.K.Wroblewski, World Scientific, 1997.

[2] B.W.Lee, C.Quigg and H.Thacker, Phys.Rev. **D16**(1977)1519.

[3] L.Maiani, G.Parisi and R.Petronzio, Nucl.Phys. **B136**(1978) 115; M.Lindner, Z.Phys. **C31**(1986)295.

[4] N.V.Krasnikov, Yad.Phys. **28**(1978)549; P.Q.Hung, Phys.Rev.Lett. **42**(1979)873; H.D.Politzer and S.wolfram, Phys.Lett. **B82**(1979)242; A.A.Anselm, JETP Lett. **29**(1979)590; M.Lindner, M.Sher and M.Zaglauer, Phys.Lett. **B228**(1989)139.

[5] For reviews and references, see: H.P.Nilles, Phys.Rep. **110** (1984)1.

[6] J.Ellis, G.Ridolfi and F.Zwirner, Phys.Lett. **B257**(1991)83;
H.Haber and R.Hempfling, Phys.Rev.Lett. **66**(1991)1815;
A.Yamada, Phys.Lett. **B263**(1991)233;
R.Barbieri, M.Frigeni and F.Caravaglios, Phys.Lett. **B258**(1991)167;
M.Carena et al., Phys.Lett. **B355**(1995)209;
M.Carena et. al, Nucl.Phys. **B461**(1996)407.

[7] N.V.Krasnikov. G.Kreyerhoff and R.Rodenberg, Mod. Phys. Lett. **A9**(1994)3663.

[8] M.Schmelling, in Proceedings of the 28 th International Conference on High Energy Physics, Editors, Z.Ajduk and A.K.Wroblewski, World Scientific, 1997.

[9] P.L.Tipton, in Proceedings of the 28 th International Conference on High Energy Physics, Editors, Z.Ajduk and A.K.Wroblewski, World Scientific, 1997.

[10] I.Antoniadis, C.Kounas and K.Tamvakis, Phys.Lett. **B119** (1982)377.

[11] M.E.Machacek and M.T.Vaughn, Nucl.Phys. B**222**(1983)83; **236**(1984)221; **249**(1985)70.

[12] Particle Data Group, Review of particle properties, Phys.Rev. **D50**(1994); P.Langacker and N.Polonsky, Phys.Rev. **D52**(11995)3081; P.Chanowski, Z.Pluciennik and S.Pokorski, Nucl.Phys. **B439**(1995)23.

[13] N.V.Krasnikov, Mod.Phys.Lett. **A9**(1994)2825.

[14] A.Sirlin and R.Zucchini, Nucl.Phys. **B266**(1986)389.

[15] N.Cray, D.S.Broadhurst, W.Craie and K.Scilcher, Z.Phys. C (1990)673.

[16] N.V.Krasnikov and S.Pokorski, Phys.Lett. **B288**(1992)284; Marco A.Diaz, Tonnis A. ter Veldhuis and Thomas J. Weiler, Phys.Rev.Lett. **74**(1995)2876.