Restoring the Correct Asymptotic Behaviour of Amplitudes for the Case of Nonzero Mass Particles Collisions

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Abstract

Fine-tuning mechanisms of the Standard Model are used for the case of non-zero mass particles collisions to the full.

It is deemed that the benchmark test of any consistent perturbative theory is its renormalisibility. Namely the proof by 't Hooft of this property for the model nowadays known as the standard one put an end to searching for unified theory of electromagnetic and weak interactions. To somewhat less extent the property of the fine-tuning unitarity cancellations to save a correct behaviour of scattering amplitudes at high energies has been mentioned. Meanwhile, both the notions are intimately connected between themselves, as it was noted long ago. For the case of models with non-abelian gauge symmetry and Higgs mechanism for mass acquiring the question was elaborated by LLewellyn-Smith [1]. Remind that in this paper it has been shown, that for amplitudes to be "well behaved" they must be: 1) of the Yang-Mills character and 2) the mass acquisition scheme must be of the Higgs type. The phenomenon of the first type is usually demonstrates for the case of reaction $e^+ + e^- \to W^+ + W^-$. Here "bad behaviour" of the Z, γ exchange amplitudes in the s-channel is canceled by those of the neutrino exchange in the $t-$ channel.

Here we demonstrate the whole machinery of fine-tuning in the Standard Model for the case of well known (Bjorken) process $l^+ + l^- \to Z^0 + H^0$, where initial leptons have nonzero masses. For definitness we consider the muon collider case.The diagrams, corresponding to this process are depicted on Fig.1.

Usually in the course of cross-section calculations one uses the diagram of Fig.1-c alone. The same we did but took into account the masses of initial muons. Then we obtain the following asymptotics of this process at $\sqrt{s} \to \infty$:

$$\sigma_{as}^{(c)}(\mu^+\mu^- \to ZH^0)|_{m_\mu \neq 0} = \frac{2\pi\ \alpha^2}{\sin^4(2\theta_W)} \cdot g_A^2 \cdot \frac{m_\mu^2}{m_Z^4}. \tag{1}$$

It is seen, that despite of the fact that this diagram is the pure s–channel one, the corresponding cross-section is not falling at high energy, but approaches a constant limit, whose value is equal to $\approx 1.2 \cdot 10^{-2}\ fb$. When considering the angular dependence of this cross-section, we can see, that this distribution is **flat**, indicating that it comes entirely from the $J = 0$ plane wave. It is obvious, that this behaviour contradicts the unitarity condition, which requires $\sigma_{J=0} \leq s^{-1}$ at high energy.

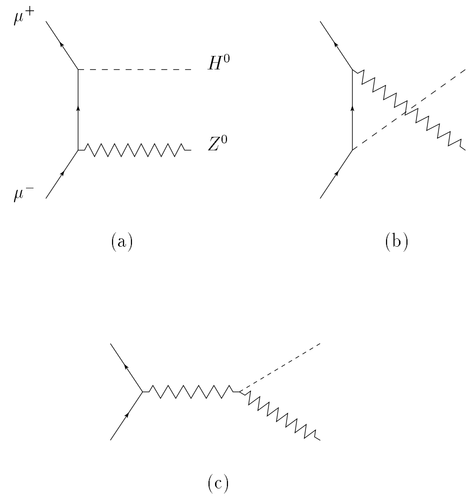

Figure 1:

Now we calculate the contribution into cross-section of the two remaining graphs of Fig.1-a and Fig.1-b.It turns out that the corresponding contribution is again equal exactly to that value of $\approx 1.2 \cdot 10^{-2}\ fb$. The corresponding angular distribution is also flat. At last, let's take into account the interference term corresponding to the diagram of Fig.1-c from the one hand side and that of joint contribution of Fig.1-a and Fig.1-b, from the other hand side. We have found that it is equal exactly to $\approx -2.4 \cdot 10^{-2}\ fb$, removing a seeming contradiction.

It might be useful to note, that all the three contributions reach their constant asymptotic values not simultaneously. The contribution stemming from the sum of Fig.1-a and Fig.1-b go to the plateau at the energy around 1 TeV. Negative contribution reaches its minimum value at $\sqrt{s} \approx 2.5 TeV$, while the cross-section, corresponding to Fig.1-c becomes constant (at finite muon mass) far away from 1 $\div$ 2 TeV region.

At last, it seems expedient to write out the asymptotic form of cross section for the process considered at $\sqrt{s} \to \infty$ in the case of zero mass initial states. It has the form

$$\sigma^{as}(\mu^+\mu^- \to H^0 Z) = \frac{1}{3} \cdot \frac{\pi\alpha^2}{sin^4(2\theta_W)} \cdot \left(g_V^2 + g_A^2\right) \cdot \frac{1}{s} \tag{2}$$

Do pay attention to the difference between factors containing the coupling constants in Eq.(1) and Eq.(2).

Remind that for the sake of simplicity we considered here the case of Standard Higgs only. In the case of a more complicated structure of Higgs sector the corresponding couplings must be arranged in the appropriate way in order to obey the fine tuning above. Evidently, this condition severely restricts the scope of possible models for enlarging the Higgs sector.

Bearing in mind that this Conference is the Lepton-Photon one let's mention shortly the process closely related to those the considered above, namely, the process $\mu^+\mu^- \rightarrow H^0\gamma$ (see Fig.2 for the Feynman graphs). In the e^+e^- collisions the contribution of those diagrams to the cross–section is extremely small in comparison with those of the higher order diagrams with heavy particles in loops [2].

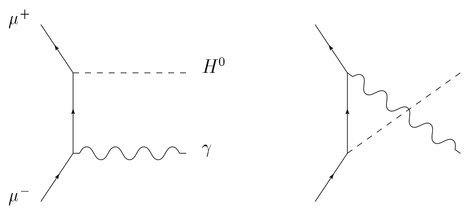

Figure 2:

For the case at hand we obtained the integrated cross-section, corresponding to graphs of Fig.2. It has the following form:

$$\sigma(\mu^+\mu^- \rightarrow H^0\gamma) = \frac{\pi\alpha^2}{2\sin^2\theta_W} \cdot \frac{m_\mu^2}{M_W^2} \cdot \frac{1}{s^2} \cdot \frac{1}{\sqrt{1 - \frac{4m_\mu^2}{s}}} \cdot \frac{1}{s - m_H^2} \times$$

$$\left\{ 6m_H^2\, s + \left[s^2 + m_H^4 + 4m_\mu^2 m_H^2 \right] \cdot \frac{1}{\sqrt{1 - \frac{4m_\mu^2}{s}}} \cdot \ln \frac{1 + \sqrt{1 - \frac{4m_\mu^2}{s}}}{1 - \sqrt{1 - \frac{4m_\mu^2}{s}}} \right\} \tag{3}$$

In view of simplicity of the expression obtained we didn't neglect the masses of initial particles. An additional reason is that this expression could be helpful for the low-energy "higgs-like" particles searches, such as, for example, axions. At high energies the cross-section is also suitable for the case of heavy-mass axion, proposed recently by Rubakov [3].

The results obtained for the both reactions are equally applicable for the case of quarks in the initial state. At first sight the tree level amplitudes could be significant for heavy quarks. However, it seems that corresponding luminocities are too small, so this mechanism is negligible in comparision with that of light quark annihilation by means of one-loop amplitudes [4].

Concerning the other processes let's remark that we representatively concidered two of them, e.g. $\mu^+\mu^- \rightarrow W^+ + W^-$ and $\mu^+\mu^- \rightarrow Z^0 + Z^0$. To the usually used sets of Feynman diagrams, corresponding to these processes we added s-channel Higgs boson exchange diagrams for both of them. Calculations show, that additional pieces of cross-sections obtained due to new graphs alone are again equal to $\approx 1.2 \cdot 10^{-2}\ fb$.

In conclusion, we have demonstrated in practice how both fine-tuning mechanisms work in the non-abelian gauge theories with spontaneous symmetry breaking. Examples are presented which might be useful for studying at next generations of colliders.

References

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